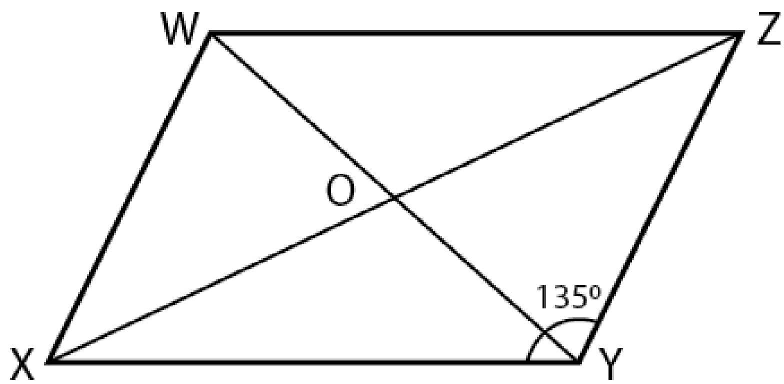
Practice Set 5.1 Page 62

1. Diagonals of a parallelogram WXYZ intersect each other at point O. If $\angle XYZ = 135°$ then what is the measure of $\angle XWZ$ and $\angle YZW$? If $l(OY)$= 5 cm then $l(WY)$= ?

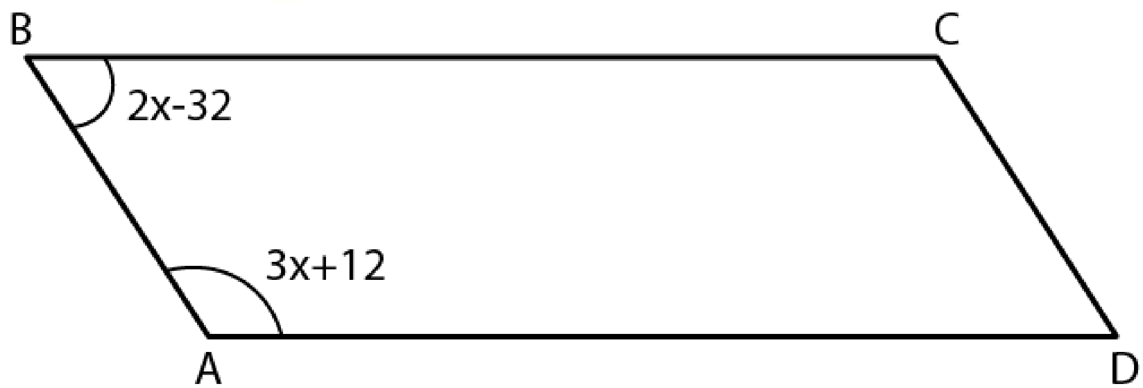
Solution:



Given $\angle XYZ = 135^\circ$
Opposite angles of a parallelogram are congruent.
$\therefore \angle XWZ = \angle XYZ = 135^\circ$
Adjacent angles of a parallelogram are supplementary.
$\therefore \angle YZW + \angle XWZ = 180^\circ$
$\therefore \angle YZW + 135^\circ = 180^\circ$
$\Rightarrow \angle YZW = 180^\circ - 135^\circ = 45^\circ$
Given $l(OY) = 5$ cm
Diagonals of a parallelogram bisect each other.
$\therefore l(WY) = 2 \times OY = 2 \times 5 = 10$ cm
$\therefore$ Measure of $\angle XWZ$ and $\angle YZW$ are 135° and 45° respectively.
Length of WY is 10 cm.

2. In a parallelogram ABCD, If $\angle A = (3x + 12)°$, $\angle B = (2x - 32)°$ then find the value of x and then find the measures of $\angle C$ and $\angle D$.

Solution:



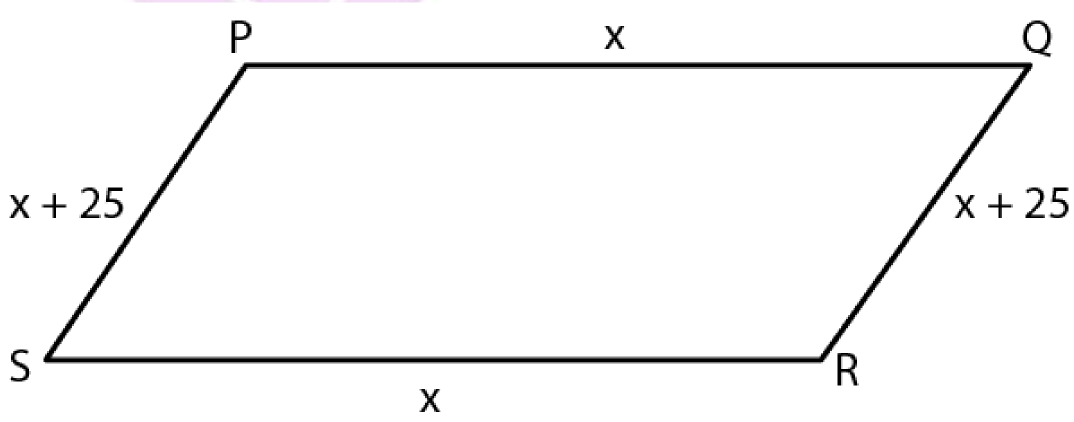
Given $\angle A = (3x+12)^\circ$

$\angle B = (2x-32)^\circ$
Opposite angles of a parallelogram are equal.
$\therefore \angle C = \angle A$ ….(i)
$\Rightarrow \angle C = (3x+12)^\circ$
$\angle D = \angle B$ ….(ii)
$\angle D = (2x-32)^\circ$
In a quadrilateral, the sum of all the angles is equal to 360˚.
$\therefore$ In □ABCD,
$\angle A+\angle B+\angle C+\angle D = 360^\circ$
$\therefore 3x+12+2x-32+3x+12+2x-32 = 360$
$10x-40 = 360$
$10x = 360+40 = 400$
$\Rightarrow x = 400/10 = 40$

$\therefore \angle A = (3x+12)^\circ$
$\therefore \angle A = 3\times40+12$
$\therefore \angle A = 120+12$
$\Rightarrow \angle A = 132^\circ$
$\therefore \angle C = 132^\circ$ [From (i)]
$\angle B = (2x-32)^\circ$
$\therefore \angle B = 2\times40-32$
$\therefore \angle B = 80-32$
$\Rightarrow \angle B = 48^\circ$
$\therefore \angle D = 48^\circ$ [From (ii)]
Hence measure of x is 40.
Also, measure of ∠C and ∠D are 132˚ and 48˚ respectively.

3. Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides.

Solution:



Let □PQRS be the parallelogram.
Let the length PQ be x.
Given other side is greater by 25cm.
$\therefore$ PS = x+25

Since opposite sides of a parallelogram are equal,
PQ = SR
PS = QR
Given perimeter of □PQRS = 150cm.
$\therefore$ PQ+QR+SR+PS = 150
x+x+25+x+x+25 = 150
$\therefore$ 4x+50 = 150
$\therefore$ 4x = 150-50
$\therefore$ 4x = 100
$\Rightarrow$ x = 100/4 = 25
$\therefore$ PQ = SR = x = 25 cm
PS = QR = x+25 = 25+25 = 50 cm
Hence the lengths of the sides of the parallelogram are 25 cm , 50 cm , 25 cm and 50 cm.

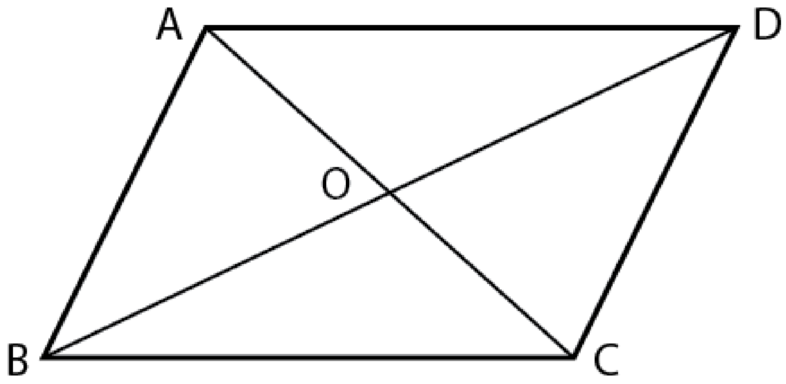
4. If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.

Solution:

Let □ABCD be the parallelogram.
Given the ratio of measures of two adjacent angles is 1:2.
Let $\angle A = x$
$\angle B = 2x$
Adjacent angles of a parallelogram are supplementary.
$\therefore \angle A+\angle B = 180^\circ$
$\Rightarrow x+2x = 180$
3x = 180
x = 180/3 = 60
$\therefore \angle A = 60^\circ$
Opposite angles of a parallelogram are congruent.
$\therefore \angle C = 60^\circ$
$\angle B = \angle D = 2x$
$\Rightarrow \angle B = \angle D = 2\times 60 = 120^\circ$
Hence the angles of the parallelogram are 60˚,120˚,60˚ and 120˚.

5* . Diagonals of a parallelogram intersect each other at point O. If AO = 5, BO = 12 and AB = 13 then show that □ABCD is a rhombus.

Solution:



Given : AO = 5, BO = 12, AB = 13
To prove : □ABCD is a rhombus.
Proof:
Given AO = 5, BO = 12, AB = 13
$AO^2+BO^2 = 5^2+12^2 = 25+144 = 169$
$AB^2 = 13^2 = 169$
$AO^2+BO^2 = AB^2$
Hence these form a Pythagorean triplet.
△AOB is a right angled triangle.
$\therefore \angle AOB = 90^\circ$
$\therefore$ seg AC ⊥ seg BD
Hence in parallelogram ABCD, seg AC⊥ seg BD.
i.e, the diagonals are perpendicular to each other.
Rhombus is a parallelogram whose diagonals meet at right angles.
Hence □ABCD is a rhombus.
Hence proved.

6. In the figure 5.12, □PQRS and □ABCR are two parallelograms. If ∠P = 110° then find the measures of all angles of □ABCR.

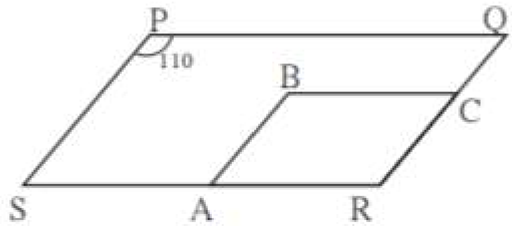


Fig. 5.12

Solution:
In □PQRS Given $\angle P = 110^\circ$
$\therefore \angle R = \angle P$ [Opposite angles of a parallelogram are equal]
$\therefore \angle R = 110^\circ$
In □ABCR , $\angle R = 110^\circ$
$\therefore \angle B = \angle R$ [Opposite angles of a parallelogram are equal]

$\therefore \angle B = 110^\circ$
$\angle A + \angle B = 180^\circ$ [Adjacent angles of a parallelogram are supplementary]
$\angle A + 110^\circ = 180^\circ$
$\angle A = 180 - 110 = 70^\circ$
$\therefore \angle C = \angle A$ [Opposite angles of a parallelogram are equal]
$\therefore \angle C = 70^\circ$
Hence, in □ABCR, $\angle A$ is 70°, $\angle B$ is 110°, $\angle C$ is 70° and $\angle R$ is 110°.

7. In figure 5.13 □ABCD is a parallelogram. Point E is on the ray AB such that BE = AB then prove that line ED bisects seg BC at point F.

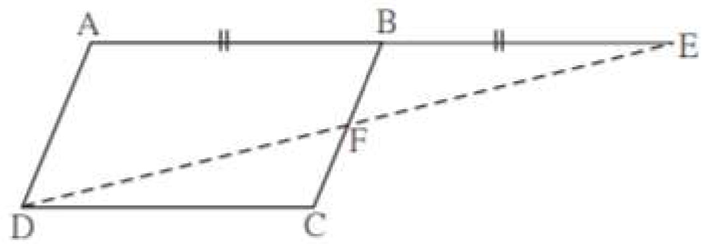


Fig. 5.13

Solution:
Given : E is on the ray AB such that BE = AB.
To prove: Line ED bisects seg BC at point F.
Proof:
AB ≅ CD …(i) [Opposite sides of a parallelogram are congruent]
AE ∥ DC, DE is the transversal.
$\angle BEF \cong \angle FDC$ [Alternate interior angles]
AE ∥ DC, BC is the transversal.
$\angle EBF = \angle FCD$ [Alternate interior angles]
Given BE = AB
$\therefore$ BE ≅ CD [From (i)]
$\therefore$ By ASA test, $\triangle BEF \cong \triangle CDF$.
$\therefore$ FB ≅ FC [c.s.c.t]
$\therefore$ Line ED bisects seg BC at point F.
Hence proved.

Practice Set 5.2 Page 67

1. In figure 5.22, □ABCD is a parallelogram, P and Q are midpoints of side AB and DC respectively, then prove □APCQ is a parallelogram

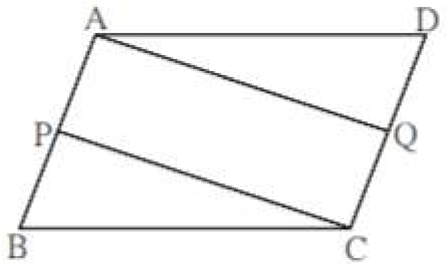
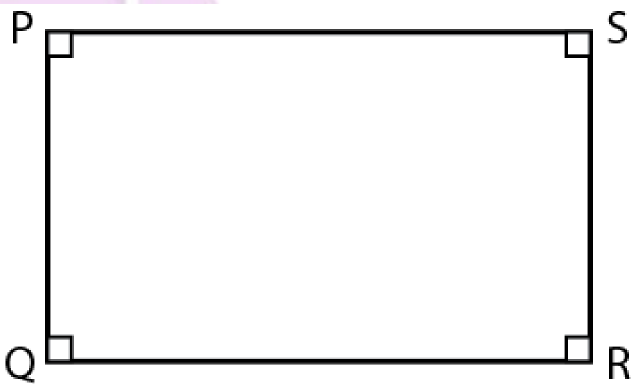


Fig. 5.22

Solution:
Proof:
AP = ½ AB [P is the midpoint of AB]
QC = ½ DC [Q is the midpoint of DC]
Given ABCD is a parallelogram
So AB = DC [Opposite sides of a parallelogram are equal]
∴ ½ AB = ½ DC
⇒ AP = QC…(iii) [from (i) and (ii)]
AB ∥ DC [Opposite sides of parallelogram are parallel]
∴ AP ∥ QC….(iv)
∴□APCQ is a parallelogram. [From (iii) and (iv)]
Hence proved.

2. Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.

Solution:



Proof:
□PQRS is a rectangle.
$\angle P = \angle R = 90^\circ$
$\angle Q = \angle S = 90^\circ$
Here opposite angles of the quadrilateral are congruent.
A quadrilateral is a parallelogram if its pairs of opposite angles are congruent.

Hence rectangle PQRS is a parallelogram.
Hence proved.

3. In figure 5.23, G is the point of concurrence of medians of △ DEF. Take point H on ray DG such that D-G-H and DG = GH, then prove that □GEHF is a parallelogram.

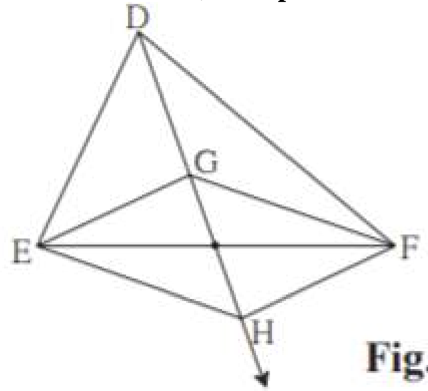
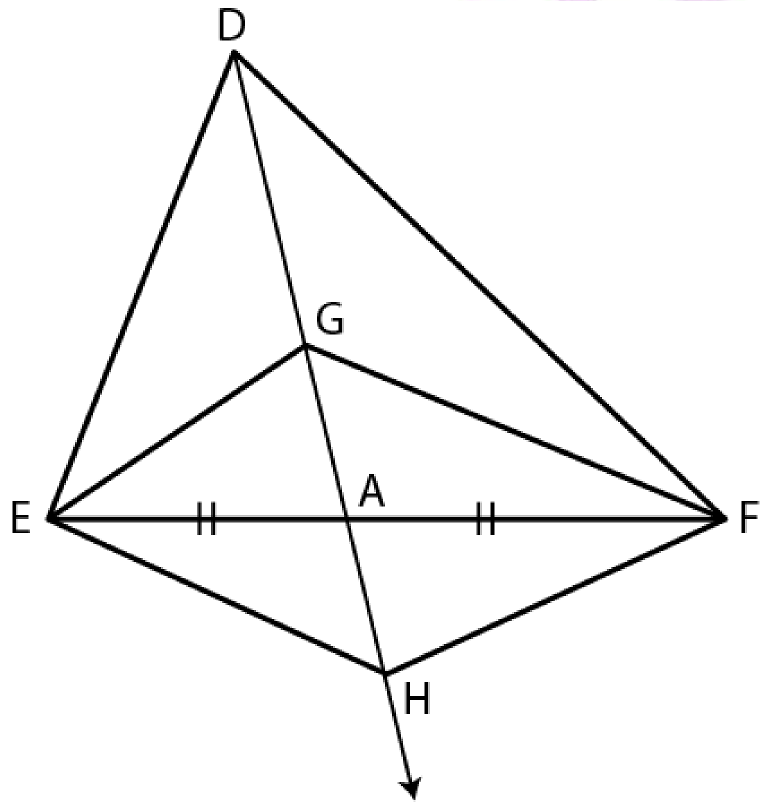


Fig. 5.23

Solution:



Let ray DH intersect seg EF at point A such that E-A-F.
∴seg DA is the median of △DEF.
∴ EA = FA ……(i)
Point G is the centroid of △DEF.
Since, the centroid divides each median in the ratio 2:1
DG/GA = 2/1
∴ DG = 2(GA)

∴ GH = 2(GA) [Given DG = GH]
∴GA+HA = 2(GA) [G-A-H]
∴ HA = 2(GA) – GA
∴HA = GA ….(ii)
A quadrilateral is a parallelogram, if its diagonals bisect each other.
From (i) and (ii),
□GEHF is a parallelogram.
Hence proved.

4. Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle. (Figure 5.24)

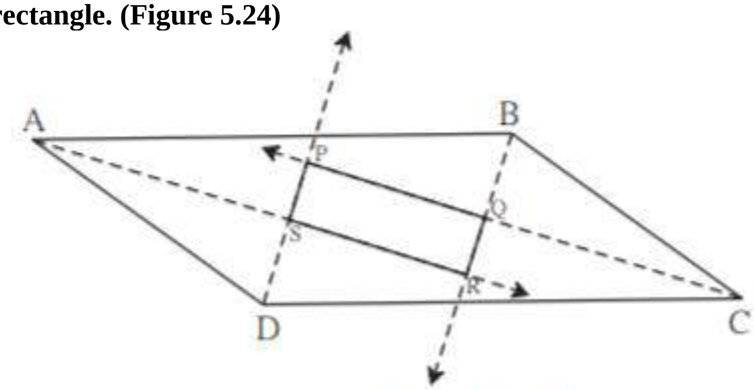


Fig. 5.24

Solution:
Given ABCD is a parallelogram.
DC ∥ AB. DA is the transversal.
∴∠A+∠D = 180˚ [Corresponding angles on same side of transversal are supplementary]
Multiply both sides by ½
½ ∠A+ ½ ∠D = 90˚….(i)
Let ½ ∠A = e and ½ ∠D = f
½ ∠A = ∠SAD = ∠RAB = e [∵ AS is the angle bisector of ∠A.]
½ ∠D = ∠SDA = ∠SDC = f [∵ DS is the angle bisector of ∠D.]
∴∠SAD+∠SDA = 90˚ [From (i)]
e+f = 90
∴∠ASD = 90˚ [Sum of all angles in △ASD is 180˚]
∴∠PSR = ∠ASD
∴∠PSR = 90˚ [Vertically opposite angles]

AD ∥ BC. AB is the transversal.
∠A+∠B = 180˚ [Corresponding angles on same side of transversal are supplementary]
Multiply both sides by ½
½ ∠A+ ½ ∠B = 90˚….(ii)
Let ½ ∠B = g
½ ∠A = ∠SAD = ∠RAB = e [∵ AS is the angle bisector of ∠A.]
½ ∠B = ∠ABR = ∠CBR = g [∵ BR is the angle bisector of ∠B.]
∴∠SAD+∠ABR = 90˚ from (ii)

i.e, $e+g = 90^\circ$
In $\triangle ARB$,
$\angle ARB+\angle RAB+\angle ABR = 180^\circ$
$\angle ARB = \angle SRQ$
$\angle SRQ+e+g = 180$
$\therefore \angle SRQ = 90^\circ$ [$\because e+g = 90$]

DC ‖ AB. BC is the transversal.
$\therefore \angle B+\angle C = 180^\circ$ [Corresponding angles on same side of transversal are supplementary]
Multiply both sides by ½
$\frac{1}{2}\angle B+\frac{1}{2}\angle C = 90^\circ$….(ii)
$\frac{1}{2}\angle B = \angle QBC = \angle ABR$ [$\because$ BQ is the angle bisector of $\angle B$.]
$\frac{1}{2}\angle C = \angle QCB$ [$\because$ CQ is the angle bisector of $\angle C$.]
$\therefore \angle QBC+\angle QCB = 90^\circ$ [From (ii)]
$\therefore \angle BQC = 90^\circ$ [Sum of all angles in $\triangle BQC$ is 180°]
$\therefore \angle PQR = \angle BQC = 90^\circ$ [Vertically opposite angles]
$\therefore \angle PQR = 90^\circ$

Similarly, we can prove that QPS = $\angle 90^\circ$

Here $\angle QRS = \angle PRS = \angle SRQ = \angle QPS = 90^\circ$
Hence PQRS is a quadrilateral whose angles are 90° each.
$\therefore$ PQRS is a rectangle.
Hence proved.

5. In figure 5.25, if points P, Q, R, S are on the sides of parallelogram such that AP = BQ = CR = DS then prove that □PQRS is a parallelogram.

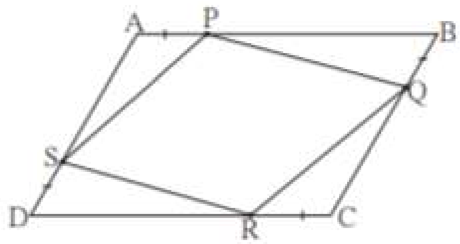


Fig. 5.25

Solution:
Given : ABCD is a parallelogram.
AP= BQ = CR = DS
To prove : PQRS is a parallelogram.
Proof:
AB = DC [Opposite sides of a parallelogram are equal]
$\therefore$ AP+BP = DR+CR
$\therefore$ AP+BP = DR+AP [Given AP = CR]
$\therefore$ BP = DR …(i)

$\angle B = \angle D$ …(ii) [Opposite angles of a parallelogram are equal]

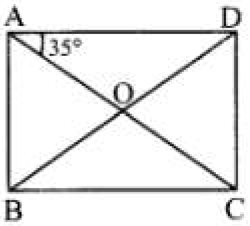
In $\triangle BPQ$ and $\triangle DRS$,
$BP \cong DR$ [from (i)]
$BQ \cong DS$ [Given]
$\angle B \cong \angle D$ [from (ii)]
$\therefore$ By SAS test $\triangle BPQ \cong \triangle DRS$.
$\therefore SR = PQ$ …(iii) [c.s.c.t]

Similarly we can prove that $\triangle PAS \cong \triangle RCQ$.
$\therefore PS = RQ$ …(iv)
From (iii) and (iv)
□PQRS is a parallelogram [If opposite sides of a quadrilateral are equal, then it is a parallelogram]
Hence proved.

Practice Set 5.3 Page 69

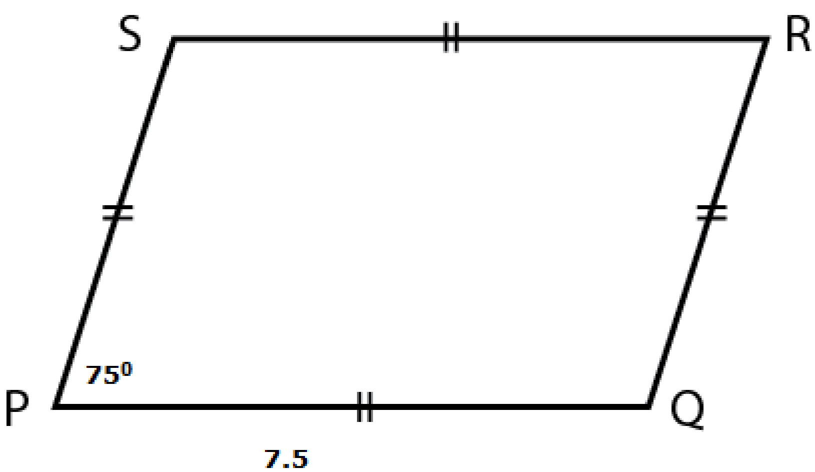
1. Diagonals of a rectangle ABCD intersect at point O. If AC = 8 cm then find BO and if ∠CAD = 35° then find ∠ACB.

Solution:



Given AC = 8cm
Diagonals of a rectangle are congruent.
∴AC = BD
⇒BD = 8cm
BO = ½ BD [Diagonals of a rectangle bisect each other]
∴BO = ½ ×8 = 4 cm
Given ∠CAD = 35˚
∠CAD = ∠ACB [Alternate interior angles]
∴∠ACB = 35˚
Hence measure of BO is 4 cm and ∠ACB is 35˚.

2. In a rhombus PQRS if PQ = 7.5 then find QR. If ∠QPS = 75° then find the measure of ∠PQR and ∠SRQ.
Solution:



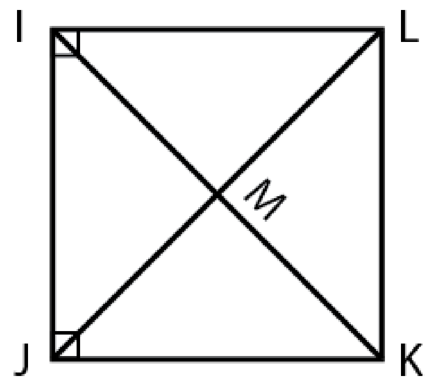
Given PQ = 7.5
∴QR = PQ = 7.5 [All sides of rhombus are equal]
∠QPS = 75˚
∴∠SRQ = ∠QPS [Opposite angles of a rhombus are equal]
∴∠SRQ = 75˚
∠QPS+∠PQR = 180˚ [Adjacent angles of a rhombus are supplementary]

$\therefore 75^\circ + \angle PQR = 180^\circ$
$\therefore \angle PQR = 180 - 75 = 105^\circ$

Hence the measure of $\angle PQR$ and $\angle SRQ$ are 105° and 75° respectively.

3. Diagonals of a square IJKL intersects at point M, Find the measures of ∠IMJ, ∠JIK and ∠LJK .

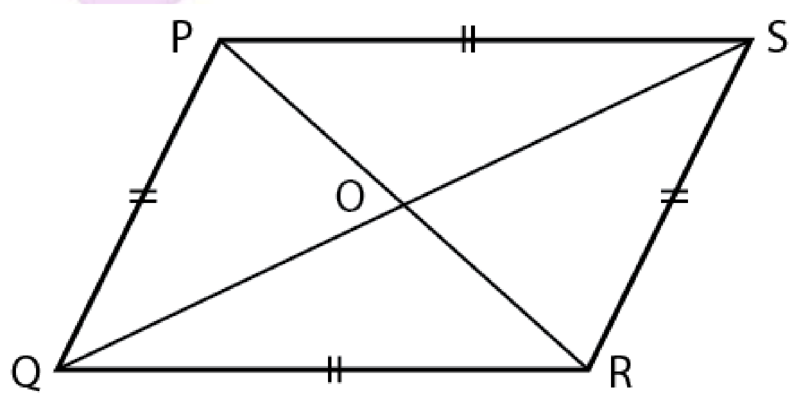
Solution:



Diagonals of a square are perpendicular bisectors of each other.
seg JL ⊥ seg IK
$\therefore \angle IMJ = 90^\circ$
$\angle JIL = 90^\circ$ [Angle of a square]
Diagonals of a square bisect opposite angles.
$\therefore \angle JIK = 90/2 = 45^\circ$
$\angle IJK = 90^\circ$ [Angle of a square]
$\therefore \angle LJK = 90/2 = 45^\circ$
Hence measures of $\angle IMJ$, $\angle JIK$, $\angle LJK$ are 90°, 45° and 45°.

4. Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.

Solution:



Let PQRS be the rhombus.
Let PR = 20 cm

QS = 21 cm
PO = PR/2   [Diagonals of a rhombus bisect each other]
$\therefore$ PO = 20/2 = 10 cm
QO = QS/2   [Diagonals of a rhombus bisect each other]
$\therefore$ QO = 21/2 = 10.5 cm
$\angle$POQ = 90°   [Diagonals of a rhombus are perpendicular bisectors of each other]
In $\triangle$POQ ,
$PQ^2 = PO^2+QO^2$ [Pythagoras theorem]
$\therefore PQ^2 = (20/2)^2+(21/2)^2$
$\therefore PQ = \sqrt{[(400+441)/4]}$
$\therefore PQ = \sqrt{(841/4)}$
$\therefore$ PQ = 29/2 = 14.5 cm
Perimeter = 4×PQ = 4×14.5 = 58 cm
Hence side and perimeter of the rhombus are 14.5 cm and 58 cm respectively.

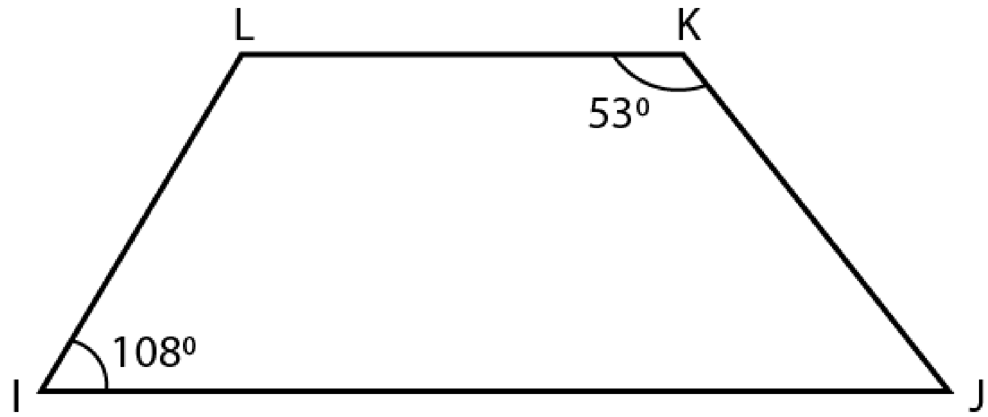
5. State with reasons whether the following statements are 'true' or 'false'.
(i) Every parallelogram is a rhombus.
(ii) Every rhombus is a rectangle.
(iii) Every rectangle is a parallelogram.
(iv) Every square is a rectangle.
(v) Every square is a rhombus.
(vi) Every parallelogram is a rectangle.

Solution:
(i) All the sides of a rhombus are congruent, while the opposite sides of a parallelogram are congruent.
Hence the statement is false.
(ii) All the angles of a rectangle are congruent, while the opposite angles of a rhombus are congruent.
Hence the statement is false.
(iii) The opposite sides of a rectangle are parallel and congruent. All its angles are congruent.
The opposite sides of a parallelogram are parallel and congruent. Its opposite angles are congruent.
Hence the statement is true.
(iv) All the sides of a square are parallel and congruent. Also, all its angles are congruent.
The opposite sides of a rectangle are parallel and congruent. Also, all its angles are congruent.
Hence the statement is true.
(v) All the sides of a square are congruent. Also, its diagonals are perpendicular bisectors of each other.
All the sides of a rhombus are congruent. Also, its diagonals are perpendicular bisectors of each other.
Hence the statement is true.
(vi) All the angles of a rectangle are congruent, while the opposite angles of a parallelogram are congruent.
Hence the statement is false.

Practice Set 5.4 Page 71

1. In □IJKL, side IJ || side KL ∠I = 108° ∠K = 53° then find the measures of ∠J and ∠L.

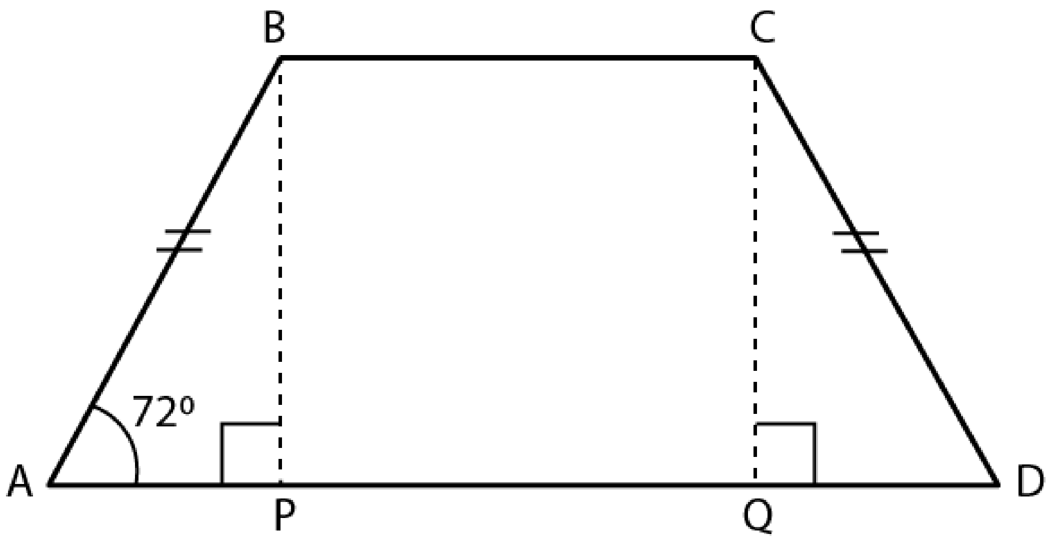
Solution:



Given IJ || KL.
IL is the transversal.
∠I = 108˚ [Given]
∴ ∠I+∠L = 180˚ [Interior angles on same side of transversal are supplementary]
∴ 108˚+∠L = 180˚
∠L = 180-108 = 72˚
Given IJ || KL.
KJ is the transversal.
∠K = 53˚ [Given]
∠K+∠J = 180˚ [Interior angles on same side of transversal are supplementary]
∴ 53+∠J = 180˚
∴ ∠J = 180-53 = 127˚
Hence measure of ∠J and ∠L are 127˚ and 72˚ respectively.

2. In □ABCD, side BC || side AD, side AB ≅ side DC If ∠A = 72° then find the measures of ∠B, and ∠D.

Solution:
Construction: Draw seg CQ ⊥ side AD.
Draw seg BP ⊥ side AD.



$\angle A = 72°$ [Given]
In □ABCD, side AD || side BC [Given]
AB is their transversal.
$\therefore \angle A + \angle B = 180°$ [Interior angles on same side of transversal]
$\therefore 72° + \angle B = 180°$
$\therefore \angle B = 180° - 72°$
$\therefore \angle B = 108°$
In $\triangle APB$ and $\triangle DQC$,
AB ≅ DC [Given]
$\angle APB \cong \angle DQC$ [Each angle is of measure 90°]
seg BP ≅ seg CQ [Perpendicular distance between two parallel lines]
∴ By hypotenuse side test
$\therefore \triangle APB \cong \triangle DQC$
$\therefore \angle A = \angle D$ [c. a. c. t.]
$\therefore \angle D = 72°$
Hence measure of $\angle B$ and $\angle D$ are 108° and 72° respectively.

3. In □ABCD, side BC < side AD (Figure 5.32) side BC || side AD and if side BA ≅ side CD then prove that $\angle ABC \cong \angle DCB$.

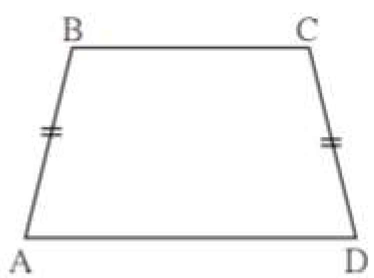
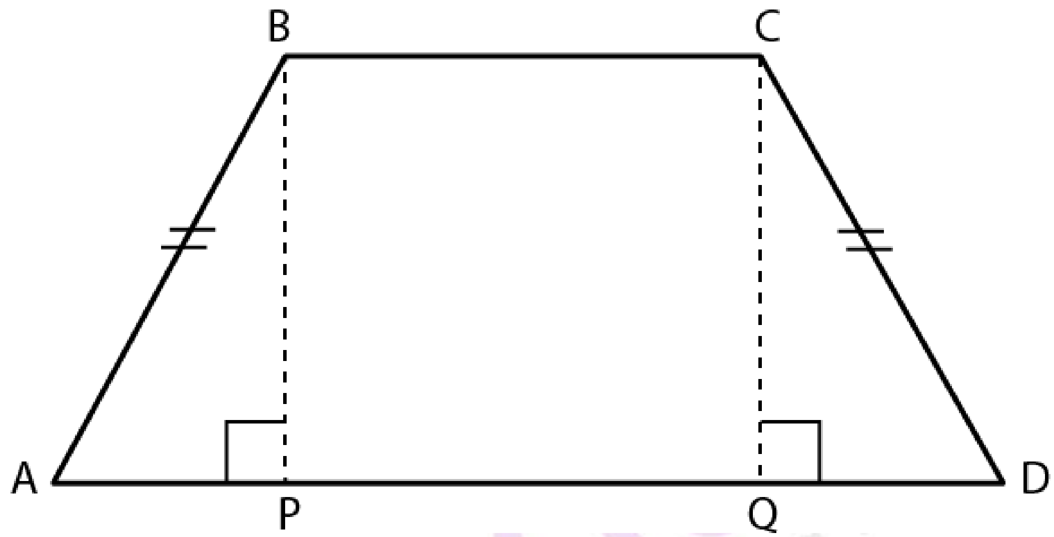


Fig. 5.32

Solution:
Given: side BC < side AD

side BC ∥ side AD
side BA = side CD
To prove: ∠ABC ≅ ∠DCB
Construction: Draw seg BP ⊥ side AD, A-P-D
seg CQ ⊥ side AD, A-Q-D



Proof:
In △BPA and △CQD,
∠BPA ≅ ∠CQB [Each angle is of measure 90°]
BA ≅ CD [Given]
seg BP ≅ seg CQ [Perpendicular distance between two parallel lines]
By Hypotenuse side test
△BPA ≅ △CQD
∴∠BAP ≅ ∠CDQ [c. a. c. t.]
∴∠A = ∠D ….(i)
Given side BC ∥ side AD
side AB is their transversal. [Given]
∠A+∠B = 180°…..(ii) [Interior angles on same side of transversal]
Given, side BC ∥ side AD
side CD is their transversal.
∴∠C+∠D = 180° …..(iii) [Interior angles on same side of transversal]
Equating (ii) and (iii)
∴∠A+∠B = ∠C+∠D [From (ii) and (iii)]
∴∠A+∠B = ∠C+∠A [From (i)]
⇒∠B = ∠C
∴∠ABC ≅ ∠DCB
Hence proved.

Practice set 5.5 Page 73

1. In figure 5.38, points X, Y, Z are the midpoints of side AB, side BC and side AC of △ ABC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find the length of XY, YZ, XZ.

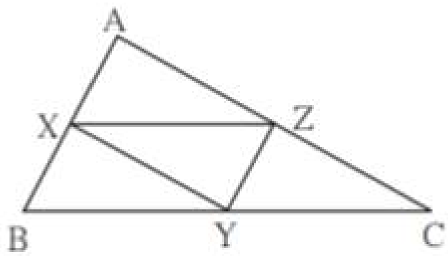


Fig. 5.38

Solution:

Given AC = 9 cm

X and Y are the midpoints of side AB and BC.

The segment joining midpoints of any two sides of a triangle is parallel to the third side and half of it.

∴XY = ½ AC

∴XY = ½ ×9 = 4.5 cm

Given AB = 5 cm

Z and Y are the midpoints of side AC and BC.

The segment joining midpoints of any two sides of a triangle is parallel to the third side and half of it.

∴YZ = ½ AB

∴YZ = ½ ×5 = 2.5 cm

Given BC = 11 cm

Z and X are the midpoints of side AC and AB.

The segment joining midpoints of any two sides of a triangle is parallel to the third side and half of it.

∴XZ = ½ BC

∴XZ = ½ ×11 = 5.5 cm

Hence the measures of XY, YZ and XZ are 4.5 cm, 2.5 cm and 5.5 cm.

2. In figure 5.39,□PQRS and □MNRL are rectangles. If point M is the midpoint of side PR then prove that,
(i) SL = LR,
(ii) LN = ½ SQ.

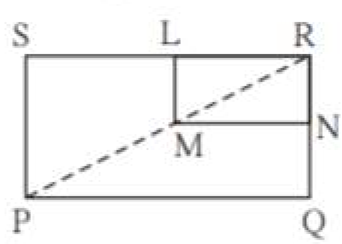


Fig. 5.39

Solution:

(i)Given PQRS and MNRL are rectangles.

∠S = 90˚ [Angle of rectangle]

$\angle L = 90^\circ$ [Angle of rectangle]
RN ॥ LM …(i) [Opposite sides of rectangle MNRL]
RN ॥ SP ….(ii) [Opposite sides of rectangle PQRS]
From (i) and (ii)
LM ॥ SP …..(iii)
In △PRS,
M is the midpoint of PR. [Given]
LM ॥ SP [From (iii)
∴L is the midpoint of SR …(iv) [Converse of midpoint theorem]
∴SL = LR
Hence proved.

(ii)Similarly for △PQR we can prove that
N is the midpoint of QR …(v).
So in △RSQ, N and L are the midpoints of RQ and SR respectively. [From (iv) and (v)]
∴LN = ½ SQ [Midpoint theorem]
Hence proved.

3. In figure 5.40, △ABC is an equilateral triangle. Points F,D and E are midpoints of side AB, side BC, side AC respectively. Show that △ FED is an equilateral triangle.

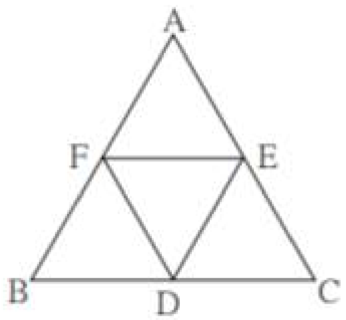


Fig. 5.40

Solution:
Proof:
Given ABC is an equilateral triangle.
∴AB = AC = BC….(i)
Points F,D and E are midpoints of side AB, side BC, side AC respectively.
D and E are the midpoints of side BC and side AC respectively.
∴DE = ½ AB…..(ii) [Midpoint theorem]
D and F are the midpoints of side BC and side AB respectively.
∴DF = ½ AC…..(iii) [Midpoint theorem]
F and E are the midpoints of side AB and side AC respectively.
∴FE = ½ BC…..(iv) [Midpoint theorem]
Since AB = AC = BC,
DE = DF = FE [From (ii),(iii) and (iv)]
∴△FED is an equilateral triangle.
Hence proved.

4. In figure 5.41, seg PD is a median of △ PQR. Point T is the mid point of seg PD. Produced QT intersects PR at M. Show that PM/PR = 1/3 . [Hint : draw DN || QM.]

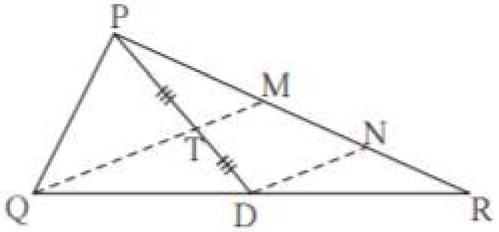


Fig. 5.41

Solution:

Given: PD is a median.
T is the midpoint of PD.
QT intersects PR at M
To prove :
PM/PR = 1/3
Proof:
In △PDN,
Point T is the midpoint of seg PD
Given seg TM ∥ seg DN [DN ∥ QM and Q-T-M]
∴Point M is the midpoint of seg PN. [Construction and Q-T-M]
∴PM = MN ….(i) [Converse of midpoint theorem]
In △QMR,
Point D is the midpoint of seg QR and seg DN ∥ seg QM [Construction]
∴ Point N is the midpoint of seg MR. [Converse of midpoint theorem]
∴ MN = NR …..(ii)
∴ PM = MN = RN …..(iii) [From (i) and (ii)]
Now, PR = PM+MN+RN [P-M-R]
∴PR = PM + PM + PM [From (iii)]
∴ PR = 3PM
PM/PR = 1/3
Hence proved.

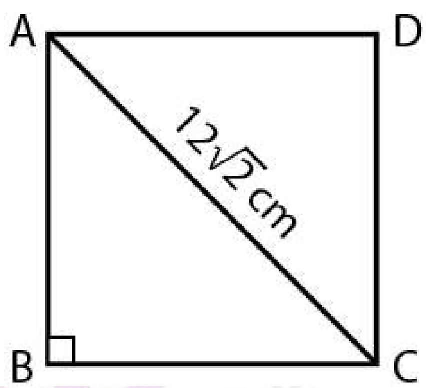
Problem set 5 Page 73

1. Choose the correct alternative answer and fill in the blanks.
(i) If all pairs of adjacent sides of a quadrilateral are congruent then it is called ....
(A) rectangle (B) parallelogram (C) trapezium, (D) rhombus

Solution:
If all the pairs of adjacent sides of a quadrilateral are congruent, then it is a rhombus.
Hence Option D is the answer.

(ii) If the diagonal of a square is 12√2 cm then the perimeter of square is ......
(A) 24 cm (B) 24 2 cm (C) 48 cm (D) 48 2 cm

Solution:



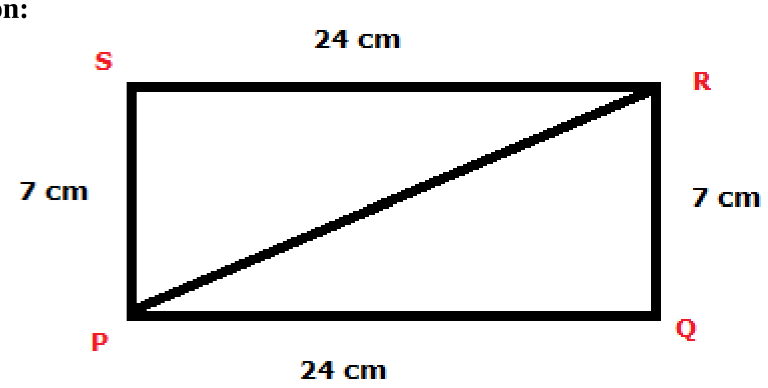
Here AC = $12\sqrt{2}$
AD = DC [sides of square are equal]
In △ADC,
$AC^2 = AD^2+DC^2$ [Pythagoras theorem]
$AC^2 = AD^2+AD^2$ [AD = DC]
$AC^2 = 2AD^2$
$AC = \sqrt{2}\times AD$
$\therefore 12\sqrt{2} = \sqrt{2}\times AD$
$\Rightarrow AD = 12$
Perimeter = 4×AD = 4×12 = 48 cm
Hence Option C is the answer.

(iii) If opposite angles of a rhombus are (2x)° and (3x - 40)° then value of x is ...
(A) 100 ° (B) 80 ° (C) 160 ° (D) 40

Solution:
2x = 3x-40 [Opposite angles of rhombus are equal]
2x-3x = -40
$\therefore x = 40^\circ$
Hence Option D is the answer.

2. Adjacent sides of a rectangle are 7 cm and 24 cm. Find the length of its diagonal.

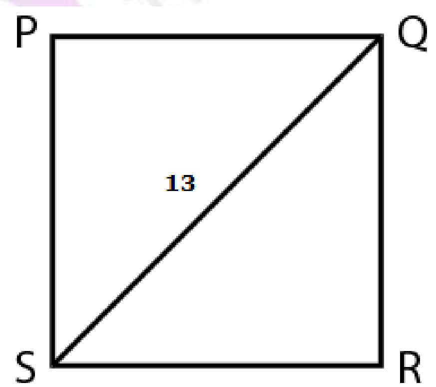
Solution:



Given PQ = 24 cm
QR = 7 cm
In △PQR,
∠Q = 90˚ [Angle in a rectangle]
$PR^2 = PQ^2+QR^2$ [Pythagoras theorem]
$PR^2 = 24^2+7^2$
$PR^2 = 576+49$
$PR^2 = 625$
Taking square root on both sides,
PR = 25
Hence the diagonal is 25 cm long.

3. If diagonal of a square is 13 cm then find its side.

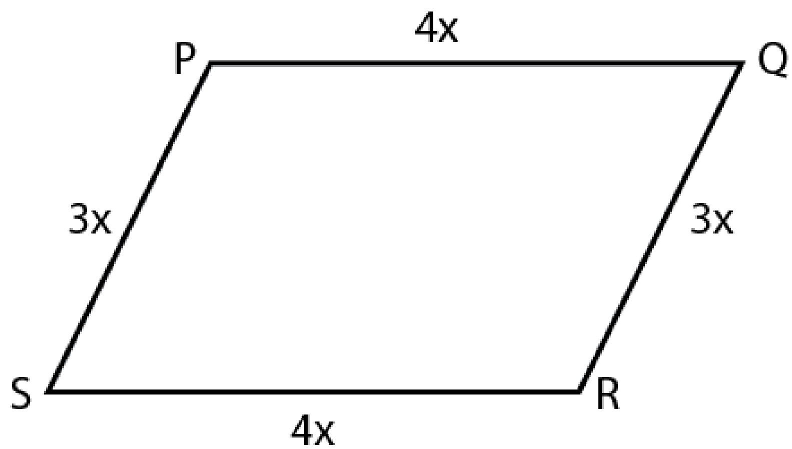
Solution:



Given SQ = 13 cm
R = 90˚ [Each angle of a square is 90˚]
SR = QR [All sides of a square are equal]
In △SRQ ,
$SQ^2 = SR^2+QR^2$
$SQ^2 = SR^2+SR^2$ [SR = QR]
$SQ^2 = 2SR^2$

Taking square root on both sides
SQ = SR×√2
13 = SR×√2
SR = 13/√2 = 13×√2/(√2×√2)
= 13√2/2 = 6.5√2 cm
Hence length of side is 6.5√2 cm.

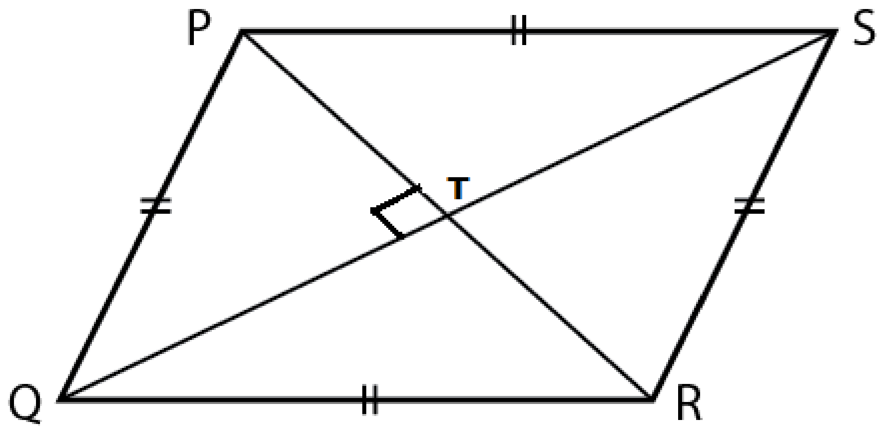
4. Ratio of two adjacent sides of a parallelogram is 3 : 4, and its perimeter is 112 cm. Find the length of its each side.
Solution:



Given ratio of adjacent sides of a parallelogram is 3:4.
Let PQ = SR = 4x
PS = QR = 3x
Perimeter = 112 [Given]
∴ 4x+3x+4x+3x+ = 112
⇒14x = 112
⇒x = 112/14 = 8
∴ PQ = SR = 4x
⇒PQ = SR = 4×8 = 32 cm
PS = QR = 3x
⇒ PS = QR = 3×8 = 24 cm
Hence lengths of the sides of the parallelogram are 32 cm, 24 cm , 32 cm and 24 cm .

5. Diagonals PR and QS of a rhombus PQRS are 20 cm and 48 cm respectively. Find the length of side PQ.

Solution:

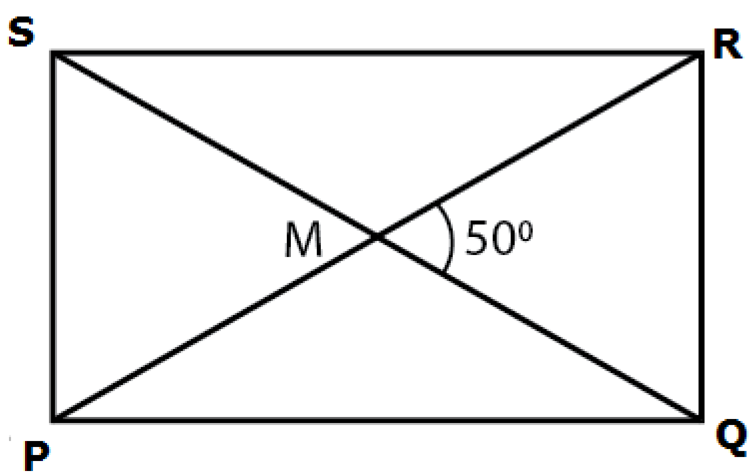


Given PR = 20
QS = 48
$\therefore$ QT = QS/2 = 48/2 = 24 [Diagonals of a rhombus bisect each other]

PT = PR/2 = 20/2 = 10 [Diagonals of a rhombus bisect each other]
In $\triangle$PQT,
$\angle$PTQ = 90° [Diagonals of a rhombus are perpendicular bisectors of each other.]
$\therefore PQ^2 = PT^2+QT^2$
$\therefore PQ^2 = 10^2+24^2$
$\therefore PQ^2 = 100+576$
$\therefore PQ^2 = 676$
Taking square root on both sides,
PQ = 26
Hence measure of side PQ is 26 cm.

6. Diagonals of a rectangle PQRS are intersecting in point M. If $\angle$QMR = 50° then find the measure of $\angle$MPS.

Solution:



Given ∠QMR = 50˚
∴∠SMP = 50˚ [Vertically opposite angles]
PR = SQ …(i) [Diagonals of a rectangle are congruent]
PM = ½ PR …(ii) [Diagonals of a rectangle bisect each other]
MS = ½ SQ….(iii) [Diagonals of a rectangle bisect each other]
∴PM = MS…(iv) [From (i) ,(ii) and (iii)]
∴∠MSP = ∠MPS…(v) [Isoceles triangle theorem]
In △SMP,
∠MSP +∠MPS+∠SMP = 180˚ [Angle sum property of triangle]
∴∠MPS+∠MPS+50˚ = 180˚ [From (i) and (v)]
2∠MPS = 180-50 = 130˚
⇒∠MPS = 130/2 = 65˚
Hence measure of ∠MPS is 65˚.

7. In the adjacent Figure 5.42, if seg AB || seg PQ , seg AB @ seg PQ, seg AC || seg PR, seg AC ≅ seg PR then prove that, seg BC || seg QR and seg BC ≅ seg QR.

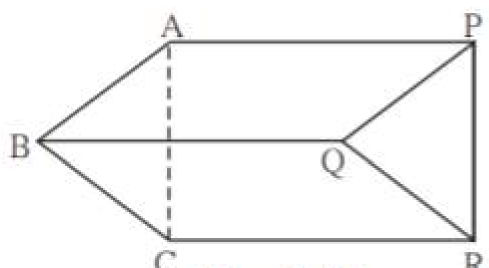


Fig. 5.42

Solution:
Given: seg AB ॥ seg PQ ,
seg AB ≅ seg PQ
seg AC ॥ seg PR
seg AC ॥ seg PR
To prove: seg BC ॥ seg QR and seg BC ≅ seg QR
Proof:
In □ABQP
seg AB ॥ seg PQ [Given]
seg AB ≅ seg PQ [Given]
∴□ ABQP is a parallelogram. [A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent]
∴ segAP ॥ segBQ …..(i)
∴seg AP ≅ seg BQ …..(ii) [Opposite sides of a parallelogram are congruent]
In □ACRP,
seg AC ॥ seg PR [Given]
seg AC ≅ seg PR [Given]
∴□ACRP is a parallelogram. [A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent]
∴seg AP ॥ seg CR …(iii)
∴ seg AP ≅ seg CR …….(iv) [Opposite sides of a parallelogram are congruent]

In □BCRQ,
seg BQ ∥ seg CR
seg BQ ≅ seg CR
∴□BCRQ is a parallelogram. [A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent]
∴ seg BC ∥ seg QR
∴seg BC ≅ seg QR [Opposite sides of a parallelogram are congruent]
Hence Proved.

8* . In the Figure 5.43,□ABCD is a trapezium. AB ∥ DC. Points P and Q are midpoints of seg AD and seg BC respectively. Then prove that, PQ || AB and PQ = ½ (AB + DC)

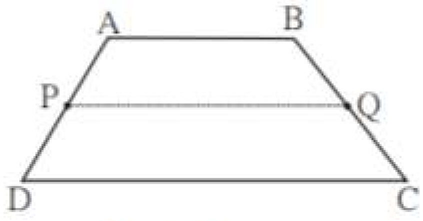
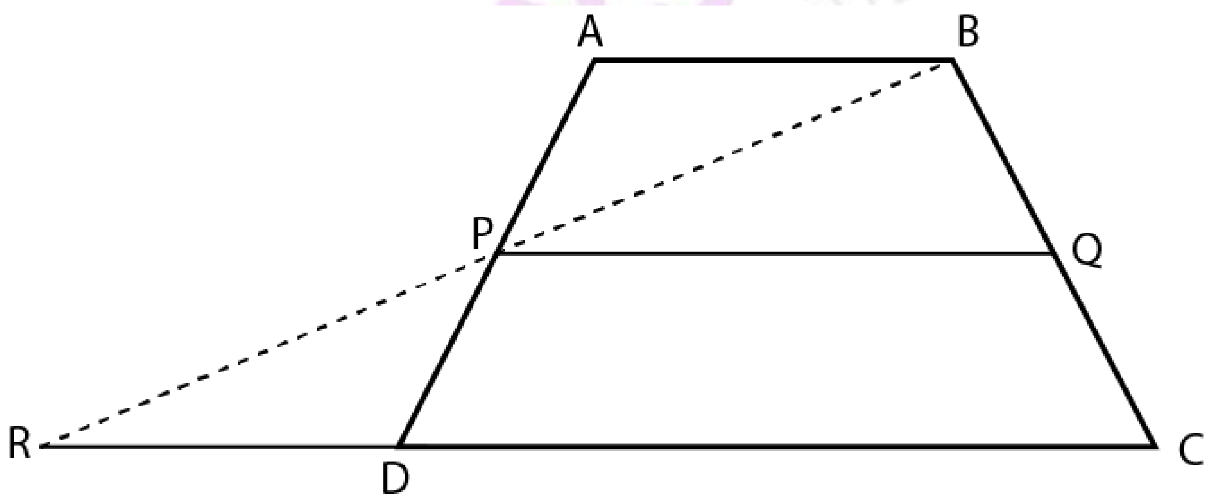


Fig. 5.43

Solution:



Construction: Join BP and extend it to meet CD produced at R.
To prove: PQ ∥ AB and PQ = ½ (AB+DC)
Proof:
In △ABP and △DRP,
∠APB =∠DPR [Vertically opposite angles]
∠PDA =∠PAB [Alternate interior angles]
AP = PD [P is the mid point of DA]
∠APB = ∠DPR [Vertically opposite angles]
∠PDA = ∠PAB [Alternate interior angles]
AP = PD [P is the mid point of DA]
By ASA test △ABP ≅ △DRP.

∴ PB = PR [c.s.c.t]
AB = RD [c.s.c.t]
In △BRC,
Given Q is the mid point of BC
P is the mid point of BR [∵ PB = PR]
So, by midpoint theorem, PQ ∥ RC
⇒⇒PQ ∥ DC
Given AB ∥ DC
So, PQ ∥ AB.
Also, PQ = ½ RC
PQ = ½ (RD+DC)
PQ = ½ (AB+DC) [∵ AB = RD]
Hence proved.

9. In the adjacent figure 5.44, �ABCD is a trapezium. AB || DC. Points M and N are midpoints of diagonal AC and DB respectively then prove that MN || AB.

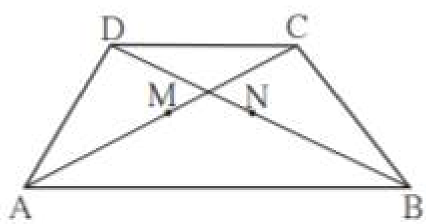
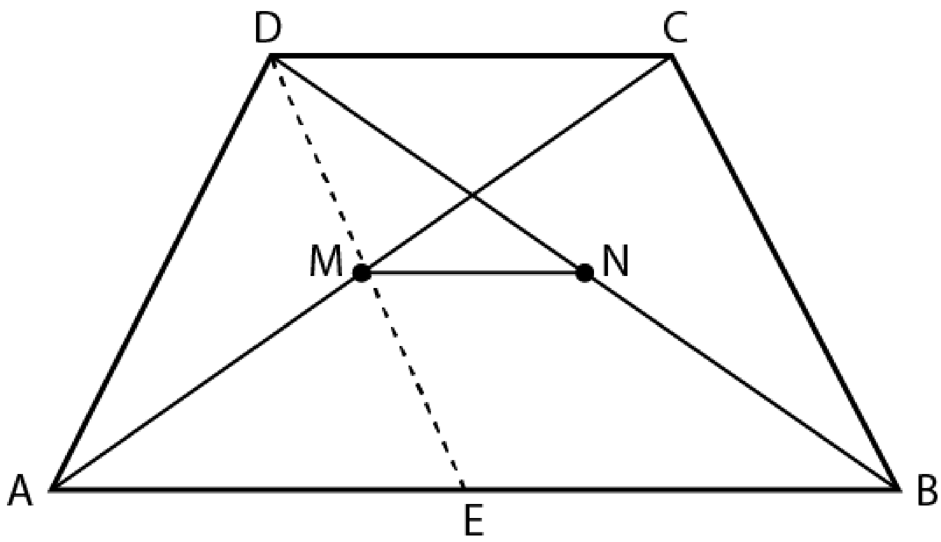
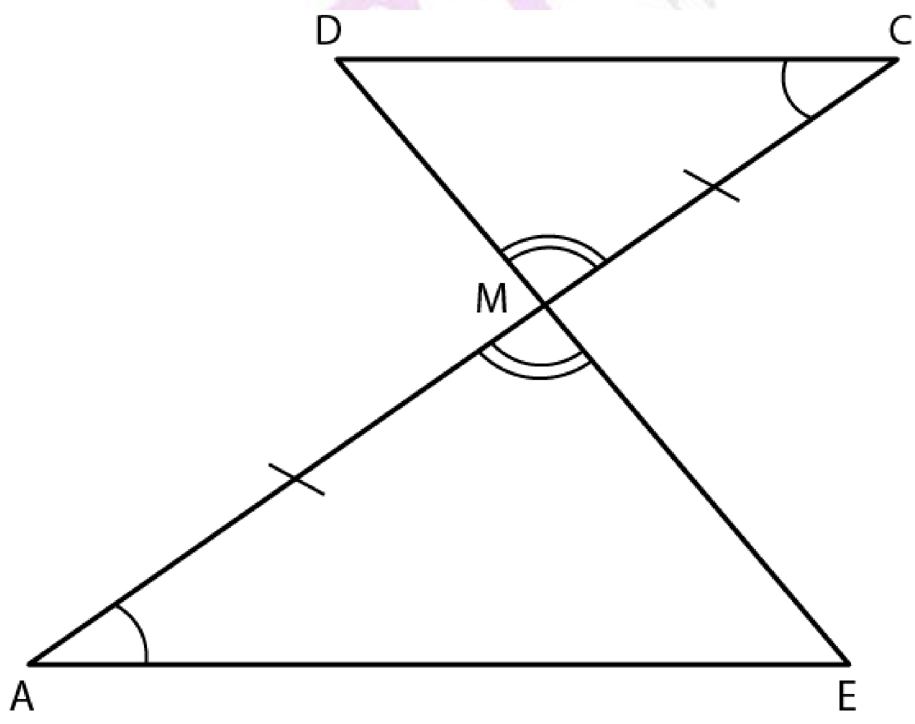


Fig. 5.44

Solution:
Given: □ABCD is a trapezium.
AB ∥ DC.
Points M and N are midpoints of diagonals AC and DB respectively.
To prove: MN ∥ AB
Construction: Join D and M. Extend seg DM to meet seg AB at point E such that A-E-B.
Proof:
Given seg AB ∥ seg DC
seg AC is their transversal.
∴ ∠CAB ≅ ∠ACD [Alternate interior angles]
∴ ∠MAE ≅ ∠MCD ….(i) [C-M-A, A-E-B]
In △AME and △CMD,



$\angle AME \cong \angle CMD$ [Vertically opposite angles]
seg $AM \cong$ seg CM [M is the midpoint of AC]
$\angle MAE \cong \angle MCD$ [From (i)]
By ASA test $\triangle AME \cong \triangle CMD$
$\therefore$ seg $ME \cong$ seg MD [c.s.c.t]
$\therefore$ Point M is the midpoint of seg DE. …(ii)
In $\triangle DEB$,



Given points M and N are the midpoints of DE and DB respectively.
$\therefore$ seg MN $\parallel$ seg EB [Midpoint theorem]
$\therefore$ seg MN $\parallel$ seg AB [A-E-B]
Hence proved.