EXERCISE 23.8 PAGE NO: 23.65

1. A line passes through a point A (1, 2) and makes an angle of 60^0 with the x–axis and intercepts the line x + y = 6 at the point P. Find AP.

Solution:

Given:

(x_1, y_1) = A (1, 2), θ = 60°

Let us find the distance AP.

By using the formula,

The equation of the line is given by:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

Now, substitute the values, we get

$$\frac{x-1}{\cos 60°} = \frac{y-2}{\sin 60°} = r$$

$$\frac{x-1}{\frac{1}{2}} = \frac{y-2}{\frac{\sqrt{3}}{2}} = r$$

Here, r represents the distance of any point on the line from point A (1, 2).

The coordinate of any point P on this line are (1 + r/2, 2 + √3r/2)

It is clear that, P lies on the line x + y = 6

So,

$$1+\frac{r}{2}+2+\frac{\sqrt{3}}{2}r = 6$$

$$\frac{\sqrt{3}}{2}r + \frac{r}{2} = 3$$

$$r(\sqrt{3}+1) = 6$$

$$r = \frac{6}{\sqrt{3}+1} = 3(\sqrt{3}-1)$$

∴ The value of AP is 3(√3 – 1)

2. If the straight line through the point P(3, 4) makes an angle π/6 with the x–axis and meets the line 12x + 5y + 10 = 0 at Q, find the length PQ.

Solution:

Given:

(x_1, y_1) = A (3, 4), θ = π/6 = 30°

Let us find the length PQ.

By using the formula,

The equation of the line is given by:

$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

Now, substitute the values, we get

$\frac{x-3}{\cos 30^\circ} = \frac{y-4}{\sin 30^\circ} = r$

$\frac{x-3}{\frac{\sqrt{3}}{2}} - \frac{y-4}{\frac{1}{2}} = r$

$x - \sqrt{3}\,y + 4\sqrt{3} - 3 = 0$

Let PQ = r

Then, the coordinate of Q are given by

$\frac{x-3}{\cos 30^\circ} = \frac{y-4}{\sin 30^\circ} = r$

$x = 3 + \frac{\sqrt{3}}{2}r$, $y = 4 + \frac{r}{2}$

The coordinate of point Q is $\left(3 + \frac{\sqrt{3}}{2}r, 4 + \frac{r}{2}\right)$

It is clear that, Q lies on the line $12x + 5y + 10 = 0$

So,

$12\left(3 + \frac{\sqrt{3}}{2}r\right) + 5\left(4 + \frac{r}{2}\right) + 10 = 0$

$66 + \frac{12\sqrt{3}+5}{2}r = 0$

$r = -\frac{132}{5+12\sqrt{3}}$

$PQ = |r| = \frac{132}{5+12\sqrt{3}}$

$\therefore$ The value of PQ is $\frac{132}{5+12\sqrt{3}}$

3. A straight line drawn through the point A (2, 1) making an angle π/4 with positive x–axis intersects another line x + 2y + 1 = 0 in the point B. Find length AB.

Solution:

Given:

$(x_1, y_1) = A\ (2, 1)$, $\theta = \pi/4 = 45^\circ$

Let us find the length AB.

By using the formula,

The equation of the line is given by:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

Now, substitute the values, we get

$$\frac{x-2}{\cos 45^\circ} = \frac{y-1}{\sin 45^\circ} = r$$

$$\frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = r$$

x – y – 1 = 0

Let AB = r

Then, the coordinate of B is given by

$$\frac{x-2}{\cos 45^\circ} = \frac{y-1}{\sin 45^\circ} = r$$

$$x = 2 + \frac{1}{\sqrt{2}}r,\ y = 1 + \frac{r}{\sqrt{2}}$$

The coordinate of point B is $\left(2 + \frac{1}{\sqrt{2}}r, 1 + \frac{r}{\sqrt{2}}\right)$

It is clear that, B lies on the line x + 2y + 1 = 0

$$2 + \frac{1}{\sqrt{2}}r + 2\left(1 + \frac{r}{\sqrt{2}}\right) + 1 = 0$$

$$5 + \frac{3r}{\sqrt{2}}r = 0$$

$$r = \frac{5\sqrt{2}}{3}$$

∴ The value of AB is $\frac{5\sqrt{2}}{3}$

4. A line a drawn through A (4, – 1) parallel to the line 3x – 4y + 1 = 0. Find the coordinates of the two points on this line which are at a distance of 5 units from A.

Solution:

Given:

(x_1, y_1) = A (4, -1)

Let us find Coordinates of the two points on this line which are at a distance of 5 units from A.

Given: Line 3x – 4y + 1 = 0

4y = 3x + 1

y = 3x/4 + 1/4

Slope tan θ = 3/4

So,

Sin θ = 3/5

Cos θ = 4/5

The equation of the line passing through A (4, −1) and having slope ¾ is
By using the formula,
The equation of the line is given by:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$$

Now, substitute the values, we get

$$\frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}}$$

3x – 4y = 16

Here, AP = r = ± 5
Thus, the coordinates of P are given by

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

$$\frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}} = r$$

$$x = \frac{4r}{5} + 4 \text{ and } y = \frac{3r}{5} - 1$$

$$x = \frac{4(\pm 5)}{5} + 4 \text{ and } y = \frac{3(\pm 5)}{5} - 1$$

x = ±4 + 4 and y = ±3 –1
x = 8, 0 and y = 2, – 4
∴ The coordinates of the two points at a distance of 5 units from A are (8, 2) and (0, −4).

5. The straight line through P(x_1, y_1) inclined at an angle θ with the x–axis meets the line ax + by + c = 0 in Q. Find the length of PQ.
Solution:
Given:
The equation of the line that passes through P(x_1, y_1) and makes an angle of θ with the x–axis.
Let us find the length of PQ.
We know that,

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$$

Let PQ = r
Then, the coordinates of Q are given by
By using the formula,
The equation of the line is given by:

$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

$x = x_1 + r\cos\theta, y = y_1 + r\sin\theta$

Thus, the coordinates of Q are $(x_1 + r\cos\theta, y_1 + r\sin\theta)$

It is clear that, Q lies on the line $ax + by + c = 0$.

So,

$a(x_1 + r\cos\theta) + b(y_1 + r\sin\theta) + c = 0$

$r = PQ = \left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$

$\therefore$ The value of PQ is $\left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$