Exercise 9.3

1. Find the solution of $\frac{dy}{dx} = 2^{y-x}$
Solution:

Given differential equation,

$$\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{dx} = \frac{2^y}{2^x}$$

On separating the variables, we have

$$\frac{dy}{2^y} = \frac{dx}{2^x} \Rightarrow 2^{-y}dy = 2^{-x}dx$$

Now, integrating both the sides, we get

$$\int 2^{-y}dy = \int 2^{-x}dx$$

$$\frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + c \Rightarrow -2^{-y} = -2^{-x} + c\log 2$$

$$-2^{-y} + 2^{-x} = c\log 2$$

$$2^{-x} - 2^{-y} = k \qquad [\text{where } c\log 2 = k]$$

Thus, the solution of the differential equation is $2^{-x} - 2^{-y} = k$

2. Find the differential equation of all non-vertical lines in a plane.
Solution:

We know that, the equation of all non-vertical lines are y = mx + c
On differentiating w.r.t. x, we get
dy/dx = m
Again, on differentiating w.r.t. x, we have
$d^2y/dx^2 = 0$
Thus, the required equation is $d^2y/dx^2 = 0$.

3. Given that $\frac{dy}{dx} = e^{-2y}$ **and *y* = 0 when *x* = 5.**

Find the value of *x* when *y* = 3.
Solution:

Given equation,

$$\frac{dy}{dx} = e^{-2y}$$

$$\frac{dy}{e^{-2y}} = dx \Rightarrow e^{2y}.dy = dx$$

Integrating both sides, we get

$$\int e^{2y}dy = \int dx \Rightarrow \frac{1}{2}e^{2y} = x + c$$

Put $y = 0$ and $x = 5$

$\Rightarrow \frac{1}{2}e^0 = 5 + c \Rightarrow c = \frac{1}{2} - 5 = -\frac{9}{2}$

So, The equation becomes $\frac{1}{2}e^{2y} = x - \frac{9}{2}$

Now putting $y = 3$, we get

$$\frac{1}{2}e^6 = x - \frac{9}{2} \Rightarrow x = \frac{1}{2}e^6 + \frac{9}{2}$$

Thus, the required value of $x = \frac{1}{2}(e^6 + 9)$.

4. Solve the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$
Solution:

Given different equation,

$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$

On dividing by $(x^2 - 1)$, we have

$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = \frac{1}{(x^2 - 1)^2}$

Clearly, it is a linear differential equation of first order and first degree.

Now, $P = \frac{2x}{x^2 - 1}$ and $Q = \frac{1}{(x^2 - 1)^2}$

Integrating factor I.E $= e^{\int P dx} = e^{\int \frac{2x}{x^2 - 1} dx} = e^{\log(x^2 - 1)} = (x^2 - 1)$.

Hence, the solution of the equation is

$y(x^2 - 1) = \frac{1}{2}\log\left|\frac{x - 1}{x + 1}\right| + C.$

5. Solve the differential equation $\frac{dy}{dx} + 2xy = y$
Solution:

Given differential equation,

dy/dx + 2xy = y

$\frac{dy}{dx} = y - 2xy \Rightarrow \frac{dy}{dx} = y(1 - 2x) \Rightarrow \frac{dy}{y} = (1 - 2x)\,dx$

Integrating both sides, we have

$\int \frac{dy}{dy} = \int (1 - 2x)dx \Rightarrow \log y = x - 2.\frac{x^2}{2} + \log c$

$\log y = x - x^2 + \log c \Rightarrow \log y - \log c = x - x^2$

$\log\frac{y}{c} = x - x^2 \Rightarrow \frac{y}{c} = e^{x-x^2}$

$\therefore\ y = y = c\,.\,e^{x-x^2}$

Thus, the required solution is $y = c\,.\,e^{x-x^2}$.

6. Find the general solution of $\frac{dy}{dx} + ay = e^{mx}$
Solution:

Given equation, dy/dx + ay = e^{mx}
Solving for linear differential equation of first order, we have
P = a and Q = e^{mx}

So, I.F $= e^{\int P dx} = e^{\int a.dx} = e^{ax}$
Solution of equation is $y \times$ I.F $= \int Q\,$ I.F $dx + c$

$$y.e^{ax} = \int e^{mx}.e^{ax}dx + c \Rightarrow y\,.\,e^{ax} = \int e^{(m+a)x}dx + c$$

$$y.e^{ax} = \frac{e^{(m+a)x}}{(m+a)} + c \Rightarrow y = \frac{e^{(m+a)x}}{(m+a)}.e^{-ax} + c.e^{-ax}$$

$$\therefore\quad y = \frac{e^{mx}}{(m+a)} + c\,.\,e^{-ax}$$

Thus, the required solution is $y = \frac{e^{mx}}{(m+a)} + c\,.\,e^{-ax}$.

7. Solve the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$
Solution:

Given differential equation, dy/dx + 1 = e^{x+y}
Substituting x + y = t and differentiating w.r.t. x, we have

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} = e^t \Rightarrow \frac{dt}{e^t} = dx \Rightarrow e^{-t}dt = dx$$

Integrating both sides, we have

$$\int e^{-t}dt = \int dx \Rightarrow -e^{-t} = x + c$$

$$-e^{-(x+y)} = x + c \Rightarrow \frac{-1}{e^{x+y}} = x + c \Rightarrow (x+c)\,e^{x+y} = -1$$

Thus, the required solution is $(x+c)\,.\,e^{x\ \ y} + 1 = 0$

8. Solve: ***ydx – xdy = x²ydx.***
Solution:

Given equation, $ydx - xdy = x^2ydx$
$y\,dx - x^2y\,dx = xdy$
$y(1 - x^2)\,dx = xdy$

$$\left(\frac{1-x^2}{x}\right)dx = \frac{dy}{y} \Rightarrow \left(\frac{1}{x} - x\right)dx = \frac{dy}{y}$$

Integrating both sides we get

$$\int\left(\frac{1}{x} - x\right)dx = \int\frac{dy}{y}$$

$$\log x - \frac{x^2}{2} = \log y + \log c$$

$$\log x - \frac{x^2}{2} = \log yc \Rightarrow \log x - \log c = \frac{x^2}{2} \Rightarrow \log\frac{x}{yc} = \frac{x^2}{2}$$

$$\Rightarrow \frac{x}{yc} = e^{x^2/2} \Rightarrow \frac{yc}{x} = e^{-x^2/2} \Rightarrow yc = xe^{-x^2/2}$$

$$\therefore y = \frac{1}{c}.xe^{-x^2/2} \Rightarrow y = kxe^{-x^2/2} \qquad \left[\because k = \frac{1}{c}\right]$$

Thus, the required solution is $y = kxe^{-x^2/2}$

9. Solve the differential equation dy/dx = 1 + x + y^2 + xy^2, when y = 0, x = 0.
Solution:

Given equation, dy/dx = 1 + x + y^2 + xy^2

$$\frac{dy}{dx} = 1(1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2) \Rightarrow \frac{dy}{1+y^2} = (1+x)\,dx$$

On integerating both side, we get

$$\int\frac{dy}{1+y^2} = \int(1+x)\,dx \Rightarrow \tan^{-1}y = x + \frac{x^2}{2} + c$$

Put $x = 0$ and $y = 0$, we get $\tan^{-1}(0) = 0 + 0 + c \Rightarrow c = 0$

$$\therefore \tan^{-1}y = x + \frac{x^2}{2} \Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$$

Thus, the required solution is $y = \tan\left(x + \frac{x^2}{2}\right)$.

10. Find the general solution of (x + 2y^3) dy/dx = y.
Solution:

Given equation, $(x+2y^3)\frac{dy}{dx}=y$

$\frac{dy}{dx}=\frac{y}{x+2y^3} \Rightarrow \frac{dx}{dy}=\frac{x+2y^3}{y}$

$\frac{dx}{dy}=\frac{x}{y}+\frac{2y^3}{y} \Rightarrow \frac{dx}{dy}-\frac{x}{y}=2y^2$

Here $P=-\frac{1}{y}$ and $Q=2y^2$.

So,

Integrating factor I.F. $= e^{\int P dy} = e^{\int -\frac{1}{y} dy} = e^{-\log y} = e^{\log \frac{1}{y}} = \frac{1}{y}$

So the solution of the equation is

$x.\text{I.F.} = \int \text{Q.I.F.} dy + c$

$x.\frac{1}{y} = \int 2y^2.\frac{1}{y} dy + c$

$\frac{x}{y} = 2\int y\, dy + c \Rightarrow \frac{x}{y} = 2.\frac{y^2}{2} + c \Rightarrow \frac{x}{y} = y^2 + c$

$x = y^3 + cy = y(y^2+c)$

Thus, the required solution is $x = y(y^2+c)$

11. If $y(x)$ is a solution of $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx}=-\cos x$ and y (0) = 1, then find the value of y(π/2).

Solution:

Given equation,

$$\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$$

$\left(\frac{2+\sin x}{\cos x}\right)\frac{dy}{dx} = -(1+y) \Rightarrow \frac{dy}{(1+y)} = -\left(\frac{\cos x}{2+\sin x}\right)dx$

Integrating both sides, we get

$$\int \frac{dy}{1+y} = -\int \frac{\cos x}{2+\sin x} dx$$

$$\log|1+y| = -\log|2+\sin x| + \log c$$

$\log|1+y| + \log|2+\sin x| = \log c$

$\log(1+y)(2+\sin x) = \log c \Rightarrow (1+y)(2+\sin x) = c$

Put $x=0$ and $y=1$, we get

$(1+1)(2+\sin 0) = c \Rightarrow 4 = c$

So, equation is $(1+y)(2+\sin x) = 4$

Now put $x = \frac{\pi}{2}$

$$(1+y)\left(2+\sin\frac{\pi}{2}\right) = 4$$

$$(1+y)(2+1) = 4 \Rightarrow 1+y=\frac{4}{3} \Rightarrow y=\frac{4}{3}-1=\frac{1}{3}$$

So, $y\left(\frac{\pi}{2}\right)=\frac{1}{3}$

Thus, the required solution is $y\left(\frac{\pi}{2}\right)=\frac{1}{3}$

12. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt}-ty=1$ and $y(0) = -1$, then show that $y(1) = -1/2$.
Solution:

Given equation,

$$(1+t)\frac{dy}{dt}-ty=1 \Rightarrow \frac{dy}{dt}-\left(\frac{t}{1+t}\right)y=\frac{1}{1+t}$$

Here, $P=\frac{-t}{1+t}$ and $Q=\frac{1}{1+t}$

Now,

Integrating factor I.F. $=e^{\int P dt}=e^{\int\frac{-t}{1+t}dt}=e^{-\int\frac{1+t-1}{1+t}dt}$

$$=e^{-\int\left(1-\frac{1}{1+t}\right)dt}=e^{-[t-\log(1+t)]}$$

$$=e^{-t+\log(1+t)}=e^{-t}.e^{\log(1+t)}$$

$\therefore$ I.F. $=e^{-t}.(1+t)$

The required solution of the given different equation is,

$$y.\text{I.F.}=\int Q.\text{I.F.}\,dt+c$$

$$y.e^{-t}(1+t)=\int\frac{1}{(1+t)}.e^{-t}.(1+t)\,dt+c$$

$$y.e^{-t}(1+t)=\int e^{-t}\,dt+c$$

$$\Rightarrow y.e^{-t}(1+t)=-e^{-t}+c$$

Puting t = 0 and y = -1 $\quad [\because y(0)=-1]$

$$-1.e^{0}.1=-e^{0}+c$$

$$-1=-1+c \Rightarrow c=0$$

So the equation becomes

$$ye^{-t}(1+t)=-e^{-t}$$

Now put $t=1$

$$y.e^{-1}(1+1)=-e^{-1}$$

$$2y=-1 \Rightarrow y=-\frac{1}{2}$$

Thus, $y(1)=-\frac{1}{2}$ is verified.

13. Form the differential equation having $y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$, where A and B are arbitrary constants, as its general solution.
Solution:

Given equation is $y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$

$$\frac{dy}{dx} = 2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}} + A \cdot \left(\frac{-1}{\sqrt{1-x^2}}\right)$$

On multiplying both sides by $\sqrt{1-x^2}$, we get

$$\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1}x - A$$

Again differentiating w.r.t x, we get

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1 \times (-2x)}{2\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

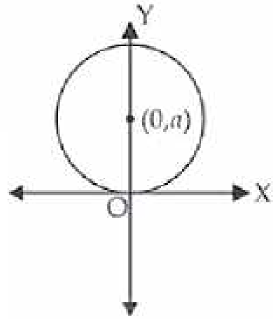
On multiplying both sides by $\sqrt{1-x^2}$, we get

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

Thus, the above is the required differential equation.

14. Form the differential equation of all circles which pass through origin and whose centres lie on y-axis.
Solution:

The equation of circles which pass through the origin and whose centre lies on the y-axis is given by,
$(x - 0)^2 + (y - a)^2 = a^2$, where $(0, a)$ is the centre
$x^2 + y^2 + a^2 - 2ay = a^2$
$x^2 + y^2 - 2ay = 0$ ……… (i)
Differentiating both sides w.r.t. x, we get



$$2x + 2y \cdot \frac{dy}{dx} - 2a \cdot \frac{dy}{dx} = 0$$

$$x + y\frac{dy}{dx} - a \cdot \frac{dy}{dx} = 0 \quad \Rightarrow \quad x + (y-a) \cdot \frac{dy}{dx} = 0$$

$$y - a = \frac{x}{dy/dx}$$

$$\therefore \quad a = y + \frac{-x}{dy/dx}$$

Putting the value of a in eq. (i), we get

$$x^2+y^2-2\left(y+\frac{x}{dy/dx}\right)y=0$$

$$\Rightarrow x^2+y^2-2y^2-\frac{2xy}{\frac{dy}{dx}}=0 \quad \Rightarrow x^2-y^2=\frac{2xy}{\frac{dy}{dx}}$$

$$\therefore \quad (x^2-y^2)\frac{dy}{dx}-2xy=0$$

Thus, the required differential equation is

$$(x^2-y^2)\frac{dy}{dx}-2xy=0$$

15. Find the equation of a curve passing through origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx}+2xy=4x^2$

Solution:

Given equation,

$$(1+x^2)\frac{dy}{dx}+2xy=4x^2$$

$$\frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{4x^2}{1+x^2}$$

Here, $P=\frac{2x}{1+x^2}$ and $Q=\frac{4x^2}{1+x^2}$

Integrating factor I.F. $= e^{\int P dx}=e^{\int \frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$

So, the solution is

$$y\times \text{I.F.}=\int Q\times \text{I.F.}\,dx+c$$

$$y(1+x^2)=\int\frac{4x^2}{1+x^2}\times(1+x^2)\,dx+c$$

$$y(1+x^2)=\int 4x^2dx+c$$

$$\Rightarrow y(1+x^2)=\frac{4}{3}x^3+c \qquad \ldots(i)$$

Since the curve is passing through origin *i.e.*, (0, 0)

Putting $y=0$ and $x=0$ in eq. (i)

$$0(1+0)=\frac{4}{3}(0)^3+c \Rightarrow c=0$$

Now,

Equation is $y(1+x^2)=\frac{4}{3}x^3 \Rightarrow y=\frac{4x^3}{3(1+x^2)}$

Thus, the required solution is $y=\frac{4x^3}{3(1+x^2)}$

16. Solve: $x^2\dfrac{dy}{dx} = x^2 + xy + y^2.$
Solution:

Given equation,

$$x^2\frac{dy}{dx} = x^2 + xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

Now, taking y = vx [As it is a homogeneous differential equation]

$$\frac{dy}{dx} = v + x.\frac{dv}{dx}$$

So,

$$v + x.\frac{dv}{dx} = \frac{x^2 + vx^2 + v^2x^2}{x^2}$$

$$v + x.\frac{dv}{dx} = \frac{x^2(1 + v + v^2)}{x^2}$$

$$v + x.\frac{dv}{dx} = 1 + v + v^2 \Rightarrow x.\frac{dv}{dx} = 1 + v + v^2 - v$$

$$\Rightarrow x.\frac{dv}{dx} = 1 + v^2 \Rightarrow \frac{dv}{1 + v^2} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int\frac{dv}{1 + v^2} = \int\frac{dx}{x}$$

$$\Rightarrow \tan^{-1}v = \log x + c \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \log x + c$$

Thus, the required solution is $\tan^{-1}\left(\dfrac{y}{x}\right) = \log|x| + c$

17. Find the general solution of the differential equation $(1 + y^2) + (x - e^{\tan-1y})\dfrac{dy}{dx} = 0.$
Solution:

Given differential equation,

$$(1 + y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$$

$$(x - e^{\tan^{-1}y})\frac{dy}{dx} = -(1 + y^2) \Rightarrow \frac{dy}{dx} = \frac{-(1 + y^2)}{x - e^{\tan^{-1}y}}$$

$$\frac{dx}{dy} = \frac{x - e^{\tan^{-1}y}}{-(1 + y^2)} \Rightarrow \frac{dx}{dy} = -\frac{x}{(1 + y^2)} + \frac{e^{\tan^{-1}y}}{1 + y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{(1 + y^2)} = \frac{e^{\tan^{-1}y}}{1 + y^2}$$ [First order linear differential equation in y]

Here, $P = \frac{1}{1+y^2}$ and $Q = \frac{e^{\tan^{-1} y}}{1+y^2}$

So,
Integrating factor I.F. $= e^{\int P dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

And, the solution is

$$x \,.\, \text{I.F.} = \int Q \,.\text{I.F.}\, dy + c$$

$$\Rightarrow \quad x \,.\, e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2} \,.\, e^{\tan^{-1} y} dy + c$$

Put $e^{\tan^{-1} y} = t$, we have

$$e^{\tan^{-1} y} . \frac{1}{1+y^2} dy = dt$$

$$x \,.\, e^{\tan^{-1} y} = \int t \,.\, dt + c$$

$$x \,.\, e^{\tan^{-1} y} = \frac{1}{2} t^2 + c$$

$$x \,.\, e^{\tan^{-1} y} = \frac{1}{2} (e^{\tan^{-1} y})^2 + c \quad \Rightarrow x = \frac{1}{2} (e^{\tan^{-1} y}) + \frac{c}{e^{\tan^{-1} y}}$$

$$2x = e^{\tan^{-1} y} + \frac{2c}{e^{\tan^{-1} y}}$$

$$\Rightarrow 2x \,.\, e^{\tan^{-1} y} = (e^{\tan^{-1} y})^2 + 2c$$

Thus, the above is the required solution of the given differential equation.

18. Find the general solution of $y^2dx + (x^2 - xy + y^2)\, dy = 0$.
Solution:

Given equation, $y^2dx + (x^2 - xy + y^2)\, dy = 0$

$$y^2 dx = -(x^2 - xy + y^2)\, dy$$

$$\Rightarrow \frac{dx}{dy} = -\frac{x^2 - xy + y^2}{y^2}$$

Since it is a homogeneous differential equation

Putting, $x = vy \Rightarrow \frac{dx}{dy} = v + y \,.\, \frac{dv}{dy}$

So, $v + y \,.\, \frac{dv}{dy} = -\left(\frac{v^2y^2 - vy^2 + y^2}{y^2}\right)$

$$v + y \,.\, \frac{dv}{dy} = -\frac{y^2(v^2 - v + 1)}{y^2}$$

$$v + y \,.\, \frac{dv}{dy} = (-v^2 + v - 1) \quad \Rightarrow y \,.\, \frac{dv}{dy} = -v^2 + v - 1 - v$$

$$y\cdot\frac{dv}{dy} = -v^2 - 1 \quad \Rightarrow \frac{dv}{(v^2+1)} = -\frac{dy}{y}$$

Integrating both sides, we get

$$\int\frac{dv}{(v^2+1)} = -\int\frac{dy}{y} \quad \Rightarrow \tan^{-1}v = -\log y + c$$

$$\Rightarrow \tan^{-1}\left(\frac{x}{y}\right) + \log y = c$$

Thus, the required solution is $\tan^{-1}\left(\frac{x}{y}\right) + \log y = c$.

19. Solve : (*x* + *y*) (*dx* – *dy*) = *dx* + *dy*. **[Hint: Substitute *x* + *y* = *z* after separating *dx* and *dy*]**
Solution:

Given differential equation, $(x + y)(dx - dy) = dx + dy$

$(x + y)\, dx - (x - y)\, dy = dx + dy$

$-(x + y)\, dy - dy = dx - (x + y)\, dx$

$-(x + y + 1)\, dy = -(x + y - 1)\, dx$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y-1}{x+y+1}$$

Putting x + y = z, we have

So, $1 + \frac{dy}{dx} = \frac{dz}{dx}$

$$\frac{dy}{dx} = \frac{dz}{dx} - 1$$

Now, $\frac{dz}{dx} - 1 = \frac{z-1}{z+1}$

$$\frac{dz}{dx} = \frac{z-1}{z+1} + 1 \quad \Rightarrow \frac{dz}{dx} = \frac{z-1+z+1}{z+1}$$

$$\frac{dz}{dx} = \frac{2z}{z+1} \quad \Rightarrow \frac{z+1}{z}\,dz = 2\,.\,dx$$

Integrating both sides, we get

$$\int\frac{z+1}{z}\,dz = 2\int dx$$

$$\int\left(1+\frac{1}{z}\right)dz = 2\int dx$$

$$z + \log|z| = 2x + \log|c|$$

$$x + y + \log|x+y| = 2x + \log|c|$$

$$y + \log|x+y| = x + \log|c|$$

$$\log|x+y| = x - y + \log|c|$$

$$\log|x+y| - \log|c| = (x-y)$$

$$\Rightarrow \log\left|\frac{x+y}{c}\right| = (x-y) \quad \Rightarrow \frac{x+y}{c} = e^{x-y}$$

$$\therefore \quad x+y = c \cdot e^{x-y}$$

Thus, the required solution is $x + y = c \cdot e^{x-y}$

20. Solve : 2 (y + 3) – xy dy/dx = 0, given that y(1) = -2.
Solution:

Given differential equation, 2 (y + 3) – xy dy/dx = 0

$$2(y+3) - xy \cdot \frac{dy}{dx} = 0$$

$$xy \cdot \frac{dy}{dx} = 2y + 6$$

$$\left(\frac{y}{2y+6}\right)dy = \frac{dx}{x} \quad \Rightarrow \frac{1}{2}\left(\frac{y}{y+3}\right)dy = \frac{dx}{x}$$

Integrating both sides, we get

$$\frac{1}{2}\int \frac{y}{y+3} \cdot dy = \int \frac{dx}{x} \quad \Rightarrow \frac{1}{2}\int \frac{y+3-3}{y+3}dy = \int \frac{dx}{x}$$

$$\frac{1}{2}\int\left(1 - \frac{3}{y+3}\right)dy = \int \frac{dx}{x}$$

$$\frac{1}{2}\int 1 \cdot dy - \frac{3}{2}\int \frac{1}{y+3}dy = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}y - \frac{3}{2}\log|y+3| = \log x + c$$

Putting x = 1 and y = -2, we have

$$\Rightarrow \frac{1}{2}(-2) - \frac{3}{2}\log|-2+3| = \log(1) + c$$

$$-1 - \frac{3}{2}\log(1) = \log(1) + c$$

$$-1 - 0 = 0 + c \qquad [\because \log(1) = 0]$$

$$\therefore \; c = -1$$

Now, the eqaution is

$$\frac{1}{2}y - \frac{3}{2}\log|y+3| = \log x - 1$$

$$y - 3\log|y+3| = 2\log x - 2$$

$$y - \log|(y+3)^3| = \log x^2 - 2$$

$$\log|(y+3)^3| + \log x^2 = y + 2$$

$$\log|x^2(y+3)^3| = y + 2 \quad \Rightarrow x^2(y+3)^3 = e^{y+2}$$

Thus, the required solution is $x^2(y+3)^3 = e^{y+2}$.

21. Solve the differential equation dy = cos x (2 – y cosec x) dx given that y = 2 when x = π/2.

Solution:

Given differential equation, $dy = \cos x\ (2 - y \text{ cosec } x)\ dx$

$$\frac{dy}{dx} = \cos x (2 - y \operatorname{cosec} x) \Rightarrow \frac{dy}{dx} = 2\cos x - y \cos x . \operatorname{cosec} x$$

$$\frac{dy}{dx} = 2\cos x - y \cot x \Rightarrow \frac{dy}{dx} + y \cot x = 2\cos x$$

Here, P = $\cot x$ and Q = $2 \cos x$.

So, Integrating factor I.F. = $e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

And, Required solution is $y \times \text{I.F} = \int Q \times \text{I.F.}\, dx + c$

$$y . \sin x = \int 2\cos x . \sin x\, dx + c$$

$$y . \sin x = \int \sin 2x\, dx + c \Rightarrow y . \sin x = -\frac{1}{2}\cos 2x + c$$

Put $x = \frac{\pi}{2}$ and $y = 2$, we get

$$2 \sin \frac{\pi}{2} = -\frac{1}{2}\cos \pi + c$$

$$2(1) = -\frac{1}{2}(-1) + c \Rightarrow 2 = \frac{1}{2} + c \Rightarrow c = 2 - \frac{1}{2} = \frac{3}{2}$$

Thus, the equation is $y \sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$.

22. Form the differential equation by eliminating A and B in Ax^2 + By^2 = 1.

Solution:

Given, Ax^2 + By^2 = 1

Differentiating w.r.t. x, we get

$$2A . x + 2By\frac{dy}{dx} = 0$$

$$Ax + By . \frac{dy}{dx} = 0 \Rightarrow By . \frac{dy}{dx} = -Ax$$

$$\Rightarrow \frac{y}{x} . \frac{dy}{dx} = -\frac{A}{B}$$

Differentiating both sides again w.r.t. x, we have

$$\frac{y}{x} . \frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{x . \frac{dy}{dx} - y.1}{x^2}\right) = 0$$

$$\frac{yx^2}{x} . \frac{d^2y}{dx^2} + x.\left(\frac{dy}{dx}\right)^2 - y.\frac{dy}{dx} = 0$$

$$xy.\frac{d^2y}{dx^2} + x.\left(\frac{dy}{dx}\right)^2 - y.\frac{dy}{dx} = 0 \Rightarrow xy.y'' + x.(y')^2 - y.y' = 0$$

Thus, the required equation is

$$xy.y'' + x.(y')^2 - y.y' = 0$$

23. Solve the differential equation $(1 + y^2)\tan^{-1}x\,dx + 2y(1 + x^2)\,dy = 0$.
Solution:

Given differential equation, $(1 + y^2)\tan^{-1}x\,dx + 2y(1 + x^2)\,dy = 0$

$2y(1 + x^2)\,dy = -(1 + y^2)\tan^{-1}x\,dx$

$$2y(1 + x^2)\,dy = -(1 + y^2).\tan^{-1}x.dx$$

$$\frac{2y}{1+y^2}dy = -\frac{\tan^{-1}x}{1+x^2}.dx$$

Integrating both sides, we get

$$\int\frac{2y}{1+y^2}dy = -\int\frac{\tan^{-1}x}{1+x^2}.dx$$

$$\log|1+y^2| = -\frac{1}{2}(\tan^{-1}x)^2 + c$$

$$\Rightarrow \frac{1}{2}(\tan^{-1}x)^2 + \log|1+y^2| = c$$

Thus, the above equation is the required solution.

24. Find the differential equation of system of concentric circles with centre (1, 2).
Solution:

The family of concentric circles with centre (1, 2) and radius 'r' is given by

$(x - 1)^2 + (y - 2)^2 = r^2$

Differentiating both sides w.r.t, x we get

$$2(x-1) + 2(y-2)\frac{dy}{dx} = 0 \Rightarrow (x-1) + (y-2)\frac{dy}{dx} = 0$$

Thus, the above is the required equation.

Long Answer (L.A.)

25. Solve : $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$
Solution:

Given differential equation,

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

$$y + x.\frac{dy}{dx} + y = x(\sin x + \log x)$$

$$x\frac{dy}{dx} = x(\sin x + \log x) - 2y$$

$$\Rightarrow \frac{dy}{dx} = (\sin x + \log x) - \frac{2y}{x} \Rightarrow \frac{dy}{dx} + \frac{2}{x}y = (\sin x + \log x)$$

Here, $P = \frac{2}{x}$ and $Q = (\sin x + \log x)$

Integrating factor I.F. $= e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2\log x} = e^{\log x^2} = x^2$

Now, the solution is

$$y \times \text{I.F.} = \int \text{Q.I.F.}\, dx + c$$

$$\Rightarrow \quad y \cdot x^2 = \int (\sin x + \log x)\, x^2 dx + c \qquad \ldots(1)$$

Let $I = \int (\sin x + \log x)\, x^2 dx$

$$= \int \underset{\text{I}}{x^2} \underset{\text{II}}{\sin x}\, dx + \int \underset{\text{II}}{x^2} \underset{\text{I}}{\log x}\, dx$$

$$= \left[x^2 . \int \sin x\, dx - \int \left(D(x^2) . \int \sin x\, dx\right) dx\right] + \left[\log x . \int x^2\, dx - \int \left(D(\log x) . \int x^2 dx\right) dx\right]$$

$$= \left[x^2(-\cos x) - 2\int -x \cos x\, dx\right] + \left[\log x . \frac{x^3}{3} - \int \frac{1}{x} . \frac{x^3}{3} dx\right]$$

$$= \left[-x^2 \cos x + 2\left(x \sin x - \int 1 . \sin x\, dx\right)\right] + \left[\frac{x^3}{3}\log x - \frac{1}{3}\int x^2 dx\right]$$

$$= -x^2 \cos x + 2x \sin x + 2\cos x + \frac{x^3}{3}\log x - \frac{1}{9}x^3$$

Now from eq (1) we get,

$$y . x^2 = -x^2 \cos x + 2x \sin x + 2\cos x + \frac{x^3}{3}\log x - \frac{1}{9}x^3 + c$$

$$\therefore \; y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x \log x}{3} - \frac{1}{9}x + c . x^{-2}$$

Thus, the required solution is

$$y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x \log x}{3} - \frac{1}{9}x + c . x^{-2}$$

26. Find the general solution of (1 + tan *y*) (*dx* – *dy*) + 2*xdy* = 0.
Solution:

Given, (1 + tan *y*) (*dx* – *dy*) + 2*xdy* = 0

$$(1 + \tan y)\, dx - (1 + \tan y)\, dy + 2x dy = 0$$

$$(1 + \tan y)\, dx - (1 + \tan y - 2x)\, dy = 0$$

$$(1 + \tan y)\frac{dx}{dy} = (1 + \tan y - 2x) \quad \Rightarrow \frac{dx}{dy} = \frac{1 + \tan y - 2x}{1 + \tan y}$$

$$\frac{dx}{dy} = 1 - \frac{2x}{1 + \tan y} \quad \Rightarrow \frac{dx}{dy} + \frac{2x}{1 + \tan y} = 1$$

Here, $P = \frac{2}{1 + \tan y}$ and $Q = 1$

Integrating factor I.F.

$$= e^{\int \frac{2}{1+\tan y} dy} = e^{\int \frac{2\cos y}{\sin y + \cos y} dy}$$

$$= e^{\int \frac{\sin y + \cos y - \sin y + \cos y}{(\sin y + \cos y)} dy} = e^{\int \left(1 + \frac{\cos y - \sin y}{\sin y + \cos y}\right) dy}$$

$$= e^{\int 1 . dy} . e^{\int \frac{\cos y - \sin y}{\sin y + \cos y} dy}$$

$$= e^{y} . e^{\log (\sin y + \cos y)} = e^{y} .(\sin y + \cos y)$$

Now, the solution is $x \times \text{I.F.} = \int Q \times \text{I.F.}\, dy + c$

$$\Rightarrow \quad x . e^{y} (\sin y + \cos y) = \int 1 . e^{y} (\sin y + \cos y) dy + c$$

$$x . e^{y} (\sin y + \cos y) = e^{y} . \sin y + c$$

$$\left[\because \int e^{x} [f(x) + f'(x)]\, dx = e^{x} f(x) + c\right]$$

$$x(\sin y + \cos y) = \sin y + c . e^{-y}$$

Thus, the required solution is $x(\sin y + \cos y) = \sin y + c . e^{-y}$.

27. Solve: *dy/dx* = cos(*x* + *y*) + sin (*x* + *y*). **[Hint: Substitute *x* + *y* = *z*]**

Solution:

Putting x + y = v and on differentiating w.r.t. x, we get

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - 1 = \cos v + \sin v$$

$$\frac{dv}{dx} = \cos v + \sin v + 1$$

So, $$\frac{dv}{\cos v + \sin v + 1} = dx$$

Integrating both sides, we have

$$\int \frac{dv}{\cos v + \sin v + 1} = \int 1 . dx$$

$$\int \frac{dv}{\left(\frac{1 - \tan^2 \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + \frac{2 \tan \frac{v}{2}}{1 + \tan^2 \frac{v}{2}} + 1\right)} = \int 1 . dx$$

$$\int \frac{\left(1 + \tan^2 \frac{v}{2}\right)}{1 - \tan^2 \frac{v}{2} + 2 \tan \frac{v}{2} + 1 + \tan^2 \frac{v}{2}} dv = \int 1 . dx$$

$$\int \frac{\sec^2 \frac{v}{2}}{2 + 2\tan \frac{v}{2}} dv = \int 1 . dx$$

Now, putting $2 + 2\tan\frac{v}{2} = t$

Differentiating, $2 . \frac{1}{2}\sec^2\frac{v}{2}dv = dt \Rightarrow \sec^2\frac{v}{2}dv = dt$

$$\int \frac{dt}{t} = \int 1 . dx$$

$$\log |t| = x + c$$

$$\log \left|2 + 2\tan\frac{v}{2}\right| = x + c$$

$$\log \left|2 + 2\tan\left(\frac{x+y}{2}\right)\right| = x + c \Rightarrow \log 2\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + c$$

$$\log 2 + \log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + c$$

$$\Rightarrow \log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + c - \log 2$$

Thus, the required solution is

$$\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + K \qquad [c - \log 2 = K]$$

28. Find the general solution of *dy/dx* – 3y = sin 2x.
Solution:

Given equation, *dy/dx* – 3y = sin 2x
It's a first order linear differential equation
Here P = -3 and Q = sin 2x
So,
Integrating factor I.F. = $e^{\int P dx} = e^{\int -3 dx} = e^{-3x}$
And, the solution is

$$y \times \text{I.F.} = \int Q . \text{I.F.} \, dx + c$$

$$\Rightarrow y . e^{-3x} = \int \sin 2x . e^{-3x} \, dx + c$$

Now let, $I = \int \underset{\text{I}}{\sin 2x} . \underset{\text{II}}{e^{-3x}} \, dx$

$$I = \sin 2x . \int e^{-3x} dx - \int \left(D(\sin 2x) . \int e^{-3x} dx\right) dx$$

$$I = \sin 2x . \frac{e^{-3x}}{-3} - \int 2\cos 2x . \frac{e^{-3x}}{-3} dx$$

$$I = \frac{e^{-3x}}{-3}\sin 2x + \frac{2}{3}\int \underset{\text{I}}{\cos 2x} . \underset{\text{II}}{e^{-3x}} \, dx$$

$$I = \frac{e^{-3x}}{-3}\sin 2x + \frac{2}{3}\left[\cos 2x . \int e^{-3x}dx - \int [D\cos 2x . \int e^{-3x}dx]\,dx\right]$$

$$I = \frac{e^{-3x}}{-3}\sin 2x + \frac{2}{3}\left[\cos 2x . \frac{e^{-3x}}{-3} - 2\sin 2x . \frac{e^{-3x}}{-3}\right]dx$$

$$I = \frac{e^{-3x}}{-3}\sin 2x - \frac{2}{9}\cos 2x . e^{-3x} - \frac{4}{9}\int \sin 2x . e^{-3x}\,dx$$

$$= \frac{e^{-3x}}{-3}\sin 2x - \frac{2}{9}e^{-3x}\cos 2x - \frac{4}{9}I$$

$$I + \frac{4}{9}I = \frac{e^{-3x}}{-3}\sin 2x - \frac{2}{9}e^{-3x}\cos 2x$$

$$\frac{13I}{9} = -\frac{1}{9}\left[3e^{-3x}\sin 2x + 2e^{-3x}\cos 2x\right]$$

$$I = -\frac{1}{13}e^{-3x}[3\sin 2x + 2\cos 2x]$$

Hence, the equation becomes

$$y . e^{-3x} = -\frac{1}{13}e^{-3x}[3\sin 2x + 2\cos 2x] + c$$

$$\therefore\ y = -\frac{1}{13}[3\sin 2x + 2\cos 2x] + c . e^{3x}$$

Thus, the required solution is

$$y = -\left[\frac{3\sin 2x + 2\cos 2x}{13}\right] + c . e^{3x}$$

29. Find the equation of a curve passing through (2, 1) if the slope of the tangent to the curve at any point (x, y) is $(x^2 + y^2)/ 2xy$.
Solution:

Given that the slope of tangent to a curve at (x, y) is $(x^2 + y^2)/ 2xy$.
It's a homogeneous differential function

So, putting $y = vx \Rightarrow \frac{dy}{dx} = v + x . \frac{dv}{dx}$

$$v + x . \frac{dv}{dx} = \frac{x^2 + v^2x^2}{2x . vx}$$

$$v + x . \frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$x \cdot \frac{dv}{dx} = \frac{1+v^2}{2v} - v \quad \Rightarrow x \cdot \frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$x \cdot \frac{dv}{dx} = \frac{1-v^2}{2v} \quad \Rightarrow \frac{2v}{1-v^2} dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{2v}{1-v^2} dv = \int \frac{dx}{x} \Rightarrow -\log |1 - v^2| = \log x + \log c$$

$$-\log \left|1 - \frac{y^2}{x^2}\right| = \log x + \log c \Rightarrow -\log \left|\frac{x^2 - y^2}{x^2}\right| = \log x + \log c$$

$$\log \left|\frac{x^2}{x^2 - y^2}\right| = \log |xc| \quad \Rightarrow \frac{x^2}{x^2 - y^2} = xc$$

As, the curve is passing through the point (2, 1)

$$\therefore \frac{(2)^2}{(2)^2 - (1)^2} = 2c \quad \Rightarrow \frac{4}{3} = 2c \quad \Rightarrow c = \frac{2}{3}$$

Thus, the required equation is

$$\frac{x^2}{x^2 - y^2} = \frac{2}{3} x \quad \Rightarrow 2(x^2 - y^2) = 3x$$

30. Find the equation of the curve through the point (1, 0) if the slope of the tangent to the curve at any point (*x*, *y*) is (y - 1)/ (x^2 + x)
Solution:

Given that the slope of the tangent to the curve at (x, y) is

$$\frac{dy}{dx} = \frac{y-1}{x^2+x} \quad \Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

Integrating both sides, we have

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x}$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2 + x + \frac{1}{4} - \frac{1}{4}} \text{ [making perfect square]}$$

$$\int \frac{dy}{y-1} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$\log |y-1| = \frac{1}{2 \times \frac{1}{2}} \log \left|\frac{x + \frac{1}{2} - \frac{1}{2}}{x + \frac{1}{2} + \frac{1}{2}}\right| + \log c$$

$$\log|y-1| = \log\left|\frac{x}{x+1}\right| + \log c$$

$$\log|y-1| = \log\left|c\left(\frac{x}{x+1}\right)\right|$$

$$y-1 = \frac{cx}{x+1} \Rightarrow (y-1)(x+1) = cx$$

As, the line is passing through the point (1, 0), then (0 - 1) (1 + 1) = c(1) ⇒ c = 2

Thus, the required solution is (y - 1) (x + 1) = 2x.

31. Find the equation of a curve passing through origin if the slope of the tangent to the curve at any point (*x*, *y*) is equal to the square of the difference of the abscissa and ordinate of the point.

Solution:

We know that,

The slope of the tangent of the curve = dy/dx

And the difference between the abscissa and ordinate = x – y

So, as given in the question we have

$dy/dx = (x - y)^2$

Taking, x – y = v

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = 1 - \frac{dv}{dx}$$

So, the equation becomes

$$1 - \frac{dv}{dx} = v^2 \Rightarrow \frac{dv}{dx} = 1 - v^2 \Rightarrow \frac{dv}{1-v^2} = dx$$

Integrating both sides, we get

$$\int \frac{dv}{1-v^2} = \int dx$$

$$\frac{1}{2}\log\left|\frac{1+v}{1-v}\right| = x + c \Rightarrow \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| = x + c \quad ...(1)$$

As, the curve is passing through (0, 0)

then $\frac{1}{2}\log\left|\frac{1+0-0}{1-0+0}\right| = 0 + c \Rightarrow c = 0$

∴ On putting $c = 0$ in eq. (1) we get

$$\frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right| = x \Rightarrow \log\left|\frac{1+x-y}{1-x+y}\right| = 2x$$

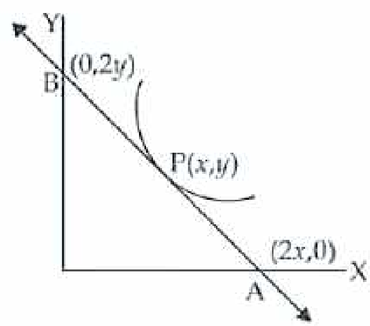
$$\frac{1+x-y}{1-x+y} = e^{2x}$$

$$\Rightarrow (1+x-y) = e^{2x}(1-x+y)$$

Thus, the required equation is $(1+x-y) = e^{2x}(1-x+y)$

32. Find the equation of a curve passing through the point (1, 1). If the tangent drawn at any point P (*x*, *y*) on the curve meets the co-ordinate axes at A and B such that P is the mid-point of AB.
Solution:

Let's take P(x , y) be any point on the curve and AB be the tangent to the given curve at P.
Also given, P is the mid-point of AB
So, the coordinates of A and B are (2x, 0) and (0, 2y) respectively.
The slope of the tangent AB = (2y - 0)/ (0 – 2x) = -y/x



So, $\frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$

Integrating both sides, we get

$$\int \frac{dy}{y} = -\int \frac{dx}{x} \Rightarrow \log y = -\log x + \log c$$

$$\log y + \log x = \log c \Rightarrow \log yx = \log c$$

$$\therefore\ yx = c$$

As, the curve passes through (1, 1)

$$1 \times 1 = c \quad \therefore \quad c = 1$$

$$\Rightarrow yx = 1$$

Thus, the required equation is xy = 1

33. Solve : $x\frac{dy}{dx} = y\ (\log y - \log x + 1)$

Solution:

Given that: $x\frac{dy}{dx} = y\,(\log y - \log x + 1)$

$$\Rightarrow x\frac{dy}{dx} = y\left[\log\left(\frac{y}{x}\right) + 1\right] \Rightarrow \frac{dy}{dx} = \frac{y}{x}\left[\log\left(\frac{y}{x}\right) + 1\right]$$

As its a homogeneous different equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\,.\frac{dv}{dx}$

$$v + x\,.\frac{dv}{dx} = \frac{vx}{x}\left[\log\left(\frac{vx}{x}\right) + 1\right]$$

$$v + x\,.\frac{dv}{dx} = v\,[\log v + 1]$$

$$x\,.\frac{dv}{dx} = v\,[\log v + 1] - v \Rightarrow x\,.\frac{dv}{dx} = v\,[\log v + 1 - 1]$$

$$\Rightarrow x\,.\frac{dv}{dx} = v\,.\log v \Rightarrow \frac{dv}{v\log v} = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{v\log v} = \int \frac{dx}{x}$$

Now, put $\log v = t$ on L.H.S.

$$\frac{1}{v}dv = dt$$

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\log|t| = \log|x| + \log c$$

$$\log|\log v| = \log xc \Rightarrow \log v = xc$$

$$\Rightarrow \log\left(\frac{y}{x}\right) = xc$$

Thus, the required solution is $\log\left(\frac{y}{x}\right) = xc$.

Objective Type

Choose the correct answer from the given four options in each of the Exercises from 34 to 75 (M.C.Q)

34. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is:

(A) 1 (B) 2 (C) 3 (D) not defined

Solution:

Correct option is (D) not defined.

Since the value of sin (dy/dx) on expansion will be in increasing power of dy/dx, the degree of the given differential equation is not defined.

35. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

(A) 4 (B) 3/2 (C) not defined (D) 2

Solution:

Correct option is (D) 2.

Given differential equation is

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \left(\frac{d^2y}{dx^2}\right)$$

Squaring both sides, we have

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Thus, the degree of the given differential equation is 2.

36. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$, **respectively, are**

(A) 2 and not defined (B) 2 and 2 (C) 2 and 3 (D) 3 and 3

Solution:

Correct option is (A) 2 and not defined.

Given differential equation is

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{1}{4}}+x^{\frac{1}{5}}=0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{1}{4}}=-x^{\frac{1}{5}}$$

As the degree of dy/dx is in fraction its undefined and the degree is 2.

37. If $y = e^{-x}$ (A cos x + B sin x), then y is a solution of

(A) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$ (B) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

(C) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$ (D) $\frac{d^2y}{dx^2}+2y=0$

Solution:

Correct option is (C).
Given equation, $y = e^{-x}$ (A cos x + B sin x)
Differentiating on both the sides, w.r.t. x, we get

$$\frac{dy}{dx}=e^{-x}(-A\sin x+B\cos x)-e^{-x}(A\cos x+B\sin x)$$

$$\frac{dy}{dx}=e^{-x}(-A\sin x+B\cos x)-y$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2}=e^{-x}(-A\cos x-B\sin x)-e^{-x}(-A\sin x+B\cos x)-\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2}=-e^{-x}(A\cos x+B\sin x)-\left[\frac{dy}{dx}+y\right]-\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2}=-y-\frac{dy}{dx}-y-\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2}=-2\frac{dy}{dx}-2y \quad\Rightarrow\quad \frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$$

38. The differential equation for y = A cos αx + B sin αx, where A and B are arbitrary constants is

(A) $\frac{d^2y}{dx^2}-\alpha^2y=0$ (B) $\frac{d^2y}{dx^2}+\alpha^2y=0$

(C) $\frac{d^2y}{dx^2}+\alpha y=0$ (D) $\frac{d^2y}{dx^2}-\alpha y=0$

Solution:

Correct option is (B).

Given equation is y = A cos ax + B sin ax
Differentiating both sides w.r.t. x, we have

$$\frac{dy}{dx} = -A \sin \alpha x \,.\, \alpha + B \cos \alpha x \,.\, \alpha$$
$$= -A \alpha \sin \alpha x + B \alpha \cos \alpha x$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -A \alpha^2 \cos \alpha x - B \alpha^2 \sin \alpha x$$
$$\frac{d^2y}{dx^2} = -\alpha^2 (A \cos \alpha x + B \sin \alpha x)$$
$$\frac{d^2y}{dx^2} = -\alpha^2 y \Rightarrow \frac{d^2y}{dx^2} + \alpha^2 y = 0$$