EXERCISE 23.5 PAGE NO: 23.35

1. Find the equation of the straight lines passing through the following pair of points:
(i) (0, 0) and (2, -2)
(ii) (a, b) and (a + c sin α, b + c cos α)
Solution:
(i) (0, 0) and (2, -2)
Given:
$(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (2, -2)$
The equation of the line passing through the two points (0, 0) and (2, −2) is
By using the formula,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Now, substitute the values, we get

$$y - 0 = \frac{-2-0}{2-0}(x - 0)$$

y = - x
∴ The equation of line is y = -x

(ii) (a, b) and (a + c sin α, b + c cos α)
Given:
$(x_1, y_1) = (a, b)$, $(x_2, y_2) = (a + c \sin \alpha, b + c \cos \alpha)$
So, the equation of the line passing through the two points (0, 0) and (2, −2) is
By using the formula,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Now, substitute the values, we get

$$y - b = \frac{b + c \cos\alpha - b}{a + c \sin \alpha - a}(x - a)$$

y – b = cot α (x - a)
∴ The equation of line is y – b = cot α (x - a)

2. Find the equations to the sides of the triangles the coordinates of whose angular points are respectively:
(i) (1, 4), (2, -3) and (-1, -2)
(ii) (0, 1), (2, 0) and (-1, -2)
Solution:
(i) (1, 4), (2, -3) and (-1, -2)
Given:

Points A (1, 4), B (2, -3) and C (-1, -2).
Let us assume,
m_1, m_2, and m_3 be the slope of the sides AB, BC and CA, respectively.
So,
The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).
Then,

$$m_1 = \frac{-3-4}{2-1},$$

$$m_2 = \frac{-2+3}{-1-2},$$

$$m_3 = \frac{4+2}{1+1}$$

$m_1 = -7$, $m_2 = -1/3$ and $m_3 = 3$
So, the equation of the sides AB, BC and CA are
By using the formula,
$y - y_1 = m(x - x_1)$
=> $y - 4 = -7(x - 1)$
$y - 4 = -7x + 7$
$y + 7x = 11$,

=> $y + 3 = (-1/3)(x - 2)$
$3y + 9 = -x + 2$
$3y + x = -7$
$x + 3y + 7 = 0$ and

=> $y + 2 = 3(x+1)$
$y + 2 = 3x + 3$
$y - 3x = 1$
So, we get
$y + 7x = 11$, $x + 3y + 7 = 0$ and $y - 3x = 1$
∴ The equation of sides are $y + 7x = 11$, $x + 3y + 7 = 0$ and $y - 3x = 1$

(ii) (0, 1), (2, 0) and (-1, -2)
Given:
Points A (0, 1), B (2, 0) and C (-1, -2).
Let us assume,
m_1, m_2, and m_3 be the slope of the sides AB, BC and CA, respectively.
So,
The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).
Then,

$m_1 = \frac{0-1}{2-0}$,

$m_2 = \frac{-2-0}{-1-2}$,

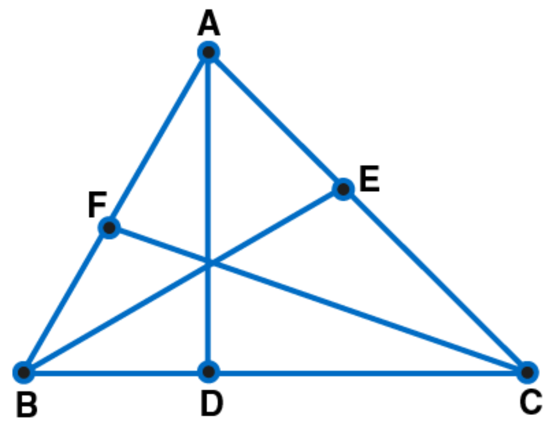
$m_3 = \frac{1+2}{1+0}$

$m_1 = -1/2$, $m_2 = -2/3$ and $m_3 = 3$
So, the equation of the sides AB, BC and CA are
By using the formula,
$y - y_1 = m (x - x_1)$
=> $y - 1 = (-1/2) (x - 0)$
$2y - 2 = -x$
$x + 2y = 2$

=> $y - 0 = (-2/3) (x - 2)$
$3y = -2x + 4$
$2x - 3y = 4$

=> $y + 2 = 3(x+1)$
$y + 2 = 3x + 3$
$y - 3x = 1$
So, we get
$x + 2y = 2$, $2x - 3y = 4$ and $y - 3x = 1$
∴ The equation of sides are $x + 2y = 2$, $2x - 3y = 4$ and $y - 3x = 1$

3. Find the equations of the medians of a triangle, the coordinates of whose vertices are (-1, 6), (-3,-9) and (5, -8).
Solution:
Given:
A (−1, 6), B (−3, −9) and C (5, −8) be the coordinates of the given triangle.
Let us assume: D, E, and F be midpoints of BC, CA and AB, respectively. So, the coordinates of D, E and F are



Median AD passes through A (-1, 6) and D (1, -17/2)
So, by using the formula,

$$y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$$

$$y - 6 = \frac{-\frac{17}{2} - 6}{1+1}(x+1)$$

4y – 24 = -29x – 29
29x + 4y + 5 = 0

Similarly, Median BE passes through B (-3,-9) and E (2,-1)
So, by using the formula,

$$y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$$

$$y + 9 = \frac{-1+9}{2+3}(x+3)$$

5y + 45 = 8x + 24
8x – 5y – 21=0

Similarly, Median CF passes through C (5,-8) and F(-2,-3/2)
So, by using the formula,

$$y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$$

$$y + 9 = \frac{-\frac{3}{2}+8}{-2-5}(x-5)$$

-14y – 112 = 13x – 65
13x + 14y + 47 = 0
∴ The equation of lines are: 29x + 4y + 5 = 0, 8x – 5y – 21=0 and 13x + 14y + 47 = 0

4. Find the equations to the diagonals of the rectangle the equations of whose sides are x = a, x = a', y = b and y = b'.
Solution:
Given:
The rectangle formed by the lines x = a, x = a', y = b and y = b'
It is clear that, the vertices of the rectangle are A (a, b), B (a', b), C (a', b') and D (a, b') .
The diagonal passing through A (a, b) and C (a', b') is
By using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - b = \frac{b' - b}{a' - a}(x - a)$$

(a' – a)y – b(a' – a) = (b' – b)x – a(b' – b)
(a' – a) – (b' – b)x = ba' – ab'

Similarly, the diagonal passing through B (a', b) and D (a, b') is
By using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - b = \frac{b' - b}{a - a'}(x - a')$$

(a' – a)y – b(a – a') = (b' – b)x – a' (b' – b)
(a' – a) + (b' – b)x = a'b' – ab
∴ The equation of diagonals are y(a' – a) – x(b' – b) = a'b – ab' and
y(a' – a) + x(b' – b) = a'b' – ab

5. Find the equation of the side BC of the triangle ABC whose vertices are A (-1, -2), B (0, 1) and C (2, 0) respectively. Also, find the equation of the median through A (-1, -2).
Solution:
Given:
The vertices of triangle ABC are A (-1, -2), B(0, 1) and C(2, 0).
Let us find the equation of median through A.
So, the equation of BC is
By using the formula,

$$y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$$

$$y - 1 = \frac{0-1}{2-0}(x - 0)$$

$$y - 1 = \frac{-1}{2}(x - 0)$$

x + 2y – 2 = 0

Let D be the midpoint of median AD,

So, $D\left(\frac{0+2}{2}, \frac{1+0}{2}\right) = \left(1, \frac{1}{2}\right)$

The equation of the median AD is

By using the formula,

$$y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$$

$$y + 2 = \frac{\frac{1}{2}+2}{1+1}(x + 1)$$

4y + 8 = 5x + 5

5x – 4y – 3 = 0

∴ The equation of line BC is x + 2y – 2 = 0

The equation of median is 5x – 4y – 3 = 0