EXERCISE 29.3 PAGE NO: 29.23

Evaluate the following limits:

1. $\lim_{x \to -5} \frac{2x^2 + 9x - 5}{x + 5}$

Solution:

Given:

The limit $\lim_{x \to -5} \frac{2x^2 + 9x - 5}{x + 5}$

By substituting the value of x, we get

$$\lim_{x \to -5} \frac{2x^2 + 9x - 5}{x + 5} = \frac{2(-5)^2 + 9(-5) - 5}{(-5) + 5}$$

$$= \frac{50 - 50}{(-5) + 5}$$

$$= \frac{0}{0}$$ [Since, it is of the form indeterminate]

By using factorization method:

$$\lim_{x \to -5} \frac{2x^2 + 9x - 5}{x + 5} = \lim_{x \to -5} \frac{2x^2 + 9x - 5}{x + 5}$$

$$= \lim_{x \to -5} \frac{2x^2 + 10x - x - 5}{x + 5}$$

$$= \lim_{x \to -5} \frac{2x(x + 5) - (x + 5)}{x + 5}$$

$$= \lim_{x \to -5} \frac{(2x - 1)(x + 5)}{x + 5}$$

$$= \lim_{x \to -5} 2x - 1$$

$$= 2(-5) - 1$$

$$= -11$$

∴ The value of the given limit is – 11.

2. $\lim_{x\to 3} \frac{x^2-4x+3}{x^2-2x-3}$

Solution:

Given:

The limit $\lim_{x\to 3} \frac{x^2-4x+3}{x^2-2x-3}$

By substituting the value of x, we get

$$\lim_{x\to 3} \frac{x^2-4x+3}{x^2-2x-3} = \frac{(3)^2-4(3)+3}{(3)^2-2(3)-3}$$

$$= \frac{12-12}{(-9)+9}$$

$$= \frac{0}{0}$$ [Since, it is of the form indeterminate]

By using factorization method:

$$\lim_{x\to 3} \frac{x^2-4x+3}{x^2-2x-3} = \lim_{x\to 3} \frac{(x^2-4x+3)}{(x^2-2x-3)}$$

$$= \lim_{x\to 3} \frac{(x^2-3x-x+3)}{(x^2-3x+x-3)}$$

$$= \lim_{x\to 3} \frac{x(x-3)-1(x-3)}{x(x-3)+1(x-3)}$$

$$= \lim_{x\to 3} \frac{(x-3)(x-1)}{(x-3)(x+1)}$$

$$= \lim_{x\to 3} \frac{(x-1)}{(x+1)}$$

$$= \frac{(3-1)}{(3+1)}$$

$$= 2/4$$

$$= 1/2$$

∴ The value of the given limit is ½.

3. $\lim_{x\to3} \frac{x^4-81}{x^2-9}$

Solution:

Given:

The limit $\lim_{x\to3} \frac{x^4-81}{x^2-9}$

By substituting the value of x, we get

$$\lim_{x\to3} \frac{x^4-81}{x^2-9} = \frac{(3)^4-81}{(3)^2-9}$$

$$= \frac{81-81}{(-9)+9}$$

$$= \frac{0}{0}$$ [Since, it is of the form indeterminate]

By using factorization method:

$$\lim_{x\to3} \frac{x^4-81}{x^2-9} = \lim_{x\to3} \frac{(x^4-81)}{(x^2-9)}$$

$$= \lim_{x\to3} \frac{(x^4-3^4)}{(x^2-3^2)}$$

$$= \lim_{x\to3} \frac{((x^2)^2-(3^2)^2)}{(x^2-3^2)}$$ [Since $a^2 - b^2 = (a + b)(a - b)$]

So,

$$= \lim_{x\to3} \frac{(x^2-3^2)(x^2+3^2)}{(x^2-3^2)}$$

$$= \lim_{x\to3}(x^2+3^2)$$

$$= 3^2 + 3^2$$

$$= 18$$

∴ The value of the given limit is 18.

4. $\lim_{x\to2} \frac{x^3-8}{x^2-4}$

Solution:

Given:

The limit $\lim_{x\to2} \frac{x^3-8}{x^2-4}$

By substituting the value of x, we get

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4} = \frac{(2)^3-8}{(2)^2-4}$$

$$= \frac{8-8}{(4)-4}$$

$$= \frac{0}{0}$$ [Since, it is of the form indeterminate]

By using factorization method:

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4} = \lim_{x\to 2}\frac{(x^3-8)}{(x^2-4)}$$

$$= \lim_{x\to 2}\frac{(x^3-2^3)}{(x^2-2^2)}$$

$$= \lim_{x\to 2}\frac{(x-2)(x^2+2^2+2x)}{(x+2)(x-2)}$$

[By using the formula, $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$ & $(a^2 - b^2) = (a + b)(a - b)$]

$$= \lim_{x\to 2}\frac{(x^2+2^2+2x)}{(x+2)}$$

$$= \frac{(2^2+2^2+2(2))}{(2+2)}$$

$$= \frac{3.4}{(4)}$$

$= 3$

∴ The value of the given limit is 3.

5. $\lim_{x\to -1/2}\frac{8x^3+1}{2x+1}$

Solution:

Given:

The limit $\lim_{x\to -1/2}\frac{8x^3+1}{2x+1}$

By substituting the value of x, we get

$$\lim_{x\to -1/2}\frac{8x^3+1}{2x+1} = \frac{8\left(-\frac{1}{2}\right)^3+1}{2\left(-\frac{1}{2}\right)+1}$$

$$= \frac{-1+1}{-1+1}$$

$$= \frac{0}{0}$$ [Since, it is of the form indeterminate]

By using factorization method:

$$\lim_{x \to -1/2} \frac{8x^3+1}{2x+1} = \lim_{x \to -\frac{1}{2}} \frac{8x^3 + 1}{2x + 1}$$

$$= \lim_{x \to -\frac{1}{2}} \frac{(2x)^3 + (1)^3}{2x + 1}$$

[By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

$$= \lim_{x \to -\frac{1}{2}} \frac{(2x + 1)((2x)^2 + (1)^2 - 2x)}{2x + 1}$$

$$= \lim_{x \to -\frac{1}{2}} (2x)^2 + (1)^2 - 2x$$

$$= (2(\frac{-1}{2}))^2 + (1)^2 - 2(-\frac{1}{2})$$

$$= 1 + 1 + 1$$

$$= 3$$

∴ The value of the given limit is 3.