Practice Set 8.1

1.In the Fig.8.12, ∠R is the right angle of △ PQR. Write the following ratios.
(i) sin P (ii) cos Q (iii) tan P (iv) tan Q

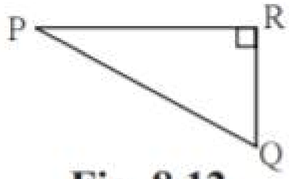

Fig. 8.12

Solution:
(i) sin P = Opposite side of ∠P / Hypotenuse = QR/PQ
(ii) cos Q = Adjacent side of ∠Q / Hypotenuse = QR/PQ
(iii) tan P = Opposite side of ∠P / Adjacent side of ∠P = QR/PR
(iii) tan Q = Opposite side of ∠Q / Adjacent side of ∠Q = PR/QR

2. In the right angled △ XYZ, ∠XYZ = 90° and a,b,c are the lengths of the sides as shown in the figure. Write the following ratios,
(i) sin X (ii) tan Z (iii) cos X (iv) tan X.

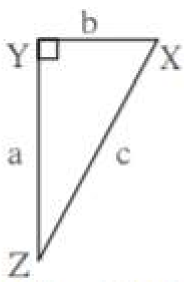

Fig. 8.13

Solution:
(i) sin X = Opposite side of ∠X / Hypotenuse = a/c
(ii) tan Z = Opposite side of ∠Z / Adjacent side of ∠Z = b/a
(iii) cos X = Adjacent side of ∠X / Hypotenuse = b/c
(iv) tan X = Opposite side of ∠X / Adjacent side of ∠X = a/b

3.In right angled △ LMN, ∠LMN = 90° ∠L = 50° and ∠N = 40°, write the following ratios.
(i) sin 50° (ii) cos 50° (iii) tan 40° (iv) cos 40°

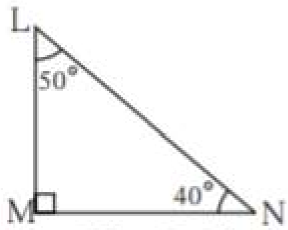

Fig. 8.14

Solution:

(i) sin 50˚ = Opposite side of 50˚ / Hypotenuse = MN/LN
(ii) cos 50˚ = Adjacent side of 50˚ / Hypotenuse = LM/LN
(iii) tan 40˚ = Opposite side of 40˚ / Adjacent side of 40˚ = LM/MN
(iv) cos 40˚ = Adjacent side of 40˚ / Hypotenuse = MN/LN

4.In the figure 8.15,.∠PQR = 90°, ∠PQS = 90°, ∠PRQ = α and ∠QPS = θ
Write the following trigonometric ratios.
(i) sin α, cos α, tan α
(ii) sin θ, cos θ, tan θ

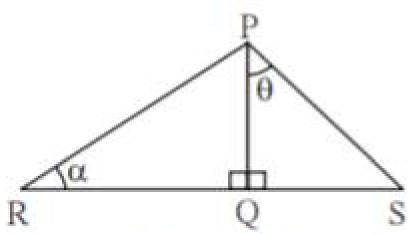

Fig. 8.15

Solution:

(i) sin α = Opposite side of α /Hypotenuse = PQ/PR
cos α = Adjacent side of α/ Hypotenuse = QR/PR
tan α = Opposite side of α/ Adjacent side of α = PQ/QR

(ii) sin θ = Opposite side of θ /Hypotenuse = QS/PS
cos θ = Adjacent side of θ/ Hypotenuse = PQ/PS
tan θ = Opposite side of θ/ Adjacent side of θ = QS/PQ

Practice Set 8.2 Page 112

1. In the following table, a ratio is given in each column. Find the remaining two ratios in the column and complete the table.

$\sin\theta$		$\frac{11}{61}$		$\frac{1}{2}$				$\frac{3}{5}$	
$\cos\theta$	$\frac{35}{37}$				$\frac{1}{\sqrt{3}}$				
$\tan\theta$			1			$\frac{21}{20}$	$\frac{8}{15}$		$\frac{1}{2\sqrt{2}}$

Solution:

(1) $\cos\theta = 35/37$

$\sin^2\theta+\cos^2\theta = 1$

$\therefore \sin^2\theta+(35/37)^2 = 1$

$\therefore \sin^2\theta = 1-(35/37)^2$

$\therefore \sin^2\theta = 1-(1225/1369)$

$\therefore \sin^2\theta = (1369-1225)/1369$

$\therefore \sin^2\theta = 144/1369$

Taking square root on both sides

$\sin\theta = 12/37$

$\tan\theta = \sin\theta/\cos\theta$

$\tan\theta = (12/37)\div(35/37)$

$\tan\theta = (12/37)\times(37/35) = 12/35$

(2) $\sin\theta = 11/61$

$\sin^2\theta+\cos^2\theta = 1$

$\therefore (11/61)^2 +\cos^2\theta = 1$

$\therefore \cos^2\theta = 1-(11/61)^2$

$\therefore \cos^2\theta = 1-(121/3721)$

$\therefore \cos^2\theta = (3721-121)/3721$

$\therefore \cos^2\theta = 3600/3721$

Taking square root on both sides

$\cos\theta = 60/61$

$\tan\theta = \sin\theta/\cos\theta$

$\tan\theta = (11/61)\div(60/61)$

$\tan\theta = (11/61)\times(61/60) = 11/60$

3. $\tan\theta = 1$

$\Rightarrow \sin\theta/\cos\theta = 1$

Let $\sin\theta = \cos\theta = k$

$\sin^2\theta+\cos^2\theta = 1$

$\Rightarrow k^2+k^2 = 1$
$\Rightarrow 2k^2 = 1$
$\Rightarrow k^2 = 1/2$
$\Rightarrow k = 1/\sqrt{2}$
$\therefore \sin\theta = k = 1/\sqrt{2}$
$\therefore \cos\theta = k = 1/\sqrt{2}$

4. $\sin\theta = 1/2$
$\sin^2\theta+\cos^2\theta = 1$
$\therefore (1/2)^2 +\cos^2\theta = 1$
$\therefore \cos^2\theta = 1-(1/2)^2$
$\therefore \cos^2\theta = 1-(1/4)$
$\therefore \cos^2\theta = (4-1)/4$
$\therefore \cos^2\theta = 3/4$
Taking square root on both sides
$\cos\theta = \sqrt{3}/2$
$\tan\theta = \sin\theta/\cos\theta$
$\tan\theta = (1/2)\div(\sqrt{3}/2)$
$\tan\theta = (1/2)\times(2/\sqrt{3}) = 1/\sqrt{3}$

(5) $\cos\theta = 1/\sqrt{3}$
$\sin^2\theta+\cos^2\theta = 1$
$\therefore \sin^2\theta+(1/\sqrt{3})^2 = 1$
$\therefore \sin^2\theta = 1-(1/\sqrt{3})^2$
$\therefore \sin^2\theta = 1-(1/3)$
$\therefore \sin^2\theta = (3-1)/3$
$\therefore \sin^2\theta = 2/3$
Taking square root on both sides
$\sin\theta = \sqrt{2}/\sqrt{3}$
$\tan\theta = \sin\theta/\cos\theta$
$\tan\theta = (\sqrt{2}/\sqrt{3})\div(1/\sqrt{3})$
$\tan\theta = (\sqrt{2}/\sqrt{3})\times(3/1) = \sqrt{2}$

(6) $\tan\theta = 21/20$
$\Rightarrow \sin\theta/\cos\theta = 21/20$
Let $\sin\theta = 21k$
$\cos\theta = 20k$
$\sin^2\theta+\cos^2\theta = 1$
$\Rightarrow (21k)^2+(20k)^2 = 1$
$\Rightarrow 441k^2+400k^2 = 1$
$\Rightarrow 841k^2 = 1$
$\Rightarrow k^2 = 1/841$
Taking square root on both sides
$\Rightarrow k = 1/29$
$\therefore \sin\theta = 21k = 21\times(1/29) = 21/29$
$\therefore \cos\theta = 20k = 20\times(1/29) = 20/29$

(7) $\tan\theta = 8/15$
$\Rightarrow \sin\theta/\cos\theta = 8/15$
Let $\sin\theta = 8k$
$\cos\theta = 15k$
$\sin^2\theta+\cos^2\theta = 1$
$\Rightarrow(8k)^2+(15k)^2 = 1$
$\Rightarrow 64k^2+225k^2 = 1$
$\Rightarrow 289k^2 = 1$
$\Rightarrow k^2 = 1/289$
Taking square root on both sides
$\Rightarrow k = 1/17$
$\therefore \sin\theta = 8k = 8\times(1/17) = 8/17$
$\therefore \cos\theta = 15k$
$\therefore \cos\theta = 15\times(1/17)$
$\therefore \cos\theta = 15/17$

(8) $\sin\theta = 3/5$
$\sin^2\theta+\cos^2\theta = 1$
$\therefore (3/5)^2 +\cos^2\theta = 1$
$\therefore \cos^2\theta = 1-(3/5)^2$
$\therefore \cos^2\theta = 1-(9/25)$
$\therefore \cos^2\theta = (25-9)/25$
$\therefore \cos^2\theta = 16/25$
Taking square root on both sides
$\cos\theta = 4/5$
$\tan\theta = \sin\theta/\cos\theta$
$\tan\theta = (3/5)\div(4/5)$
$\tan\theta = (3/5)\times(5/4)$
$\tan\theta = 3/4$

(9) $\tan\theta = 1/2\sqrt{2}$
$\Rightarrow \sin\theta/\cos\theta = 1/2\sqrt{2}$
Let $\sin\theta = k$
$\cos\theta = 2\sqrt{2}k$
$\sin^2\theta+\cos^2\theta = 1$
$\Rightarrow(k)^2+(2\sqrt{2}k)^2 = 1$
$\Rightarrow k^2+8k^2 = 1$
$\Rightarrow 9k^2 = 1$
$\Rightarrow k^2 = 1/9$
Taking square root on both sides
$\Rightarrow k = 1/3$
$\therefore \sin\theta = k = 1/3$
$\therefore \cos\theta = 2\sqrt{2}k$
$\therefore \cos\theta = 2\sqrt{2}\times(1/3)$
$\therefore \cos\theta = 2\sqrt{2}/3$

The completed table is given below.

Sl.No	1	2	3	4	5	6	7	8	9
sin θ	12/37	11/61	1/√2	1/2	√2/√3	21/29	8/17	3/5	1/3
cos θ	35/37	60/61	1/√2	√3/2	1/√3	20/29	15/17	4/5	2√2/3
tan θ	12/35	11/60	1	1/√3	√2	21/20	8/15	3/4	1/2√2

2. Find the values of -
(i) 5 sin 30˚+3tan 45˚
(ii) 4/5 $\tan^2$ 60˚+3 $\sin^2$ 60
(iii) 2 sin 30 °+ cos 0˚+ 3 sin 90°
(iv) tan 60/ (sin 60+cos 60)
(v) $\cos^2$ 45˚ + $\sin^2$ 30˚
(vi) cos 60°× cos 30° + sin 60°× sin 30°

Solution:
(i) sin 30˚ = 1/2
tan 45˚ = 1
∴ 5 sin 30˚+3tan 45˚ = (5×1/2) +(3×1)
= 5/2 +3
= (5+6)/3
= 11/3
Hence 5 sin 30˚+3tan 45˚ = 11/3

(ii) tan 60˚ = √3
sin 60˚ = √3/2
∴ 4/5 $\tan^2$ 60˚+3 $\sin^2$ 60 = (4/5)×$\sqrt{3}^2$ + 3×$(\sqrt{3}/2)^2$
= (12/5)+(9/4)
= (48+45)/20
= 93/20
Hence 4/5 $\tan^2$ 60˚+3 $\sin^2$ 60 = 93/20

(iii) sin 30˚ = 1/2
cos 0˚ = 1
sin 90˚ = 1
∴2 sin 30 °+cos 0˚+3 sin 90° = 2×(1/2)+1+3×1
= 1+1+3
= 5
Hence 2 sin 30 °+cos 0˚+3 sin 90° = 5

(iv) tan 60 = √3
sin 60 = √3/2
cos 60 = 1/2
∴ tan 60/ (sin 60+cos 60) = √3 ÷[(√3/2)+(1/2)]
= √3 ÷[(√3+1)/2]
= √3×2/(√3+1)
= 2√3/(√3+1)

Hence tan 60/ (sin 60+cos 60) = 2√3/(√3+1)

(v) cos 45˚ = 1/√2
sin 30˚ = ½
$\cos^2 45˚ + \sin^2 30˚$ =(1/√2)^2 +(1/2)^2
= (½)+(1/4)
= 3/4
Hence $\cos^2 45˚ + \sin^2 30˚$ = 3/4

(vi) cos 60° = 1/2
cos 30° = √3/2
sin 60° = √3/2
sin 30° = 1/2
cos 60°× cos 30° + sin 60°× sin 30° =(½)×(√3/2)+(√3/2)×(1/2)
= (√3/4)+ (√3/4)
= 2√3/4
= √3/2
Hence cos 60°× cos 30° + sin 60°× sin 30° = √3/2

3. If sin θ = 4/5 then find cos θ.

Solution:
sin θ = 4/5
$\sin^2θ+\cos^2θ = 1$
∴(4/5)^2+ $\cos^2θ = 1$
∴$\cos^2θ$ = 1-(4/5)^2
∴$\cos^2θ$ = 1-(16/25)
∴$\cos^2θ$ = (25-16)/25
∴ $\cos^2θ$ = 9/25
Taking square root on both sides
cos θ = 3/5
Hence cos θ is 3/5

4. If cos θ = 15/17 then find sin θ

Solution:
cos θ = 15/17
$\sin^2θ+\cos^2θ = 1$
∴$\sin^2 θ$+ (15/17)^2 = 1
∴$\sin^2θ$ = 1-(15/17)^2
∴$\sin^2θ$ = 1-(225/289)
∴$\sin^2θ$ = (289-225)/289
∴ $\sin^2θ$ = 64/289
Taking square root on both sides
sin θ = 8/17
Hence sin θ is 8/17

Problem Set 8 Page 113

1. Choose the correct alternative answer for following multiple choice questions.
(i) Which of the following statements is true ?
(A) sin θ = cos (90-θ) (B) cos θ = tan (90- θ)
(C) sin θ = tan (90- θ) (D) tan θ = tan (90- θ)
(ii) Which of the following is the value of sin 90° ?
(A)$\sqrt{3}/2$ (B) 0 (C) 1/2 (D) 1
(iii) 2 tan 45° + cos 45° - sin 45° = ?
(A) 0 (B) 1 (C) 2 (D) 3
(iv) cos 28° /sin 62 ° = ?
(A) 2 (B) -1 (C) 0 (D) 1

Solution:
(i) sin θ = cos (90-θ)
Hence Option A is the answer.

(ii) sin 90˚ = 1
Hence Option D is the answer.

(iii) tan 45° = 1
cos 45° = $1/\sqrt{2}$
sin 45° = $1/\sqrt{2}$
∴ 2 tan 45° + cos 45° - sin 45° = $(2\times1)+(1/\sqrt{2})-(1/\sqrt{2}) = 2$
Hence Option C is the answer.

(iv) cos 28° = sin (90-28) = sin 62˚
cos 28°/sin 62 ° = sin 62˚/ sin 62˚ = 1
Hence Option D is the answer.

2. In right angled △ TSU, TS = 5, ∠S = 90°, SU = 12 then find sin T, cos T, tan T. Similarly find sin U, cos U, tan U.

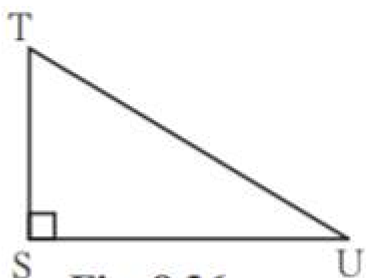

Fig. 8.26

Solution:
Given TS = 5
SU = 12
∠S = 90˚
$\therefore TU^2 = TS^2+SU^2$ [Pythagoras theorem]

$\therefore TU^2 = 5^2+12^2$
$\therefore TU^2 = 25+144$
$\therefore TU^2 = 169$
Taking square root on both sides
TU = 13
$\therefore$ sin T = Opposite side of ∠T/Hypotenuse = SU/TU = 12/13
cos T = Adjacent side of ∠T / Hypotenuse = TS/TU = 5/13
tan T = Opposite side of ∠T/Adjacent side of ∠T = SU/TS = 12/5
sin U = Opposite side of ∠U/Hypotenuse = TS/TU = 5/13
cos U = Adjacent side of ∠U / Hypotenuse = SU/TU = 12/13
tan U = Opposite side of ∠U/Adjacent side of ∠U = TS/SU = 5/12

3. In right angled △ YXZ, ∠X = 90°, XZ = 8 cm, YZ = 17 cm, find sin Y, cos Y, tan Y, sin Z, cos Z, tan Z.

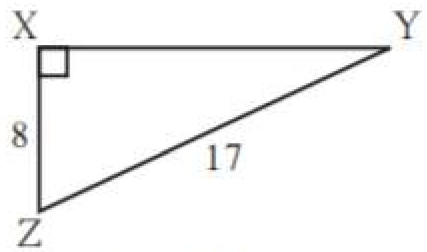

Fig. 8.27

Solution:
Given ∠X = 90°
XZ = 8 cm
YZ = 17 cm
$\therefore YZ^2 = XZ^2+XY^2$ [Pythagoras theorem]
$17^2 = 8^2+XY^2$
$XY^2 = 17^2-8^2$
$XY^2 = 289-64$
$XY^2 = 225$
Taking square root on both sides
XY = 15
$\therefore$ sin Y = Opposite side of ∠Y/Hypotenuse = XZ/YZ = 8/17
cos Y = Adjacent side of ∠Y / Hypotenuse = XY/YZ = 15/17
tan Y = Opposite side of ∠Y/Adjacent side of ∠Y = XZ/XY = 8/15
sin Z = Opposite side of ∠Z/Hypotenuse = XY/YZ = 15/17
cos Z = Adjacent side of ∠Z/ Hypotenuse = XZ/YZ = 8/17
tan Z = Opposite side of ∠Z/Adjacent side of ∠Z = XY/XZ = 15/8

4. In right angled △ LMN, if ∠N = θ, ∠M = 90°, cos θ = 24/ 25 , find sin θ and tan θ Similarly, find ($\sin^2 \theta$) and ($\cos^2 \theta$)

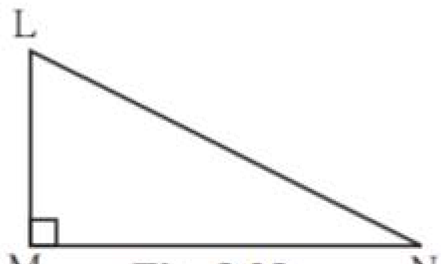

Fig. 8.28

Solution:

Given ∠N = θ

∠M = 90°

cos θ = 24/25

∴ $\cos^2\theta = (24/25)^2 = 576/625$

$\sin^2\theta + \cos^2\theta = 1$

∴ $\sin^2\theta + (24/25)^2 = 1$

∴ $\sin^2\theta = 1-(24/25)^2$

∴ $\sin^2\theta = 1-(576/625)$

∴ $\sin^2\theta = (625-576)/625$

∴ $\sin^2\theta = 49/625$

Taking square root on both sides

sin θ = 7/25

∴tan θ = sin θ/cos θ

∴tan θ = (7/25) ÷ (24/25)

∴tan θ = (7/25) × (25/24)

∴tan θ = 7/24

Hence sin θ = 7/25

tan θ = 7/24

$\sin^2\theta = 49/625$

$\cos^2\theta = (24/25)^2 = 576/625$

5. Fill in the blanks.

(i) sin20° = cos ___°

(ii) tan30°× tan ___° =1

(iii) cos40° = sin ___°

Solution:

(i) sin θ = cos (90-θ)

sin 20°= cos (90-20) = cos 70°

Hence sin 20°= cos 70°

(ii)tan θ×tan (90-θ) = 1

∴tan 30°×tan (90-30) = 1

∴ tan 30°×tan (60)° = 1

Hence tan 30°×tan 60° = 1

(iii) cos θ = sin (90-θ)

∴ cos 40° = sin (90-40)°

$\therefore$ $\cos 40^\circ = \sin 50^\circ$

Hence $\cos 40^\circ = \sin 50^\circ$