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Differentiate the following functions with respect to x:

1. $\frac{x^2+1}{x+1}$

Solution:

Let us consider

$y = \frac{x^2+1}{x+1}$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$u = x^2 + 1$ and $v = x + 1$

$\therefore y = u/v$

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ … Equation (1)

As, $u = x^2 + 1$

$\frac{du}{dx} = 2x^{2-1} + 0 = 2x$ … Equation (2) {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

As, $v = x + 1$

$\frac{dv}{dx} = \frac{d}{dx}(x + 1) = 1$ … Equation (3) {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

Now, from equation 1, we can find dy/dx

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(x+1)(2x)-(x^2+1)(1)}{(x+1)^2}$ {Using equation 2 and 3}

$= \frac{2x^2+2x-x^2-1}{(x+1)^2}$

$= \frac{x^2+2x-1}{(x+1)^2}$

$\therefore \frac{dy}{dx} = \frac{x^2 + 2x - 1}{(x+1)^2}$

2. $\frac{2x-1}{x^2+1}$

Solution:

Let us consider

$y = \frac{2x-1}{x^2+1}$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$u = 2x - 1$ and $v = x^2 + 1$

$\therefore y = u/v$

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \text{ ... Equation (1)}$$

As, $u = 2x - 1$

$$\frac{du}{dx} = 2x^{1-1} - 0 = 2 \text{ ... Equation (2) } \{\text{Since, } \frac{d}{dx}(x^n) = nx^{n-1}\}$$

As, $v = x^2 + 1$

$$\frac{dv}{dx} = \frac{d}{dx}(x^2 + 1) = 2x \text{ ... Equation (3) } \{\text{Since, } \frac{d}{dx}(x^n) = nx^{n-1}\}$$

Now, from equation 1, we can find dy/dx

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(x^2+1)(2) - (2x-1)(2x)}{(x^2+1)^2} \{\text{Using equation 2 and 3}\}$$

$$= \frac{2x^2 + 2 - 4x^2 + 2x}{(x^2+1)^2}$$

$$= \frac{-2x^2 + 2x + 2}{(x^2+1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{2(1 + x - x^2)}{(x^2+1)^2}$$

3. $\frac{x + e^x}{1 + \log x}$

Solution:

Let us consider

$y = \frac{x + e^x}{1 + \log x}$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$u = x + e^x$ and $v = 1 + \log x$

$\therefore y = u/v$

Now let us apply quotient rule of differentiation.
By using quotient rule, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ … Equation 1

As, $u = x + e^x$

$\frac{du}{dx} = \frac{d}{dx}(x + e^x)$ {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}(e^x) = e^x$}

$\frac{du}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(e^x) = 1 + e^x$ … Equation 2

As, $v = 1 + \log x$

$\frac{dv}{dx} = \frac{d}{dx}(\log x + 1)$

$= \frac{d}{dx}(1) + \frac{d}{dx}(\log x)$

$\frac{dv}{dx} = 0 + \frac{1}{x} = \frac{1}{x}$ … Equation 3 {Since, $\frac{d}{dx}(\log x) = \frac{1}{x}$ }

Now, from equation 1, we can find dy/dx

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{dy}{dx} = \frac{(1 + \log x)(1 + e^x) - (x + e^x)\left(\frac{1}{x}\right)}{(\log x + 1)^2}$ {Using equation 2 and 3}

$= \frac{1 + e^x + \log x + e^x \log x - 1 - \frac{e^x}{x}}{(\log x + 1)^2}$

$= \frac{x\log x(1 + e^x) + e^x(x-1)}{x(\log x + 1)^2}$

$\therefore \frac{dy}{dx} = \frac{x \log x(1 + e^x) - e^x(1 - x)}{x(1 + \log x)^2}$

4. $\frac{e^x - \tan x}{\cot x - x^n}$

Solution:

Let us consider

$y = \frac{e^x - \tan x}{\cot x - x^n}$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$u = e^x - \tan x$ and $v = \cot x - x^n$

$\therefore y = u/v$

Now let us apply quotient rule of differentiation.
By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \ldots \text{Equation (1)}$$

As, $u = e^x - \tan x$

$$\frac{du}{dx} = \frac{d}{dx}(e^x - \tan x) \quad \left\{\text{Since, } \frac{d}{dx}(\tan x) = \sec^2 x \;\&\; \frac{d}{dx}(e^x) = e^x\right\}$$

$$\frac{du}{dx} = -\frac{d}{dx}(\tan x) + \frac{d}{dx}(e^x) = \sec^2 x + e^x \quad \ldots \text{Equation (2)}$$

As, $v = \cot x - x^n$

$$\frac{dv}{dx} = \frac{d}{dx}(\cot x - x^n)$$

$$= \frac{d}{dx}(\cot x) - \frac{d}{dx}(x^n) \quad \left\{\text{Since, } \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \;\&\; \frac{d}{dx}(x^n) = nx^{n-1}\right\}$$

$$\frac{dv}{dx} = -\operatorname{cosec}^2 x - nx^{n-1} \quad \ldots \text{Equation (3)}$$

Now, from equation 1, we can find dy/dx

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \{\text{Using equation 2 and 3, we get}\}$$

$$\frac{dy}{dx} = \frac{(\cot x - x^n)(\sec^2 x + e^x) - (e^x - \tan x)(-\operatorname{cosec}^2 x - nx^{n-1})}{(\cot x - x^n)^2}$$

$$\therefore \frac{dy}{dx} = \frac{(\cot x - x^n)(e^x - \sec^2 x) + (e^x - \tan x)(\operatorname{cosec}^2 x + nx^{n-1})}{(\cot x - x^n)^2}$$

5. $\dfrac{ax^2 + bx + c}{px^2 + qx + r}$

Solution:

Let us consider

$$y = \frac{ax^2 + bx + c}{px^2 + qx + r}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$u = ax^2 + bx + c$ and $v = px^2 + qx + r$

$\therefore y = u/v$

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ … Equation (1)

As, $u = ax^2 + bx + c$

$\frac{du}{dx} = 2ax + b$ … Equation (2) {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

As, $v = px^2 + qx + r$

$\frac{dv}{dx} = \frac{d}{dx}(px^2 + qx + r) = 2px + q$ … Equation (3)

Now, from equation 1, we can find dy/dx

$$\frac{dy}{dx} = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$= \frac{(px^2 + qx + r)(2ax + b) - (ax^2 + bx + c)(2px + q)}{(px^2 + qx + r)^2}$$ {Using equation 2 and 3}

$$= \frac{2apx^3 + bpx^2 + 2aqx^2 + bqx + 2arx + br - 2apx^3 - aqx^2 - 2bpx^2 - bqx - 2pcx - qc}{(px^2 + qx + r)^2}$$

$$= \frac{aqx^2 - bpx^2 + 2arx + br - 2pcx - qc}{(px^2 + qx + r)^2}$$

$$= \frac{x^2(aq - bp) + 2x(ar - pc) + br - qc}{(px^2 + qx + r)^2}$$

$$\therefore \frac{dy}{dx} = \frac{x^2(aq - bp) + 2x(ar - pc) + br - qc}{(px^2 + qx + r)^2}$$