EXERCISE 23.1 PAGE NO: 23.12

1. Find the slopes of the lines which make the following angles with the positive direction of x - axis:
(i) – π/4
(ii) 2π/3
Solution:
(i) – π/4
Let the slope of the line be 'm'
Where, m = tan θ
So, the slope of Line is m = tan (– π/4)
= – 1
∴ The slope of the line is – 1.

(ii) 2π/3
Let the slope of the line be 'm'
Where, m = tan θ
So, the slope of Line is m = tan (2π/3)

$$\tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{\pi}{3}\right)$$
$$\tan\left(\frac{2\pi}{3}\right) = \tan\left(-\frac{\pi}{3}\right)$$
$$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

∴ The slope of the line is -√3

2. Find the slopes of a line passing through the following points :
(i) (–3, 2) and (1, 4)
(ii) $(at^2_1, 2at_1)$ and $(at^2_2, 2at_2)$
Solution:
(i) (–3, 2) and (1, 4)
By using the formula,

$$\text{Slope of line, m} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{So, the slope of the line, m} = \frac{4-2}{1-(-3)}$$

= 2 / 4
= 1 / 2

∴ The slope of the line is ½.

(ii) $(at^2_1, 2at_1)$ and $(at^2_2, 2at_2)$
By using the formula,

Slope of line, $m = \frac{y_2 - y_1}{x_2 - x_1}$

Now, substitute the values
The slope of the line, $m = \frac{2at_2 - 2at_1}{at_2^2 - at_1^2}$

$= \frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)}$

$= \frac{2a(t_2 - t_1)}{a(t_2 - t_1)(t_2 + t_1)}$ [Since, $(a^2 - b^2 = (a - b)(a + b)$]

$= \frac{2}{t_2 + t_1}$

∴ The slope of the line is $\frac{2}{t_2 + t_1}$

3. State whether the two lines in each of the following are parallel, perpendicular or neither:
(i) Through (5, 6) and (2, 3); through (9, –2) and (6, –5)
(ii) Through (9, 5) and (– 1, 1); through (3, –5) and (8, –3)
Solution:
(i) Through (5, 6) and (2, 3); through (9, –2) and (6, –5)
By using the formula,

Slope of line, $m = \frac{y_2 - y_1}{x_2 - x_1}$

The slope of the line whose Coordinates are (5, 6) and (2, 3)

$m_1 = \frac{3 - 6}{2 - 5}$

$= \frac{-3}{-3}$

$= 1$

So, $m_1 = 1$

The slope of the line whose Coordinates are (9, – 2) and (6, – 5)

$m_2 = \frac{-5 - (-2)}{6 - 9}$

$= \frac{-3}{-3}$

So, $m_2 = 1$

Here, $m_1 = m_2 = 1$
∴ The lines are parallel to each other.

(ii) Through (9, 5) and (– 1, 1); through (3, –5) and (8, –3)
By using the formula,
Slope of line, $m = \frac{y_2 - y_1}{x_2 - x_1}$
The slope of the line whose Coordinates are (9, 5) and (– 1, 1)

$$m_1 = \frac{1-5}{-1-9}$$
$$= \frac{-4}{-10}$$
$$= 2/5$$

So, $m_1 = 2/5$
The slope of the line whose Coordinates are (3, – 5) and (8, – 3)

$$m_2 = \frac{-3-(-5)}{8-3}$$
$$= 2/5$$

So, $m_2 = 2/5$
Here, $m_1 = m_2 = 2/5$
∴ The lines are parallel to each other.

4. Find the slopes of a line
(i) which bisects the first quadrant angle
(ii) which makes an angle of 30^0 with the positive direction of y - axis measured anticlockwise.
Solution:
(i) Which bisects the first quadrant angle?
Given: Line bisects the first quadrant
We know that, if the line bisects in the first quadrant, then the angle must be between line and the positive direction of x - axis.
Since, angle = 90/2 = 45^o
By using the formula,
The slope of the line, m = tan θ
The slope of the line for a given angle is m = tan 45^o
So, m = 1
∴ The slope of the line is 1.

(ii) Which makes an angle of 30^0 with the positive direction of y - axis measured

anticlockwise?
Given: The line makes an angle of 30^o with the positive direction of y – axis.
We know that, angle between line and positive side of axis => $90^o + 30^o = 120^o$
By using the formula,
The slope of the line, m = tan θ
The slope of the line for a given angle is m = tan 120^o
So, m = – √3
∴ The slope of the line is – √3.

5. Using the method of slopes show that the following points are collinear:
(i) A (4, 8), B (5, 12), C (9, 28)
(ii) A(16, – 18), B(3, – 6), C(– 10, 6)
Solution:
(i) A (4, 8), B (5, 12), C (9, 28)
By using the formula,
The slope of the line = $[y_2 – y_1] / [x_2 – x_1]$
So,
The slope of line AB = [12 - 8] / [5 - 4]
= 4 / 1

The slope of line BC = [28 - 12] / [9 - 5]
= 16 / 4
= 4

The slope of line CA = [8 - 28] / [4 - 9]
= -20 / -5
= 4
Here, AB = BC = CA
∴ The Given points are collinear.

(ii) A(16, – 18), B(3, – 6), C(– 10, 6)
By using the formula,
The slope of the line = $[y_2 – y_1] / [x_2 – x_1]$
So,
The slope of line AB = [-6 – (-18)] / [3 - 16]
= 12 / -13

The slope of line BC = [6 – (-6)] / [-10 - 3]
= 12 / -13

The slope of line CA = [6 – (-18)] / [-10 - 16]
= 12 / -13
= 4

Here, AB = BC = CA

∴ The Given points are collinear.

EXERCISE 23.2 PAGE NO: 23.17

1. Find the equation of the parallel to x–axis and passing through (3, –5).

Solution:

Given: A line which is parallel to x–axis and passing through (3, –5)
By using the formula,
The equation of line: $[y - y_1 = m(x - x_1)]$
We know that the parallel lines have equal slopes
And, the slope of x–axis is always 0
Then
The slope of line, m = 0
Coordinates of line are $(x_1, y_1) = (3, -5)$
The equation of line = $y - y_1 = m(x - x_1)$
Now, substitute the values, we get
$y - (-5) = 0(x - 3)$
$y + 5 = 0$
∴ The equation of line is $y + 5 = 0$

2. Find the equation of the line perpendicular to x–axis and having intercept – 2 on x–axis.

Solution:

Given: A line which is perpendicular to x–axis and having intercept –2
By using the formula,
The equation of line: $[y - y_1 = m(x - x_1)]$
We know that, the line is perpendicular to the x–axis, then x is 0 and y is –1.
The slope of line is, m = y/x
= -1/0

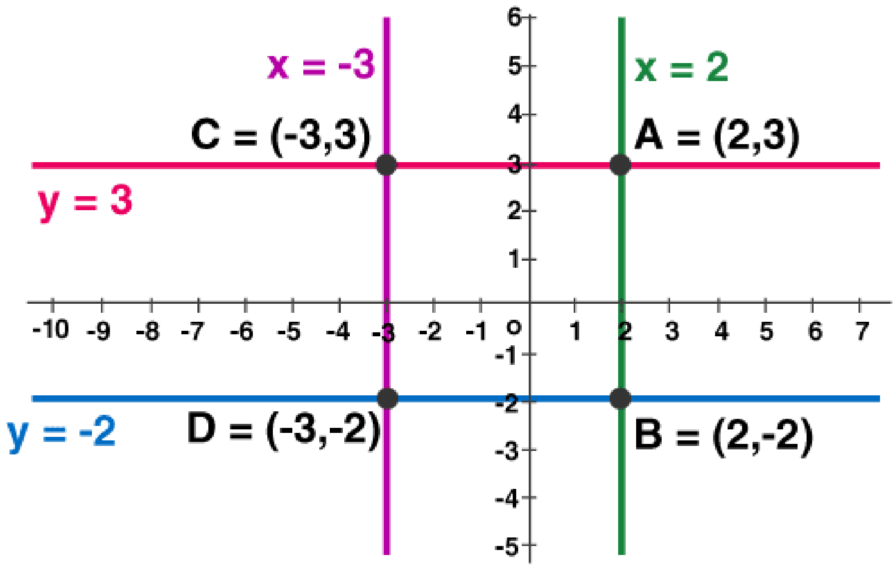
It is given that x–intercept is –2, so, y is 0.
Coordinates of line are $(x_1, y_1) = (-2, 0)$
The equation of line = $y - y_1 = m(x - x_1)$
Now, substitute the values, we get
$y - 0 = (-1/0)(x - (-2))$
$x + 2 = 0$
∴ The equation of line is $x + 2 = 0$

3. Find the equation of the line parallel to x–axis and having intercept – 2 on y – axis.

Solution:

Given: A line which is parallel to x–axis and having intercept –2 on y – axis
By using the formula,
The equation of line: $[y – y_1 = m(x – x_1)]$
The parallel lines have equal slopes,
And, the slope of x–axis is always 0
Then
The slope of line, m = 0
It is given that intercept is –2, on y – axis then
Coordinates of line are $(x_1, y_1) = (0, – 2)$
The equation of line is $y – y_1 = m(x – x_1)$
Now, substitute the values, we get
$y – (– 2) = 0 (x – 0)$
$y + 2 = 0$
∴ The equation of line is y + 2 = 0

4. Draw the lines x = –3, x = 2, y = –2, y = 3 and write the coordinates of the vertices of the square so formed.
Solution:
Given: x = –3, x = 2, y = –2 and y = 3



∴ The Coordinates of the square are: A(2, 3), B(2, –2), C(–3, 3), and D(–3, –2).

5. Find the equations of the straight lines which pass through (4, 3) and are respectively parallel and perpendicular to the x–axis.
Solution:

Given: A line which is perpendicular and parallel to x–axis respectively and passing through (4, 3)
By using the formula,
The equation of line: $[y - y_1 = m(x - x_1)]$
Let us consider,

Case 1: When Line is parallel to x–axis
The parallel lines have equal slopes,
And, the slope of x–axis is always 0, then
The slope of line, m = 0
Coordinates of line are $(x_1, y_1) = (4, 3)$
The equation of line is $y - y_1 = m(x - x_1)$
Now substitute the values, we get
$y - (3) = 0(x - 4)$
$y - 3 = 0$

Case 2: When line is perpendicular to x–axis
The line is perpendicular to the x–axis, then x is 0 and y is – 1.
The slope of the line is, m = y/x
= -1/0
Coordinates of line are $(x_1, y_1) = (4, 3)$
The equation of line = $y - y_1 = m(x - x_1)$
Now substitute the values, we get
$y - 3 = (-1/0)(x - 4)$
$x = 4$
∴ The equation of line when it is parallel to x – axis is y = 3 and it is perpendicular is x = 4.

EXERCISE 23.3 PAGE NO: 23.21

1. Find the equation of a line making an angle of 150° with the x–axis and cutting off an intercept 2 from y–axis.

Solution:

Given: A line which makes an angle of 150^o with the x–axis and cutting off an intercept at 2

By using the formula,

The equation of a line is y = mx + c

We know that angle, θ = 150^o

The slope of the line, m = tan θ

Where, m = tan 150^o

= -1/ √3

Coordinate of y–intercept is (0, 2)

The required equation of the line is y = mx + c

Now substitute the values, we get

y = -x/√3 + 2

√3y - 2√3 + x = 0

x + √3y = 2√3

∴ The equation of line is x + √3y = 2√3

2. Find the equation of a straight line:

(i) with slope 2 and y – intercept 3;

(ii) with slope – 1/ 3 and y – intercept – 4.

(iii) with slope – 2 and intersecting the x–axis at a distance of 3 units to the left of origin.

Solution:

(i) With slope 2 and y – intercept 3

The slope is 2 and the coordinates are (0, 3)

Now, the required equation of line is

y = mx + c

Substitute the values, we get

y = 2x + 3

(ii) With slope – 1/ 3 and y – intercept – 4

The slope is – 1/3 and the coordinates are (0, – 4)

Now, the required equation of line is

y = mx + c

Substitute the values, we get
$y = -1/3x - 4$
$3y + x = -12$

(iii) With slope – 2 and intersecting the x–axis at a distance of 3 units to the left of origin
The slope is – 2 and the coordinates are (– 3, 0)
Now, the required equation of line is $y - y_1 = m(x - x_1)$
Substitute the values, we get
$y - 0 = -2(x + 3)$
$y = -2x - 6$
$2x + y + 6 = 0$

3. Find the equations of the bisectors of the angles between the coordinate axes.
Solution:
There are two bisectors of the coordinate axes.
Their inclinations with the positive x-axis are 45^o and 135^o
The slope of the bisector is $m = \tan 45^o$ or $m = \tan 135^o$
i.e., m = 1 or m = -1, c = 0
By using the formula, $y = mx + c$
Now, substitute the values of m and c, we get
$y = x + 0$
$x - y = 0$ or $y = -x + 0$
$x + y = 0$
∴ The equation of the bisector is $x \pm y = 0$

4. Find the equation of a line which makes an angle of $\tan^{-1}(3)$ with the x–axis and cuts off an intercept of 4 units on the negative direction of y–axis.
Solution:
Given:
The equation which makes an angle of $\tan^{-1}(3)$ with the x–axis and cuts off an intercept of 4 units on the negative direction of y–axis
By using the formula,
The equation of the line is $y = mx + c$
Here, angle $\theta = \tan^{-1}(3)$
So, $\tan \theta = 3$
The slope of the line is, m = 3
And, Intercept in the negative direction of y–axis is (0, -4)
The required equation of the line is $y = mx + c$
Now, substitute the values, we get

$y = 3x - 4$

∴ The equation of the line is $y = 3x - 4$.

5. Find the equation of a line that has y – intercept – 4 and is parallel to the line joining (2, –5) and (1, 2).

Solution:

Given:

A line segment joining (2, – 5) and (1, 2) if it cuts off an intercept – 4 from y–axis

By using the formula,

The equation of line is $y = mx + C$

It is given that, $c = -4$

Slope of line joining $(x_1 - x_2)$ and $(y_1 - y_2)$,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

So, Slope of line joining (2, – 5) and (1, 2),

$$m = \frac{2-(-5)}{1-2} = \frac{7}{-1}$$

$m = -7$

The equation of line is $y = mx + c$

Now, substitute the values, we get

$y = -7x - 4$

$y + 7x + 4 = 0$

∴ The equation of line is $y + 7x + 4 = 0$.

EXERCISE 23.4 PAGE NO: 23.29

1. Find the equation of the straight line passing through the point (6, 2) and having slope – 3.

Solution:

Given, A straight line passing through the point (6, 2) and the slope is – 3

By using the formula,

The equation of line is $[y – y_1 = m(x – x_1)]$

Here, the line is passing through (6, 2)

It is given that, the slope of line, m = –3

Coordinates of line are $(x_1, y_1) = (6,2)$

The equation of line = $y – y_1 = m(x – x_1)$

Now, substitute the values, we get

$y – 2 = – 3(x – 6)$

$y – 2 = – 3x + 18$

$y + 3x – 20 = 0$

∴ The equation of line is $3x + y – 20 = 0$

2. Find the equation of the straight line passing through (–2, 3) and indicated at an angle of 45° with the x – axis.

Solution:

Given:

A line which is passing through (–2, 3), the angle is 45^o.

By using the formula,

The equation of line is $[y – y_1 = m(x – x_1)]$

Here, angle, $\theta = 45^o$

The slope of the line, $m = \tan \theta$

$m = \tan 45^o$

$= 1$

The line passing through $(x_1, y_1) = (–2, 3)$

The required equation of line is $y – y_1 = m(x – x_1)$

Now, substitute the values, we get

$y – 3 = 1(x – (– 2))$

$y – 3 = x + 2$

$x – y + 5 = 0$

∴The equation of line is $x – y + 5 = 0$

3. Find the equation of the line passing through (0, 0) with slope m

Solution:

Given:
A straight line passing through the point (0, 0) and slope is m.
By using the formula,
The equation of line is $[y - y_1 = m(x - x_1)]$
It is given that, the line is passing through (0, 0) and the slope of line, m = m
Coordinates of line are $(x_1, y_1) = (0, 0)$
The equation of line = $y - y_1 = m(x - x_1)$
Now, substitute the values, we get
$y - 0 = m(x - 0)$
$y = mx$
∴ The equation of line is y = mx.

4. Find the equation of the line passing through (2, 2√3) and inclined with x – axis at an angle of 75°.
Solution:
Given:
A line which is passing through (2, 2√3), the angle is 75°.
By using the formula,
The equation of line is $[y - y_1 = m(x - x_1)]$
Here, angle, $\theta = 75^\circ$
The slope of the line, $m = \tan\theta$
$m = \tan 75^\circ$
$= 3.73 = 2 + \sqrt{3}$
The line passing through $(x_1, y_1) = (2, 2\sqrt{3})$
The required equation of the line is $y - y_1 = m(x - x_1)$
Now, substitute the values, we get
$y - 2\sqrt{3} = 2 + \sqrt{3}\,(x - 2)$
$y - 2\sqrt{3} = (2 + \sqrt{3})x - 7.46$
$(2 + \sqrt{3})x - y - 4 = 0$
∴ The equation of the line is $(2 + \sqrt{3})x - y - 4 = 0$

5. Find the equation of the straight line which passes through the point (1, 2) and makes such an angle with the positive direction of x – axis whose sine is 3/5.
Solution:
A line which is passing through (1, 2)
To Find: The equation of a straight line.
By using the formula,
The equation of line is $[y - y_1 = m(x - x_1)]$
Here, $\sin\theta = 3/5$

We know, sin θ = perpendicular/hypotenuse
= 3/5

So, according to Pythagoras theorem,
$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$
$(5)^2 = (\text{Base})^2 + (3)^2$
(Base) = √(25 - 9)
$(\text{Base})^2 = \sqrt{16}$
Base = 4
Hence, tan θ = perpendicular/base
= 3/4
The slope of the line, m = tan θ
= 3/4
The line passing through $(x_1,y_1) = (1,2)$
The required equation of line is $y – y_1 = m(x – x_1)$
Now, substitute the values, we get
y – 2 = (¾) (x – 1)
4y – 8 = 3x – 3
3x – 4y + 5 = 0
∴ The equation of line is 3x – 4y + 5 = 0

EXERCISE 23.5

1. Find the equation of the straight lines passing through the following pair of points:
(i) (0, 0) and (2, -2)
(ii) (a, b) and (a + c sin α, b + c cos α)
Solution:
(i) (0, 0) and (2, -2)
Given:
$(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (2, -2)$
The equation of the line passing through the two points (0, 0) and (2, −2) is
By using the formula,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Now, substitute the values, we get

$$y - 0 = \frac{-2-0}{2-0}(x - 0)$$

y = - x
∴ The equation of line is y = -x

(ii) (a, b) and (a + c sin α, b + c cos α)
Given:
$(x_1, y_1) = (a, b)$, $(x_2, y_2) = (a + c \sin α, b + c \cos α)$
So, the equation of the line passing through the two points (0, 0) and (2, −2) is
By using the formula,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Now, substitute the values, we get

$$y - b = \frac{b + c \cos α - b}{a + c \sin α - a}(x - a)$$

y – b = cot α (x - a)
∴ The equation of line is y – b = cot α (x - a)

2. Find the equations to the sides of the triangles the coordinates of whose angular points are respectively:
(i) (1, 4), (2, -3) and (-1, -2)
(ii) (0, 1), (2, 0) and (-1, -2)
Solution:
(i) (1, 4), (2, -3) and (-1, -2)
Given:

Points A (1, 4), B (2, -3) and C (-1, -2).
Let us assume,
m_1, m_2, and m_3 be the slope of the sides AB, BC and CA, respectively.
So,
The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).
Then,

$m_1 = \frac{-3-4}{2-1}$,

$m_2 = \frac{-2+3}{-1-2}$,

$m_3 = \frac{4+2}{1+1}$

$m_1 = -7$, $m_2 = -1/3$ and $m_3 = 3$
So, the equation of the sides AB, BC and CA are
By using the formula,
$y – y_1 = m (x – x_1)$
=> y – 4 = -7 (x – 1)
y – 4 = -7x + 7
y + 7x = 11,

=> y + 3 = (-1/3) (x - 2)
3y + 9 = -x + 2
3y + x = - 7
x + 3y + 7 = 0 and

=> y + 2 = 3(x+1)
y + 2 = 3x + 3
y – 3x = 1
So, we get
y + 7x =11, x+ 3y + 7 =0 and y – 3x = 1
∴ The equation of sides are y + 7x =11, x+ 3y + 7 =0 and y – 3x = 1

(ii) (0, 1), (2, 0) and (-1, -2)
Given:
Points A (0, 1), B (2, 0) and C (-1, -2).
Let us assume,
m_1, m_2, and m_3 be the slope of the sides AB, BC and CA, respectively.
So,
The equation of the line passing through the two points (x_1, y_1) and (x_2, y_2).
Then,

$m_1 = \frac{0-1}{2-0}$,

$m_2 = \frac{-2-0}{-1-2}$,

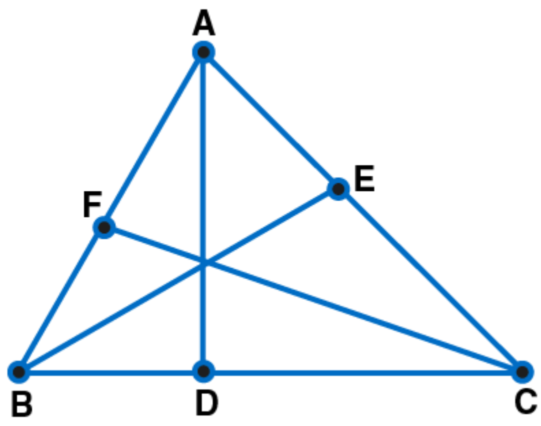
$m_3 = \frac{1+2}{1+0}$

m_1 = -1/2, m_2 = -2/3 and m_3= 3
So, the equation of the sides AB, BC and CA are
By using the formula,
$y – y_1 = m (x – x_1)$
=> y – 1 = (-1/2) (x - 0)
2y – 2 = -x
x + 2y = 2

=> y – 0 = (-2/3) (x - 2)
3y = -2x + 4
2x - 3y = 4

=> y + 2 = 3(x+1)
y + 2 = 3x + 3
y – 3x = 1
So, we get
x + 2y = 2, 2x – 3y =4 and y – 3x = 1
∴ The equation of sides are x + 2y = 2, 2x – 3y =4 and y – 3x = 1

3. Find the equations of the medians of a triangle, the coordinates of whose vertices are (-1, 6), (-3,-9) and (5, -8).
Solution:
Given:
A (−1, 6), B (−3, −9) and C (5, −8) be the coordinates of the given triangle.
Let us assume: D, E, and F be midpoints of BC, CA and AB, respectively. So, the coordinates of D, E and F are



Median AD passes through A (-1, 6) and D (1, -17/2)
So, by using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - 6 = \frac{-\frac{17}{2} - 6}{1 + 1}(x + 1)$$

4y – 24 = -29x – 29
29x + 4y + 5 = 0

Similarly, Median BE passes through B (-3,-9) and E (2,-1)
So, by using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y + 9 = \frac{-1+9}{2+3}(x + 3)$$

5y + 45 = 8x + 24
8x – 5y – 21=0

Similarly, Median CF passes through C (5,-8) and F(-2,-3/2)
So, by using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y + 9 = \frac{-\frac{3}{2}+8}{-2-5}(x - 5)$$

-14y – 112 = 13x – 65
13x + 14y + 47 = 0
∴ The equation of lines are: 29x + 4y + 5 = 0, 8x – 5y – 21=0 and 13x + 14y + 47 = 0

4. Find the equations to the diagonals of the rectangle the equations of whose sides are x = a, x = a', y = b and y = b'.

Solution:

Given:

The rectangle formed by the lines x = a, x = a', y = b and y = b'

It is clear that, the vertices of the rectangle are A (a, b), B (a', b), C (a', b') and D (a, b') .

The diagonal passing through A (a, b) and C (a', b') is

By using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - b = \frac{b' - b}{a' - a}(x - a)$$

(a' – a)y – b(a' – a) = (b' – b)x – a(b' – b)

(a' – a) – (b' – b)x = ba' – ab'

Similarly, the diagonal passing through B (a', b) and D (a, b') is

By using the formula,

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - b = \frac{b' - b}{a - a'}(x - a')$$

(a' – a)y – b(a – a') = (b' – b)x – a' (b' – b)

(a' – a) + (b' – b)x = a'b' – ab

∴ The equation of diagonals are y(a' – a) – x(b' – b) = a'b – ab' and y(a' – a) + x(b' – b) = a'b' – ab

5. Find the equation of the side BC of the triangle ABC whose vertices are A (-1, -2), B (0, 1) and C (2, 0) respectively. Also, find the equation of the median through A (-1, -2).

Solution:

Given:

The vertices of triangle ABC are A (-1, -2), B(0, 1) and C(2, 0).

Let us find the equation of median through A.

So, the equation of BC is

By using the formula,

$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$y - 1 = \frac{0-1}{2-0}(x - 0)$

$y - 1 = \frac{-1}{2}(x - 0)$

x + 2y – 2 = 0

Let D be the midpoint of median AD,

So, $D\left(\frac{0+2}{2}, \frac{1+0}{2}\right) = \left(1, \frac{1}{2}\right)$

The equation of the median AD is

By using the formula,

$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$y + 2 = \frac{\frac{1}{2} + 2}{1 + 1}(x + 1)$

4y + 8 = 5x + 5

5x – 4y – 3 = 0

∴ The equation of line BC is x + 2y – 2 = 0

The equation of median is 5x – 4y – 3 = 0

EXERCISE 23.6 PAGE NO: 23.46

1. Find the equation to the straight line
(i) cutting off intercepts 3 and 2 from the axes.
(ii) cutting off intercepts -5 and 6 from the axes.
Solution:
(i) Cutting off intercepts 3 and 2 from the axes.
Given:
a = 3, b = 2
Let us find the equation of line cutoff intercepts from the axes.
By using the formula,
The equation of the line is x/a + y/b = 1
x/3 + y/2 = 1
By taking LCM,
2x + 3y = 6
∴ The equation of line cut off intercepts 3 and 2 from the axes is 2x + 3y = 6

(ii) Cutting off intercepts -5 and 6 from the axes.
Given:
a = -5, b = 6
Let us find the equation of line cutoff intercepts from the axes.
By using the formula,
The equation of the line is x/a + y/b = 1
x/-5 + y/6 = 1
By taking LCM,
6x – 5y = -30
∴ The equation of line cut off intercepts 3 and 2 from the axes is 6x – 5y = -30

2. Find the equation of the straight line which passes through (1, -2) and cuts off equal intercepts on the axes.
Solution:
Given:
A line passing through (1, -2)
Let us assume, the equation of the line cutting equal intercepts at coordinates of length 'a' is
By using the formula,
The equation of the line is x/a + y/b = 1
x/a + y/a = 1
x + y = a

The line x + y = a passes through (1, -2)
Hence, the point satisfies the equation.
1 -2 = a
a = -1
∴ The equation of the line is x+ y = -1

3. Find the equation to the straight line which passes through the point (5, 6) and has intercepts on the axes
(i) Equal in magnitude and both positive
(ii) Equal in magnitude but opposite in sign
Solution:
(i) Equal in magnitude and both positive
Given:
a = b
Let us find the equation of line cutoff intercepts from the axes.
By using the formula,
The equation of the line is x/a + y/b = 1
x/a + y/a = 1
x + y = a
The line passes through the point (5, 6)
Hence, the equation satisfies the points.
5 + 6 = a
a = 11
∴ The equation of the line is x + y = 11

(ii) Equal in magnitude but opposite in sign
Given:
b = -a
Let us find the equation of line cutoff intercepts from the axes.
By using the formula,
The equation of the line is x/a + y/b = 1
x/a + y/-a = 1
x - y = a
The line passes through the point (5, 6)
Hence, the equation satisfies the points.
5 - 6 = a
a = -1
∴ The equation of the line is x - y = -1

4. For what values of a and b the intercepts cut off on the coordinate axes by the line ax + by + 8 = 0 are equal in length but opposite in signs to those cut off by the line 2x – 3y + 6 = 0 on the axes.

Solution:

Given:

Intercepts cut off on the coordinate axes by the line ax + by +8 = 0 …… (i)

And are equal in length but opposite in sign to those cut off by the line

2x – 3y +6 = 0 ……(ii)

We know that, the slope of two lines is equal

The slope of the line (i) is –a/b

The slope of the line (ii) is 2/3

So let us equate,

-a/b = 2/3

a = -2b/3

The length of the perpendicular from the origin to the line (i) is

By using the formula,

$$d = \left|\frac{ax+by+d}{\sqrt{a^2+b^2}}\right|$$

$$d_1 = \left|\frac{a(0)+b(0)+8}{\sqrt{a^2+b^2}}\right|$$

$$= \frac{8\times3}{\sqrt{13b^2}}$$

The length of the perpendicular from the origin to the line (ii) is

By using the formula,

$$d = \left|\frac{ax+by+d}{\sqrt{a^2+b^2}}\right|$$

$$d_2 = \left|\frac{2(0)-3(0)+6}{\sqrt{2^2+3^2}}\right|$$

It is given that, $d_1 = d_2$

$$\frac{8\times3}{\sqrt{13b^2}} = \frac{6}{\sqrt{13}}$$

b = 4

So, a = -2b/3

= -8/3

∴ The value of a is -8/3 and b is 4.

5. Find the equation to the straight line which cuts off equal positive intercepts on the axes and their product is 25.

Solution:

Given:

a = b and ab = 25

Let us find the equation of the line which cutoff intercepts on the axes.

∴ $a^2 = 25$

a = 5 [considering only positive value of intercepts]

By using the formula,

The equation of the line with intercepts a and b is x/a + y/b = 1

x/5 + y/5 = 1

By taking LCM

x + y = 5

∴ The equation of line is x + y = 5

EXERCISE 23.7 PAGE NO: 23.53

1. Find the equation of a line for which
(i) p = 5, α = 60°
(ii) p = 4, α = 150°
Solution:
(i) p = 5, α = 60°
Given:
p = 5, α = 60°
The equation of the line in normal form is given by
Using the formula,
$x \cos \alpha + y \sin \alpha = p$
Now, substitute the values, we get
$x \cos 60° + y \sin 60° = 5$
$x/2 + \sqrt{3}y/2 = 5$
$x + \sqrt{3}y = 10$
∴ The equation of line in normal form is $x + \sqrt{3}y = 10$.

(ii) p = 4, α = 150°
Given:
p = 4, α = 150°
The equation of the line in normal form is given by
Using the formula,
$x \cos \alpha + y \sin \alpha = p$
Now, substitute the values, we get
$x \cos 150° + y \sin 150° = 4$
$\cos (180° – \theta) = – \cos \theta$, $\sin (180° – \theta) = \sin \theta$
$x \cos(180° – 30°) + y \sin(180° – 30°) = 4$
$– x \cos 30° + y \sin 30° = 4$
$-\sqrt{3}x/2 + y/2 = 4$
$-\sqrt{3}x + y = 8$
∴ The equation of line in normal form is $-\sqrt{3}x + y = 8$.

2. Find the equation of the line on which the length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of x–axis is 30°.
Solution:
Given:
p = 4, α = 30°

The equation of the line in normal form is given by
Using the formula,
x cos α + y sin α = p
Now, substitute the values, we get
x cos 30° + y sin 30° = 4
x√3/2 + y1/2 = 4
√3x + y = 8
∴ The equation of line in normal form is √3x + y = 8.

3. Find the equation of the line whose perpendicular distance from the origin is 4 units and the angle which the normal makes with the positive direction of x–axis is 15°.

Solution:

Given:
p = 4, α = 15°
The equation of the line in normal form is given by
We know that, cos 15° = cos (45° – 30°) = cos45°cos30° + sin45°sin30°
Cos (A – B) = cos A cos B + sin A sin B
So,

$$\cos 15 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

And sin 15 = sin (45° – 30°) = sin 45° cos 30° – cos 45° sin 30°
Sin (A – B) = sin A cos B – cos A sin B
So,

$$\sin 15 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

Now, by using the formula,
x cos α + y sin α = p
Now, substitute the values, we get

$$\frac{\sqrt{3} + 1}{2\sqrt{2}} x + \frac{\sqrt{3} - 1}{2\sqrt{2}} y = 4$$

(√3+1)x +(√3-1) y = 8√2
∴ The equation of line in normal form is (√3+1)x +(√3-1) y = 8√2.

4. Find the equation of the straight line at a distance of 3 units from the origin such that the perpendicular from the origin to the line makes an angle α given by tan α =

5/12 with the positive direction of x–axis.

Solution:

Given:

$p = 3, \alpha = \tan^{-1} (5/12)$

So, $\tan \alpha = 5/12$

$\sin \alpha = 5/13$

$\cos \alpha = 12/13$

The equation of the line in normal form is given by

By using the formula,

$x \cos \alpha + y \sin \alpha = p$

Now, substitute the values, we get

$12x/13 + 5y/13 = 3$

$12x + 5y = 39$

∴ The equation of line in normal form is $12x + 5y = 39$.

5. Find the equation of the straight line on which the length of the perpendicular from the origin is 2 and the perpendicular makes an angle α with x–axis such that sin α = 1/3.

Solution:

Given:

$p = 2, \sin \alpha = 1/3$

We know that $\cos \alpha = \sqrt{(1 - \sin^2 \alpha)}$

$= \sqrt{(1 - 1/9)}$

$= 2\sqrt{2}/3$

The equation of the line in normal form is given by

By using the formula,

$x \cos \alpha + y \sin \alpha = p$

Now, substitute the values, we get

$x 2\sqrt{2}/3 + y/3 = 2$

$2\sqrt{2}x + y = 6$

∴ The equation of line in normal form is $2\sqrt{2}x + y = 6$.

EXERCISE 23.8 PAGE NO: 23.65

1. A line passes through a point A (1, 2) and makes an angle of 60^0 with the x–axis and intercepts the line x + y = 6 at the point P. Find AP.

Solution:

Given:

(x_1, y_1) = A (1, 2), θ = 60°

Let us find the distance AP.

By using the formula,

The equation of the line is given by:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

Now, substitute the values, we get

$$\frac{x-1}{\cos 60^\circ} = \frac{y-2}{\sin 60^\circ} = r$$

$$\frac{x-1}{\frac{1}{2}} = \frac{y-2}{\frac{\sqrt{3}}{2}} = r$$

Here, r represents the distance of any point on the line from point A (1, 2).

The coordinate of any point P on this line are (1 + r/2, 2 + √3r/2)

It is clear that, P lies on the line x + y = 6

So,

$$1+\frac{r}{2}+2+\frac{\sqrt{3}}{2}r = 6$$

$$\frac{\sqrt{3}}{2}r + \frac{r}{2} = 3$$

$$r(\sqrt{3}+1) = 6$$

$$r = \frac{6}{\sqrt{3}+1} = 3(\sqrt{3}-1)$$

∴ The value of AP is 3(√3 – 1)

2. If the straight line through the point P(3, 4) makes an angle π/6 with the x–axis and meets the line 12x + 5y + 10 = 0 at Q, find the length PQ.

Solution:

Given:

(x_1, y_1) = A (3, 4), θ = π/6 = 30°

Let us find the length PQ.

By using the formula,

The equation of the line is given by:

$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

Now, substitute the values, we get

$\frac{x-3}{\cos 30°} = \frac{y-4}{\sin 30°} = r$

$\frac{x-3}{\frac{\sqrt{3}}{2}} - \frac{y-4}{\frac{1}{2}} = r$

$x - \sqrt{3}y + 4\sqrt{3} - 3 = 0$

Let PQ = r

Then, the coordinate of Q are given by

$$\frac{x-3}{\cos 30°} = \frac{y-4}{\sin 30°} = r$$

$$x = 3 + \frac{\sqrt{3}}{2}r, y = 4 + \frac{r}{2}$$

The coordinate of point Q is $\left(3 + \frac{\sqrt{3}}{2}r, 4 + \frac{r}{2}\right)$

It is clear that, Q lies on the line 12x + 5y + 10 = 0

So,

$$12\left(3 + \frac{\sqrt{3}}{2}r\right) + 5\left(4 + \frac{r}{2}\right) + 10 = 0$$

$$66 + \frac{12\sqrt{3}+5}{2}r = 0$$

$$r = -\frac{132}{5+12\sqrt{3}}$$

$$PQ = |r| = \frac{132}{5+12\sqrt{3}}$$

∴ The value of PQ is $\frac{132}{5+12\sqrt{3}}$

3. A straight line drawn through the point A (2, 1) making an angle π/4 with positive x–axis intersects another line x + 2y + 1 = 0 in the point B. Find length AB.

Solution:

Given:

(x_1, y_1) = A (2, 1), θ = π/4 = 45°

Let us find the length AB.

By using the formula,

The equation of the line is given by:

$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

Now, substitute the values, we get

$\frac{x-2}{\cos 45°} = \frac{y-1}{\sin 45°} = r$

$\frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}} = r$

x – y – 1 = 0

Let AB = r

Then, the coordinate of B is given by

$\frac{x-2}{\cos 45°} = \frac{y-1}{\sin 45°} = r$

$x = 2 + \frac{1}{\sqrt{2}}r, y = 1 + \frac{r}{\sqrt{2}}$

The coordinate of point B is $\left(2 + \frac{1}{\sqrt{2}}r, 1 + \frac{r}{\sqrt{2}}\right)$

It is clear that, B lies on the line x + 2y + 1 = 0

$2 + \frac{1}{\sqrt{2}}r + 2\left(1 + \frac{r}{\sqrt{2}}\right) + 1 = 0$

$5 + \frac{3r}{\sqrt{2}}r = 0$

$r = \frac{5\sqrt{2}}{3}$

∴ The value of AB is $\frac{5\sqrt{2}}{3}$

4. A line a drawn through A (4, – 1) parallel to the line 3x – 4y + 1 = 0. Find the coordinates of the two points on this line which are at a distance of 5 units from A.

Solution:

Given:

(x_1, y_1) = A (4, -1)

Let us find Coordinates of the two points on this line which are at a distance of 5 units from A.

Given: Line 3x – 4y + 1 = 0

4y = 3x + 1

y = 3x/4 + 1/4

Slope tan θ = 3/4

So,

Sin θ = 3/5

Cos θ = 4/5

The equation of the line passing through A (4, −1) and having slope ¾ is
By using the formula,
The equation of the line is given by:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$$

Now, substitute the values, we get

$$\frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}}$$

3x – 4y = 16

Here, AP = r = ± 5
Thus, the coordinates of P are given by

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

$$\frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}} = r$$

$$x = \frac{4r}{5} + 4 \text{ and } y = \frac{3r}{5} - 1$$

$$x = \frac{4(\pm 5)}{5} + 4 \text{ and } y = \frac{3(\pm 5)}{5} - 1$$

x = ±4 + 4 and y = ±3 –1
x = 8, 0 and y = 2, – 4
∴ The coordinates of the two points at a distance of 5 units from A are (8, 2) and (0, −4).

5. The straight line through P(x_1, y_1) inclined at an angle θ with the x–axis meets the line ax + by + c = 0 in Q. Find the length of PQ.
Solution:
Given:
The equation of the line that passes through P(x_1, y_1) and makes an angle of θ with the x–axis.
Let us find the length of PQ.
We know that,

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$$

Let PQ = r
Then, the coordinates of Q are given by
By using the formula,
The equation of the line is given by:

$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

$x = x_1 + r\cos\theta, y = y_1 + r\sin\theta$

Thus, the coordinates of Q are $(x_1 + r\cos\theta, y_1 + r\sin\theta)$

It is clear that, Q lies on the line $ax + by + c = 0$.

So,

$a(x_1 + r\cos\theta) + b(y_1 + r\sin\theta) + c = 0$

$r = PQ = \left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$

∴ The value of PQ is $\left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$

EXERCISE 23.9 PAGE NO: 23.72

1. Reduce the equation √3x + y + 2 = 0 to:
(i) slope - intercept form and find slope and y - intercept;
(ii) Intercept form and find intercept on the axes
(iii) The normal form and find p and α.
Solution:
(i) Given:
√3x + y + 2 = 0
y = - √3x – 2
This is the slope intercept form of the given line.
∴ The slope = - √3 and y - intercept = -2

(ii) Given:
√3x + y + 2 = 0
√3x + y = -2
Divide both sides by -2, we get
√3x/-2 + y/-2 = 1
∴ The intercept form of the given line. Here, x - intercept = - 2/√3 and y - intercept = -2

(iii) Given:
√3x + y + 2 = 0
-√3x - y = 2

$$-\frac{\sqrt{3}x}{\sqrt{(-\sqrt{3})^2+(-1)^2}}-\frac{y}{\sqrt{(-\sqrt{3})^2+(-1)^2}}=\frac{2}{\sqrt{(-\sqrt{3})^2+(-1)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$-\frac{\sqrt{3}x}{2}-\frac{y}{2}=1$$

This is the normal form of the given line.
So, p = 1 cos α = -√3/2 and sin α = - 1/2
∴ p = 1 and α = 210

2. Reduce the following equations to the normal form and find p and α in each case:
(i) x + √3y – 4 = 0
(ii) x + y + √2 = 0
Solution:
(i) x + √3y – 4 = 0

$x + \sqrt{3}y = 4$

$$\frac{x}{\sqrt{1^2 + (\sqrt{3})^2}} + \frac{\sqrt{3}y}{\sqrt{1^2 + (\sqrt{3})^2}} = \frac{4}{\sqrt{1^2 + (\sqrt{3})^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$$\frac{x}{2} + \frac{\sqrt{3}y}{2} = 2$$

The normal form of the given line, where p = 2, cos α = 1/2 and sin α = √3/2

∴ p = 2 and α = π/3

(ii) $x + y + \sqrt{2} = 0$

$-x - y = \sqrt{2}$

$$\frac{-x}{\sqrt{(-1)^2 + (-1)^2}} + \frac{y}{\sqrt{(-1)^2 + (-1)^2}} = \frac{\sqrt{2}}{\sqrt{(-1)^2 + (-1)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{coefficient of } y)^2}$

$$-\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 1$$

The normal form of the given line, where p = 1, cos α = -1/√2 and sin α = -1/√2

∴ p = 1 and α = 225^o

3. Put the equation x/a + y/b = 1 the slope intercept form and find its slope and y - intercept.

Solution:

Given: the equation is x/a + y/b = 1

We know that,

General equation of line y = mx + c.

bx + ay = ab

ay = - bx + ab

y = -bx/a + b

The slope intercept form of the given line.

∴ Slope = - b/a and y - intercept = b

4. Reduce the lines 3x – 4y + 4 = 0 and 2x + 4y – 5 = 0 to the normal form and hence find which line is nearer to the origin.

Solution:

Given:

The normal forms of the lines 3x − 4y + 4 = 0 and 2x + 4y − 5 = 0.
Let us find, in given normal form of a line, which is nearer to the origin.
-3x + 4y = 4

$$-\frac{3x}{\sqrt{(-3)^2 + (4)^2}} + 4\frac{y}{\sqrt{(-3)^2 + (4)^2}} = \frac{4}{\sqrt{(-3)^2 + (4)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$-\frac{3}{5}x + \frac{4}{5}y = \frac{4}{5} \;\ldots\ldots (1)$$

Now 2x + 4y = - 5
-2x – 4y = 5

$$-\frac{2x}{\sqrt{(-2)^2 + (-4)^2}} - 4\frac{y}{\sqrt{(-2)^2 + (-4)^2}} = \frac{5}{\sqrt{(-2)^2 + (-4)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$-\frac{2}{2\sqrt{5}}x - \frac{4}{2\sqrt{5}}y = \frac{5}{2\sqrt{5}} \;\ldots\ldots (2)$$

From equations (1) and (2):
45 < 525
∴ The line 3x − 4y + 4 = 0 is nearer to the origin.

5. Show that the origin is equidistant from the lines 4x + 3y + 10 = 0; 5x – 12y + 26 = 0 and 7x + 24y = 50.

Solution:

Given:
The lines 4x + 3y + 10 = 0; 5x – 12y + 26 = 0 and 7x + 24y = 50.
We need to prove that, the origin is equidistant from the lines 4x + 3y + 10 = 0; 5x – 12y + 26 = 0 and 7x + 24y = 50.
Let us write down the normal forms of the given lines.
First line: 4x + 3y + 10 = 0
-4x - 3y = 10

$$-\frac{4x}{\sqrt{(-4)^2 + (-3)^2}} - 3\frac{y}{\sqrt{(-4)^2 + (-3)^2}} = \frac{10}{\sqrt{(-4)^2 + (-3)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$-\frac{4}{5}x - \frac{3}{5}y = 2$$

So, p = 2

Second line: 5x − 12y + 26 = 0
-5x + 12y = 26

$$-\frac{5\,x}{\sqrt{(-5)^2 + (12)^2}} + 12\frac{y}{\sqrt{(-5)^2 + (12)^2}} = \frac{26}{\sqrt{(-5)^2 + (12)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$-\frac{5}{13}x + \frac{12}{13}y = 2$$

So, p = 2

Third line: 7x + 24y = 50

$$\frac{7\,x}{\sqrt{(7)^2 + (24)^2}} + 24\frac{y}{\sqrt{(7)^2 + (24)^2}} = \frac{50}{\sqrt{(7)^2 + (24)^2}}$$

Divide both sides by $\sqrt{(\text{coefficient of x})^2 + (\text{coefficient of y})^2}$

$$\frac{7}{25}x + \frac{24}{25}y = 2$$

So, p = 2
∴ The origin is equidistant from the given lines.

EXERCISE 23.10 PAGE NO: 23.77

1. Find the point of intersection of the following pairs of lines:
(i) 2x – y + 3 = 0 and x + y – 5 = 0
(ii) bx + ay = ab and ax + by = ab
Solution:
(i) 2x – y + 3 = 0 and x + y – 5 = 0
Given:
The equations of the lines are as follows:
2x − y + 3 = 0 … (1)
x + y − 5 = 0 … (2)
Let us find the point of intersection of pair of lines.
By solving (1) and (2) using cross - multiplication method, we get

$$\frac{x}{5-3} = \frac{y}{3+10} = \frac{1}{2+1}$$

$$\frac{x}{2} = \frac{y}{13} = \frac{1}{3}$$

x = 2/3 and y = 13/3
∴ The point of intersection is (2/3, 13/3)

(ii) bx + ay = ab and ax + by = ab
Given:
The equations of the lines are as follows:
bx + ay − ab = 0… (1)
ax + by = ab ⇒ ax + by − ab = 0 … (2)
Let us find the point of intersection of pair of lines.
By solving (1) and (2) using cross - multiplication method, we get

$$\frac{x}{-a^2b + ab^2} = \frac{y}{-a^2b + ab^2} = \frac{1}{b^2 - a^2}$$

$$\frac{x}{ab(b-a)} = \frac{y}{ab(b-a)} = \frac{1}{(a+b)(b-a)}$$

$x = \frac{ab}{a+b}$ and $y = \frac{ab}{a+b}$

∴ The point of intersection is (ab/a+b, ab/a+b)

2. Find the coordinates of the vertices of a triangle, the equations of whose sides are:
(i) x + y – 4 = 0, 2x – y + 3 0 and x – 3y + 2 = 0
(ii) $y(t_1 + t_2) = 2x + 2at_1t_2$, $y(t_2 + t_3) = 2x + 2at_2t_3$ and, $y(t_3 + t_1) = 2x + 2at_1t_3$.
Solution:

(i) x + y – 4 = 0, 2x – y + 3 0 and x – 3y + 2 = 0
Given:
x + y − 4 = 0, 2x − y + 3 = 0 and x − 3y + 2 = 0
Let us find the point of intersection of pair of lines.
x + y − 4 = 0 … (1)
2x − y + 3 = 0 … (2)
x − 3y + 2 = 0 … (3)
By solving (1) and (2) using cross - multiplication method, we get

$$\frac{x}{3-4} = \frac{y}{-8-3} = \frac{1}{-1-2}$$

x = 1/3, y = 11/3

Solving (1) and (3) using cross - multiplication method, we get

$$\frac{x}{2-12} = \frac{y}{-4-2} = \frac{1}{-3-1}$$

x = 5/2, y = 3/2

Similarly, solving (2) and (3) using cross - multiplication method, we get

$$\frac{x}{-2+9} = \frac{y}{3-4} = \frac{1}{-6+1}$$

x = - 7/5, y = 1/5
∴ The coordinates of the vertices of the triangle are (1/3, 11/3), (5/2, 3/2) and (-7/5, 1/5)

(ii) $y(t_1 + t_2) = 2x + 2at_1t_2$, $y(t_2 + t_3) = 2x + 2at_2t_3$ and, $y(t_3 + t_1) = 2x + 2at_1t_3$.
Given:
$y(t_1 + t_2) = 2x + 2a\, t_1t_2$, $y(t_2 + t_3) = 2x + 2a\, t_2t_3$ and $y(t_3 + t_1) = 2x + 2a\, t_1t_3$
Let us find the point of intersection of pair of lines.
$2x - y(t_1 + t_2) + 2a\, t_1t_2 = 0$ … (1)
$2x - y(t_2 + t_3) + 2a\, t_2t_3 = 0$ … (2)
$2x - y(t_3 + t_1) + 2a\, t_1t_3 = 0$ … (3)
By solving (1) and (2) using cross - multiplication method, we get

$$\frac{x}{-(t_1 + t_2) \times 2at_2t_3 + (t_2 + t_3)2at_1t_2} = \frac{-y}{4at_2t_3 - 4at_1t_2}$$
$$= \frac{1}{-2(t_2 + t_3) + 2(t_1 + t_2)}$$
$$x = \frac{-(t_1 + t_2) \times 2at_2t_3 + (t_2 + t_3)2at_1t_2}{-2(t_2 + t_3) + 2(t_1 + t_2)} = at_2^2$$
$$y = -\frac{4at_2t_3 - 4at_1t_2}{-2(t_2 + t_3) + 2(t_1 + t_2)} = 2at_2$$

Solving (1) and (3) using cross - multiplication method, we get

$$\frac{x}{-(t_1 + t_2) \times 2at_1t_3 + (t_3 + t_1)2at_1t_2} = \frac{-y}{4at_1t_3 - 4at_1t_2}$$
$$= \frac{1}{-2(t_3 + t_1) + 2(t_1 + t_2)}$$
$$x = \frac{-(t_1 + t_2) \times 2at_1t_3 + (t_3 + t_1)2at_1t_2}{-2(t_3 + t_1) + 2(t_1 + t_2)} = at_1^2$$
$$y = -\frac{4at_1t_3 - 4at_1t_2}{-2(t_3 + t_1) + 2(t_1 + t_2)} = 2at_1$$

Solving (2) and (3) using cross - multiplication method, we get

$$\frac{x}{-(t_2 + t_3) \times 2at_1t_3 + (t_3 + t_1)2at_2t_3} = \frac{-y}{4at_1t_3 - 4at_2t_3}$$
$$= \frac{1}{-2(t_3 + t_1) + 2(t_2 + t_3)}$$
$$x = \frac{-(t_2 + t_3) \times 2at_1t_3 + (t_3 + t_1)2at_2t_3}{-2(t_3 + t_1) + 2(t_2 + t_3)} = at_3^2$$
$$y = -\frac{4at_1t_3 - 4at_2t_3}{-2(t_3 + t_1) + 2(t_2 + t_3)} = 2at_3$$

∴ The coordinates of the vertices of the triangle are $(at^2_1, 2at_1)$, $(at^2_2, 2at_2)$ and $(at^2_3, 2at_3)$.

3. Find the area of the triangle formed by the lines
$y = m_1x + c_1$, $y = m_2x + c_2$ and $x = 0$

Solution:

Given:

$y = m_1x + c_1$ … (1)

$y = m_2x + c_2$ … (2)

x = 0 … (3)
In triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.
Solving (1) and (2), we get

$$x = \frac{c_2 - c_1}{m_1 - m_2}, y = \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2}$$

Thus, AB and BC intersect at B $\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2}\right)$
Solving (1) and (3):
$x = 0, y = c_1$
Thus, AB and CA intersect at A $0, c_1$.

Similarly, solving (2) and (3):
$x = 0, y = c_2$
Thus, BC and CA intersect at C $0, c_2$.

$$\therefore \text{Area of triangle ABC} = \frac{1}{2}\begin{vmatrix} 0 & c_1 & 1 \\ 0 & c_2 & 1 \\ \frac{c_2 - c_1}{m_1 - m_2} & \frac{m_1 c_2 - m_2 c_1}{m_1 - m_2} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left(\frac{c_2 - c_1}{m_1 - m_2}\right)(c_1 - c_2)$$

$$= \frac{\frac{1}{2}(c_1 - c_2)^2}{m_2 - m_1}$$

4. Find the equations of the medians of a triangle, the equations of whose sides are: 3x + 2y + 6 = 0, 2x – 5y + 4 = 0 and x – 3y – 6 = 0

Solution:

Given:
3x + 2y + 6 = 0 … (1)
2x − 5y + 4 = 0 … (2)
x − 3y − 6 = 0 … (3)
Let us assume, in triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.
Solving equations (1) and (2), we get
x = −2, y = 0
Thus, AB and BC intersect at B (−2, 0).

Now, solving (1) and (3), we get
x = - 6/11, y = - 24/11
Thus, AB and CA intersect at A (-6/11, -24/11)

Similarly, solving (2) and (3), we get
x = −42, y = −16
Thus, BC and CA intersect at C (−42, −16).

Now, let D, E and F be the midpoints the sides BC, CA and AB, respectively.
Then, we have:

$$D = \left(\frac{-2-42}{2}, \frac{0-16}{2}\right) = (-22, -8)$$

$$E = \left(\frac{-\frac{6}{11}-42}{2}, \frac{-\frac{24}{11}-16}{2}\right) = \left(-\frac{234}{11}, -\frac{100}{11}\right)$$

$$F = \left(\frac{-\frac{6}{11}-2}{2}, \frac{-\frac{24}{11}+0}{2}\right) = \left(-\frac{14}{11}, -\frac{12}{11}\right)$$

Now, the equation of the median AD is

$$y + \frac{24}{11} = \frac{-8 + \frac{24}{11}}{-22 + \frac{6}{11}}\left(x + \frac{6}{11}\right)$$

$16x - 59y - 120 = 0$

The equation of median CF is

$$y + 16 = \frac{-\frac{12}{11} + 16}{-\frac{14}{11} + 42}(x + 42)$$

$41x - 112y - 70 = 0$

And, the equation of the median BE is

$$y - 0 = \frac{-\frac{100}{11} - 0}{-\frac{234}{11} + 2}(x + 2)$$

$25x - 53y + 50 = 0$

∴ The equations of the medians of a triangle are: 41x – 112y – 70 = 0, 16x – 59y – 120 = 0, 25x – 53y + 50 = 0

5. Prove that the lines y = √3x + 1, y = 4 and y = -√3x + 2 form an equilateral triangle.

Solution:

Given:

y = √3x + 1…… (1)

y = 4 ……. (2)

y = - √3x + 2……. (3)

Let us assume in triangle ABC, let equations (1), (2) and (3) represent the sides AB, BC and CA, respectively.

By solving equations (1) and (2), we get

x = √3, y = 4

Thus, AB and BC intersect at B(√3,4)

Now, solving equations (1) and (3), we get

x = 1/2√3, y = 3/2

Thus, AB and CA intersect at A (1/2√3, 3/2)

Similarly, solving equations (2) and (3), we get

x = -2/√3, y = 4

Thus, BC and AC intersect at C (-2/√3,4)

Now, we have:

$$AB = \sqrt{\left(\frac{1}{2\sqrt{3}} - \sqrt{3}\right)^2 + \left(\frac{3}{2} - 4\right)^2} = \frac{5}{\sqrt{3}}$$

$$BC = \sqrt{\left(\frac{1}{2\sqrt{3}} + \frac{2}{\sqrt{3}}\right)^2 + \left(\frac{3}{2} - 4\right)^2} = \frac{5}{\sqrt{3}}$$

$$AC = \sqrt{\left(\frac{1}{2\sqrt{3}} + \frac{2}{\sqrt{3}}\right)^2 + \left(\frac{3}{2} - 4\right)^2} = \frac{5}{\sqrt{3}}$$

Hence proved, the given lines form an equilateral triangle.

EXERCISE 23.11 PAGE NO: 23.83

1. Prove that the following sets of three lines are concurrent:
(i) 15x – 18y + 1 = 0, 12x + 10y – 3 = 0 and 6x + 66y – 11 = 0
(ii) 3x – 5y – 11 = 0, 5x + 3y – 7 = 0 and x + 2y = 0
Solution:
(i) 15x – 18y + 1 = 0, 12x + 10y – 3 = 0 and 6x + 66y – 11 = 0
Given:
15x – 18y + 1 = 0 …… (i)
12x + 10y – 3 = 0 …… (ii)
6x + 66y – 11 = 0 …… (iii)
Now, consider the following determinant:

$$\begin{vmatrix} 15 & -18 & 1 \\ 12 & 19 & -3 \\ 6 & 66 & -11 \end{vmatrix} = 15(-110+198)+18(-132+18)+1(792-60)$$

=> 1320 – 2052 + 732 = 0
Hence proved, the given lines are concurrent.

(ii) 3x – 5y – 11 = 0, 5x + 3y – 7 = 0 and x + 2y = 0
Given:
3x − 5y − 11 = 0 …… (i)
5x + 3y − 7 = 0 …… (ii)
x + 2y = 0 …… (iii)
Now, consider the following determinant:

$$\begin{vmatrix} 3 & -5 & -11 \\ 5 & 3 & -7 \\ 1 & 2 & 0 \end{vmatrix} = 3 \times 14 + 5 \times 7 - 11 \times 7 = 0$$

Hence, the given lines are concurrent.

2. For what value of λ are the three lines 2x – 5y + 3 = 0, 5x – 9y + λ = 0 and x – 2y + 1 = 0 concurrent?
Solution:
Given:
2x − 5y + 3 = 0 … (1)
5x − 9y + λ = 0 … (2)
x − 2y + 1 = 0 … (3)
It is given that the three lines are concurrent.
Now, consider the following determinant:

$$\therefore \begin{vmatrix} 2 & -5 & 3 \\ 5 & -9 & \lambda \\ 1 & -2 & 1 \end{vmatrix} = 0$$

$2(-9 + 2\lambda) + 5(5 - \lambda) + 3(-10 + 9) = 0$
$-18 + 4\lambda + 25 - 5\lambda - 3 = 0$
$\lambda = 4$
∴ The value of λ is 4.

3. Find the conditions that the straight lines $y = m_1x + c_1$, $y = m_2x + c_2$ and $y = m_3x + c_3$ may meet in a point.
Solution:
Given:
$m_1x - y + c_1 = 0$ … (1)
$m_2x - y + c_2 = 0$ … (2)
$m_3x - y + c_3 = 0$ … (3)
It is given that the three lines are concurrent.
Now, consider the following determinant:

$$\therefore \begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix} = 0$$

$m_1(-c_3 + c_2) + 1(m_2c_3 - m_3c_2) + c_1(-m_2 + m_3) = 0$
$m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$
∴ The required condition is $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$

4. If the lines $p_1x + q_1y = 1$, $p_2x + q_2y = 1$ and $p_3x + q_3y = 1$ be concurrent, show that the points (p_1, q_1), (p_2, q_2) and (p_3, q_3) are collinear.
Solution:
Given:
$p_1x + q_1y = 1$
$p_2x + q_2y = 1$
$p_3x + q_3y = 1$
The given lines can be written as follows:
$p_1 x + q_1 y - 1 = 0$ … (1)
$p_2 x + q_2 y - 1 = 0$ … (2)
$p_3 x + q_3 y - 1 = 0$ … (3)
It is given that the three lines are concurrent.
Now, consider the following determinant:

$$\begin{vmatrix} p_1 & q_1 & -1 \\ p_2 & q_2 & -1 \\ p_3 & q_3 & -1 \end{vmatrix} = 0$$

$$-\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} p_1 & q_1 & 1 \\ p_2 & q_2 & 1 \\ p_3 & q_3 & 1 \end{vmatrix} = 0$$

Hence proved, the given three points, (p_1, q_1), (p_2, q_2) and (p_3, q_3) are collinear.

5. Show that the straight lines $L_1 = (b + c)x + ay + 1 = 0$, $L_2 = (c + a)x + by + 1 = 0$ and $L_3 = (a + b)x + cy + 1 = 0$ are concurrent.

Solution:

Given:

$L_1 = (b + c)x + ay + 1 = 0$

$L_2 = (c + a)x + by + 1 = 0$

$L_3 = (a + b)x + cy + 1 = 0$

The given lines can be written as follows:

$(b + c) x + ay + 1 = 0$ … (1)

$(c + a) x + by + 1 = 0$ … (2)

$(a + b) x + cy + 1 = 0$ … (3)

Consider the following determinant.

$$\begin{vmatrix} b + c & a & 1 \\ c + a & b & 1 \\ a + b & c & 1 \end{vmatrix}$$

Let us apply the transformation $C_1 \to C_1 + C_2$, we get

$$\begin{vmatrix} b + c & a & 1 \\ c + a & b & 1 \\ a + b & c & 1 \end{vmatrix} = \begin{vmatrix} a + b + c & a & 1 \\ c + a + b & b & 1 \\ a + b + c & c & 1 \end{vmatrix}$$

$$\begin{vmatrix} b + c & a & 1 \\ c + a & b & 1 \\ a + b & c & 1 \end{vmatrix} = (a + b + c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$$\begin{vmatrix} b + c & a & 1 \\ c + a & b & 1 \\ a + b & c & 1 \end{vmatrix} = 0$$

Hence proved, the given lines are concurrent.

EXERCISE 23.12 PAGE NO: 23.92

1. Find the equation of a line passing through the point (2, 3) and parallel to the line 3x– 4y + 5 = 0.

Solution:

Given:

The equation is parallel to $3x - 4y + 5 = 0$ and pass through (2, 3)

The equation of the line parallel to $3x - 4y + 5 = 0$ is

$3x - 4y + \lambda = 0$,

Where, λ is a constant.

It passes through (2, 3).

Substitute the values in above equation, we get

$3(2) - 4(3) + \lambda = 0$

$6 - 12 + \lambda = 0$

$\lambda = 6$

Now, substitute the value of $\lambda = 6$ in $3x - 4y + \lambda = 0$, we get

$3x - 4y + 6$

∴ The required line is $3x - 4y + 6 = 0$.

2. Find the equation of a line passing through (3, -2) and perpendicular to the line x – 3y + 5 = 0.

Solution:

Given:

The equation is perpendicular to $x - 3y + 5 = 0$ and passes through (3,-2)

The equation of the line perpendicular to $x - 3y + 5 = 0$ is

$3x + y + \lambda = 0$,

Where, λ is a constant.

It passes through (3, − 2).

Substitute the values in above equation, we get

$3(3) + (-2) + \lambda = 0$

$9 - 2 + \lambda = 0$

$\lambda = -7$

Now, substitute the value of $\lambda = -7$ in $3x + y + \lambda = 0$, we get

$3x + y - 7 = 0$

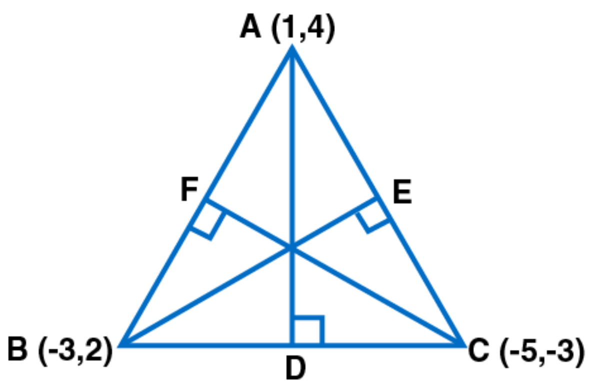
∴ The required line is $3x + y - 7 = 0$.

3. Find the equation of the perpendicular bisector of the line joining the points (1, 3) and (3, 1).

Solution:

Given:
A (1, 3) and B (3, 1) be the points joining the perpendicular bisector
Let C be the midpoint of AB.
So, coordinates of C = [(1+3)/2, (3+1)/2]
= (2, 2)
Slope of AB = [(1-3) / (3-1)]
= -1
Slope of the perpendicular bisector of AB = 1
Thus, the equation of the perpendicular bisector of AB is given as,
y – 2 = 1(x – 2)
y = x
x – y = 0
∴ The required equation is y = x.

4. Find the equations of the altitudes of a ΔABC whose vertices are A (1, 4), B (-3, 2) and C (-5, -3).
Solution:
Given:
The vertices of ΔABC are A (1, 4), B (− 3, 2) and C (− 5, − 3).
Now let us find the slopes of ΔABC.



Slope of AB = [(2 - 4) / (-3-1)]
= ½

Slope of BC = [(-3 - 2) / (-5+3)]
= 5/2

Slope of CA = [(4 + 3) / (1 + 5)]
= 7/6
Thus, we have:

Slope of CF = -2
Slope of AD = -2/5
Slope of BE = -6/7
Hence,
Equation of CF is:
$y + 3 = -2(x + 5)$
$y + 3 = -2x - 10$
$2x + y + 13 = 0$

Equation of AD is:
$y – 4 = (-2/5)(x - 1)$
$5y – 20 = -2x + 2$
$2x + 5y – 22 = 0$

Equation of BE is:
$y – 2 = (-6/7)(x + 3)$
$7y – 14 = -6x - 18$
$6x + 7y + 4 = 0$
∴ The required equations are $2x + y + 13 = 0$, $2x + 5y – 22 = 0$, $6x + 7y + 4 = 0$.

5. Find the equation of a line which is perpendicular to the line $\sqrt{3}x – y + 5 = 0$ and which cuts off an intercept of 4 units with the negative direction of y-axis.
Solution:
Given:
The equation is perpendicular to $\sqrt{3}x – y + 5 = 0$ equation and cuts off an intercept of 4 units with the negative direction of y-axis.
The line perpendicular to $\sqrt{3}x – y + 5 = 0$ is $x + \sqrt{3}y + \lambda = 0$

It is given that the line $x + \sqrt{3}y + \lambda = 0$ cuts off an intercept of 4 units with the negative direction of the y-axis.
This means that the line passes through (0,-4).
So,
Let us substitute the values in the equation $x + \sqrt{3}y + \lambda = 0$, we get
$0 - \sqrt{3}(4) + \lambda = 0$
$\lambda = 4\sqrt{3}$
Now, substitute the value of λ back, we get
$x + \sqrt{3}y + 4\sqrt{3} = 0$
∴ The required equation of line is $x + \sqrt{3}y + 4\sqrt{3} = 0$.

EXERCISE 23.13

1. Find the angles between each of the following pairs of straight lines:
(i) 3x + y + 12 = 0 and x + 2y – 1 = 0
(ii) 3x – y + 5 = 0 and x – 3y + 1 = 0
Solution:
(i) 3x + y + 12 = 0 and x + 2y – 1 = 0
Given:
The equations of the lines are
3x + y + 12 = 0 … (1)
x + 2y − 1 = 0 … (2)
Let m_1 and m_2 be the slopes of these lines.
$m_1 = -3$, $m_2 = -1/2$
Let θ be the angle between the lines.
Then, by using the formula
$\tan\theta = [(m_1 – m_2) / (1 + m_1m_2)]$
$= [(-3 + 1/2) / (1 + 3/2)]$
$= 1$
So,
$\theta = \pi/4$ or 45^o
∴ The acute angle between the lines is 45°

(ii) 3x – y + 5 = 0 and x – 3y + 1 = 0
Given:
The equations of the lines are
3x − y + 5 = 0 … (1)
x − 3y + 1 = 0 … (2)
Let m_1 and m_2 be the slopes of these lines.
$m_1 = 3$, $m_2 = 1/3$
Let θ be the angle between the lines.
Then, by using the formula
$\tan\theta = [(m_1 – m_2) / (1 + m_1m_2)]$
$= [(3 + 1/3) / (1 + 1)]$
$= 4/3$
So,
$\theta = \tan^{-1} (4/3)$
∴ The acute angle between the lines is $\tan^{-1} (4/3)$.

2. Find the acute angle between the lines 2x – y + 3 = 0 and x + y + 2 = 0.
Solution:
Given:
The equations of the lines are
$2x - y + 3 = 0$ … (1)
$x + y + 2 = 0$ … (2)
Let m_1 and m_2 be the slopes of these lines.
$m_1 = 2, m_2 = -1$
Let θ be the angle between the lines.
Then, by using the formula
$\tan \theta = [(m_1 - m_2) / (1 + m_1 m_2)]$
$= [(2 + 1) / (1 + 2)]$
$= 3$
So,
$\theta = \tan^{-1} (3)$
∴ The acute angle between the lines is $\tan^{-1} (3)$.

3. Prove that the points (2, -1), (0, 2), (2, 3) and (4, 0) are the coordinates of the vertices of a parallelogram and find the angle between its diagonals.
Solution:
To prove:
The points (2, -1), (0, 2), (2, 3) and (4, 0) are the coordinates of the vertices of a parallelogram
Let us assume the points, A (2, − 1), B (0, 2), C (2, 3) and D (4, 0) be the vertices.
Now, let us find the slopes

Slope of AB = [(2+1) / (0-2)]
= -3/2

Slope of BC = [(3-2) / (2-0)]
= ½

Slope of CD = [(0-3) / (4-2)]
= -3/2

Slope of DA = [(-1-0) / (2-4)]
= ½
Thus, AB is parallel to CD and BC is parallel to DA.
Hence proved, the given points are the vertices of a parallelogram.

Now, let us find the angle between the diagonals AC and BD.
Let m_1 and m_2 be the slopes of AC and BD, respectively.
$m_1 = [(3+1) / (2-2)]$
$= \infty$

$m_2 = [(0-2) / (4-0)]$
$= -1/2$

Thus, the diagonal AC is parallel to the y-axis.
$\angle ODB = \tan^{-1} (1/2)$

In triangle MND,
$\angle DMN = \pi/2 - \tan^{-1} (1/2)$
∴ The angle between the diagonals is $\pi/2 - \tan^{-1} (1/2)$.

4. Find the angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1.

Solution:

Given:
Points (2, 0), (0, 3) and the line $x + y = 1$.
Let us assume A (2, 0), B (0, 3) be the given points.
Now, let us find the slopes
Slope of AB = m_1
$= [(3-0) / (0-2)]$
$= -3/2$

Slope of the line $x + y = 1$ is -1
∴ $m_2 = -1$

Let θ be the angle between the line joining the points (2, 0), (0, 3) and the line x + y =
$\tan \theta = |[(m_1 - m_2) / (1 + m_1m_2)]|$
$= [(-3/2 + 1) / (1 + 3/2)]$
$= 1/5$
$\theta = \tan^{-1} (1/5)$
∴ The acute angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1 is $\tan^{-1} (1/5)$.

EXERCISE 23.13

1. Find the angles between each of the following pairs of straight lines:
(i) 3x + y + 12 = 0 and x + 2y – 1 = 0
(ii) 3x – y + 5 = 0 and x – 3y + 1 = 0
Solution:
(i) 3x + y + 12 = 0 and x + 2y – 1 = 0
Given:
The equations of the lines are
3x + y + 12 = 0 … (1)
x + 2y − 1 = 0 … (2)
Let m_1 and m_2 be the slopes of these lines.
$m_1 = -3$, $m_2 = -1/2$
Let θ be the angle between the lines.
Then, by using the formula
$\tan\theta = [(m_1 – m_2) / (1 + m_1m_2)]$
$= [(-3 + 1/2) / (1 + 3/2)]$
$= 1$
So,
$\theta = \pi/4$ or 45^o
∴ The acute angle between the lines is 45°

(ii) 3x – y + 5 = 0 and x – 3y + 1 = 0
Given:
The equations of the lines are
3x − y + 5 = 0 … (1)
x − 3y + 1 = 0 … (2)
Let m_1 and m_2 be the slopes of these lines.
$m_1 = 3$, $m_2 = 1/3$
Let θ be the angle between the lines.
Then, by using the formula
$\tan\theta = [(m_1 – m_2) / (1 + m_1m_2)]$
$= [(3 + 1/3) / (1 + 1)]$
$= 4/3$
So,
$\theta = \tan^{-1}(4/3)$
∴ The acute angle between the lines is $\tan^{-1}(4/3)$.

2. Find the acute angle between the lines 2x – y + 3 = 0 and x + y + 2 = 0.

Solution:

Given:

The equations of the lines are

$2x - y + 3 = 0$ … (1)

$x + y + 2 = 0$ … (2)

Let m_1 and m_2 be the slopes of these lines.

$m_1 = 2$, $m_2 = -1$

Let θ be the angle between the lines.

Then, by using the formula

$\tan \theta = [(m_1 - m_2) / (1 + m_1m_2)]$

$= [(2 + 1) / (1 + 2)]$

$= 3$

So,

$\theta = \tan^{-1} (3)$

∴ The acute angle between the lines is $\tan^{-1} (3)$.

3. Prove that the points (2, -1), (0, 2), (2, 3) and (4, 0) are the coordinates of the vertices of a parallelogram and find the angle between its diagonals.

Solution:

To prove:

The points (2, -1), (0, 2), (2, 3) and (4, 0) are the coordinates of the vertices of a parallelogram

Let us assume the points, A (2, − 1), B (0, 2), C (2, 3) and D (4, 0) be the vertices.

Now, let us find the slopes

Slope of AB = [(2+1) / (0-2)]

= -3/2

Slope of BC = [(3-2) / (2-0)]

= ½

Slope of CD = [(0-3) / (4-2)]

= -3/2

Slope of DA = [(-1-0) / (2-4)]

= ½

Thus, AB is parallel to CD and BC is parallel to DA.

Hence proved, the given points are the vertices of a parallelogram.

Now, let us find the angle between the diagonals AC and BD.
Let m_1 and m_2 be the slopes of AC and BD, respectively.
$m_1 = [(3+1) / (2-2)]$
$= \infty$

$m_2 = [(0-2) / (4-0)]$
$= -1/2$

Thus, the diagonal AC is parallel to the y-axis.
$\angle ODB = \tan^{-1} (1/2)$

In triangle MND,
$\angle DMN = \pi/2 - \tan^{-1} (1/2)$
∴ The angle between the diagonals is $\pi/2 - \tan^{-1} (1/2)$.

4. Find the angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1.

Solution:
Given:
Points (2, 0), (0, 3) and the line x + y = 1.
Let us assume A (2, 0), B (0, 3) be the given points.
Now, let us find the slopes
Slope of AB = m_1
$= [(3-0) / (0-2)]$
$= -3/2$

Slope of the line x + y = 1 is -1
∴ $m_2 = -1$

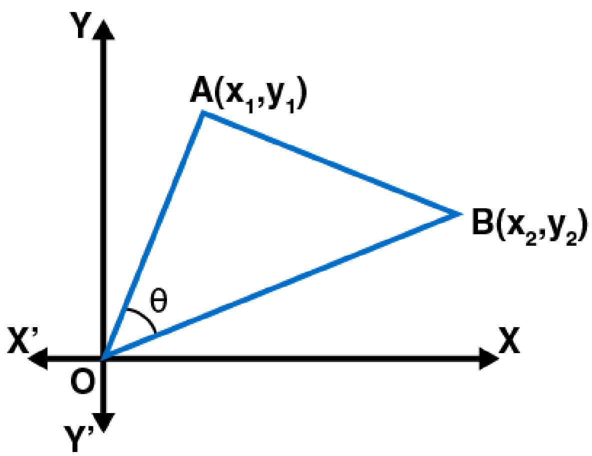
Let θ be the angle between the line joining the points (2, 0), (0, 3) and the line x + y =
$\tan \theta = |[(m_1 – m_2) / (1 + m_1m_2)]|$
$= [(-3/2 + 1) / (1 + 3/2)]$
$= 1/5$
$\theta = \tan^{-1} (1/5)$
∴ The acute angle between the line joining the points (2, 0), (0, 3) and the line x + y = 1 is $\tan^{-1} (1/5)$.

5. If θ is the angle which the straight line joining the points (x_1, y_1) and (x_2, y_2) subtends at the origin, prove that $\tan\theta = \frac{x_2y_1 - x_1y_2}{x_1x_2 + y_1y_2}$ **and** $\cos\theta = \frac{x_1y_2 + y_1y_2}{\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}}$

Solution:

We need to prove:

$\tan\theta = \frac{x_2y_1 - x_1y_2}{x_1x_2 + y_1y_2}$ and $\cos\theta = \frac{x_1x_2 + y_1y_2}{\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}}$.



Let us assume A (x_1, y_1) and B (x_2, y_2) be the given points and O be the origin.

Slope of OA = $m_1 = y_{1x1}$

Slope of OB = $m_2 = y_{2x2}$

It is given that θ is the angle between lines OA and OB.

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Now, substitute the values, we get

$$= \frac{\frac{y_1}{x_1} - \frac{y_2}{x_2}}{1 + \frac{y_1}{x_1} \times \frac{y_2}{x_2}}$$

$$\tan\theta = \frac{x_2 y_1 - x_1 y_2}{x_1 x_2 + y_1 y_2}$$

Now,

As we know that $\cos\theta = \sqrt{\frac{1}{1 + \tan^2\theta}}$

Now, substitute the values, we get

$$\cos\theta = \frac{x_1 x_2 + y_1 y_2}{\sqrt{(x_2 y_1 - x_1 y_2)^2 + (x_1 x_2 + y_1 y_2)^2}}$$

$$\cos\theta = \frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 y_1^2 + x_1^2 y_2^2 + x_1^2 x_2^2 + y_1^2 y_2^2}}$$

$$\cos\theta = \frac{x_1 x_2 + y_1 y_2}{\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}}.$$

Hence proved.

EXERCISE 23.14 PAGE NO: 23.102

1. Find the values of α so that the point P(α^2, α) lies inside or on the triangle formed by the lines x – 5y + 6 = 0, x – 3y + 2 = 0 and x – 2y – 3 = 0.

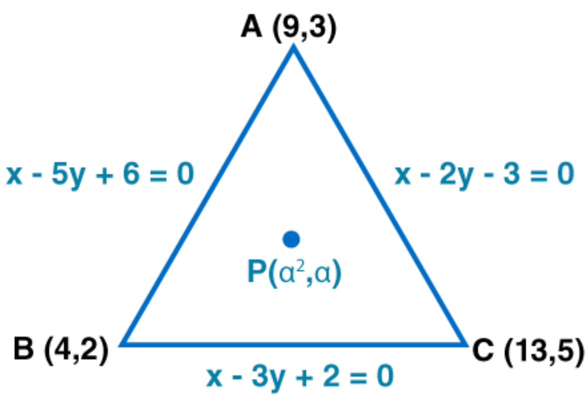
Solution:

Given:

x – 5y + 6 = 0, x – 3y + 2 = 0 and x – 2y – 3 = 0 forming a triangle and point P(α^2, α) lies inside or on the triangle

Let ABC be the triangle of sides AB, BC and CA whose equations are x − 5y + 6 = 0, x − 3y + 2 = 0 and x − 2y − 3 = 0, respectively.

On solving the equations, we get A (9, 3), B (4, 2) and C (13, 5) as the coordinates of the vertices.



It is given that point P (α^2, α) lies either inside or on the triangle. The three conditions are given below.

(i) A and P must lie on the same side of BC.
(ii) B and P must lie on the same side of AC.
(iii) C and P must lie on the same side of AB.

If A and P lie on the same side of BC, then

$(9 – 9 + 2)(\alpha^2 – 3\alpha + 2) \geq 0$

$(\alpha – 2)(\alpha – 1) \geq 0$

$\alpha \in (-\infty, 1] \cup [2, \infty)$ … (1)

If B and P lie on the same side of AC, then

$(4 – 4 – 3)(\alpha^2 – 2\alpha – 3) \geq 0$

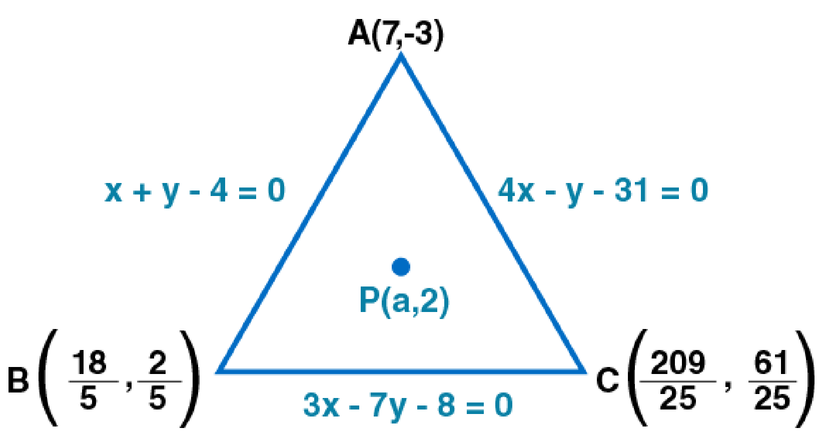
$(\alpha – 3)(\alpha + 1) \leq 0$

$\alpha \in [-1, 3]$ … (2)

If C and P lie on the same side of AB, then
$(13 - 25 + 6)(\alpha^2 - 5\alpha + 6) \geq 0$
$(\alpha - 3)(\alpha - 2) \leq 0$
$\alpha \in [2, 3]$ … (3)

From equations (1), (2) and (3), we get
$\alpha \in [2, 3]$
$\therefore \alpha \in [2, 3]$

2. Find the values of the parameter a so that the point (a, 2) is an interior point of the triangle formed by the lines x + y – 4 = 0, 3x – 7y – 8 = 0 and 4x – y – 31 = 0.
Solutions:
Given:
x + y – 4 = 0, 3x – 7y – 8 = 0 and 4x – y – 31 = 0 forming a triangle and point (a, 2)is an interior point of the triangle
Let ABC be the triangle of sides AB, BC and CA whose equations are x + y − 4 = 0, 3x − 7y − 8 = 0 and 4x − y − 31 = 0, respectively.
On solving them, we get A (7, - 3), B (18/5, 2/5) and C (209/25, 61/25) as the coordinates of the vertices.
Let P (a, 2) be the given point.



It is given that point P (a, 2) lies inside the triangle. So, we have the following:
(i) A and P must lie on the same side of BC.
(ii) B and P must lie on the same side of AC.
(iii) C and P must lie on the same side of AB.

Thus, if A and P lie on the same side of BC, then
21 + 21 – 8 – 3a – 14 – 8 > 0

a > 22/3 … (1)

If B and P lie on the same side of AC, then

$$4 \times \frac{18}{5} - \frac{2}{5} - 31 - 4a - 2 - 31 > 0$$

a < 33/4 … (2)

If C and P lie on the same side of AB, then

$$\frac{209}{25} + \frac{61}{25} - 4 - a + 2 - 4 > 0$$

$$\frac{34}{5} - 4 - a + 2 - 4 > 0$$

a > 2 … (3)

From (1), (2) and (3), we get:

A ∈ (22/3, 33/4)

∴ A ∈ (22/3, 33/4)

3. Determine whether the point (-3, 2) lies inside or outside the triangle whose sides are given by the equations x + y – 4 = 0, 3x – 7y + 8 = 0, 4x – y – 31 = 0.

Solution:

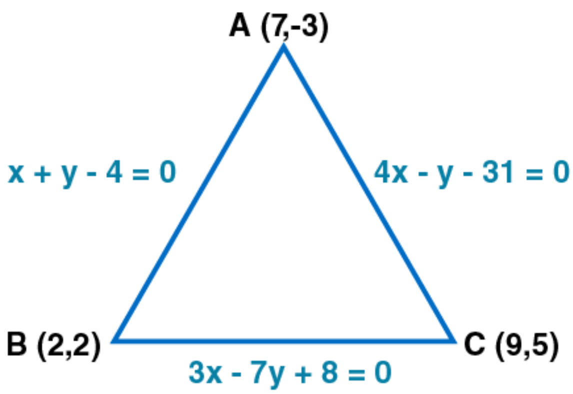
Given:

x + y – 4 = 0, 3x – 7y + 8 = 0, 4x – y – 31 = 0 forming a triangle and point (-3, 2)

Let ABC be the triangle of sides AB, BC and CA, whose equations x + y − 4 = 0, 3x − 7y + 8 = 0 and 4x − y − 31 = 0, respectively.

On solving them, we get A (7, - 3), B (2, 2) and C (9, 5) as the coordinates of the vertices.

Let P (− 3, 2) be the given point.



The given point P (− 3, 2) will lie inside the triangle ABC, if

(i) A and P lies on the same side of BC

(ii) B and P lies on the same side of AC
(iii) C and P lies on the same side of AB

Thus, if A and P lie on the same side of BC, then
$21 + 21 + 8 – 9 – 14 + 8 > 0$
$50 \times - 15 > 0$
$-750 > 0$,
This is false
∴ The point (−3, 2) lies outside triangle ABC.

EXERCISE 23.15 PAGE NO: 23.107

1. Find the distance of the point (4, 5) from the straight line 3x – 5y + 7 = 0.
Solution:
Given:
The line: 3x – 5y + 7 = 0
Comparing ax + by + c = 0 and 3x − 5y + 7 = 0, we get:
a = 3, b = − 5 and c = 7
So, the distance of the point (4, 5) from the straight line 3x − 5y + 7 = 0 is

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{3 \times 4 - 5 \times 5 + 7}{\sqrt{3^2 + (-5^2)}}\right|$$

$$= \frac{6}{\sqrt{34}}$$

∴ The required distance is 6/√34

2. Find the perpendicular distance of the line joining the points (cos θ, sin θ) and (cos ϕ, sin ϕ) from the origin.
Solution:
Given:
The points (cos θ, sin θ) and (cos ϕ, sin ϕ) from the origin.
The equation of the line joining the points (cos θ, sin θ) and (cos ϕ, sin ϕ) is given below:

$$y - \sin\theta = \frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$$

$$(\cos\phi - \cos\theta)y - \sin\theta(\cos\phi - \cos\theta) = (\sin\phi - \sin\theta)x - (\sin\phi - \sin\theta)\cos\theta$$

$$(\sin\phi - \sin\theta)x - (\cos\phi - \cos\theta)y + \sin\theta\cos\phi - \sin\phi\cos\theta = 0$$

Let d be the perpendicular distance from the origin to the line
$(\sin\phi - \sin\theta)x - (\cos\phi - \cos\theta)y + \sin\theta\cos\phi - \sin\phi\cos\theta = 0$

$$d = \left|\frac{\sin\theta - \phi}{\sqrt{(\sin\phi - \sin\theta)^2 + (\cos\phi - \cos\theta)^2}}\right|$$

$$= \left|\frac{\sin\theta - \phi}{\sqrt{\sin^2\phi + \sin^2\theta - 2\sin\phi\sin\theta + \cos^2\phi + \cos^2\theta - 2\cos\phi\cos\theta}}\right|$$

$$= \left|\frac{\sin\theta - \phi}{\sqrt{\sin^2\phi + \cos^2\phi + \sin^2\theta + \cos^2\phi + \cos^2\theta - 2\cos(\theta-\phi)}}\right|$$

$$= \left|\frac{\frac{1}{\sqrt{2}}(\sin(\theta-\phi))}{\sqrt{1-\cos(\theta-\phi)}}\right|$$

$$= \frac{1}{\sqrt{2}}\left|\frac{\sin(\theta-\phi)}{\sqrt{2\sin^2\left(\frac{\theta-\phi}{2}\right)}}\right|$$

$$= \frac{1}{2}\left|\frac{2\sin\left(\frac{\theta-\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)}{\sin\left(\frac{\theta-\phi}{2}\right)}\right|$$

$$= \cos\left(\frac{\theta-\phi}{2}\right)$$

∴ The required distance is $\cos\left(\frac{\theta-\phi}{2}\right)$

3. Find the length of the perpendicular from the origin to the straight line joining the two points whose coordinates are (a cos α, a sin α) and (a cos β, a sin β).

Solution:

Given:

Coordinates are (a cos α, a sin α) and (a cos β, a sin β).

Equation of the line passing through (a cos α, a sin α) and (a cos β, a sin β) is

$$y - a\sin\alpha = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha}(x - a\cos\alpha)$$

$$y - a\sin\alpha = \frac{\sin\beta - \sin\alpha}{\cos\beta - \cos\alpha}(x - a\cos\alpha)$$

$$y - a\sin\alpha = \frac{2\cos\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)}{2\sin\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}(x - a\cos\alpha)$$

$$y - a\sin\alpha = -\cot\left(\frac{\beta+\alpha}{2}\right)(x - a\cos\alpha)$$

$$y - a\sin\alpha = -\cot\left(\frac{\alpha+\beta}{2}\right)(x - a\cos\alpha)$$

$$x\cot\left(\frac{\alpha+\beta}{2}\right) + y - a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha+\beta}{2}\right) = 0$$

The distance of the line from the origin is

$$d = \left|\frac{-a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha+\beta}{2}\right)}{\sqrt{\cot^2\left(\frac{(\alpha+\beta)}{2}\right)+1}}\right|$$

$$d = \left|\frac{-a\sin\alpha - a\cos\alpha\cot\left(\frac{\alpha+\beta}{2}\right)}{\sqrt{\mathrm{cosec}^2\left(\frac{(\alpha+\beta)}{2}\right)}}\right| \because \mathrm{cosec}^2\theta = 1 + \cot^2\theta$$

$$= a\left|\sin\left(\frac{\alpha+\beta}{2}\right)\sin\alpha + \cos\alpha\cos\left(\frac{\alpha+\beta}{2}\right)\right|$$

$$= a\left|\sin\alpha\sin\left(\frac{\alpha+\beta}{2}\right) + \cos\alpha\cos\left(\frac{\alpha+\beta}{2}\right)\right|$$

$$= a\left|\cos\left(\frac{\alpha+\beta}{2} - \alpha\right)\right| = a\cos\left(\frac{\beta-\alpha}{2}\right)$$

∴ The required distance is $a\cos\left(\frac{\beta-\alpha}{2}\right)$

4. Show that the perpendicular let fall from any point on the straight line 2x + 11y – 5 = 0 upon the two straight lines 24x + 7y = 20 and 4x – 3y – 2 = 0 are equal to each other.

Solution:

Given:

The lines 24x + 7y = 20 and 4x – 3y – 2 = 0

Let us assume, P(a, b) be any point on 2x + 11y − 5 = 0

So,

2a + 11b − 5 = 0

$b = \frac{5-2a}{11}$ …….. (1)

Let d_1 and d_2 be the perpendicular distances from point P on the lines 24x + 7y = 20 and 4x − 3y − 2 = 0, respectively.

$$d_1 = \left|\frac{24a+7b-20}{\sqrt{24^2+7^2}}\right| = \left|\frac{24a+7b-20}{25}\right|$$

$$= \left|\frac{24a + 7\times\frac{5-2a}{11} - 20}{25}\right|$$

From (1)

$d_1 = \left|\frac{50a - 37}{55}\right|$

Similarly,

$d_2 = \left|\frac{4a - 3b - 2}{\sqrt{3^2 + (-4)^2}}\right| = \left|\frac{4a - 3 \times \frac{5-2a}{11} - 2}{5}\right|$

$= \left|\frac{44a - 15 + 6a - 22}{11 \times 5}\right|$

From (1)

$d_2 = \left|\frac{50a - 37}{55}\right|$

$\therefore d_1 = d_2$

Hence proved.

5. Find the distance of the point of intersection of the lines 2x + 3y = 21 and 3x – 4y + 11 = 0 from the line 8x + 6y + 5 = 0.

Solution:

Given:

The lines 2x + 3y = 21 and 3x – 4y + 11 = 0

Solving the lines 2x + 3y = 21 and 3x − 4y + 11 = 0 we get:

$$\frac{x}{33 - 84} = \frac{y}{-63 - 22} = \frac{1}{-8 - 9}$$

x = 3, y = 5

So, the point of intersection of 2x + 3y = 21 and 3x − 4y + 11 = 0 is (3, 5).

Now, the perpendicular distance d of the line 8x + 6y + 5 = 0 from the point (3, 5) is

$$d = \left|\frac{24 + 30 + 5}{\sqrt{8^2 + 6^2}}\right| = \frac{59}{10}$$

∴ The distance is 59/10.

EXERCISE 23.16

1. Determine the distance between the following pair of parallel lines:
(i) 4x – 3y – 9 = 0 and 4x – 3y – 24 = 0
(ii) 8x + 15y – 34 = 0 and 8x + 15y + 31 = 0
Solution:
(i) 4x – 3y – 9 = 0 and 4x – 3y – 24 = 0
Given:
The parallel lines are
4x − 3y − 9 = 0 … (1)
4x − 3y − 24 = 0 … (2)
Let d be the distance between the given lines.
So,

$$d = \left|\frac{-9 + 24}{\sqrt{4^2 + (-3)^2}}\right| = \frac{15}{5} = 3 \text{ units}$$

∴ The distance between givens parallel line is 3units.

(ii) 8x + 15y – 34 = 0 and 8x + 15y + 31 = 0
Given:
The parallel lines are
8x + 15y − 34 = 0 … (1)
8x + 15y + 31 = 0 … (2)
Let d be the distance between the given lines.
So,

$$d = \left|\frac{-34 - 31}{\sqrt{8^2 + 15^2}}\right| = \frac{65}{17} \text{units}$$

∴ The distance between givens parallel line is 65/17 units.

2. The equations of two sides of a square are 5x – 12y – 65 = 0 and 5x – 12y + 26 = 0. Find the area of the square.
Solution:
Given:
Two side of square are 5x – 12y – 65 = 0 and 5x – 12y + 26 = 0
The sides of a square are
5x − 12y − 65 = 0 … (1)
5x − 12y + 26 = 0 … (2)
We observe that lines (1) and (2) are parallel.
So, the distance between them will give the length of the side of the square.

Let d be the distance between the given lines.

$$d = \left|\frac{-65 - 26}{\sqrt{5^2 + (-12)^2}}\right| = \frac{91}{13} = 7$$

∴ Area of the square = 7^2 = 49 square units

3. Find the equation of two straight lines which are parallel to x + 7y + 2 = 0 and at unit distance from the point (1, -1).

Solution:

Given:

The equation is parallel to x + 7y + 2 = 0 and at unit distance from the point (1, -1)

The equation of given line is

x + 7y + 2 = 0 … (1)

The equation of a line parallel to line x + 7y + 2 = 0 is given below:

x + 7y + λ = 0 … (2)

The line x + 7y + λ = 0 is at a unit distance from the point (1, − 1).

So,

1 = 1 – 7 + λ1 + 49

λ – 6 = ± 5√2

λ = 6 + 5√2, 6 - 5√2

now, substitute the value of λ back in equation x + 7y + λ = 0, we get

x + 7y + 6 + 5√2 = 0 and x + 7y + 6 - 5√2

∴ The required lines:

x + 7y + 6 + 5√2 = 0 and x + 7y + 6 - 5√2

4. Prove that the lines 2x + 3y = 19 and 2x + 3y + 7 = 0 are equidistant from the line 2x + 3y = 6.

Solution:

Given:

The lines A, 2x + 3y = 19 and B, 2x + 3y + 7 = 0 also a line C, 2x + 3y = 6.

Let d_1 be the distance between lines 2x + 3y = 19 and 2x + 3y = 6,

While d_2 is the distance between lines 2x + 3y + 7 = 0 and 2x + 3y = 6

$$d_1 = \left|\frac{-19 - (-6)}{\sqrt{2^2 + 3^2}}\right| \text{ and } d_2 = \left|\frac{7 - (-6)}{\sqrt{2^2 + 3^2}}\right|$$

$$d_1 = \left|-\frac{13}{\sqrt{13}}\right| = \sqrt{13} \text{ and } d_2 = \left|\frac{13}{\sqrt{13}}\right| = \sqrt{13}$$

Hence proved, the lines 2x + 3y = 19 and 2x + 3y + 7 = 0 are equidistant from the line 2x + 3y = 6

5. Find the equation of the line mid-way between the parallel lines 9x + 6y – 7 = 0 and 3x + 2y + 6 = 0.

Solution:

Given:

9x + 6y – 7 = 0 and 3x + 2y + 6 = 0 are parallel lines

The given equations of the lines can be written as:

3x + 2y – 7/3 = 0 … (1)

3x + 2y + 6 = 0 … (2)

Let the equation of the line midway between the parallel lines (1) and (2) be

3x + 2y + λ = 0 … (3)

The distance between (1) and (3) and the distance between (2) and (3) are equal.

$$\left|\frac{-\frac{7}{3}-\lambda}{\sqrt{3^2+2^2}}\right| = \left|\frac{6-\lambda}{\sqrt{3^2+2^2}}\right|$$

$$\left|-\lambda+\frac{7}{3}\right| = |6-\lambda|$$

$$6-\lambda = \lambda+\frac{7}{3}$$

$$\lambda = \frac{11}{6}$$

Now substitute the value of λ back in equation 3x + 2y + λ = 0, we get

3x + 2y + 11/6 = 0

By taking LCM

18x + 12y + 11 = 0

∴ The required equation of line is 18x + 12y + 11 = 0

EXERCISE 23.17

1. Prove that the area of the parallelogram formed by the lines
$a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_2x + b_2y + d_2 = 0$
is $\left|\frac{(d_1 - c_1)(d_2 - c_2)}{a_1b_2 - a_2b_1}\right|$ sq. units.
Deduce the condition for these lines to form a rhombus.
Solution:
Given:
The given lines are
$a_1x + b_1y + c_1 = 0$ … (1)
$a_1x + b_1y + d_1 = 0$ … (2)
$a_2x + b_2y + c_2 = 0$ … (3)
$a_2x + b_2y + d_2 = 0$ … (4)
Let us prove, the area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_2x + b_2y + d_2 = 0$ is $\left|\frac{(d_1 - c_1)(d_2 - c_2)}{(a_1b_2 - a_2b_1)}\right|$ sq. units.

The area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_2x + b_2y + d_2 = 0$ is given below:

$$\text{Area} = \left|\frac{(c_1 - d_1)(c_2 - d_2)}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}\right|$$

Since, $\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$

$$\therefore \text{Area} = \left|\frac{(c_1 - d_1)(c_2 - d_2)}{(a_1b_2 - a_2b_1)}\right| = \left|\frac{(d_1 - c_1)(d_2 - c_2)}{(a_1b_2 - a_2b_1)}\right|$$

If the given parallelogram is a rhombus, then the distance between the pair of parallel lines is equal.

$$\therefore \left|\frac{c_1 - d_1}{\sqrt{a_1^2 + b_1^2}}\right| = \left|\frac{c_2 - d_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

Hence proved.

2. Prove that the area of the parallelogram formed by the lines 3x – 4y + a = 0, 3x – 4y + 3a = 0, 4x – 3y – a = 0 and 4x – 3y – 2a = 0 is $2a^2/7$ sq. units.

Solution:

Given:

The given lines are

3x − 4y + a = 0 … (1)

3x − 4y + 3a = 0 … (2)

4x − 3y − a = 0 … (3)

4x − 3y − 2a = 0 … (4)

Let us prove, the area of the parallelogram formed by the lines 3x – 4y + a = 0, 3x – 4y + 3a = 0, 4x – 3y – a = 0 and 4x – 3y – 2a = 0 is $2a^2/7$ sq. units.

From above solution, we know that

$$\text{Area of the parallelogram} = \left|\frac{(c_1 - d_1)(c_2 - d_2)}{(a_1 b_2 - a_2 b_1)}\right|$$

$$\text{Area of the parallelogram} = \left|\frac{(a - 3a)(2a - a)}{(-9 + 16)}\right| = \frac{2a^2}{7} \text{ square units}$$

Hence proved.

3. Show that the diagonals of the parallelogram whose sides are lx + my + n = 0, lx + my + n’ = 0, mx + ly + n = 0 and mx + ly + n’ = 0 include an angle π/2.

Solution:

Given:

The given lines are

lx + my + n = 0 … (1)

mx + ly + n’ = 0 … (2)

lx + my + n’ = 0 … (3)

mx + ly + n = 0 … (4)

Let us prove, the diagonals of the parallelogram whose sides are lx + my + n = 0, lx + my + n’ = 0, mx + ly + n = 0 and mx + ly + n’ = 0 include an angle π/2.

By solving (1) and (2), we get

$$B = \left(\frac{mn' - ln}{l^2 - m^2}, \frac{mn - ln'}{l^2 - m^2}\right)$$

Solving (2) and (3), we get,

$$C = \left(-\frac{n'}{m + l}, -\frac{n'}{m + l}\right)$$

Solving (3) and (4), we get,

$$D = \left(\frac{mn - ln'}{l^2 - m^2}, \frac{mn' - ln}{l^2 - m^2}\right)$$

Solving (1) and (4), we get,

$$A = \left(\frac{-n}{m+l}, \frac{-n}{m+l}\right)$$

Let m_1 and m_2 be the slope of AC and BD.
Now,

$$m_1 = \frac{\frac{-n'}{m+l} + \frac{n}{m+l}}{\frac{-n'}{m+l} + \frac{n}{m+l}} = 1$$

$$m_2 = \frac{\frac{mn' - ln}{l^2 - m^2} - \frac{mn - ln'}{l^2 - m^2}}{\frac{mn - ln'}{l^2 - m^2} - \frac{mn' - ln}{l^2 - m^2}} = -1$$

$\therefore m_1m_2 = -1$

Hence proved.

EXERCISE 23.18 PAGE NO: 23.124

1. Find the equation of the straight lines passing through the origin and making an angle of 45° with the straight line √3x + y = 11.

Solution:

Given:

Equation passes through (0, 0) and make an angle of 45° with the line √3x + y = 11.

We know that, the equations of two lines passing through a point x1,y1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,

$x_1 = 0$, $y_1 = 0$, $\alpha = 45°$ and $m = -\sqrt{3}$

So, the equations of the required lines are

$$y - 0 = \frac{-\sqrt{3} + \tan 45°}{1 + \sqrt{3}\tan 45°}(x - 0) \text{ and } y - 0$$

$$= \frac{-\sqrt{3} - \tan 45°}{1 - \sqrt{3}\tan 45°}(x - 0)$$

$$= \frac{-\sqrt{3} + 1}{1 + \sqrt{3}}x \text{ and } y = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}x$$

$$= -\frac{3 + 1 - 2\sqrt{3}}{3 - 1}x \text{ and } y = \frac{3 + 1 + 2\sqrt{3}}{3 - 1}x$$

$$= (\sqrt{3} - 2)x \text{ and } y = (\sqrt{3} + 2)x$$

∴ The equation of given line is $y = (\sqrt{3} - 2)x$ and $y = (\sqrt{3} + 2)x$

2. Find the equations to the straight lines which pass through the origin and are inclined at an angle of 75° to the straight line x + y + √3(y - x) = a.

Solution:

Given:

The equation passes through (0,0) and make an angle of 75° with the line x + y + √3(y - x) = a.

We know that the equations of two lines passing through a point x1,y1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here, equation of the given line is,

$x + y + \sqrt{3}(y - x) = a$

$(\sqrt{3} + 1)y = (\sqrt{3} - 1)x + a$

$y = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}x + \frac{a}{\sqrt{3} + 1}$

Comparing this equation with y = mx + c

We get,

$m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$\therefore x_1 = 0, y_1 = 0, \alpha = 75°,$

$m = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$ and $\tan 75° = 2 + \sqrt{3}$

So, the equations of the required lines are

$$y - 0 = \frac{2 - \sqrt{3} + \tan 75°}{1 - (2 - \sqrt{3})\tan 75°}(x - 0) \text{ and } y - 0$$

$$= \frac{2 - \sqrt{3} - \tan 75°}{1 + (2 - \sqrt{3})\tan 75°}(x - 0)$$

$$y = \frac{2 - \sqrt{3} + 2 + \sqrt{3}}{1 - (2 - \sqrt{3})(2 + \sqrt{3})}x \text{ and } y = \frac{2 - \sqrt{3} - 2 - \sqrt{3}}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}x$$

$$y = \frac{4}{1 - 1}x \text{ and } y = -\sqrt{3}x$$

$x = 0$ and $\sqrt{3}x + y = 0$

$\therefore$ The equation of given line is $x = 0$ and $\sqrt{3}x + y = 0$

3. Find the equations of straight lines passing through (2, -1) and making an angle of 45^{o} with the line 6x + 5y – 8 = 0.

Solution:

Given:

The equation passes through (2,-1) and make an angle of 45° with the line 6x + 5y – 8 = 0

We know that the equations of two lines passing through a point x_1, y_1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here, equation of the given line is,
6x + 5y – 8 = 0
5y = - 6x + 8
y = -6x/5 + 8/5
Comparing this equation with y = mx + c
We get, m = -6/5
Where, $x_1 = 2$, $y_1 = -1$, α = 45°, m = -6/5
So, the equations of the required lines are

$$y + 1 = \frac{\left(-\frac{6}{5} + \tan 45°\right)}{\left(1 + \frac{6}{5}\tan 45°\right)}(x - 2) \text{ and } y + 1 = \frac{\left(-\frac{6}{5} - \tan 45°\right)}{\left(1 - \frac{6}{5}\tan 45°\right)}(x - 2)$$

$$y + 1 = \frac{\left(-\frac{6}{5} + 1\right)}{\left(1 + \frac{6}{5}\right)}(x - 2) \text{ and } y + 1 = \frac{\left(-\frac{6}{5} - 1\right)}{\left(1 - \frac{6}{5}\right)}(x - 2)$$

$$y + 1 = -\frac{1}{11}(x - 2) \text{ and } y + 1 = -\frac{11}{-1}(x - 2)$$

x + 11y + 9 = 0 and 11x – y – 23 = 0
∴ The equation of given line is x + 11y + 9 = 0 and 11x – y – 23 = 0

4. Find the equations to the straight lines which pass through the point (h, k) and are inclined at angle $\tan^{-1}$ m to the straight line y = mx + c.
Solution:
Given:
The equation passes through (h, k) and make an angle of $\tan^{-1}$ m with the line y = mx + c
We know that the equations of two lines passing through a point x_1, y_1 and making an angle α with the given line y = mx + c are
m′ = m
So,

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,
$x_1 = h$, $y_1 = k$, $\alpha = \tan^{-1} m$, m′ = m.
So, the equations of the required lines are

$$y - k = \frac{m + m}{1 - m^2}(x - h) \text{ and } y - k = \frac{m - m}{1 + m^2}(x - h)$$

$$y - k = \frac{2m}{1 - m^2}(x - h) \text{ and } y - k = 0$$

$$(y - k)(1 - m^2) = 2m(x - h) \text{ and } y = k$$

∴ The equation of given line is $(y - k)(1 - m^2) = 2m(x - h)$ and $y = k$.

5. Find the equations to the straight lines passing through the point (2, 3) and inclined at an angle of 45^0 to the lines 3x + y – 5 = 0.

Solution:

Given:

The equation passes through (2, 3) and make an angle of $45^0$with the line 3x + y – 5 = 0.

We know that the equations of two lines passing through a point x1,y1 and making an angle α with the given line y = mx + c are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here,

Equation of the given line is,

3x + y – 5 = 0

y = - 3x + 5

Comparing this equation with y = mx + c we get, m = - 3

$x_1 = 2$, $y_1 = 3$, α = 45∘, m = - 3.

So, the equations of the required lines are

$$y - 3 = \frac{-3 + \tan 45°}{1 + 3\tan 45°}(x - 2) \text{ and } y - 3 = \frac{-3 - \tan 45°}{1 - 3\tan 45°}(x - 2)$$

$$y - 3 = \frac{-3 + 1}{1 + 3}(x - 2) \text{ and } y - 3 = \frac{-3 - 1}{1 - 3}(x - 2)$$

$$y - 3 = \frac{-1}{2}(x - 2) \text{ and } y - 3 = 2(x - 2)$$

x + 2y – 8 = 0 and 2x – y – 1 = 0

∴ The equation of given line is x + 2y – 8 = 0 and 2x – y – 1 = 0

EXERCISE 23.19 PAGE NO: 23.124

1. Find the equation of a straight line through the point of intersection of the lines 4x – 3y = 0 and 2x – 5y + 3 = 0 and parallel to 4x + 5 y + 6 = 0.

Solution:

Given:

Lines 4x – 3y = 0 and 2x – 5y + 3 = 0 and parallel to 4x + 5 y + 6 = 0

The equation of the straight line passing through the points of intersection of 4x − 3y = 0 and 2x − 5y + 3 = 0 is given below:

$4x - 3y + \lambda(2x - 5y + 3) = 0$

$(4 + 2\lambda)x + (-3 - 5\lambda)y + 3\lambda = 0$

$$y = \left(\frac{4 + 2\lambda}{3 + 5\lambda}\right)x + \frac{3\lambda}{(3 + 5\lambda)}$$

The required line is parallel to 4x + 5y + 6 = 0 or, y = -4x/5 – 6/5

$$\frac{4 + 2\lambda}{3 + 5\lambda} = -\frac{4}{5}$$

$$\frac{4 + 2\lambda}{3 + 5\lambda} = -\frac{4}{5}$$

$\lambda = -16/15$

∴ The required equation is

$$\left(4 - \frac{32}{15}\right)x - \left(3 - \frac{80}{15}\right)y - \frac{48}{15} = 0$$

28x + 35y – 48 = 0

2. Find the equation of a straight line passing through the point of intersection of x + 2y + 3 = 0 and 3x + 4y + 7 = 0 and perpendicular to the straight line x – y + 9 = 0.

Solution:

Given:

x + 2y + 3 = 0 and 3x + 4y + 7 = 0

The equation of the straight line passing through the points of intersection of x + 2y + 3 = 0 and 3x + 4y + 7 = 0 is

$x + 2y + 3 + \lambda(3x + 4y + 7) = 0$

$(1 + 3\lambda)x + (2 + 4\lambda)y + 3 + 7\lambda = 0$

$$y = -\left(\frac{1 + 3\lambda}{2 + 4\lambda}\right)x - \left(\frac{3 + 7\lambda}{2 + 4\lambda}\right)$$

The required line is perpendicular to x − y + 9 = 0 or, y = x + 9

3. Find the equation of the line passing through the point of intersection of 2x – 7y + 11 = 0 and x + 3y – 8 = 0 and is parallel to (i) x = axis (ii) y-axis.

Solution:

Given:

The equations, $2x – 7y + 11 = 0$ and $x + 3y – 8 = 0$

The equation of the straight line passing through the points of intersection of $2x − 7y + 11 = 0$ and $x + 3y − 8 = 0$ is given below:

$2x − 7y + 11 + \lambda(x + 3y − 8) = 0$

$(2 + \lambda)x + (− 7 + 3\lambda)y + 11 − 8\lambda = 0$

(i) The required line is parallel to the x-axis. So, the coefficient of x should be zero.

$2 + \lambda = 0$

$\lambda = -2$

Now, substitute the value of λ back in equation, we get

$0 + (− 7 − 6)y + 11 + 16 = 0$

$13y − 27 = 0$

∴ The equation of the required line is $13y − 27 = 0$

(ii) The required line is parallel to the y-axis. So, the coefficient of y should be zero.

$-7 + 3\lambda = 0$

$\lambda = 7/3$

Now, substitute the value of λ back in equation, we get

$(2 + 7/3)x + 0 + 11 – 8(7/3) = 0$

$13x – 23 = 0$

∴ The equation of the required line is $13x – 23 = 0$

4. Find the equation of the straight line passing through the point of intersection of 2x + 3y + 1 = 0 and 3x – 5y – 5 = 0 and equally inclined to the axes.

Solution:

Given:

The equations, $2x + 3y + 1 = 0$ and $3x – 5y – 5 = 0$

The equation of the straight line passing through the points of intersection of $2x + 3y + 1 = 0$ and $3x − 5y − 5 = 0$ is

$2x + 3y + 1 + \lambda(3x − 5y − 5) = 0$

$(2 + 3\lambda)x + (3 − 5\lambda)y + 1 − 5\lambda = 0$

$y = - [(2 + 3\lambda) / (3 - 5\lambda)] – [(1 - 5\lambda) / (3 - 5\lambda)]$

The required line is equally inclined to the axes. So, the slope of the required line is either 1 or − 1.

So,
$-[(2 + 3\lambda) / (3 - 5\lambda)] = 1$ and $-[(2 + 3\lambda) / (3 - 5\lambda)] = -1$
$-2 - 3\lambda = 3 - 5\lambda$ and $2 + 3\lambda = 3 - 5\lambda$
$\lambda = 5/2$ and $1/8$

Now, substitute the values of λ in $(2 + 3\lambda)x + (3 - 5\lambda)y + 1 - 5\lambda = 0$, we get the equations of the required lines as:
$(2 + 15/2)x + (3 - 25/2)y + 1 - 25/2 = 0$ and $(2 + 3/8)x + (3 - 5/8)y + 1 - 5/8 = 0$
$19x - 19y - 23 = 0$ and $19x + 19y + 3 = 0$
∴ The required equation is $19x - 19y - 23 = 0$ and $19x + 19y + 3 = 0$

5. Find the equation of the straight line drawn through the point of intersection of the lines x + y = 4 and 2x – 3y = 1 and perpendicular to the line cutting off intercepts 5, 6 on the axes.
Solution:
Given:
The lines $x + y = 4$ and $2x - 3y = 1$
The equation of the straight line passing through the point of intersection of $x + y = 4$ and $2x - 3y = 1$ is
$x + y - 4 + \lambda(2x - 3y - 1) = 0$
$(1 + 2\lambda)x + (1 - 3\lambda)y - 4 - \lambda = 0$ … (1)
$y = -[(1 + 2\lambda) / (1 - 3\lambda)]x + [(4 + \lambda) / (1 - 3\lambda)]$
The equation of the line with intercepts 5 and 6 on the axis is
$x/5 + y/6 = 1$ …. (2)
So, the slope of this line is $-6/5$
The lines (1) and (2) are perpendicular.

∴ $-6/5 \times [(-1+2\lambda) / (1 - 3\lambda)] = -1$
$\lambda = 11/3$
Now, substitute the values of λ in (1), we get the equation of the required line.
$(1 + 2(11/3))x + (1 - 3(11/3))y - 4 - 11/3 = 0$
$(1 + 22/3)x + (1 - 11)y - 4 - 11/3 = 0$
$25x - 30y - 23 = 0$
∴ The required equation is $25x - 30y - 23 = 0$