EXERCISE 29.5

PAGE NO: 29.33

Evaluate the following limits:

1. $\lim_{x\to a}\frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$

Solution:

Given:

The limit $\lim_{x\to a}\frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$

When $x = a$, the expression $\lim_{x\to a}\frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$ assumes the form (0/0).

So let, $Z = \lim_{x\to a}\frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$

By using the formula: $\lim_{x\to a}\frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$$Z = \lim_{x\to a}\frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$$

$$Z = \lim_{x\to a}\frac{(x+2)^{5/2}-(a+2)^{5/2}}{x+2-(a+2)}$$

Let $x + 2 = y$ and $a + 2 = k$

As $x \to a$; $y \to k$

So,

$$Z = \lim_{y\to k}\frac{(y)^{5/2}-(k)^{5/2}}{y-k}$$

By using the formula: $\lim_{x\to a}\frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

$$Z = \frac{5}{2}k^{\frac{5}{2}-1}$$

$$= \frac{5}{2}k^{\frac{3}{2}}$$

$$= \frac{5}{2}(a+2)^{\frac{3}{2}}$$

$$\therefore \lim_{x\to a}\frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a} = \frac{5}{2}(a+2)^{\frac{3}{2}}$$

2. $\lim_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

Solution:

Given:

The limit $\lim_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

When $x = a$, the expression $\lim_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$ assumes the form (0/0).

So let, $Z = \lim_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

By using the formula: $\lim_{x\to a}\frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$Z = \lim_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

$Z = \lim_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x+2-(a+2)}$

Let $x + 2 = y$ and $a+2 = k$

As $x \to a$; $y \to k$

$Z = \lim_{y\to k}\frac{(y)^{3/2}-(k)^{3/2}}{y-k}$

By using the formula: $\lim_{x\to a}\frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

$Z = \frac{3}{2}k^{\frac{3}{2}-1}$

$= \frac{3}{2}k^{\frac{1}{2}}$

$= \frac{3}{2}(a+2)^{\frac{1}{2}}$

$\therefore \lim_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a} = \frac{3}{2}\sqrt{a+2}$

3. $\lim_{x\to a}\frac{(1+x)^6-1}{(1+x)^2-1}$

Solution:

Given:

The limit $\lim_{x\to a}\frac{(1+x)^6-1}{(1+x)^2-1}$

When x = a, the expression $\lim_{x\to a}\frac{(1+x)^6-1}{(1+x)^2-1}$ assumes the form (0/0).

So let, Z = $\lim_{x\to a}\frac{(1+x)^6-1}{(1+x)^2-1}$

Z = $\frac{(1+a)^6-1}{(1+a)^2-1} = \frac{\{(1+a)^2\}^3-1}{(1+a)^2-1}$

[This can be further simplified using the formula: $a^3 - 1 = (a-1)(a^2 + a + 1)$]

Z = $\frac{\{(1+a)^2-1\}((1+a)^4+(1+a)^2+1)}{(1+a)^2-1}$

$= (1+a)^4 + (1+a)^2 + 1$

$= 1 + 1 + 1$

$= 3$

$\therefore \lim_{x\to a}\frac{(1+x)^6-1}{(1+x)^2-1} = 3$

4. $\lim_{x\to a}\frac{x^{2/7}-a^{2/7}}{x-a}$

Solution:

Given:

The limit $\lim_{x\to a}\frac{x^{2/7}-a^{2/7}}{x-a}$

When x = a, the expression $\lim_{x\to a}\frac{x^{2/7}-a^{2/7}}{x-a}$ assumes the form (0/0).

So let, Z = $\lim_{x\to a}\frac{x^{2/7}-a^{2/7}}{x-a}$

By using the formula: $\lim_{x\to a}\frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

Z = $\lim_{x\to a}\frac{x^{2/7}-a^{2/7}}{x-a}$

By using the formula: $\lim_{x\to a}\frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

Z = $\frac{2}{7}\, a^{\frac{2}{7}-1}$

$= \frac{2}{7}\, a^{-\frac{5}{7}}$

$\therefore \lim_{x\to a}\frac{x^{2/7}-a^{2/7}}{x-a} = \frac{2}{7}\, a^{-\frac{5}{7}}$

5. $\lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

Solution:

Given: $\lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

The limit $\lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

When x = a, the expression $\lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$ assumes the form (0/0).

So let, $Z = \lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

By using the formula: $\lim_{x \to a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$$Z = \lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}}$$

Let us divide the numerator and denominator by (x – a), we get

$$Z = \lim_{x \to a} \frac{\frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}}$$

By using algebra of limits, we have

$$Z = \frac{\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}}$$

So now again, by using the formula: $\lim_{x \to a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

$$Z = \frac{\frac{5}{7} a^{\frac{5}{7} - 1}}{\frac{2}{7} a^{\frac{2}{7} - 1}}$$

$$= \frac{5a^{-\frac{2}{7}}}{2a^{-\frac{5}{7}}}$$

$$= \frac{5}{2} a^{\frac{3}{7}}$$

$$\therefore \lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}} = \frac{5}{2} a^{\frac{3}{7}}$$