EXERCISE 32.4

1. Find the mean, variance and standard deviation for the following data:
(i) 2, 4, 5, 6, 8, 17

Let Mean be,

$\overline{X} = \frac{2+4+5+6+8+17}{6}$

$\overline{X} = \frac{42}{6} = 7$

X_i	$(x_i\text{-}X) = (x_i\text{-}7)$	$(x_i\text{-}7)^2$
2	-3	25
4	-3	9
5	-2	4
6	-1	1
8	1	1
17	10	100
		$\sum_1^6 (x_i - \bar{X})^2 = 140$

N = 6

Variance (X) = $\frac{1}{n} \sum_{i=1}^{6} (x_i - \overline{X})$

= 140/6

= 23.33

Variance = 23.33

Standard deviation = $\sqrt{\text{Var}(X)}$

$\sigma = \sqrt{23.33}$

Standard deviation = 4.83

(ii) 6, 7, 10, 12, 13, 4, 8, 12

Let Mean be,

$\overline{X} = \frac{6+7+10+12+13+4+8+12}{8}$

$\overline{X} = \frac{72}{8} = 9$

x_i	$(x_i-X)=(x_i-7)$	$(x_i-7)^2$
6	-3	9
7	-2	4
10	1	1
12	3	9
13	4	16
4	-5	25
12	3	9
		$\sum_{1}^{8}(x_i-\bar{X})^2=74$

N = 8

Variance (X) = $\frac{1}{n}\sum_{i=1}^{8}(x_i-\bar{X})$

= 74/8

= 9.25

Variance = 9.25

Standard deviation = $\sqrt{\text{Var}(X)}$

$\sigma = \sqrt{9.25}$

Standard deviation = 3.04

2. The variance of 20 observations is 4. If each observation is multiplied by 2, find the variance of the resulting observations.

Solution:

Let Assume, $x_1, x_2, x_3, \ldots, x_{20}$ be the given observations.

Given: Variance (X) = 5

$X = \frac{1}{n}\times\sum(x_i-\bar{X})^2$

Now, Let $u_1, u_2, \ldots u_{20}$ be the new observation,

When we multiply the new observation by 2, then

$U_i=2x_i$ (for i=1, 2, 3…, 20) …. (i)

Now,

Mean:

$\bar{U} = \frac{\sum_{1}^{20} U_i}{n}$

$= \frac{\sum_{i=1}^{20} 2x_i}{20}$

Mean = $2\bar{X}$

Since, $u_i - \bar{U} = 2x_i - 2\bar{X}$

$= 2(x_i - \overline{X})$

Now, $(u_i - \overline{U})^2 = (2(x_i - \overline{X}))$

$4(x_i - \overline{X})^2$

Comparing both the observations

$$\frac{\sum_{20}^{i=1}(u_i - \overline{U})^2}{20} = \frac{\sum_{20}^{i=1} 4(x_i - \overline{X})^2}{20}$$

$$= 4 \times \frac{\sum_{20}^{i=1}(x_i - \overline{X})^2}{20}$$

Variance (U) = 4×Variance (X)

= 4×5

= 20

∴ The variance of new observations is 20.

3. The variance of 15 observations is 4. If each observation is increased by 9, find the variance of the resulting observations.

Solution:

Let Assume, $x_1, x_2, x_3, \ldots, x_{15}$ be the given observations.

Given: Variance (X) = 4

$X = \frac{1}{n} \times \sum(x_i - \overline{X})^2$

Now, Let $u_1, u_2, \ldots u_{20}$ be the new observation,

When new observation increase by 9, then

$U_i = x_i + 9$ (for i=1, 2, 3…, 20) …. (i)

Now,

$$\overline{U} = \frac{1}{n}\sum_{i=1}^{15} u_i$$

$$= \frac{1}{15}\sum_{i=1}^{15}(x_i + 9)$$

$$= \frac{1}{15}\sum_{i=1}^{15} x_i + \frac{9 \times 15}{15}$$

$\overline{U} = 9 + \overline{X}$

$u_i - \overline{U} = (x_i + 9) - (9 + \overline{X})$

$u_i - \overline{U} = x_i - \overline{X}$

$$\frac{\sum_{i=1}^{15}(u_i - \overline{U})^2}{15} = \frac{\sum_{i=1}^{15} 4(x_i - \overline{X})^2}{15}$$

$$= \frac{\sum_{i=1}^{15}(u_i - \overline{U})^2}{15} = 4$$

Variance (U) = 4

∴ The variance of new observations is 4.

4. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6, find the other two observations.

Solution:

Let x and y be the other two observation. And Mean is 4.4

Let Mean = $\frac{1+2+6+x+y}{5} = 4.4$

=> $9 + x + y = 22$

$x + y = 13$ …… (1)

Now, Let Variance (X) is the variance of this observation which is to be 8.24

If $\overline{X}$ is the mean than we get,

$8.24 = \frac{1}{5}(1^2 + 2^2 + 6^2 + x^2 + y^2) - (\bar{x})^2$

$8.24 = \frac{1}{5}(1^2 + 2^2 + 6^2 + x^2 + y^2) - (4.4)^2$

$8.24 = \frac{1}{5}(41 + x^2 + y^2) - 19.36$

$x^2 + y^2 = 97$ …… (2)

$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

By substituting the value we get,

$13^2 + (x - y)^2 = 2 \times 97$

$(x - y)^2 = 194 - 169$

$(x - y)^2 = 25$

$x - y = \pm 5$ …… (3)

On solving equations (1) and (3) we get,

$2x = 18$

$x = 9$ and $y = 4$

∴ The other two observations are 9 and 4.

5. The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution:

Let Assume, $x_1, x_2, x_3, \ldots, x_6$ be the given observations.
Given: Variance (X) = 8
N = 6 and $\sigma = 4$ (SD)

$X = \frac{1}{n} \times \sum x_i$

$$8 = \frac{1}{6} \times \sum_{i=1}^{6} x_i$$

Now, Let $u_1, u_2, \ldots u_{20}$ be the new observation,
When we multiply the new observation by 3, then
$U_i = 3x_i$ (for i =1, 2, 3…, 6) …… (1)
Now,

$$\bar{U} = \frac{1}{n} \sum_{i=1}^{15} u_i$$

$$= \frac{1}{6} \sum_{i=1}^{6} (3x_i)$$

$$= 3 \times \frac{1}{6} \sum_{i=1}^{6} (x_i)$$

$\bar{U} = 3\bar{X}$

$= 3 \times 8 = 24$

U = 24
So, the Mean of new observation is 24
Now,
Standard Deviation $\sigma_x = 4$
$\sigma_x^2 = \text{Variance X}$
Since, Variance (X) =16

$$\text{Variance (U)} = \frac{1}{6} \sum_{i=1}^{6} (3x_i - 3X)$$

$$= 3^2 \times \frac{1}{6} \times \sum (x_i - X)^2$$

$= 9 \times 16$

$\sigma_u^2 = \text{Variance (U)}$

$\sigma_u^2 = 144$

$\sigma = 12$

∴ The mean of new observation is 24 and Standard deviation of new observation is 12.

6. The mean and variance of 8 observations are 9 and 9.25 respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution:

Let x and y be the other two observation. And Mean is 9

Let Mean = $\frac{6+7+10+12+12+13+x+y}{8} = 9$

=> $60 + x + y = 72$

$x + y = 12$ …… (1)

Now, let Variance (X) be the variance of this observation which is to be 9.25

If $\overline{X}$ is the mean than we get,

$9.25 = \frac{1}{8}(6^2 + 7^2 + 10^2 + 12^2 + 12^2 + 13^2 + x^2 + y^2) - (\bar{x})^2$

$9.25 = \frac{1}{8}(6^2 + 7^2 + 10^2 + 12^2 + 12^2 + 13^2 + x^2 + y^2) - (9)^2$

$642 + x^2 + y^2 = 722$

$x^2 + y^2 = 80$ …. (2)

$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

By substituting the value we get,

$12^2 + (x - y)^2 = 2 \times 80$

$(x - y)^2 = 160 - 144$

$(x - y)^2 = 14$

$X - y = \pm 4$ …… (3)

On solving equations (1) and (3) we get,

x = 8, 4 and y = 4, 8

∴ The other two observations are 8 and 4.