EXERCISE 23.1 PAGE NO: 23.12

1. Find the slopes of the lines which make the following angles with the positive direction of x - axis:
(i) – π/4
(ii) 2π/3
Solution:
(i) – π/4
Let the slope of the line be 'm'
Where, m = tan θ
So, the slope of Line is m = tan (– π/4)
= – 1
∴ The slope of the line is – 1.

(ii) 2π/3
Let the slope of the line be 'm'
Where, m = tan θ
So, the slope of Line is m = tan (2π/3)

$$\tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{\pi}{3}\right)$$

$$\tan\left(\frac{2\pi}{3}\right) = \tan\left(-\frac{\pi}{3}\right)$$

$$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

∴ The slope of the line is -√3

2. Find the slopes of a line passing through the following points :
(i) (–3, 2) and (1, 4)
(ii) $(at^2_1, 2at_1)$ and $(at^2_2, 2at_2)$
Solution:
(i) (–3, 2) and (1, 4)
By using the formula,

$$\text{Slope of line, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{So, the slope of the line, } m = \frac{4-2}{1-(-3)}$$

$$= 2 / 4$$
$$= 1 / 2$$

∴ The slope of the line is ½.

(ii) $(at^2_1, 2at_1)$ and $(at^2_2, 2at_2)$
By using the formula,

$$\text{Slope of line, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Now, substitute the values

$$\text{The slope of the line, } m = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}$$

$$= \frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)}$$

$$= \frac{2a(t_2 - t_1)}{a(t_2 - t_1)t_2 + t_1)} \quad [\text{Since, } (a^2 - b^2 = (a - b)(a + b)]$$

$$= \frac{2}{t_2 + t_1}$$

∴ The slope of the line is $\frac{2}{t_2 + t_1}$

3. State whether the two lines in each of the following are parallel, perpendicular or neither:
(i) Through (5, 6) and (2, 3); through (9, –2) and (6, –5)
(ii) Through (9, 5) and (– 1, 1); through (3, –5) and (8, –3)
Solution:
(i) Through (5, 6) and (2, 3); through (9, –2) and (6, –5)
By using the formula,

$$\text{Slope of line, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line whose Coordinates are (5, 6) and (2, 3)

$$m_1 = \frac{3 - 6}{2 - 5}$$

$$= \frac{-3}{-3}$$

$$= 1$$

So, $m_1 = 1$
The slope of the line whose Coordinates are (9, – 2) and (6, – 5)

$$m_2 = \frac{-5 - (-2)}{6 - 9}$$

$$= \frac{-3}{-3}$$

So, $m_2 = 1$

Here, $m_1 = m_2 = 1$
∴ The lines are parallel to each other.

(ii) Through (9, 5) and (– 1, 1); through (3, –5) and (8, –3)
By using the formula,

$$\text{Slope of line, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line whose Coordinates are (9, 5) and (– 1, 1)

$$m_1 = \frac{1-5}{-1-9}$$
$$= \frac{-4}{-10}$$
$$= 2/5$$

So, $m_1 = 2/5$
The slope of the line whose Coordinates are (3, – 5) and (8, – 3)

$$m_2 = \frac{-3-(-5)}{8-3}$$
$$= 2/5$$

So, $m_2 = 2/5$
Here, $m_1 = m_2 = 2/5$
∴ The lines are parallel to each other.

4. Find the slopes of a line
(i) which bisects the first quadrant angle
(ii) which makes an angle of 30^0 with the positive direction of y - axis measured anticlockwise.
Solution:
(i) Which bisects the first quadrant angle?
Given: Line bisects the first quadrant
We know that, if the line bisects in the first quadrant, then the angle must be between line and the positive direction of x - axis.
Since, angle = 90/2 = 45^o
By using the formula,
The slope of the line, m = tan θ
The slope of the line for a given angle is m = tan 45^o
So, m = 1
∴ The slope of the line is 1.

(ii) Which makes an angle of 30^0 with the positive direction of y - axis measured

anticlockwise?
Given: The line makes an angle of 30^o with the positive direction of y – axis.
We know that, angle between line and positive side of axis => $90^o + 30^o = 120^o$
By using the formula,
The slope of the line, m = tan θ
The slope of the line for a given angle is m = tan 120^o
So, m = – √3
∴ The slope of the line is – √3.

5. Using the method of slopes show that the following points are collinear:
(i) A (4, 8), B (5, 12), C (9, 28)
(ii) A(16, – 18), B(3, – 6), C(– 10, 6)
Solution:
(i) A (4, 8), B (5, 12), C (9, 28)
By using the formula,
The slope of the line = $[y_2 – y_1] / [x_2 – x_1]$
So,
The slope of line AB = [12 - 8] / [5 - 4]
= 4 / 1

The slope of line BC = [28 - 12] / [9 - 5]
= 16 / 4
= 4

The slope of line CA = [8 - 28] / [4 - 9]
= -20 / -5
= 4
Here, AB = BC = CA
∴ The Given points are collinear.

(ii) A(16, – 18), B(3, – 6), C(– 10, 6)
By using the formula,
The slope of the line = $[y_2 – y_1] / [x_2 – x_1]$
So,
The slope of line AB = [-6 – (-18)] / [3 - 16]
= 12 / -13

The slope of line BC = [6 – (-6)] / [-10 - 3]
= 12 / -13

The slope of line CA = [6 – (-18)] / [-10 - 16]
= 12 / -13
= 4

Here, AB = BC = CA

∴ The Given points are collinear.