

**EXERCISE 29.10**
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**Evaluate the following limits:**

$$1. \lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$$

**Solution:**

Given:  $\lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$

The limit  $\lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$  assumes the form  $(0/0)$ .

So,  $\lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$

As  $Z = \lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$

Now, multiply both numerator and denominator by  $\sqrt{4+x} + 2$  so that we can remove the indeterminate form.

$$\begin{aligned} Z &= \lim_{x \rightarrow 0} \frac{(5^x - 1)\sqrt{4+x} + 2}{(\sqrt{4+x})^2 - 2^2} \\ &= \lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4+x} - 2} \times \frac{\sqrt{4+x} + 2}{\sqrt{4+x} + 2} \end{aligned}$$

{By using  $a^2 - b^2 = (a + b)(a - b)$ }

$$\begin{aligned} Z &= \lim_{x \rightarrow 0} \frac{(5^x - 1)\sqrt{4+x} + 2}{4+x-4} \\ &= \lim_{x \rightarrow 0} \frac{(5^x - 1)\sqrt{4+x} + 2}{x} \end{aligned}$$

By using basic algebra of limits, we get

$$\begin{aligned} Z &= \lim_{x \rightarrow 0} \frac{(5^x - 1)}{x} \times \lim_{x \rightarrow 0} \sqrt{4+x} + 2 = \{\sqrt{4+0} + 2\} \lim_{x \rightarrow 0} \frac{(5^x - 1)}{x} \\ &= 4 \lim_{x \rightarrow 0} \frac{(5^x - 1)}{x} \quad [\text{By using the formula: } \lim_{x \rightarrow 0} \frac{(a^x - 1)}{x} = \log a] \\ Z &= 4 \log 5 \end{aligned}$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} \frac{5^x - 1}{\sqrt{4+x} - 2} = 4 \log 5$$

$$2. \lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$$

**Solution:**

Given:  $\lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$

The limit  $\lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$

When  $x = 0$ , the expression  $\lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$  assumes the form  $(0/0)$ .

So,

As  $Z = \lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1}$

Let us divide numerator and denominator by  $x$ , we get

$$Z = \lim_{x \rightarrow 0} \frac{\frac{\log(1+x)}{x}}{\frac{3^x - 1}{x}} = \frac{\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}}{\lim_{x \rightarrow 0} \frac{3^x - 1}{x}} \quad \{\text{by using basic limit algebra}\}$$

[By using the formula:  $\lim_{x \rightarrow 0} \frac{(a^x - 1)}{x} = \log a$ ]

$$= \frac{1}{\log 3}$$

$\therefore$  The value of  $\lim_{x \rightarrow 0} \frac{\log(1+x)}{3^x - 1} = \frac{1}{\log 3}$

$$3. \lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$$

**Solution:**

Given:  $\lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$

The limit  $\lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$

When  $x = 0$ , the expression  $\lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$  assumes the form  $(0/0)$ .

So,

As  $Z = \lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2}$

$$= \lim_{x \rightarrow 0} \frac{a^{-x}(a^{2x} - 2a^x + 1)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(a^{2x} - 2a^x + 1)}{a^x x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(a^x - 1)^2}{a^x x^2} \quad \{\text{By using } (a + b)^2 = a^2 + b^2 + 2ab\}$$

Let us use algebra of limit, we get

$$Z = \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} \right)^2 \times \lim_{x \rightarrow 0} \frac{1}{a^x}$$

[By using the formula:  $\lim_{x \rightarrow 0} \frac{(a^x - 1)}{x} = \log a$ ]

$$Z = (\log a)^2 \frac{1}{a^0} = (\log a)^2$$

$\therefore$  The value of  $\lim_{x \rightarrow 0} \frac{a^x + a^{-x} - 2}{x^2} = (\log a)^2$

4.  $\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1}, n \neq 0$

**Solution:**

Given:  $\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1}, n \neq 0$

The limit  $\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1}, n \neq 0$

When  $x = 0$ , the expression  $\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1}, n \neq 0$  assumes the form  $(0/0)$ .

So, let us include  $mx$  and  $nx$  as follows:

$$\begin{aligned} Z &= \lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1} = \lim_{x \rightarrow 0} \frac{\frac{a^{mx} - 1}{mx} \times mx}{\frac{b^{nx} - 1}{nx} \times nx} \\ &= \frac{m}{n} \lim_{x \rightarrow 0} \frac{\frac{a^{mx} - 1}{mx}}{\frac{b^{nx} - 1}{nx}} \end{aligned}$$

By using algebra of limits, we get

$$Z = \frac{m}{n} \frac{\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{mx}}{\lim_{x \rightarrow 0} \frac{b^{nx} - 1}{nx}}$$

[By using the formula:  $\lim_{x \rightarrow 0} \frac{(a^x - 1)}{x} = \log a$ ]

$$Z = \frac{m}{n} \frac{\log a}{\log b}, n \neq 0$$

$\therefore$  The value of  $\lim_{x \rightarrow 0} \frac{a^{mx} - 1}{b^{nx} - 1} = \frac{m}{n} \frac{\log a}{\log b}, n \neq 0$

$$5. \lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x}$$

**Solution:**

Given:  $\lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x}$

The limit  $\lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x}$

When  $x = 0$ , the expression  $\lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x}$  assumes the form  $(0/0)$ .

So,

$$\begin{aligned} \text{As } Z &= \lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x} \\ &= \lim_{x \rightarrow 0} \frac{a^x - 1 + b^x - 1}{x} \end{aligned}$$

By using algebra of limits, we get

$$Z = \lim_{x \rightarrow 0} \frac{a^x - 1}{x} + \lim_{x \rightarrow 0} \frac{b^x - 1}{x}$$

[By using the formula:  $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$ ]

$$Z = \log a + \log b = \log ab$$

$$\therefore \text{The value of } \lim_{x \rightarrow 0} \frac{a^x + b^x - 2}{x} = \log ab$$