EXERCISE 29.7 PAGE NO: 29.49

Evaluate the following limits:

1. $\lim_{x\to 0}\frac{\sin 3x}{5x}$

Solution:

Given:

The limit $\lim_{x\to 0}\frac{\sin 3x}{5x}$

Let us consider the limit:

$$\lim_{x\to 0}\frac{\sin 3x}{5x} = \frac{1}{5}\lim_{x\to 0}\frac{\sin 3x}{x}$$

Now let us multiply and divide the expression by 3, we get

$$= \frac{1}{5}\lim_{x\to 0}\frac{\sin 3x}{3x}\times 3$$

$$= \frac{3}{5}\lim_{x\to 0}\frac{\sin 3x}{3x}$$

Now, put $3x = y$

$$= \frac{3}{5}\lim_{y\to 0}\frac{\sin y}{y} \quad \left[\text{We know that, } \lim_{y\to 0}\frac{\sin y}{y} = 1\right]$$

So,

$$\lim_{x\to 0}\frac{\sin 3x}{5x} = \frac{3}{5}\lim_{y\to 0}\frac{\sin y}{y}$$

$$= \frac{3}{5}\times 1$$

$$= \frac{3}{5}$$

∴ The value of $\lim_{x\to 0}\frac{\sin 3x}{5x} = \frac{3}{5}$

2. $\lim_{x\to 0}\frac{\sin x^0}{x}$

Solution:

Given:

The limit $\lim_{x\to 0}\frac{\sin x^0}{x}$

We know, $1° = \frac{\pi}{180}$ radians

So,

$x^\circ = \frac{\pi x}{180}$ radians

Let us consider the limit,

$$\lim_{x\to 0}\frac{\sin x^\circ}{x} = \lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{x}$$

Now let us multiply and divide the expression by $\frac{\pi}{180}$, we get

$$= \lim_{x\to 0}\frac{\sin\frac{\pi x}{180}\times\frac{\pi}{180}}{x\times\frac{\pi}{180}}$$

$$= \frac{\pi}{180}\lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}}$$

Now, put $\frac{\pi x}{180} = y$

$$= \frac{\pi}{180}\lim_{y\to 0}\frac{\sin y}{y}$$ [We know that, $\lim_{y\to 0}\frac{\sin y}{y} = 1$]

$$\lim_{x\to 0}\frac{\sin x^\circ}{x} = \frac{\pi}{180}\lim_{y\to 0}\frac{\sin y}{y}$$

$$= \frac{\pi}{180}\times 1$$

$$= \frac{\pi}{180}$$

∴ The value of $\lim_{x\to 0}\frac{\sin x^\circ}{x} = \frac{\pi}{180}$

3. $\lim_{x\to 0}\frac{x^2}{\sin x^2}$

Solution:

Given:

The limit $\lim_{x\to 0}\frac{x^2}{\sin x^2}$

Let us consider the limit and divide the expression by x^2, we get

$$\lim_{x\to 0}\frac{x^2}{\sin x^2} = \lim_{x\to 0}\frac{1}{\frac{\sin x^2}{x^2}}$$

Now, put $x^2 = y$

$$\lim_{x\to 0}\frac{1}{\frac{\sin x^2}{x^2}} = \frac{1}{\lim_{y\to 0}\frac{\sin y}{y}}$$ [We know that, $\lim_{y\to 0}\frac{\sin y}{y} = 1$]

$$= \frac{1}{1}$$

$$=1$$

∴ The value of $\lim_{x\to 0}\frac{x^2}{\sin x^2} = 1$

4. $\lim_{x\to 0}\frac{\sin x\ \cos x}{3x}$

Solution:

Given:

The limit $\lim_{x\to 0}\frac{\sin x\ \cos x}{3x}$

Let us consider the limit

$$\lim_{x\to 0}\frac{\sin x\ \cos x}{3x} = \frac{1}{3}\lim_{x\to 0}\frac{\sin x \cos x}{x}$$

$$= \frac{1}{3}\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\cos x$$

We know,

$$\lim_{x\to 0} A(x).B(x) = \lim_{x\to 0} A(x) \times \lim_{x\to 0} B(x)$$

So,

$$= \frac{1}{3}\lim_{x\to 0}\frac{\sin x}{x} \times \lim_{x\to 0}\cos x$$ [We know that, $\lim_{x\to 0}\frac{\sin x}{x} = 1$]

$$\lim_{x\to 0}\frac{\sin x \cos x}{3x} = \frac{1}{3} \times 1 \times \cos 0$$

$$= \frac{1}{3} \times 1 \times 1$$ [Since, cos 0 = 1]

$$= \frac{1}{3}$$

∴ The value of $\lim_{x\to 0}\frac{\sin x \cos x}{3x} = \frac{1}{3}$

5. $\lim_{x\to 0} \frac{3\sin x - 4\sin^3 x}{x}$

Solution:

Given:

The limit $\lim_{x\to 0} \frac{3\sin x - 4\sin^3 x}{x}$

We know that, $\sin 3x = 3\sin x - 4\sin^3 x$

So,

$$\lim_{x\to 0} \frac{3\sin x - 4\sin^3 x}{x} = \lim_{x\to 0} \frac{\sin 3x}{x}$$

Now multiply and divide the expression by 3, we get

$$\lim_{x\to 0} \frac{\sin 3x}{x} = \lim_{x\to 0} \frac{\sin 3x \times 3}{3x}$$

$$= 3\lim_{x\to 0} \frac{\sin 3x}{3x}$$

Now, put $3x = y$

$$= 3\lim_{y\to 0} \frac{\sin y}{y} \quad \left[\text{We know that, } \lim_{y\to 0} \frac{\sin y}{y} = 1\right]$$

$$\lim_{x\to 0} \frac{3\sin x - 4\sin^3 x}{x} = 3\lim_{y\to 0} \frac{\sin y}{y}$$

$$= 3 \times 1$$

$$= 3$$

$\therefore$ The value of $\lim_{x\to 0} \frac{3\sin x - 4\sin^3 x}{x} = 3$