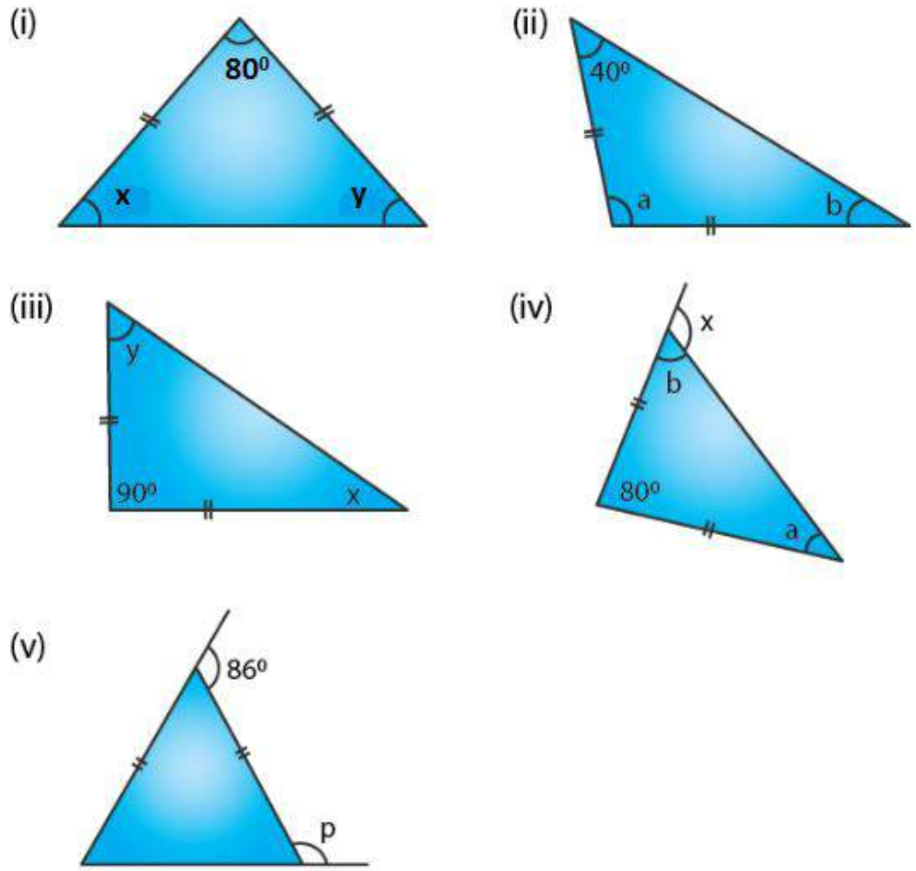
EXERCISE 15B

1. Find the unknown angles in the given figures:



Solution:

(i) From the figure (i)
x = y as the angles opposite to equal sides
In a triangle
$x + y + 80^0 = 180^0$
Substituting the values
$x + x + 80^0 = 180^0$
By further calculation
$2x = 180^0 – 80^0 = 100^0$
$x = 100^0/2 = 50^0$

Therefore, $x = y = 50^0$.

(ii) From the figure (ii)
$b = 40^0$ as the angles opposite to equal sides
In a triangle
$a + b + 40^0 = 180^0$
Substituting the values
$a + 40^0 + 40^0 = 180^0$
By further calculation

$a = 180 – 80 = 100^0$

Therefore, $a = 100^0$ and $b = 40^0$.

(iii) From the figure (iii)
x = y as the angles opposite to equal sides
In a triangle
$x + y + 90^0 = 180^0$
Substituting the values
$x + x + 90^0 = 180^0$
By further calculation
$2x = 180 – 90 = 90^0$
$x = 90/2 = 45^0$

Therefore, $x = y = 45^0$.

(iv) From the figure (iv)
a = b as the angles opposite to equal sides are equal
In a triangle
$a + b + 80^0 = 180^0$
Substituting the values
$a + a + 80^0 = 180^0$
By further calculation
$2a = 180 – 80 = 100^0$
$a = 100^0/2 = 50^0$

Here $a = b = 50^0$
We know that in a triangle the exterior angle is equal to sum of its opposite interior angles
$x = a + 80^0$
So we get
$x = 50 + 80 = 130^0$

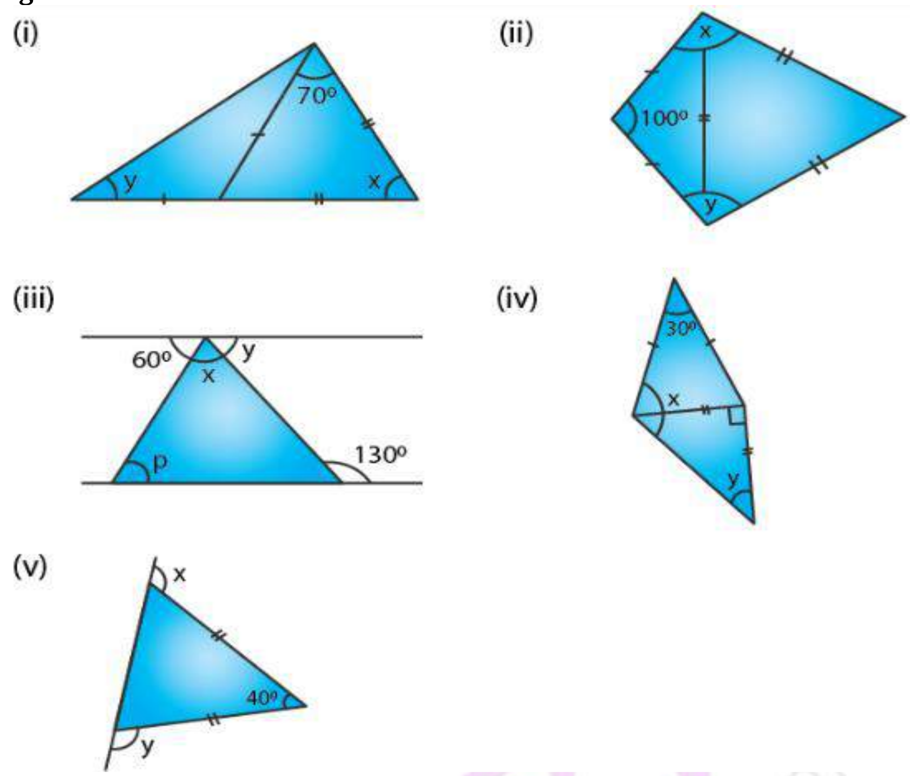
Therefore, $a = 50^0$, $b = 50^0$ and $x = 130^0$.

(v) From the figure (v)
In an isosceles triangle consider each equal angle = x
$x + x = 86^0$
$2x = 86^0$
So we get
$x = 86^0/2 = 43^0$

For a linear pair
$p + x = 180^0$
Substituting the values
$p + 43^0 = 180^0$
By further calculation
$p = 180 – 43 = 137^0$

Therefore, $p = 137^0$.

2. Apply the properties of isosceles and equilateral triangles to find the unknown angles in the given figures:



Solution:

(i) $a = 70^0$ as the angles opposite to equal sides are equal
In a triangle
$a + 70^0 + x = 180^0$
Substituting the values
$70^0 + 70^0 + x = 180^0$
By further calculation
$x = 180 – 140 = 40^0$

y = b as the angles opposite to equal sides are equal
Here a = y + b as the exterior angle is equal to sum of interior opposite angles
$70^0 = y + y$
So we get
$2y = 70^0$
$y = 70^0/2 = 35^0$

Therefore, $x = 40^0$ and $y = 35^0$.

(ii) From the figure (ii)
Each angle is 60^0 in an equilateral triangle
In a isosceles triangle
Consider each base angle = a
$a + a + 100^0 = 180^0$
By further calculation

$2a = 180 – 100 = 80^0$
So we get
$a = 80^0/2 = 40^0$
$x = 60^0 + 40^0 = 100^0$
$y = 60^0 + 40^0 = 100^0$

(iii) From the figure (iii)
$130^0 = x + p$ as the exterior angle is equal to the sum of interior opposite angles
It is given that the lines are parallel
Here $p = 60^0$ is the alternate angles and $y = a$

In a linear pair
$a + 130^0 = 180^0$
By further calculation
$a = 180 – 130 = 50^0$

Here $x + p = 130^0$
Substituting the values
$x + 60^0 = 130^0$
By further calculation
$x = 130 – 60 = 70^0$

Therefore, $x = 70^0$, $y = 50^0$ and $p = 60^0$.

(iv) From the figure (iv)
$x = a + b$
Here $b = y$ and $a = c$ as the angles opposite to equal sides are equal
$a + c + 30^0 = 180^0$
Substituting the values
$a + a + 30^0 = 180^0$
By further calculation
$2a = 180 – 30 = 150^0$
$a = 150/2 = 75^0$

We know that
$b + y = 90^0$
Substituting the values
$y + y = 90^0$
$2y = 90^0$
$y = 90/2 = 45^0$
where $b = 45^0$

Therefore, $x = a + b = 75 + 45 = 120^0$ and $y = 45^0$.

(v) From the figure (v)
$a + b + 40^0 = 180^0$
So we get
$a + b = 180 – 40 = 140^0$

The angles opposite to equal sides are equal

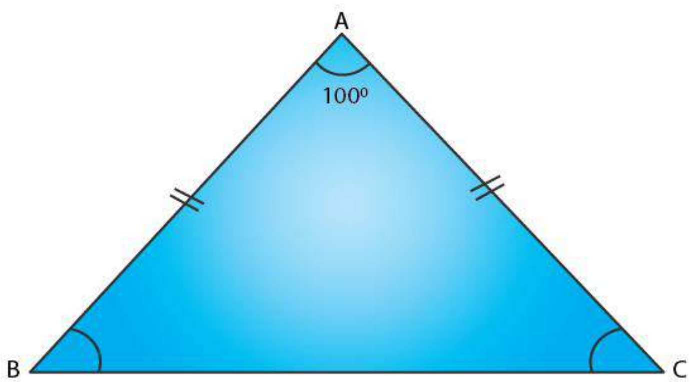
$a = b = 140/2 = 70^0$
$x = b + 40^0 = 70^0 + 40^0 = 110^0$
Here the exterior angle of a triangle is equal to the sum of its interior opposite angles

In the same way
$y = a + 40^0$
Substituting the values
$y = 70^0 + 40^0 = 110^0$

Therefore, $x = y = 110^0$.

3. The angle of vertex of an isosceles triangle is 100°. Find its base angles.
Solution:

Consider Δ ABC



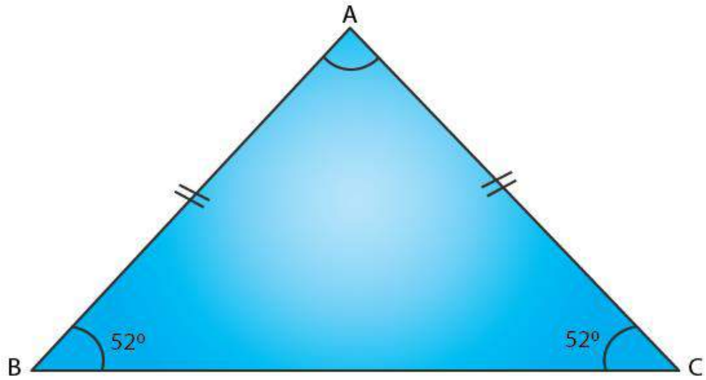
Here AB = AC and ∠B = ∠C
We know that
$\angle A = 100^0$

In a triangle
$\angle A + \angle B + \angle C = 180^0$
Substituting the values
$100^0 + \angle B + \angle B = 180^0$
By further calculation
$2\angle B = 180^0 - 100^0 = 80^0$
$\angle B = 80/2 = 40^0$

Therefore, $\angle B = \angle C = 40^0$.

4. One of the base angles of an isosceles triangle is 52°. Find its angle of vertex.
Solution:

It is given that the base angles of isosceles triangle Δ ABC = 52^0
Here $\angle B = \angle C = 52^0$



In a triangle
$\angle A + \angle B + \angle C = 180^0$
Substituting the values
$\angle A + 52^0 + 52^0 = 180^0$
By further calculation
$\angle A = 180 - 104 = 76^0$

Therefore, $\angle A = 76^0$.

5. In an isosceles triangle, each base angle is four times of its vertical angle. Find all the angles of the triangle.
Solution:

Consider the vertical angle of an isosceles triangle = x
So the base angle – 4x
In a triangle
$x + 4x + 4x = 180^0$
By further calculation
$9x = 180^0$
$x = 180/9 = 20^0$
So the vertical angle = 20^0
Each base angle = $4x = 4 \times 20^0 = 80^0$

6. The vertical angle of an isosceles triangle is 15° more than each of its base angles. Find each angle of the triangle.
Solution:

Consider the angle of the base of isosceles triangle = x^0
So the vertical angle = $x + 15^0$

In a triangle
$x + x + x + 15^0 = 180^0$
By further calculation
$3x = 180 - 15 = 165^0$
$x = 165/3 = 55^0$

Therefore, the base angle = 55^0
Vertical angle = $55 + 15 = 70^0$.

7. The base angle of an isosceles triangle is 15° more than its vertical angle. Find its each angle.
Solution:

Consider the vertical angle of the isosceles triangle = x^0
Here each base angle = $x + 15^0$

In a triangle
$x + 15^0 + x + 15^0 + x = 180^0$
By further calculation
$3x + 30^0 = 180^0$
$3x = 180 – 30 = 150^0$
$x = 150/3 = 50^0$

Therefore, vertical angle = 50^0 and each base angle = $50 + 15 = 65^0$.

8. The vertical angle of an isosceles triangle is three times the sum of its base angles. Find each angle.
Solution:

Consider each base of an isosceles triangle = x
Vertical angle = 3 (x + x) = 3 (2x) = 6x

In a triangle
$6x + x + x = 180^0$
By further calculation
$8x = 180^0$
$x = 180/8 = 22.5^0$

Therefore, each base angle = 22.5^0 and vertical angle = $3 (22.5 + 22.5) = 3 \times 45 = 135^0$.

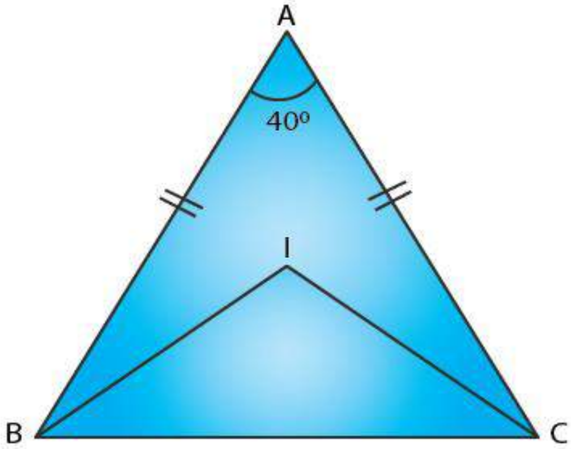
9. The ratio between a base angle and the vertical angle of an isosceles triangle is 1 : 4. Find each angle of the triangle.
Solution:

It is given that the ratio between a base angle and the vertical angle of an isosceles triangle = 1: 4
Consider base angle = x
Vertical angle = 4x

In a triangle
$x + x + 4x = 180^0$
By further calculation
$6x = 180^0$
$x = 180/6 = 30^0$

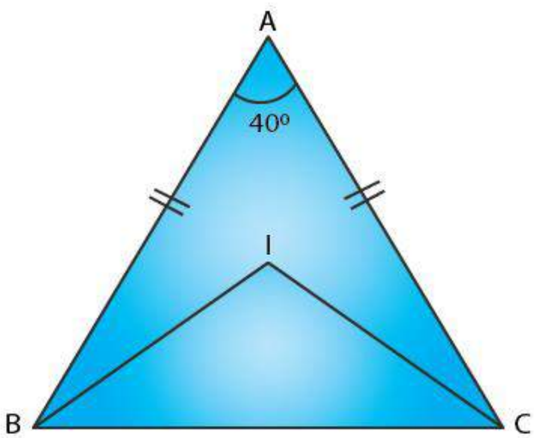
Therefore, each base angle = $x = 30^0$ and vertical angle = $4x = 4 \times 30^0 = 120^0$.

10. In the given figure, BI is the bisector of ∠ABC and CI is the bisector of ∠ACB. Find ∠BIC.



Solution:

In ∆ ABC
BI is the bisector of ∠ABC and CI is the bisector of ∠ACB
Here AB = AC
∠B = ∠C as the angles opposite to equal sides are equal
We know that $\angle A = 40^0$



In a triangle
$\angle A + \angle B + \angle C = 180^0$
Substituting the values
$40^0 + \angle B + \angle B = 180^0$
By further calculation
$40^0 + 2\angle B = 180^0$
$2\angle B = 180 - 40 = 140^0$
$\angle B = 140/2 = 70^0$

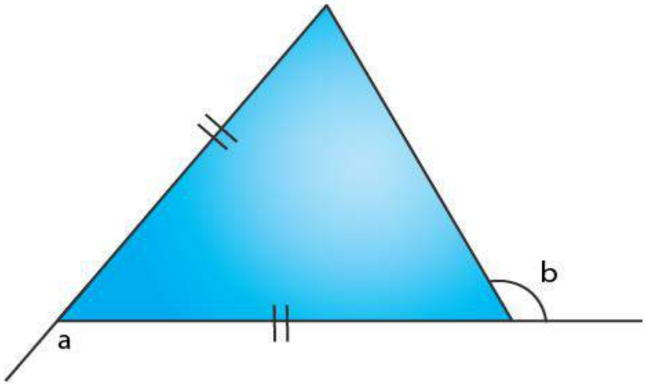
Here BI and CI are the bisectors of ∠ABC and ∠ACB
$\angle IBC = ½ \angle ABC = ½ \times 70^0 = 35^0$
$\angle ICB = ½ \angle ACB = ½ \times 70^0 = 35^0$

In ∆ IBC
$\angle BIC + \angle IBC + \angle ICB = 180^0$
Substituting the values

$\angle BIC + 35^0 + 35^0 = 180^0$
By further calculation
$\angle BIC = 180 - 70 = 110^0$

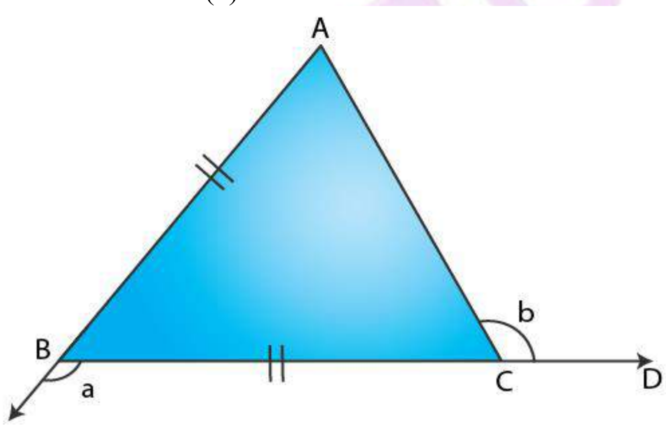
Therefore, $\angle BIC = 110^0$.

11. In the given figure, express a in terms of b.



Solution:

From the Δ ABC
BC = BA
$\angle BCA = \angle BAC$
Here the exterior $\angle CBE = \angle BCA + \angle BAC$
$a = \angle BCA + \angle BCA$
$a = 2\angle BCA$ …… (1)

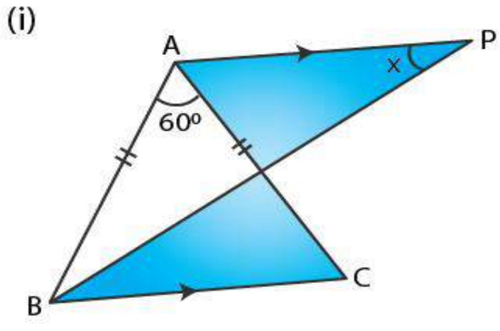


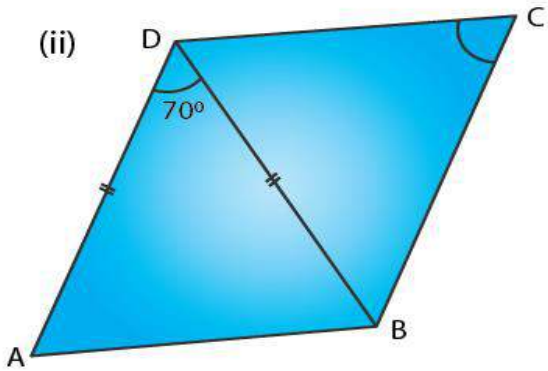
Here $\angle ACB = 180^0 - b$
Where $\angle ACD$ and $\angle ACB$ are linear pair
$\angle BCA = 180^0 - b$ ……. (2)

We get
$a = 2\angle BCA$
Substituting the values
$a = 2(180^0 - b)$
$a = 360^0 - 2b$

12. (a) In Figure (i) BP bisects ∠ABC and AB = AC. Find x.

(b) Find x in Figure (ii) Given: DA = DB = DC, BD bisects ∠ABC and ∠ADB = 70°.





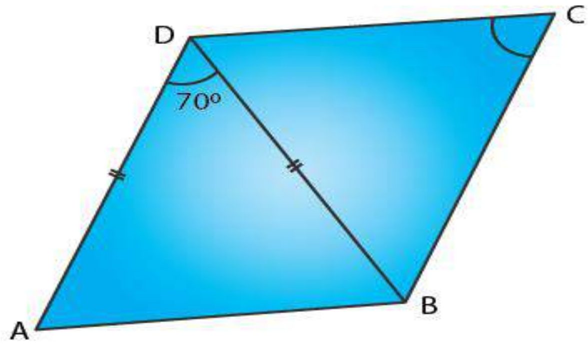
Solution:

(a) From the figure (i)
AB = AC and BP bisects ∠ABC
AP is drawn parallel to BC
Here PB is the bisector of ∠ABC
∠PBC = ∠PBA
∠APB = ∠PBC are alternate angles
x = ∠PBC ….. (1)

In Δ ABC
∠A = 60^0
Since AB = AC we get ∠B = ∠C

In a triangle
∠A + ∠B + ∠C = 180^0
Substituting the values
60^0 + ∠B + ∠C = 180^0
We get
60^0 + ∠B + ∠B = 180^0
By further calculation
2∠B = 180 – 60 = 120^0
∠B = 120/2 = 60^0
½ ∠B = 60/2 = 30^0
∠PBC = 30^0
So from figure (i) x = 30^0

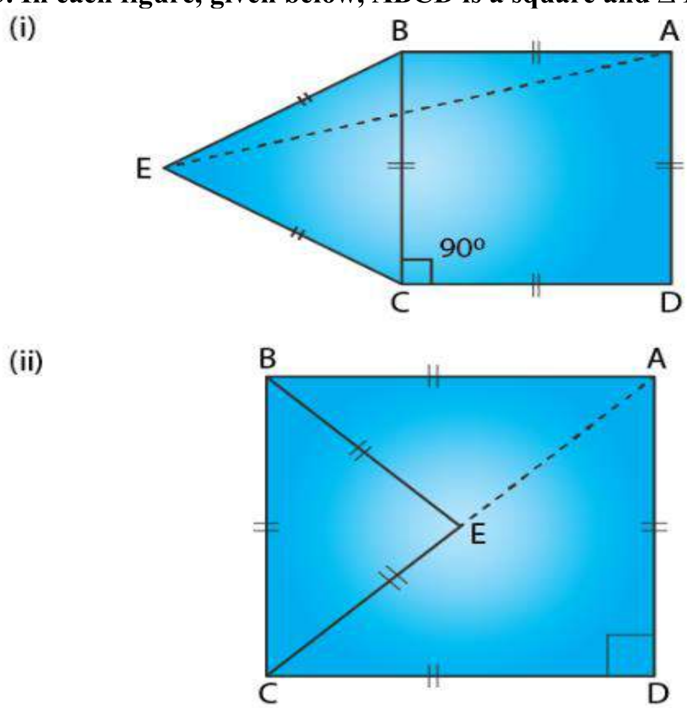
(b) From the figure (ii)
DA = DB = DC
Here BD bisects ∠ABC and ∠ADB = 70^0



In a triangle
∠ADB + ∠DAB + ∠DBA = 180^0
Substituting the values
70^0 + ∠DBA + ∠DBA = 180^0
By further calculation
70^0 + 2∠DBA = 180^0
2∠DBA = 180 – 70 = 110^0
∠DBA = 110/2 = 55^0

Here BD is the bisector of ∠ABC
So ∠DBA = ∠DBC = 55^0
In Δ DBC
DB = DC
∠DCB = ∠DBC
Hence, x = 55^0.

13. In each figure, given below, ABCD is a square and Δ BEC is an equilateral triangle.



Find, in each case: (i) ∠ABE (ii) ∠BAE
Solution:

The sides of a square are equal and each angle is 90^0
In an equilateral triangle three sides and angles are 60^0
In figure (i) ABCD is a square and Δ BEC is an equilateral triangle

(i) ∠ABE = ∠ABC + ∠CBE
Substituting the values
∠ABE = $90^0 + 60^0 = 150^0$

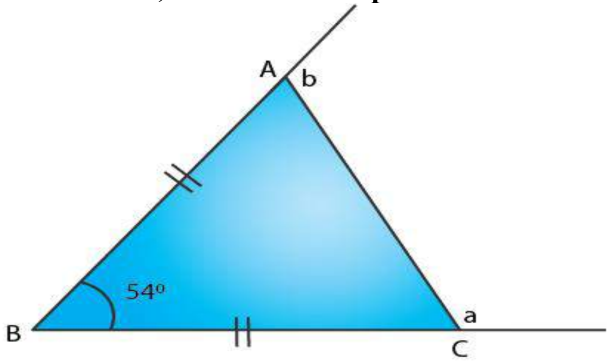
(ii) In Δ ABE
∠ABE + ∠BEA + ∠BAE = 180^0
Substituting the values
150^0 + ∠BAE + ∠BAE = 180^0
By further calculation
2∠BAE = 180 – 150 = 30^0
∠BAE = 30/2 = 15^0

In figure (ii) ABCD is a square and Δ BEC is an equilateral triangle

(i) ∠ABE = ∠ABC - ∠CBE
Substituting the values
∠ABE = $90^0 – 60^0 = 30^0$

(ii) In Δ ABE
∠ABE + ∠BEA + ∠BAE = 180^0
Substituting the values
30^0 + ∠BAE + ∠BAE = 180^0
By further calculation
2∠BAE = 180 – 30 = 150^0
∠BAE = 150/2 = 75^0

14. In Δ ABC, BA and BC are produced. Find the angles a and h. if AB = BC.



Solution:

In Δ ABC, BA and BC are produced
∠ABC = 54^0 and AB = BC

In Δ ABC
∠BAC + ∠BCA + ∠ABC = 180^0

Substituting the values
$\angle BAC + \angle BAC + 54^0 = 180^0$
$2\angle BAC = 180 – 54 = 126^0$
$\angle BAC = 126/2 = 63^0$
$\angle BCA = 63^0$

In a linear pair
$\angle BAC + b = 180^0$
Substituting the value
$63^0 + b = 180^0$
So we get
$b = 180 – 63 = 117^0$

In a linear pair
$\angle BCA + a = 180^0$
Substituting the value
$63^0 + a = 180^0$
So we get
$a = 180 – 63 = 117^0$

Therefore, $a = b = 117^0$.