EXERCISE 23.15 PAGE NO: 23.107

1. Find the distance of the point (4, 5) from the straight line 3x – 5y + 7 = 0.

Solution:

Given:

The line: 3x – 5y + 7 = 0

Comparing ax + by + c = 0 and 3x − 5y + 7 = 0, we get:

a = 3, b = − 5 and c = 7

So, the distance of the point (4, 5) from the straight line 3x − 5y + 7 = 0 is

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{3 \times 4 - 5 \times 5 + 7}{\sqrt{3^2 + (-5^2)}}\right|$$

$$= \frac{6}{\sqrt{34}}$$

∴ The required distance is 6/√34

2. Find the perpendicular distance of the line joining the points (cos θ, sin θ) and (cos ϕ, sin ϕ) from the origin.

Solution:

Given:

The points (cos θ, sin θ) and (cos ϕ, sin ϕ) from the origin.

The equation of the line joining the points (cos θ, sin θ) and (cos ϕ, sin ϕ) is given below:

$$y - \sin\theta = \frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$$

$$(\cos\phi - \cos\theta)y - \sin\theta(\cos\phi - \cos\theta) = (\sin\phi - \sin\theta)x - (\sin\phi - \sin\theta)\cos\theta$$

$$(\sin\phi - \sin\theta)x - (\cos\phi - \cos\theta)y + \sin\theta\cos\phi - \sin\phi\cos\theta = 0$$

Let d be the perpendicular distance from the origin to the line

$$(\sin\phi - \sin\theta)x - (\cos\phi - \cos\theta)y + \sin\theta\cos\phi - \sin\phi\cos\theta = 0$$

$$d = \left|\frac{\sin\theta - \phi}{\sqrt{(\sin\phi - \sin\theta)^2 + (\cos\phi - \cos\theta)^2}}\right|$$

$$= \left|\frac{\sin\theta - \phi}{\sqrt{\sin^2\phi + \sin^2\theta - 2\sin\phi\sin\theta + \cos^2\phi + \cos^2\theta - 2\cos\phi\cos\theta}}\right|$$

$$= \left|\frac{\sin\theta - \phi}{\sqrt{\sin^2\phi + \cos^2\phi + \sin^2\theta + \cos^2\phi + \cos^2\theta - 2\cos(\theta - \phi)}}\right|$$

$$= \left|\frac{\frac{1}{\sqrt{2}}(\sin(\theta - \phi))}{\sqrt{1 - \cos(\theta - \phi)}}\right|$$

$$= \frac{1}{\sqrt{2}}\left|\frac{\sin(\theta - \phi)}{\sqrt{2\sin^2\left(\frac{\theta - \phi}{2}\right)}}\right|$$

$$= \frac{1}{2}\left|\frac{2\sin\left(\frac{\theta - \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)}{\sin\left(\frac{\theta - \phi}{2}\right)}\right|$$

$$= \cos\left(\frac{\theta - \phi}{2}\right)$$

∴ The required distance is $\cos\left(\frac{\theta - \phi}{2}\right)$

3. Find the length of the perpendicular from the origin to the straight line joining the two points whose coordinates are (a cos α, a sin α) and (a cos β, a sin β).

Solution:

Given:

Coordinates are (a cos α, a sin α) and (a cos β, a sin β).

Equation of the line passing through (a cos α, a sin α) and (a cos β, a sin β) is

$$y - a\sin\alpha = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha}(x - a\cos\alpha)$$

$$y - a\sin\alpha = \frac{\sin\beta - \sin\alpha}{\cos\beta - \cos\alpha}(x - a\cos\alpha)$$

$$y - a\sin\alpha = \frac{2\cos\left(\frac{\beta + \alpha}{2}\right)\sin\left(\frac{\beta - \alpha}{2}\right)}{2\sin\left(\frac{\beta + \alpha}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)}(x - a\cos\alpha)$$

$$y - a\sin\alpha = -\cot\left(\frac{\beta + \alpha}{2}\right)(x - a\cos\alpha)$$

$$y - a\sin\alpha = -\cot\left(\frac{\alpha + \beta}{2}\right)(x - a\cos\alpha)$$

$$x\cot\left(\frac{\alpha + \beta}{2}\right) + y - a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha + \beta}{2}\right) = 0$$

The distance of the line from the origin is

$$d = \left|\frac{-a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha+\beta}{2}\right)}{\sqrt{\cot^2\left(\frac{(\alpha+\beta)}{2}\right)+1}}\right|$$

$$d = \left|\frac{-a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha+\beta}{2}\right)}{\sqrt{\mathrm{cosec}^2\left(\frac{(\alpha+\beta)}{2}\right)}}\right| \because \mathrm{cosec}^2\theta = 1 + \cot^2\theta$$

$$= a\left|\sin\left(\frac{\alpha+\beta}{2}\right)\sin\alpha + \cos\alpha\cos\left(\frac{\alpha+\beta}{2}\right)\right|$$

$$= a\left|\sin\alpha\sin\left(\frac{\alpha+\beta}{2}\right) + \cos\alpha\cos\left(\frac{\alpha+\beta}{2}\right)\right|$$

$$= a\left|\cos\left(\frac{\alpha+\beta}{2} - \alpha\right)\right| = a\cos\left(\frac{\beta-\alpha}{2}\right)$$

∴ The required distance is $a\cos\left(\frac{\beta-\alpha}{2}\right)$

4. Show that the perpendicular let fall from any point on the straight line 2x + 11y – 5 = 0 upon the two straight lines 24x + 7y = 20 and 4x – 3y – 2 = 0 are equal to each other.

Solution:

Given:

The lines 24x + 7y = 20 and 4x – 3y – 2 = 0

Let us assume, P(a, b) be any point on 2x + 11y − 5 = 0

So,

2a + 11b − 5 = 0

$b = \frac{5-2a}{11}$ …….. (1)

Let d_1 and d_2 be the perpendicular distances from point P on the lines 24x + 7y = 20 and 4x − 3y − 2 = 0, respectively.

$$d_1 = \left|\frac{24a + 7b - 20}{\sqrt{24^2 + 7^2}}\right| = \left|\frac{24a + 7b - 20}{25}\right|$$

$$= \left|\frac{24a + 7 \times \frac{5-2a}{11} - 20}{25}\right|$$

From (1)

$d_1 = \left|\frac{50a - 37}{55}\right|$

Similarly,

$d_2 = \left|\frac{4a - 3b - 2}{\sqrt{3^2 + (-4)^2}}\right| = \left|\frac{4a - 3 \times \frac{5-2a}{11} - 2}{5}\right|$

$= \left|\frac{44a - 15 + 6a - 22}{11 \times 5}\right|$

From (1)

$d_2 = \left|\frac{50a - 37}{55}\right|$

$\therefore d_1 = d_2$

Hence proved.

5. Find the distance of the point of intersection of the lines 2x + 3y = 21 and 3x – 4y + 11 = 0 from the line 8x + 6y + 5 = 0.

Solution:

Given:

The lines 2x + 3y = 21 and 3x – 4y + 11 = 0

Solving the lines 2x + 3y = 21 and 3x − 4y + 11 = 0 we get:

$$\frac{x}{33 - 84} = \frac{y}{-63 - 22} = \frac{1}{-8 - 9}$$

x = 3, y = 5

So, the point of intersection of 2x + 3y = 21 and 3x − 4y + 11 = 0 is (3, 5).

Now, the perpendicular distance d of the line 8x + 6y + 5 = 0 from the point (3, 5) is

$$d = \left|\frac{24 + 30 + 5}{\sqrt{8^2 + 6^2}}\right| = \frac{59}{10}$$

∴ The distance is 59/10.