

EXERCISE 11C

PAGE: 129

1. Multiply:

(i) $3x$, $5x^2y$ and $2y$

(ii) 5 , $3a$ and $2ab^2$

(iii) $5x + 2y$ and $3xy$

(iv) $6a - 5b$ and $-2a$

(v) $4a + 5b$ and $4a - 5b$

Solution:

(i) $3x$, $5x^2y$ and $2y$

Product = $3x \times 5x^2y \times 2y$

We can write it as

$$= 3 \times 5 \times 2 \times x \times x^2 \times y \times y$$

So we get

$$= 30x^3y^2$$

(ii) 5 , $3a$ and $2ab^2$

Product = $5 \times 3a \times 2ab^2$

We can write it as

$$= 5 \times 3 \times 2 \times a \times ab^2$$

So we get

$$= 30a^2b^2$$

(iii) $5x + 2y$ and $3xy$

Product = $3xy(5x + 2y)$

We can write it as

$$= 3xy \times 5x + 3xy \times 2y$$

So we get

$$= 15x^2y + 6xy^2$$

(iv) $6a - 5b$ and $-2a$

Product = $-2a(6a - 5b)$

We can write it as

$$= -2a \times 6a + 2a \times 5b$$

So we get

$$= -12a^2 + 10ab$$

(v) $4a + 5b$ and $4a - 5b$

Product = $(4a + 5b)(4a - 5b)$

So we get

$$= 16a^2 - 25b^2$$

$$4a + 5b$$

$$\times 4a - 5b$$

$$16a^2 + 20ab$$

$$- 20ab - 25b^2$$

$$16a^2 \quad - 25b^2$$

2. Copy and complete the following multiplications:

$$\begin{array}{r} \text{(i)} \quad 3a + 2b \\ \quad \quad x - 3xy \\ \hline \end{array} \quad \begin{array}{r} \text{(ii)} \quad 9x - 5y \\ \quad \quad x - 3xy \\ \hline \end{array} \quad \begin{array}{r} \text{(iii)} \quad 3xy - 2x^2 - 6x \\ \quad \quad x - 5x^2y \\ \hline \end{array}$$

$$\begin{array}{r} \text{(iv)} \quad a + b \\ \quad \quad x a + b \\ \hline \end{array} \quad \begin{array}{r} \text{(v)} \quad ax - b \\ \quad \quad x 2ax + 2 b^2 \\ \hline \end{array}$$

Solution:

$$\begin{array}{r} \text{(i)} \quad 3a + 2b \\ \quad \quad x - 3xy \\ \hline -9axy - 6bxy \\ \hline \end{array}$$

$$\begin{array}{r} \text{(ii)} \quad 9x - 5y \\ \quad \quad x - 3xy \\ \hline -27x^2y + 15xy^2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(iii)} \quad 3xy - 2x^2 - 6x \\ \quad \quad x - 5x^2y \\ \hline -15x^3y^2 + 10x^4y + 30x^3y \\ \hline \end{array}$$

$$\begin{array}{r} \text{(iv)} \quad a + b \\ \quad \quad x a + b \\ \hline a^2 + ab \\ \quad \quad ab + b^2 \\ \hline a^2 + 2ab + b^2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(v)} \quad ax - b \\ \quad \quad x 2ax + 2 b^2 \\ \hline 2a^2x^2 - 2abx + 2b^2x - 2b^3 \\ \hline \end{array}$$

3. Evaluate:

(i) $(c + 5)(c - 3)$

(ii) $(3c - 5d)(4c - 6d)$

(iii) $(1/2a + 1/2b)(1/2a - 1/2b)$

(iv) $(a^2 + 2ab + b^2)(a + b)$

(v) $(3x - 1)(4x^3 - 2x^2 + 6x - 3)$

Solution:

(i) $(c + 5)(c - 3)$

It can be written as

$$= c(c - 3) + 5(c - 3)$$

By further calculation

$$= c^2 - 3c + 5c - 15$$

$$= c^2 + 2c - 15$$

(ii) $(3c - 5d)(4c - 6d)$

It can be written as

$$= 3c(4c - 6d) - 5d(4c - 6d)$$

By further calculation

$$= 12c^2 - 18cd - 20cd + 30d^2$$

$$= 12c^2 - 38d + 30d^2$$

(iii) $(1/2a + 1/2b)(1/2a - 1/2b)$

It can be written as

$$= 1/2a(1/2a - 1/2b) + 1/2b(1/2a - 1/2b)$$

By further calculation

$$= 1/4a^2 - 1/4ab + 1/4ab - 1/4b^2$$

$$= 1/4a^2 - 1/4b^2$$

(iv) $(a^2 + 2ab + b^2)(a + b)$

It can be written as

$$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$$

By further calculation

$$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$

$$= a^3 + b^3 + 3a^2b + 3ab^2$$

(v) $(3x - 1)(4x^3 - 2x^2 + 6x - 3)$

It can be written as

$$= 3x(4x^3 - 2x^2 + 6x - 3) - 1(4x^3 - 2x^2 + 6x - 3)$$

By further calculation

$$= 12x^4 - 6x^3 + 18x^2 - 9x - 4x^3 + 2x^2 - 6x + 3$$

$$= 12x^4 - 6x^3 - 4x^3 + 18x^2 + 2x^2 - 9x - 6x + 3$$

So we get

$$= 12x^4 - 10x^3 + 20x^2 - 15x + 3$$

4. Evaluate:

(i) $(a + b)(a - b)$.

(ii) $(a^2 + b^2)(a + b)(a - b)$, using the result of (i).

(iii) $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$, using the result of (ii).

Solution:

(i) $(a + b)(a - b)$.

It can be written as

$$= a(a - b) + b(a - b)$$

By further calculation

$$= a^2 - ab + ab - b^2$$

$$= a^2 - b^2$$

(ii) $(a^2 + b^2)(a + b)(a - b)$

Substituting the result of (i)

$$= (a^2 + b^2)(a^2 - b^2)$$

It can be written as

$$= a^2(a^2 - b^2) + b^2(a^2 - b^2)$$

So we get

$$= a^4 - a^2b^2 + a^2b^2 - b^4$$

$$= a^4 - b^4$$

$$(iii) (a^4 + b^4) (a^2 + b^2) (a + b) (a - b)$$

Substituting the result of (ii)

$$= (a^4 + b^4) (a^4 - b^4)$$

It can be written as

$$= a^4 (a^4 - b^4) + b^4 (a^4 - b^4)$$

By further calculation

$$= a^8 - a^4 b^4 + a^4 b^4 - b^8$$

$$= a^8 - b^8$$

5. Evaluate:

$$(i) (3x - 2y) (4x + 3y)$$

$$(ii) (3x - 2y) (4x + 3y) (8x - 5y)$$

$$(iii) (a + 5) (3a - 2) (5a + 1)$$

$$(iv) (a + 1) (a^2 - a + 1) \text{ and } (a - 1) (a^2 + a + 1); \text{ and then: } (a + 1) (a^2 - a + 1) + (a - 1) (a^2 + a + 1)$$

$$(v) (5m - 2n) (5m + 2n) (25m^2 + 4n^2)$$

Solution:

$$(i) (3x - 2y) (4x + 3y)$$

It can be written as

$$= 3x (4x + 3y) - 2y (4x + 3y)$$

By further calculation

$$= 12x^2 + 9xy - 8xy - 6y^2$$

So we get

$$= 12x^2 + xy - 6y^2$$

$$(ii) (3x - 2y) (4x + 3y) (8x - 5y)$$

Substituting result of (i)

$$= (12x^2 + xy - 6y^2) (8x - 5y)$$

It can be written as

$$= 8x (12x^2 + xy - 6y^2) - 5y (12x^2 + xy - 6y^2)$$

By further calculation

$$= 96x^3 + 8x^2y - 48xy^2 - 60x^2y - 5xy^2 + 30y^3$$

So we get

$$= 96x^3 + 8x^2y - 60x^2y - 48xy^2 - 5xy^2 + 30y^3$$

$$= 96x^3 - 52x^2y - 53xy^2 + 30y^3$$

$$(iii) (a + 5) (3a - 2) (5a + 1)$$

It can be written as

$$= a (3a - 2) + 5 (3a - 2) (5a + 1)$$

By further calculation

$$= (3a^2 - 2a + 15a - 10) (5a + 1)$$

So we get

$$= (3a^2 + 13a - 10) (5a + 1)$$

We can write it as

$$= 5a (3a^2 + 13a - 10) + 1 (3a^2 + 13a - 10)$$

By further calculation

$$= 15a^3 + 65a^2 - 50a + 3a^2 + 13a - 10$$

$$= 15a^3 + 68a^2 - 37a - 10$$

(iv) $(a + 1)(a^2 - a + 1)$ and $(a - 1)(a^2 + a + 1)$; and then: $(a + 1)(a^2 - a + 1) + (a - 1)(a^2 + a + 1)$

Consider

$$(a + 1)(a^2 - a + 1)$$

It can be written as

$$= a(a^2 - a + 1) + 1(a^2 - a + 1)$$

By further calculation

$$= a^3 - a^2 + a + a^2 - a + 1$$

So we get

$$= a^3 + 1$$

$$(a - 1)(a^2 + a + 1)$$

It can be written as

$$= a(a^2 + a + 1) - 1(a^2 + a + 1)$$

By further calculation

$$= a^3 + a^2 + a - a^2 - a - 1$$

So we get

$$= a^3 - 1$$

Here

$$(a + 1)(a^2 - a + 1) + (a - 1)(a^2 + a + 1)$$

$$= a^3 + 1 + a^3 - 1$$

$$= 2a^3$$

$$(v) (5m - 2n)(5m + 2n)(25m^2 + 4n^2)$$

It can be written as

$$= [5m(5m + 2n) - 2n(5m + 2n)](25m^2 + 4n^2)$$

By further calculation

$$= (25m^2 + 10mn - 10mn - 4n^2)(25m^2 + 4n^2)$$

So we get

$$= (25m^2 - 4n^2)(25m^2 + 4n^2)$$

We can write it as

$$= 25m^2(25m^2 + 4n^2) - 4n^2(25m^2 + 4n^2)$$

By multiplying the terms

$$= 625m^4 + 100m^2n^2 - 100m^2n^2 - 16n^4$$

$$= 625m^4 - 16n^4$$

6. Multiply:

(i) mn^4 , m^3n and $5m^2n^3$

(ii) $2mnpq$, $4mnpq$ and $5mnpq$

(iii) $pq - pm$ and p^2m

(iv) $x^3 - 3y^3$ and $4x^2y^2$

(v) $a^3 - 4ab$ and $2a^2b$

Solution:

(i) mn^4 , m^3n and $5m^2n^3$

It can be written as

$$= 5m^2n^3 \times mn^4 \times m^3n$$

By further calculation

$$= 5m^{(2+1+3)}n^{(3+4+1)}$$

$$= 5m^6n^8$$

(ii) $2mnpq$, $4mnpq$ and $5mnpq$

It can be written as

$$= 5mnpq \times 2mnpq \times 4mnpq$$

By further calculation

$$= 5 \times 2 \times 4 m^{(1+1+1)} n^{(1+1+1)} p^{(1+1+1)} q^{(1+1+1)}$$
$$= 40m^3n^3p^3q^3$$

(iii) $pq - pm$ and p^2m

It can be written as

$$= p^2m \times (pq - pm)$$

So we get

$$= p^3qm - p^3m^2$$

(iv) $x^3 - 3y^3$ and $4x^2y^2$

It can be written as

$$= 4x^2y^2 \times (x^3 - 3y^3)$$

By further calculation

$$= 4x^5y^2 - 12x^2y^5$$

(v) $a^3 - 4ab$ and $2a^2b$

It can be written as

$$= 2a^2b \times (a^3 - 4ab)$$

By further calculation

$$= 2a^5b - 8a^3b^2$$

7. Multiply:

(i) $(2x + 3y)(2x + 3y)$

(ii) $(2x - 3y)(2x + 3y)$

(iii) $(2x + 3y)(2x - 3y)$

(iv) $(2x - 3y)(2x - 3y)$

(v) $(-2x + 3y)(2x - 3y)$

Solution:

(i) $(2x + 3y)(2x + 3y)$

It can be written as

$$= 2x(2x + 3y) + 3y(2x + 3y)$$

By further calculation

$$= 4x^2 + 6xy + 6xy + 9y^2$$

$$= 4x^2 + 12xy + 9y^2$$

(ii) $(2x - 3y)(2x + 3y)$

It can be written as

$$= 2x(2x + 3y) - 3y(2x + 3y)$$

By further calculation

$$= 4x^2 + 6xy - 6xy - 9y^2$$

$$= 4x^2 - 9y^2$$

(iii) $(2x + 3y)(2x - 3y)$

It can be written as

$$= 2x(2x - 3y) + 3y(2x - 3y)$$

By further calculation
$$= 4x^2 - 6xy + 6xy - 9y^2$$
$$= 4x^2 - 9y^2$$

(iv) $(2x - 3y)(2x - 3y)$

It can be written as

$$= 2x(2x - 3y) - 3y(2x - 3y)$$

By further calculation

$$= 4x^2 - 6xy - 6xy + 9y^2$$
$$= 4x^2 - 12xy + 9y^2$$

(v) $(-2x + 3y)(2x - 3y)$

It can be written as

$$= -2x(2x - 3y) + 3y(2x - 3y)$$

By further calculation

$$= -4x^2 + 6xy + 6xy - 9y^2$$
$$= -4x^2 + 12xy - 9y^2$$

