Practice Set 9.1 Page 115

1. Length, breadth and height of a cuboid shape box of medicine is 20cm, 12 cm and 10 cm respectively. Find the surface area of vertical faces and total surface area of this box.

Solution:
Given length of the cuboid shape box of medicine, l = 20 cm
Breadth, b = 12 cm
Height, h = 10 cm
$\therefore$ Surface area of vertical faces = 2(lh +bh)
= 2(20×10+12×10) = 2(200+120) = 2×320 = 640 cm^2
Total surface area of a cuboid = 2(lb+bh+lh)
$\therefore$ Total surface area of the box = 2(20×12+12×10+20×10)
= 2(240+120+200) = 2×560 = 1120 cm^2
Hence the surface area of vertical faces and total surface area of the box are 640 cm^2 and 1120 cm^2.

2. Total surface area of a box of cuboid shape is 500 sq. unit. Its breadth and height is 6 unit and 5 unit respectively. What is the length of that box ?

Solution:
Given Total surface area of the cuboid box = 500
Breadth of the box, b = 6
Height of the box, h = 5
Total surface area of the box = 2(lb+bh+lh)
$\therefore 500 = 2(l\times6+6\times5+l\times5)$
$\therefore 500 = 2(6l+30+5l)$
$\therefore 500 = 2(11l+30)$
$\therefore 500 = 22l+60$
$\therefore 22l = 500-60 = 440$
$l = 440/22 = 20$
Hence the length of the box is 20 units.

3. Side of a cube is 4.5 cm. Find the surface area of all vertical faces and total surface area of the cube.

Solution:
Given side of the cube, l = 4.5 cm
Surface area of all vertical faces of the cube = $4l^2$
= 4×4.5^2
= 4×20.25
= 81 cm^2
Total surface area of the cube = $6l^2$
= 6×4.5^2 = 6×20.25
= 121.5 cm^2
Hence the surface area of all vertical faces of the cube and Total surface area of the cube are 81 cm^2 and 121.5 cm^2 respectively.

4. Total surface area of a cube is 5400 sq. cm. Find the surface area of all vertical faces of the cube.

Solution:

Given total surface area of the cube, = 5400 cm^2
Total surface area of the cube = $6l^2$
$\therefore 6l^2 = 5400$
$l^2 = 5400/6 = 900$
$\Rightarrow l = \sqrt{900} = 30$
$\therefore$ Surface area of all vertical faces of the cube = $4l^2$
$\therefore$ Surface area of all vertical faces of the cube = 4×30^2
= 4×900
= 3600 cm^2
Hence the Surface area of all vertical faces of the cube is 3600 cm^2

5. Volume of a cuboid is 34.50 cubic metre. Breadth and height of the cuboid is 1.5m and 1.15m respectively. Find its length.

Solution:
Given volume of the cuboid, V = 34.50 m^3
Breadth of the cuboid, b = 1.5 m
Height of the cuboid, h = 1.15 m
Volume of th cuboid, V = l×b×h
$\therefore 34.50 = l\times1.5\times1.15$
$\therefore 34.50 = 1.725\ l$
$\Rightarrow l = 34.50/1.725 = 20$ m
Hence the length of the cuboid is 20 m.

6. What will be the volume of a cube having length of edge 7.5 cm ?

Solution:
Given length of the cube, l = 7.5 cm
$\therefore$ Volume of the cube, $V = l^3 = 7.5^3 = 7.5\times7.5\times7.5 = 421.875$ cm^3
Hence the volume of the cube is 421.875 cm^3

7. Radius of base of a cylinder is 20cm and its height is 13cm, find its curved surface area and total surface area. (π = 3.14)

Solution:
Given radius of the cylinder, r = 20 cm
Height of the cylinder, h = 13 cm
Curved surface area of the cylinder = 2πrh
= 2×3.14×20×13
= 1632.8 cm^2
Total surface area of a cylinder = 2πr(r+h)
= 2×3.14×20×(20+13)
= 2×3.14×20×33
= 4144.8 cm^2
Hence the curved surface area of the cylinder and total surface area of a cylinder are 1632.8 cm^2 and 4144.8 cm^2 respectively.

8. Curved surface area of a cylinder is 1980 cm^2 and radius of its base is 15cm. Find the height of the cylinder. (π = 22/7).

Solution:

Given curved surface area of the cylinder = 1980 cm^2

Radius of the cylinder, r = 15 cm

Curved surface area of the cylinder = $2\pi rh$

$\therefore 1980 = 2\times(22/7)\times15\times h$

$\Rightarrow h = 1980\times7/(2\times22\times15)$

$\Rightarrow h = 21$

Hence the height of the cylinder is 21 cm.

Practice Set 9.2 Page 119

1. Perpendicular height of a cone is 12 cm and its slant height is 13 cm. Find the radius of the base of the cone.

Solution:
Given perpendicular height of a cone, h = 12 cm
Slant height, l = 13 cm
Relation between the slant height, radius and height of a cone is given by the following equation.
$l^2 = r^2+h^2$
$\therefore 13^2 = r^2+12^2$
$\Rightarrow r^2 = 13^2-12^2 = 169-144 = 25$
Taking square root on both sides,
r = 5
Hence the radius of the base of the cone is 5 cm.

2. Find the volume of a cone, if its total surface area is 7128 sq.cm and radius of base is 28 cm. (π = 22/7).

Solution:
Given total surface area of cone = 7128 cm^2
Radius of the cone, r = 28 cm
total surface area of cone = $\pi r(l+r)$
$\therefore 7128 = (22/7) \times 28(l+28)$
$\therefore 7128 = 88(l+28)$
$\therefore l+28 = 7128/88$
$\therefore l+28 = 81$
$\Rightarrow l = 81-28 = 53$ cm
$\therefore$ slant height, l = 53 cm
We know that, $l^2 = r^2+h^2$
$\therefore 53^2 = 28^2+h^2$
$\therefore h^2 = 53^2-28^2$
$\therefore h^2 = 53^2-28^2$
$\therefore h^2 = 2809-784 = 2025$
Taking square root on both sides
h = 45
Volume of the cone, V = $(1/3)\pi r^2 h$
$\therefore V = (1/3)\times(22/7)\times 28^2\times 45$
$\therefore V = 36960\ cm^3$
Hence the volume of the cube is 36960 cm^3.

3. Curved surface area of a cone is 251.2 cm^2 and radius of its base is 8cm. Find its slant height and perpendicular height. (π = 3.14)

Solution:
Given curved surface area of the cone = 251.2 cm^2
Radius of the cone, r = 8 cm
Curved surface area of the cone = $\pi r l$

$\therefore 251.2 = 3.14 \times 8 \times l$
$\Rightarrow l = 251.2/(3.14 \times 8)$
$\Rightarrow l = 10$
$\therefore$ slant height, $l = 10$ cm
We know that, $l^2 = r^2+h^2$
$10^2 = 8^2+h^2$
$\therefore h^2 = 10^2 - 8^2$
$\therefore h^2 = 100-64 = 36$
$\Rightarrow h = 6$ cm
$\therefore$ perpendicular height = 6 cm
Hence the slant height and perpendicular height of the cone is 10cm and 6 cm respectively.

4. What will be the cost of making a closed cone of tin sheet having radius of base 6 m and slant height 8 m if the rate of making is Rs.10 per sq.m ?

Solution:
Given radius of the cone, r = 6m
slant height, $l = 8$ m
To find the cost of making a closed cone, we need to find the total surface area of the cone.
Total surface area of the cone = $\pi r(l+r)$
$= (22/7) \times 6(8+6)$
$= (22/7) \times 6 \times 14$
$= 22 \times 6 \times 2$
$= 264\ m^2$
Cost of making the cone per sq.m = Rs.10
Total cost = total surface area × rate of making the cone per sq.m
= 10×264 = Rs. 2640
Hence the cost of making cone is Rs.2640.

5. Volume of a cone is 6280 cubic cm and base radius of the cone is 30 cm. Find its perpendicular height. (π = 3.14)

Solution:
Given volume of cone, V = 6280 cm^3
Base radius of the cone, r = 30 cm
Volume of cone, $V = (1/3)\pi r^2 h$
$\therefore 6280 = (1/3) \times 3.14 \times 30^2 \times h$
$\therefore h = (3 \times 6280)/(3.14 \times 900)$
$\therefore h = 6.67$
Hence the perpendicular height of the cone is 6.67 approximately.

6. Surface area of a cone is 188.4 sq.cm and its slant height is 10cm. Find its perpendicular height (π = 3.14)

Solution:
Given surface area of a cone = 188.4 cm^2
Slant height, $l = 10$ cm
Surface area of the cone = $\pi r l$
$\therefore 188.4 = 3.14 \times r \times 10$
$\therefore r = 188.4/(3.14 \times 10)$

$\therefore r = 6$ cm
We know that, $l^2 = r^2+h^2$
$\therefore 10^2 = 6^2+h^2$
$\Rightarrow h^2 = 10^2-6^2$
$\Rightarrow h^2 = 100-36$
$\Rightarrow h^2 = 64$
Taking square root on both sides
h = 8 cm
Hence the perpendicular height of the cone is 8 cm.

7. Volume of a cone is 1232 cm^3 and its height is 24cm. Find the surface area of the cone. ($\pi = 22/7$)

Solution:
Given volume of the cone, V = 1232 cm^3
Height of the cone, h = 24 cm
Volume of the cone, $V = (1/3)\pi r^2 h$
$\therefore 1232 = (1/3)\times(22/7)\times r^2\times 24$
$r^2 = (1232\times3\times7)/(22\times24)$
$r^2 = 49$
$\Rightarrow r = 7$
We know that $l^2 = r^2+h^2$
$\therefore l^2 = 7^2+24^2$
$\therefore l^2 = 49+576 = 625$
$\Rightarrow l = 25$
Surface area of the cone = $\pi r l$
$= (22/7)\times7\times25 = 550$
Hence the surface area of the cone is 550 cm^2.

8. The curved surface area of a cone is 2200 sq.cm and its slant height is 50 cm. Find the total surface area of cone. ($\pi = 22/7$)

Solution:
Given curved surface area of a cone = 2200 cm^2
Slant height, $l = 50$ cm
Curved surface area of the cone = $\pi r l$
$\therefore 2200 = (22/7)\times r\times 50$
$\therefore r = (2200\times7)/(22\times50)$
$\therefore r = 14$ cm
Total surface area of the cone = $\pi r(l+r)$
$= (22/7)\times14\times(50+14)$
$= (22/7)\times14\times64$
$= 2816\ cm^2$
Hence the total surface area of the cone is 2816 cm^2.

9. There are 25 persons in a tent which is conical in shape. Every person needs an area of 4 sq.m. of the ground inside the tent. If height of the tent is 18m, find the volume of the tent.

Solution:
Given height of the tent, h = 18 m

Number of persons in tent = 25
Area needed for one person = 4 m^2
Hence area needed for 25 persons = 25×4 = 100 m^2
Here base area of the tent = 100 m^2
i.e, $\pi r^2 = 100$
Volume of the conical tent = $(1/3)\pi r^2 h$
= (1/3)×100×18 = 600 m^3
Hence the volume of the tent is 600 m^3.

10. In a field, dry fodder for the cattle is heaped in a conical shape. The height of the cone is 2.1m. and diameter of base is 7.2 m. Find the volume of the fodder. if it is to be covered by polythene in rainy season then how much minimum polythene sheet is needed ? (π = 22/7 and $\sqrt{17.37}$ = 4.17)

Solution:
Given the height of the cone, h = 2.1 m
Diameter of the base = 7.2 m
$\therefore$ Radius, r = diameter/2 = 7.2/2 = 3.6
Volume of the cone, $V = (1/3)\pi r^2 h$
$\therefore V = (1/3)\times(22/7)\times 3.6^2\times 2.1$
= 28.51 m^3
We know that $l^2 = r^2+h^2$
$\therefore\ l^2 = 3.6^2+2.1^2$
$\therefore\ l^2 = 12.96+4.41$
$\therefore\ l^2 = 17.37$
$\Rightarrow l = \sqrt{17.31} = 4.17$
To find how much polythene is required to cover the dry fodder, we need to find the curved surface area of the cone.
Curved surface area of the cone = $\pi r l$
= (22/7)×3.6×4.17
= 47.18 m^2
Amount of polythene required = 47.18 m^2
Hence the volume of the fodder is 28.51 m^3 and the amount of polythene required is 47.18 m^2.

Practice Set 9.3 Page 123

1. Find the surface areas and volumes of spheres of the following radii.
(i) 4 cm (ii) 9 cm (iii) 3.5 cm. (π = 3.14)

Solution:
(i) Given radius of the sphere, r = 4 cm
$\therefore$ Surface area of the sphere = $4\pi r^2$
= $4\times3.14\times4^2$
= $4\times3.14\times16$
= 200.96 cm^2
Volume of the sphere, V = $(4/3)\pi r^3$
= $(4/3)\times3.14\times4^3$
= $(4/3)\times3.14\times64$
= 267.946
= 267.95 cm^3
Hence the surface area and volume of the cone are 200.96 cm^2 and 267.95 cm^3 respectively.

(ii) Given radius of the sphere, r = 9 cm
$\therefore$ Surface area of the sphere = $4\pi r^2$
= $4\times3.14\times9^2$
= $4\times3.14\times81$
= 1017.36 cm^2
Volume of the sphere, V = $(4/3)\pi r^3$
= $(4/3)\times3.14\times9^3$
= $(4/3)\times3.14\times729$
= 3052.08 cm^3
Hence the surface area and volume of the cone are 1017.36 $cm^2$and 3052.08 cm^3 respectively.

(iii) Given radius of the sphere, r = 3.5 cm
$\therefore$ Surface area of the sphere = $4\pi r^2$
= $4\times3.14\times3.5^2$
= $4\times3.14\times12.25$
= 153.86 cm^2
Volume of the sphere, V = $(4/3)\pi r^3$
= $(4/3)\times3.14\times3.5^3$
= $(4/3)\times3.14\times42.875$
= 179.50 cm^3
Hence the surface area and volume of the cone are 153.86 $cm^2$and 179.50 cm^3 respectively.

2. If the radius of a solid hemisphere is 5cm, then find its curved surface area and total surface area. (π = 3.14)

Solution:
Given radius of the solid hemisphere = 5 cm
Curved surface area of the hemisphere = $2\pi r^2$
= $2\times3.14\times5^2$
= 157 cm^2
Total surface area = $3\pi r^2$

$= 3\times3.14\times5^2$
$= 3\times3.14\times25$
$= 235.5\ cm^2$

Hence the Curved surface area and total surface area of the hemisphere are 157 cm^2 and 235.5 cm^2.

3. If the surface area of a sphere is 2826 cm^2 then find its volume. ($\pi = 3.14$)

Solution:

Given surface area of a sphere = 2826 cm^2
Surface area of the sphere = $4\pi r^2$
$\therefore 4\pi r^2 = 2826\ cm^2$
$\therefore\ 4\times3.14\times r^2 = 2826$
$\Rightarrow r^2 = 2826/(4\times3.14)$
$\Rightarrow r^2 = 225$
Taking square root on both sides
r = 15 cm
Volume of the sphere, $V = (4/3)\pi r^3$
$= (4/3)\times3.14\times15^3$
$= (4/3)\times3.14\times15\times15\times15$
$= 14130\ cm^3$

Hence the volume of the sphere is 14130 cm^3.

4. Find the surface area of a sphere, if its volume is 38808 cubic cm. ($\pi = 22/7$)

Solution:

Given volume of the sphere = 38808 cm^3
Volume of the sphere, $V = (4/3)\pi r^3$
$\therefore (4/3)\pi r^3 = 38808$
$\therefore (4/3)\times(22/7)r^3 = 38808$
$\Rightarrow r^3 = (38808\times3\times7)/(4\times22)$
$\Rightarrow r^3 = 9261$
Taking cube root on both sides
r = 21
Surface area of the sphere = $4\pi r^2$
$= 4\times(22/7)\times21^2$
$= 4\times(22/7)\times441$
$= 5544\ cm^2$

Hence the surface area of the sphere is 5544 cm^2.

5. Volume of a hemisphere is 18000 π cubic cm. Find its diameter.

Solution:

Given volume of hemisphere = 18000 π cm^3.
Volume of the hemisphere, $V = (2/3)\pi r^3$
$\therefore (2/3)\pi r^3 = 18000\ \pi$
$\Rightarrow r^3 = 18000\times3/2$
$\Rightarrow r^3 = 27000$
Taking cube root on both sides

$\Rightarrow r = 30$
$\therefore$ radius = 30 cm
$\therefore$ Diameter = 2×radius = 2×30 = 60 cm
Hence the diameter of the hemisphere is 60cm.

Problem Set 9 Page 123

1. If diameter of a road roller is 0.9 m and its length is 1.4 m, how much area of a field will be pressed in its 500 rotations ?

Solution:

Given diameter of the road roller = 0.9 m
∴radius, r = diameter/2 = 0.9/2 = 0.45 m
Length, h = 1.4 m
The area of the field pressed in one rotation of the road roller is equal to the curved surface area of the road roller.
Curved surface area of the road roller = $2\pi rh$
= $2\times(22/7)\times0.45\times1.4$
= $3.96\ m^2$
∴Area of the field pressed in 500 rotation = $500\times3.96 = 1980\ m^2$
Hence the area of the field pressed in 500 rotation is $1980\ m^2$.

2. To make an open fish tank, a glass sheet of 2 mm gauge is used. The outer length, breadth and height of the tank are 60.4 cm, 40.4 cm and 40.2 cm respectively. How much maximum volume of water will be contained in it ?

Solution:

Given thickness of the glass = 2 mm = 0.2 cm [1 cm = 10 mm]
Outer length of the tank = 60.4 cm
Thickness of glass on both sides = 0.2+0.2 = 0.4 cm
∴Inner length of the tank, *l* = Outer length of the tank- Thickness of glass on both sides
= 60.4-0.4 = 60 cm
Outer breadth of the tank = 40.4 cm
∴Inner breadth of the tank, b = Outer breadth of the tank- Thickness of glass on both sides
= 40.4 -0.4 = 40 cm
Inner height of the tank, h = inner height -thickness of glass on bottom side
= 40.2-0.2 = 40 cm [Tank is open at the top]
∴Maximum volume of water contained in the tank
= Inner volume of the tank
= $l\times b\times h$
= $60\times40\times40$
= $96000\ cm^3$
Hence the maximum volume of water contained in the tank is $96000\ cm^3$.

3. If the ratio of radius of base and height of a cone is 5:12 and its volume is 314 cubic metre. Find its perpendicular height and slant height (π = 3.14)

Solution:

Given the ratio of the radius and the height of the cone = 5:12
Let x be a common multiple.
∴Radius, r = 5x
Height, h = 12x
Volume of the cone = $314\ m^3$
Volume of the cone = $(1/3)\pi r^2h$
∴$(1/3)\pi r^2h = 314$

$\therefore (1/3)\pi(5x)^2 \times 12x = 314$
$\therefore (1/3) \times 3.14 \times 25x^2 \times 12x = 314$
$\Rightarrow x^3 = 314 \times 3/(3.14 \times 25 \times 12) = 1$
$\Rightarrow x = 1$
$\therefore$ Radius, r = 5x
= 5×1 = 5 m
Height, h = 12x
= 12×1 = 12 m
We know that $l^2 = r^2 + h^2$
$\therefore\ l^2 = 5^2 + 12^2 = 25 + 144 = 169$
Taking square root on both sides
$l = 13$
$\therefore$ slant height , l = 13 m
Hence the perpendicular height and slant height of the cone is 12 m and 13 m respectively.

4. Find the radius of a sphere if its volume is 904.32 cubic cm. (π = 3.14)

Solution:
Given volume of a sphere, V = 904.32 cm^3
Volume of a sphere, $V = (4/3)\pi r^3$
$\therefore (4/3)\pi r^3 = 904.32$
$\therefore (4/3) \times 3.14 \times r^3 = 904.32$
$\Rightarrow r^3 = (3 \times 904.32)/(4 \times 3.14)$
$\Rightarrow r^3 = 216$
Taking cube root on both sides
r = 6
Hence the radius of the sphere is 6 cm.

5. Total surface area of a cube is 864 sq.cm. Find its volume.

Solution:
Given total surface area of a cube = 864 cm^2
Total surface area of a cube = $6l^2$
$\therefore 6l^2 = 864$
$\therefore l^2 = 864/6 = 144$
Taking square root on both sides
$l = 12$
Volume of a cube = l^3
$= 12^3$
= 12×12×12
= 1728 cm^3
Hence the volume of the cube is 1728 cm^3.

6. Find the volume of a sphere, if its surface area is 154 sq.cm.

Solution:
Given surface area of a sphere = 154 cm^2
Surface area of a sphere = $4\pi r^2$
$\therefore 4\pi r^2 = 154$

$\therefore 4\times(22/7)\times r^2 = 154$ [$\pi = 22/7$]
$\therefore r^2 = 154\times7/(4\times22)$
$\therefore r^2 = 49/4$
Taking square root on both sides
$r = 7/2$
$\therefore$ Volume of the sphere, $V = (4/3)\pi r^3$
$= (4/3)\times(22/7)\times(7/2)^3$
$= 179.67\ cm^3$
Hence the volume of the sphere is $179.67\ cm^3$.

7. Total surface area of a cone is 616 sq.cm. If the slant height of the cone is three times the radius of its base, find its slant height.

Solution:
Given slant height is three times the radius of the base.
Let radius be r.
Slant height, $l = 3r$
Total surface area of the cone $= \pi r(r+l)$
$= \pi r(r+3r)$
$= \pi r(4r)$
$= 4\pi r^2$
Given total surface area of a cone $= 616\ cm^2$
$\therefore 4\pi r^2 = 616$
$\therefore 4\times(22/7)\times r^2 = 616$
$\therefore r^2 = 616\times7/(4\times22)$
$\therefore r^2 = 49$
Taking square root on both sides,
$\Rightarrow r = 7$ cm
Slant height, $l = 3r$
$= 3\times7$
$= 21$ cm
Hence the slant height of the cone is 21 cm.

8. The inner diameter of a well is 4.20 metre and its depth is 10 metre. Find the inner surface area of the well. Find the cost of plastering it from inside at the rate Rs.52 per sq.m.

Solution:
Given inner diameter = 4.20 m
$\therefore$ radius, $r = 4.20/2 = 2.10$ m
Depth of the well, $h = 10$ m
Inner surface area of the wall $= 2\pi rh$
$= 2\times(22/7)\times2.10\times10$
$= 132\ m^2$
Cost of plastering per square metre = Rs. 52
$\therefore$ Cost of plastering 132 square metre $= 132\times 52 = 6864$
Hence the cost of plastering the well from inside is Rs. 6864.

9. The length of a road roller is 2.1m and its diameter is 1.4m. For levelling a ground 500 rotations of the road roller were required. How much area of ground was levelled by the road roller? Find the cost of levelling at the rate of Rs. 7 per sq. m.

Solution:
Given diameter of the road roller = 1.4m
$\therefore$ radius, r = diameter/2 = 1.4/2 = 0.7 m
Length, h = 2.1 m
The area of the field pressed in one rotation of the road roller is equal to the curved surface area of the road roller.
Curved surface area of the road roller = $2\pi rh$
= $2\times(22/7)\times0.7\times2.1$
= $9.24\ m^2$
$\therefore$ Area of the field pressed in 500 rotation = $500\times9.24 = 4620\ m^2$
Hence the area of the field pressed in 500 rotation is $4620\ m^2$.

Rate of leveling = Rs. $7/m^2$
$\therefore$ Total cost of leveling $4620\ m^2 = 4620\times7$ = Rs. 32340
Hence the total cost of leveling ground is Rs. 32340.