EXERCISE 23.14

1. Find the values of α so that the point P(α^2, α) lies inside or on the triangle formed by the lines x – 5y + 6 = 0, x – 3y + 2 = 0 and x – 2y – 3 = 0.

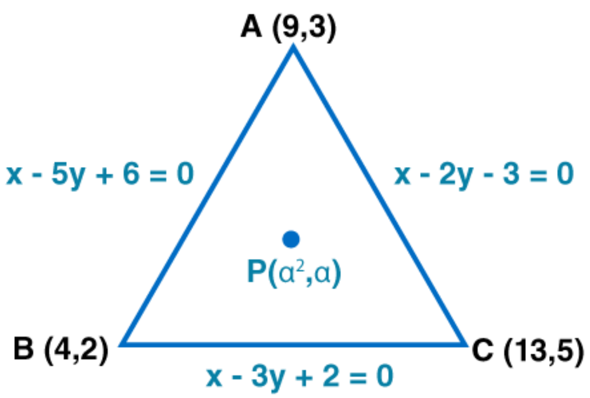
Solution:

Given:

x – 5y + 6 = 0, x – 3y + 2 = 0 and x – 2y – 3 = 0 forming a triangle and point P(α^2, α) lies inside or on the triangle

Let ABC be the triangle of sides AB, BC and CA whose equations are x − 5y + 6 = 0, x − 3y + 2 = 0 and x − 2y − 3 = 0, respectively.

On solving the equations, we get A (9, 3), B (4, 2) and C (13, 5) as the coordinates of the vertices.



It is given that point P (α^2, α) lies either inside or on the triangle. The three conditions are given below.

(i) A and P must lie on the same side of BC.

(ii) B and P must lie on the same side of AC.

(iii) C and P must lie on the same side of AB.

If A and P lie on the same side of BC, then

$(9 - 9 + 2)(\alpha^2 - 3\alpha + 2) \geq 0$

$(\alpha - 2)(\alpha - 1) \geq 0$

$\alpha \in (-\infty, 1] \cup [2, \infty)$ … (1)

If B and P lie on the same side of AC, then

$(4 - 4 - 3)(\alpha^2 - 2\alpha - 3) \geq 0$

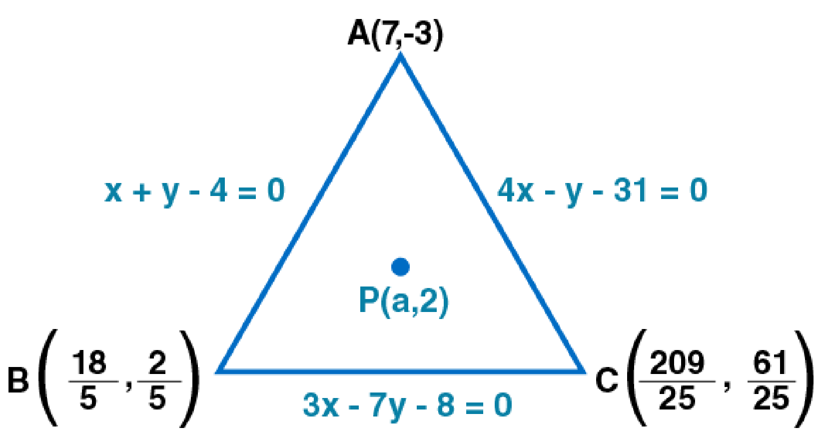
$(\alpha - 3)(\alpha + 1) \leq 0$

$\alpha \in [-1, 3]$ … (2)

If C and P lie on the same side of AB, then
$(13 - 25 + 6)(\alpha^2 - 5\alpha + 6) \geq 0$
$(\alpha - 3)(\alpha - 2) \leq 0$
$\alpha \in [2, 3]$ … (3)

From equations (1), (2) and (3), we get
$\alpha \in [2, 3]$
$\therefore \alpha \in [2, 3]$

2. Find the values of the parameter a so that the point (a, 2) is an interior point of the triangle formed by the lines x + y – 4 = 0, 3x – 7y – 8 = 0 and 4x – y – 31 = 0.
Solutions:
Given:
$x + y - 4 = 0$, $3x - 7y - 8 = 0$ and $4x - y - 31 = 0$ forming a triangle and point (a, 2)is an interior point of the triangle
Let ABC be the triangle of sides AB, BC and CA whose equations are $x + y - 4 = 0$, $3x - 7y - 8 = 0$ and $4x - y - 31 = 0$, respectively.
On solving them, we get A (7, - 3), B (18/5, 2/5) and C (209/25, 61/25) as the coordinates of the vertices.
Let P (a, 2) be the given point.



It is given that point P (a, 2) lies inside the triangle. So, we have the following:
(i) A and P must lie on the same side of BC.
(ii) B and P must lie on the same side of AC.
(iii) C and P must lie on the same side of AB.

Thus, if A and P lie on the same side of BC, then
$21 + 21 - 8 - 3a - 14 - 8 > 0$

a > 22/3 … (1)

If B and P lie on the same side of AC, then

$4 \times \frac{18}{5} - \frac{2}{5} - 31 - 4a - 2 - 31 > 0$

a < 33/4 … (2)

If C and P lie on the same side of AB, then

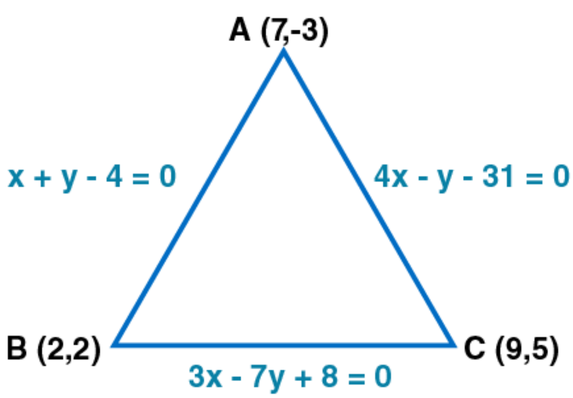
$\frac{209}{25} + \frac{61}{25} - 4 - a + 2 - 4 > 0$

$\frac{34}{5} - 4 - a + 2 - 4 > 0$

a > 2 … (3)

From (1), (2) and (3), we get:
A ∈ (22/3, 33/4)
∴ A ∈ (22/3, 33/4)

3. Determine whether the point (-3, 2) lies inside or outside the triangle whose sides are given by the equations x + y – 4 = 0, 3x – 7y + 8 = 0, 4x – y – 31 = 0.
Solution:
<u>Given:</u>
x + y – 4 = 0, 3x – 7y + 8 = 0, 4x – y – 31 = 0 forming a triangle and point (-3, 2)
Let ABC be the triangle of sides AB, BC and CA, whose equations x + y − 4 = 0, 3x − 7y + 8 = 0 and 4x − y − 31 = 0, respectively.
On solving them, we get A (7, - 3), B (2, 2) and C (9, 5) as the coordinates of the vertices.
Let P (− 3, 2) be the given point.



The given point P (− 3, 2) will lie inside the triangle ABC, if
(i) A and P lies on the same side of BC

(ii) B and P lies on the same side of AC
(iii) C and P lies on the same side of AB

Thus, if A and P lie on the same side of BC, then
$21 + 21 + 8 - 9 - 14 + 8 > 0$
$50 \times - 15 > 0$
$-750 > 0$,
This is false
∴ The point (−3, 2) lies outside triangle ABC.