## EXERCISE 23.17



**1. Prove that the area of the parallelogram formed by the lines**
**$a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_2x + b_2y + d_2 = 0$**
**is $\left|\frac{(d_1 - c_1)(d_2 - c_2)}{a_1b_2 - a_2b_1}\right|$ sq. units.**
**Deduce the condition for these lines to form a rhombus.**
**Solution:**
Given:
The given lines are
$a_1x + b_1y + c_1 = 0$ … (1)
$a_1x + b_1y + d_1 = 0$ … (2)
$a_2x + b_2y + c_2 = 0$ … (3)
$a_2x + b_2y + d_2 = 0$ … (4)
Let us prove, the area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_2x + b_2y + d_2 = 0$ is $\left|\frac{(d_1 - c_1)(d_2 - c_2)}{(a_1b_2 - a_2b_1)}\right|$ sq. units.

The area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_2x + b_2y + d_2 = 0$ is given below:

$$\text{Area} = \left|\frac{(c_1 - d_1)(c_2 - d_2)}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}\right|$$

Since, $\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$

$$\therefore \text{Area} = \left|\frac{(c_1 - d_1)(c_2 - d_2)}{(a_1b_2 - a_2b_1)}\right| = \left|\frac{(d_1 - c_1)(d_2 - c_2)}{(a_1b_2 - a_2b_1)}\right|$$

If the given parallelogram is a rhombus, then the distance between the pair of parallel lines is equal.

$$\therefore \left|\frac{c_1 - d_1}{\sqrt{a_1^2 + b_1^2}}\right| = \left|\frac{c_2 - d_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

Hence proved.

**2. Prove that the area of the parallelogram formed by the lines 3x – 4y + a = 0, 3x – 4y + 3a = 0, 4x – 3y – a = 0 and 4x – 3y – 2a = 0 is $2a^2/7$ sq. units.**

**Solution:**

Given:

The given lines are

$3x - 4y + a = 0$ … (1)

$3x - 4y + 3a = 0$ … (2)

$4x - 3y - a = 0$ … (3)

$4x - 3y - 2a = 0$ … (4)

Let us prove, the area of the parallelogram formed by the lines 3x – 4y + a = 0, 3x – 4y + 3a = 0, 4x – 3y – a = 0 and 4x – 3y – 2a = 0 is $2a^2/7$ sq. units.

From above solution, we know that

$$\text{Area of the parallelogram} = \left|\frac{(c_1 - d_1)(c_2 - d_2)}{(a_1 b_2 - a_2 b_1)}\right|$$

$$\text{Area of the parallelogram} = \left|\frac{(a - 3a)(2a - a)}{(-9 + 16)}\right| = \frac{2a^2}{7} \text{ square units}$$

Hence proved.

**3. Show that the diagonals of the parallelogram whose sides are lx + my + n = 0, lx + my + n’ = 0, mx + ly + n = 0 and mx + ly + n’ = 0 include an angle π/2.**

**Solution:**

Given:

The given lines are

$lx + my + n = 0$ … (1)

$mx + ly + n' = 0$ … (2)

$lx + my + n' = 0$ … (3)

$mx + ly + n = 0$ … (4)

Let us prove, the diagonals of the parallelogram whose sides are lx + my + n = 0, lx + my + n’ = 0, mx + ly + n = 0 and mx + ly + n’ = 0 include an angle π/2.

By solving (1) and (2), we get

$$B = \left(\frac{mn' - ln}{l^2 - m^2}, \frac{mn - ln'}{l^2 - m^2}\right)$$

Solving (2) and (3), we get,

$$C = \left(-\frac{n'}{m + l}, -\frac{n'}{m + l}\right)$$

Solving (3) and (4), we get,

$$D = \left(\frac{mn - ln'}{l^2 - m^2}, \frac{mn' - ln}{l^2 - m^2}\right)$$

Solving (1) and (4), we get,

$$A = \left(\frac{-n}{m+l}, \frac{-n}{m+l}\right)$$

Let $m_1$ and $m_2$ be the slope of AC and BD.
Now,

$$m_1 = \frac{\frac{-n'}{m+l} + \frac{n}{m+l}}{\frac{-n'}{m+l} + \frac{n}{m+l}} = 1$$

$$m_2 = \frac{\frac{mn' - ln}{l^2 - m^2} - \frac{mn - ln'}{l^2 - m^2}}{\frac{mn - ln'}{l^2 - m^2} - \frac{mn' - ln}{l^2 - m^2}} = -1$$

$\therefore m_1 m_2 = -1$
Hence proved.