

Exercise 6.4

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1. Let $\triangle ABC \sim \triangle DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2 . If $EF = 15.4 \text{ cm}$, find BC .

Solution: Given, $\triangle ABC \sim \triangle DEF$,
Area of $\triangle ABC = 64 \text{ cm}^2$
Area of $\triangle DEF = 121 \text{ cm}^2$
 $EF = 15.4 \text{ cm}$

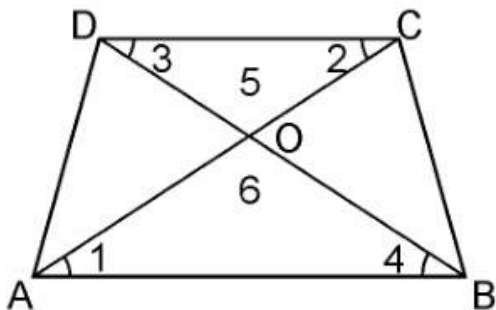
$$\therefore \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{AB^2}{DE^2}$$

As we know, if two triangles are similar, ratio of their areas are equal to the square of the ratio of their corresponding sides,
 $= AC^2/DF^2 = BC^2/EF^2$

$$\begin{aligned}\therefore 64/121 &= BC^2/EF^2 \\ \Rightarrow (8/11)^2 &= (BC/15.4)^2 \\ \Rightarrow 8/11 &= BC/15.4 \\ \Rightarrow BC &= 8 \times 15.4/11 \\ \Rightarrow BC &= 8 \times 1.4 \\ \Rightarrow BC &= 11.2 \text{ cm}\end{aligned}$$

2. Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O . If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD .

Solution: Given, $ABCD$ is a trapezium with $AB \parallel DC$. Diagonals AC and BD intersect each other at point O .



In $\triangle AOB$ and $\triangle COD$, we have
 $\angle 1 = \angle 2$ (Alternate angles)
 $\angle 3 = \angle 4$ (Alternate angles)
 $\angle 5 = \angle 6$ (Vertically opposite angle)
 $\therefore \triangle AOB \sim \triangle COD$ [AAA similarity criterion]

As we know, If two triangles are similar then the ratio of their areas are equal to the square of the ratio of their corresponding sides. Therefore,

$$\text{Area of } (\triangle AOB) / \text{Area of } (\triangle COD) = AB^2 / CD^2$$

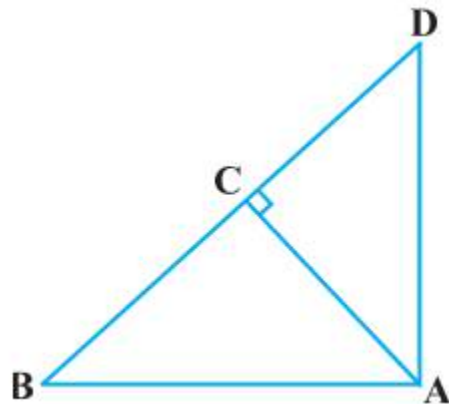
$$= (2CD)^2 / CD^2 [\because AB = 2CD]$$

$$\therefore \text{Area of } (\triangle AOB) / \text{Area of } (\triangle COD)$$

$$= 4CD^2 / CD^2 = 4/1$$

Hence, the required ratio of the area of $\triangle AOB$ and $\triangle COD = 4:1$

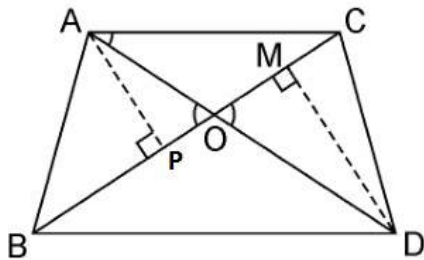
3. In the figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that area $(\triangle ABC) / \text{area } (\triangle DBC) = AO / DO$.



Solution: Given, ABC and DBC are two triangles on the same base BC. AD intersects BC at O.

We have to prove: $\text{Area } (\triangle ABC) / \text{Area } (\triangle DBC) = AO / DO$

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$$

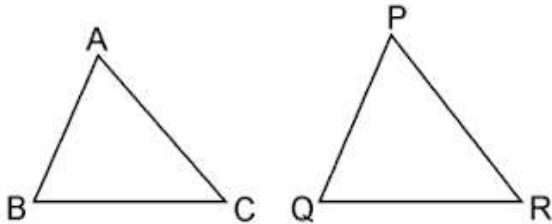
In $\triangle APO$ and $\triangle DMO$,

$\angle APO = \angle DMO$ (Each 90°)

$\angle AOP = \angle DOM$ (Vertically opposite angles)
 $\therefore \triangle APO \sim \triangle DMO$ (AA similarity criterion)
 $\therefore AP/DM = AO/DO$
 $\Rightarrow \text{Area}(\triangle ABC)/\text{Area}(\triangle DBC) = AO/DO$.

4. If the areas of two similar triangles are equal, prove that they are congruent.

Solution: Say $\triangle ABC$ and $\triangle PQR$ are two similar triangles and equal in area



Now let us prove $\triangle ABC \cong \triangle PQR$.

Since, $\triangle ABC \sim \triangle PQR$

$\therefore \text{Area of } (\triangle ABC)/\text{Area of } (\triangle PQR) = BC^2/QR^2$

$\Rightarrow BC^2/QR^2 = 1$ [Since, $\text{Area}(\triangle ABC) = \text{Area}(\triangle PQR)$]

$\Rightarrow BC^2/QR^2 = 1$

$\Rightarrow BC = QR$

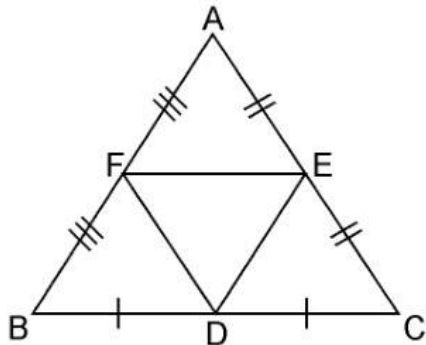
Similarly, we can prove that

$AB = PQ$ and $AC = PR$

Thus, $\triangle ABC \cong \triangle PQR$ [SSS criterion of congruence]

5. D, E and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$. Find the ratio of the area of $\triangle DEF$ and $\triangle ABC$.

Solution: Given, D, E and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$.



In $\triangle ABC$,

F is the mid point of AB (Already given)

E is the mid-point of AC (Already given)

So, by the mid-point theorem, we have,
 $FE \parallel BC$ and $FE = \frac{1}{2}BC$
 $\Rightarrow FE \parallel BC$ and $FE \parallel BD$ [$BD = \frac{1}{2}BC$]
 Since, opposite sides of parallelogram are equal and parallel
 $\therefore$ BDEF is parallelogram.

Similarly in $\triangle FBD$ and $\triangle DEF$, we have
 $FB = DE$ (Opposite sides of parallelogram BDEF)
 $FD = FD$ (Common sides)
 $BD = FE$ (Opposite sides of parallelogram BDEF)
 $\therefore \triangle FBD \cong \triangle DEF$

Similarly, we can prove that
 $\triangle AFE \cong \triangle DEF$
 $\triangle EDC \cong \triangle DEF$

As we know, if triangles are congruent, then they are equal in area.

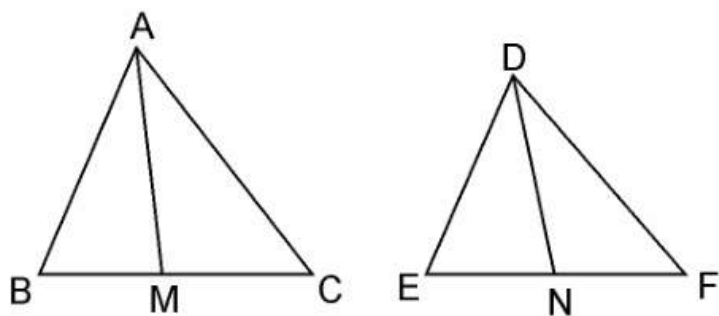
So,
 $\text{Area}(\triangle FBD) = \text{Area}(\triangle DEF)$ (i)
 $\text{Area}(\triangle AFE) = \text{Area}(\triangle DEF)$ (ii)
 and,
 $\text{Area}(\triangle EDC) = \text{Area}(\triangle DEF)$ (iii)

Now,
 $\text{Area}(\triangle ABC) = \text{Area}(\triangle FBD) + \text{Area}(\triangle DEF) + \text{Area}(\triangle AFE) + \text{Area}(\triangle EDC)$ (iv)
 $\text{Area}(\triangle ABC) = \text{Area}(\triangle DEF) + \text{Area}(\triangle DEF) + \text{Area}(\triangle DEF) + \text{Area}(\triangle DEF)$
 From equation (i), (ii) and (iii),
 $\Rightarrow \text{Area}(\triangle DEF) = \frac{1}{4}\text{Area}(\triangle ABC)$
 $\Rightarrow \text{Area}(\triangle DEF)/\text{Area}(\triangle ABC) = 1/4$

Hence, $\text{Area}(\triangle DEF):\text{Area}(\triangle ABC) = 1:4$

6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution: Given: AM and DN are the medians of triangles ABC and DEF respectively and $\triangle ABC \sim \triangle DEF$.



We have to prove: $\text{Area}(\triangle ABC)/\text{Area}(\triangle DEF) = AM^2/DN^2$

Since, $\triangle ABC \sim \triangle DEF$ (Given)

$$\therefore \text{Area}(\triangle ABC)/\text{Area}(\triangle DEF) = (AB^2/DE^2) \dots\dots\dots(\text{i})$$

$$\text{and, } AB/DE = BC/EF = CA/FD \dots\dots\dots(\text{ii})$$

$$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{CD}{FD}$$

In $\triangle ABM$ and $\triangle DEN$,

Since $\triangle ABC \sim \triangle DEF$

$$\therefore \angle B = \angle E$$

$$AB/DE = BM/EN \text{ [Already Proved in equation (i)]}$$

$$\therefore \triangle ABC \sim \triangle DEF \text{ [SAS similarity criterion]}$$

$$\Rightarrow AB/DE = AM/DN \dots\dots\dots(\text{iii})$$

$$\therefore \triangle ABM \sim \triangle DEN$$

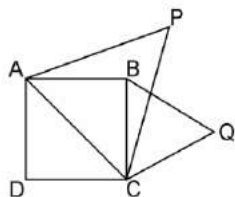
As the areas of two similar triangles are proportional to the squares of the corresponding sides.

$$\therefore \text{area}(\triangle ABC)/\text{area}(\triangle DEF) = AB^2/DE^2 = AM^2/DN^2$$

Hence, proved.

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Solution: Given, ABCD is a square whose one diagonal is AC. $\triangle APC$ and $\triangle BQC$ are two equilateral



triangles described on the diagonals AC and side BC of the square ABCD.

$$\text{Area}(\triangle BQC) = \frac{1}{2} \text{Area}(\triangle APC)$$

Since, $\triangle APC$ and $\triangle BQC$ are both equilateral triangles, as per given,

$\therefore \triangle APC \sim \triangle BQC$ [AAA similarity criterion]

$\therefore \text{area}(\triangle APC)/\text{area}(\triangle BQC) = (AC^2/BC^2) = AC^2/BC^2$

Since, Diagonal = $\sqrt{2}$ side = $\sqrt{2}$ BC

$$\left(\frac{\sqrt{2}BC}{BC}\right)^2 = 2$$

$\Rightarrow \text{area}(\triangle APC) = 2 \times \text{area}(\triangle BQC)$

$\Rightarrow \text{area}(\triangle BQC) = 1/2 \text{area}(\triangle APC)$

Hence, proved.

Tick the correct answer and justify:

8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

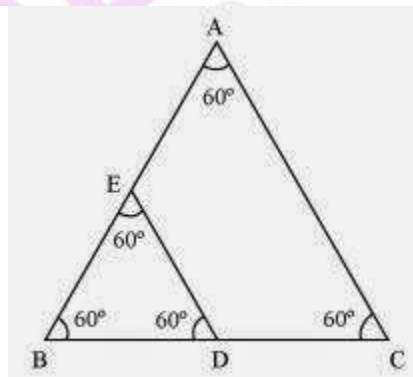
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

Solution: Given, $\triangle ABC$ and $\triangle BDE$ are two equilateral triangle. D is the midpoint of BC.



$\therefore BD = DC = 1/2 BC$

Let each side of triangle is $2a$.

As, $\triangle ABC \sim \triangle BDE$

$\therefore \text{Area}(\triangle ABC)/\text{Area}(\triangle BDE) = AB^2/BD^2 = (2a)^2/(a)^2 = 4a^2/a^2 = 4/1 = 4:1$

Hence, the correct answer is (C).

9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

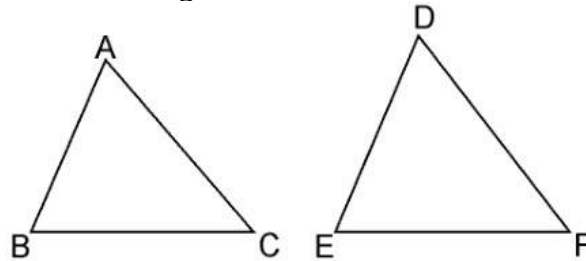
(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

Solution: Given, Sides of two similar triangles are in the ratio 4 : 9.



Let ABC and DEF are two similar triangles, such that,

$\triangle ABC \sim \triangle DEF$

And $AB/DE = AC/DF = BC/EF = 4/9$

As, the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides,

$$\therefore \text{Area}(\triangle ABC)/\text{Area}(\triangle DEF) = AB^2/DE^2$$

$$\therefore \text{Area}(\triangle ABC)/\text{Area}(\triangle DEF) = (4/9)^2 = 16/81 = 16:81$$

Hence, the correct answer is (D).