EXERCISE 29.4 PAGE NO: 29.28

Evaluate the following limits:

1. $\lim_{x\to 0} \frac{\sqrt{1+x+x^2}-1}{x}$

Solution:

Given:

The limit $\lim_{x\to 0} \frac{\sqrt{1+x+x^2}-1}{x}$

We need to find the limit of the given equation when x => 0

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x\to 0} \frac{\sqrt{1+x+x^2}-1}{x} = \lim_{x\to 0} \frac{(\sqrt{1+x+x^2}-1)}{x} \frac{(\sqrt{1+x+x^2}+1)}{(\sqrt{1+x+x^2}+1)}$$

[By using the formula: $(a+b)(a-b) = a^2 - b^2$]

$$= \lim_{x\to 0} \frac{1+x+x^2-1}{x(\sqrt{1+x+x^2}+1)}$$

$$= \lim_{x\to 0} \frac{x(1+x)}{x(\sqrt{1+x+x^2}+1)}$$

$$= \lim_{x\to 0} \frac{(1+x)}{(\sqrt{1+x+x^2}+1)}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\lim_{x\to 0} \frac{\sqrt{1+x+x^2}-1}{x} = \frac{1}{1+1}$$

$$= \frac{1}{2}$$

∴ The value of the given limit is ½.

2. $\lim_{x\to 0} \frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$

Solution:

Given:

The limit $\lim_{x\to 0} \frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$

We need to find the limit of the given equation when x => 0

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}} = \lim_{x\to 0}\frac{2x}{(\sqrt{a+x}-\sqrt{a-x})}\frac{(\sqrt{a+x}+\sqrt{a-x})}{(\sqrt{a+x}+\sqrt{a-x})}$$

[By using the formula: $(a + b)(a - b) = a^2 - b^2$]

$$= \lim_{x\to 0}\frac{2x(\sqrt{a+x}+\sqrt{a-x})}{a+x-a+x}$$

$$= \lim_{x\to 0}\frac{2x(\sqrt{a+x}+\sqrt{a-x})}{2x}$$

$$= \lim_{x\to 0}\frac{(\sqrt{a+x}+\sqrt{a-x})}{1}$$

Now we can see that the indeterminate form is removed,
So, now we can substitute the value of x as 0, we get

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}} = \sqrt{a}+\sqrt{a}$$

$$= 2\sqrt{a}$$

∴ The value of the given limit is $2\sqrt{a}$

3. $\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2}$

Solution:

Given:

The limit $\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2}$

We need to find the limit of the given equation when x => 0

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2} = \lim_{x\to 0}\frac{(\sqrt{a^2+x^2}-a)}{x^2}\frac{(\sqrt{a^2+x^2}+a)}{(\sqrt{a^2+x^2}+a)}$$

[By using the formula: $(a + b)(a - b) = a^2 - b^2$]

$$= \lim_{x\to 0}\frac{(a^2+x^2-a^2)}{x^2(\sqrt{a^2+x^2}+a)}$$

$$= \lim_{x\to 0}\frac{x^2}{x^2(\sqrt{a^2+x^2}+a)}$$

$$= \lim_{x\to 0}\frac{1}{(\sqrt{a^2+x^2}+a)}$$

Now we can see that the indeterminate form is removed,
So, now we can substitute the value of x as 0, we get

$$\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2}=\frac{1}{a+a}$$

$$=\frac{1}{2a}$$

∴ The value of the given limit is $\frac{1}{2a}$.

4. $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$

Solution:

Given:

The limit $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$

We need to find the limit of the given equation when x => 0

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{(1-x)}}{2x}=\lim_{x\to 0}\frac{\left(\sqrt{1+x}-\sqrt{(1-x)}\right)}{2x}\frac{\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}{\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}$$

[By using the formula: (a + b) (a - b) = a^2 - b^2]

$$=\lim_{x\to 0}\frac{1+x-1+x}{2x\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}$$

$$=\lim_{x\to 0}\frac{2x}{2x\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}$$

Now we can see that the indeterminate form is removed,
So, now we can substitute the value of x as 0, we get

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{(1-x)}}{2x}=\frac{1}{1+1}$$

$$=\frac{1}{2}$$

∴ The value of the given limit is ½.

5. $\lim_{x\to 2} \frac{\sqrt{3-x}-1}{2-x}$

Solution:

Given:

The limit $\lim_{x\to 2} \frac{\sqrt{3-x}-1}{2-x}$

We need to find the limit of the given equation when x => 0

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x\to 2}\frac{\sqrt{3-x}-1}{2-x} = \lim_{x\to 2}\frac{(\sqrt{3-x}-1)}{(2-x)}\frac{(\sqrt{3-x}+1)}{(\sqrt{3-x}+1)}$$

[By using the formula: (a + b) (a - b) = a^2 - b^2]

$$= \lim_{x\to 2}\frac{(3-x-1)}{(2-x)(\sqrt{3-x}+1)}$$

$$= \lim_{x\to 2}\frac{(2-x)}{(2-x)(\sqrt{3-x}+1)}$$

$$= \lim_{x\to 2}\frac{1}{(\sqrt{3-x}+1)}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\lim_{x\to 2}\frac{\sqrt{3-x}-1}{2-x} = \frac{1}{1+1}$$

$$= \frac{1}{2}$$

∴ The value of the given limit is ½.