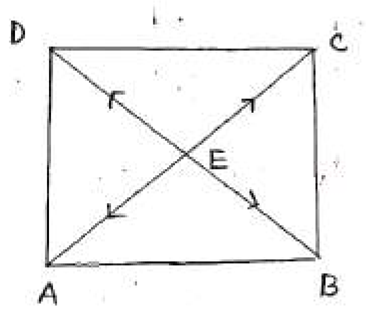
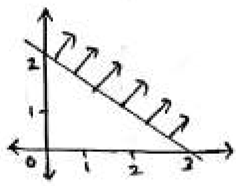
CLASS XII
MATHS SET – I 65/5/1

S.NO	SOLUTION	MARK
1	**(C)** $A(adj\ A) = \|A\| I$ $\Rightarrow \|A\| I = 10 I$ $\Rightarrow \|A\| = 10$ $\|adj\ A\| = \|A\|^{n-1} = 10^{3-1} = 10^2 = 100$	1
2	**(D)** $\|KA\| = K^n.\|A\|$ $\|3A\| = 3^3.\|A\| = 27 \times 8 = 216$	1
3	**(A)** $y = Ae^{5x} + Be^{-5x}$ $\frac{dy}{dx} = 5Ae^{5x} - 5Be^{-5x}$ $\frac{d^2y}{dx^2} = 25Ae^{5x} + 25Be^{-5x}$ $= 25\left(Ae^{5x} + Be^{-5x}\right) = 25y$	1
4	**(A)** $\int x^2.e^{x^3}.dx$ Put $x^3 = t \Rightarrow 3x^2.dx = dt \Rightarrow x^2.dx = \frac{1}{3}dt$ $\int x^2.e^{x^3}.dx = \frac{1}{3}\int e^t.dt = \frac{1}{3}e^t + c = \frac{1}{3}e^{x^3} + c$	1
5	**(C)** If two vectors are perpendicular then their scalar product is zero. $\therefore \hat{i}.\hat{k} = 0$	1
6	**(A)** $\|\vec{EA}\| = \|\vec{EC}\|$ $\overrightarrow{EA} = -\overrightarrow{EC} \Rightarrow \overrightarrow{EA} + \overrightarrow{EC} = 0$ $\overrightarrow{EB} = -\overrightarrow{ED} \Rightarrow \overrightarrow{EB} + \overrightarrow{ED} = 0$ $\therefore \overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = 0 + 0$	1

	$=0$ 	
7	**(A)** Given that the two lines are perpendicular. $i.e \quad a_1a_2+b_1b_2+c_1c_2=0$ $\Rightarrow 1(K)+1(2)+(-K)(-2)=0 \quad \Rightarrow \quad 3K+2=0 \Rightarrow K=\frac{-2}{3}$	1
8	**(B)** $2x+3y>6$ $2(0)+3(0)>6 \quad \Rightarrow \quad 0>6$ 	1
9	**(C)** E: Number of spade cards F: Number of Queen cards $P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{1/52}{4/52}=\frac{1}{4}$	1
10	**(D)** $A=\{4,5,6\}$ $B=\{1,2,3,4,\}$, $\quad A\cap B=\{4\}$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $=\frac{3}{6}+\frac{4}{6}-\frac{1}{6}=\frac{6}{6}=1$	1
11	Identity relation	1
12	$A+B=\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} \quad \Rightarrow \quad 2A+2B=\begin{bmatrix}2 & 0\\2 & 2\end{bmatrix} \longrightarrow (i)$	1

	$A-2B=\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}\longrightarrow(ii)$ Add (i) and (ii) $3A=\begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix} \Rightarrow A=\begin{bmatrix}1/3 & 1/3\\ 2/3 & 1/3\end{bmatrix}$	
13	$A.M \geq GM$ $\frac{1}{2}\left(ax+\frac{b}{x}\right)\geq\sqrt{ax.\frac{b}{x}}$ $ax+\frac{b}{x}\geq 2\sqrt{ab}$ $\therefore$ minimum value $=2\sqrt{ab}$	1
14	$x.\frac{dy}{dx}+2y=x^2$ $\frac{dy}{dx}+\left(\frac{2}{x}\right)y=x$ $P=\frac{2}{x}, Q=x$ $I.F=e^{\int P.dx}=e^{\int\frac{2}{x}.dx}=e^{2\log x}=x^2$	1
	(OR) The degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2=x$ is $\underline{2}$	1
15	$\vec{a}=3i+4j-7k$ $\vec{b}=i-j+6k$ $\vec{r}=\vec{a}+\lambda(b-a)$ $=(3i+4j-7k)+\lambda(i-j+6k-3i-4j+7k)$ $=(3i+4j-7k)+\lambda(-2i-5j+13k)$	1
	(OR) The line of shortest distance between two skew lines is **perpendicular** (normal) to both the lines.	1
16	$\sin^{-1}\left[\sin\left(\frac{-17\pi}{8}\right)\right]=-\sin^{-1}\left[\sin\left(\frac{17\pi}{8}\right)\right]$	½

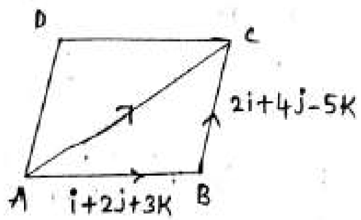
	$= -\sin^{-1}\left[\sin\left(2\pi + \frac{\pi}{8}\right)\right]$ $= -\frac{\pi}{8}$	½
17	$\det A = ad - bc = -3 + 4 = 1$ $A^{-1} = \frac{1}{\lvert A\rvert}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$	½ ½
18	$f(3) = \underset{x\to 3}{Lt} \frac{x^2-9}{x-3} \Rightarrow f(3) = \underset{x\to 3}{Lt} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$ $K = 3+3$ $K = 6$	½ ½
19	We have $f(x) = x^4 - 10$ $f'(x) = 4x^3$ $f(x+\Delta x) = f(x) + \Delta x . f'(x)$ $x = 2 \quad , \quad \Delta x = 0.1$ $f(2.1) = f(2) + (0.1)4(2)^3$ $= 6 + 3.2$ $= 9.2$	½ ½
	(OR) $y = 2.\sin^2(3x)$ $\frac{dy}{dx} = 2(2.\sin 3x)(\cos 3x)(3)$ At $x = \frac{\pi}{6}, \frac{dy}{dx} = 12.\sin\left(\frac{\pi}{2}\right).\cos\left(\frac{\pi}{2}\right) = 0$	½ ½
20	$\lvert x-a\rvert = x - a$ if $x \geq a$ $= -(x-a)$ if $x < a$ $\lvert x-5\rvert = -(x-5)$ if $x < 5$	½

	$\int_1^4 \lvert x-5\rvert .dx = -\int_1^4 (x-5).dx = -\left[\frac{x^2}{2}-5x\right]_1^4 = \left[5x-\frac{x^2}{2}\right]_1^4$ $=(20-8)-\left(5-\frac{1}{2}\right)=\frac{15}{2}$	½
21	$fof(x)=f[f(x)]$ $=f\left(\frac{4x+3}{6x-4}\right)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{16x+12+18x-12}{24x+18-24x+16}$ $=\frac{34x}{34}=x$	1
	Let $y=f(x)\Rightarrow y=\frac{4x+3}{6x-4}$ $\Rightarrow 6xy=4y=4x+3$ $\Rightarrow x=\frac{4y+3}{6y-4}$ $\Rightarrow f^{-1}(y)=\frac{4y+3}{6y-4}=f(y)$	1
	$\therefore$ Inverse of $f=f$.	
	(OR) (i) **Symmetric:** Let $a,b\in R$ and $(a,b)\in R$ Consider $a<b$ does not imply $b<a$ $\Rightarrow (a,b)\in R$ but $(b,a)\notin R$ $\therefore R$ is not symmetric	1
	(ii) **Transitive:** Let $a,b,c\in R$ If $(a,b)\in R$ and $(b,c)\in R\Rightarrow a<b$ and $b<c$ $\Rightarrow a<c\Rightarrow (a,c)\in R$ $\therefore$ R is Transitive.	1
22	Let $I=\int \frac{x}{x^2+3x+2}.dx$	½

	$\frac{x}{x^2+3x+2}=\frac{x}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$ $x=A(x+2)+B(x+1)$ Put $x=-1\Rightarrow A=-1$ Put $x=-2\Rightarrow B=2$ $\int\frac{x}{x^2+3x+2}=\int\frac{-1}{x+1}.dx+{}^{2}\int\frac{1}{x+2}.dx$ $=-\log(x+1)+2\log(x+2)+C$	½ 1
23	$x=a\cos\theta\Rightarrow\frac{dx}{d\theta}=-a\sin\theta$ $y=b\sin\theta\Rightarrow\frac{dy}{d\theta}=b\cos\theta$ $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{b\cos\theta}{-a\sin\theta}=\frac{-b}{a}.\cot\theta$ $\frac{d^2y}{dx^2}=\frac{-b}{a}.(\cos ec^2\theta).\frac{d\theta}{dx}$ $=\frac{b}{a}.\cos ec^2\theta.\frac{-1}{a\sin\theta}=\frac{-b}{a^2}\cos ec^3\theta$	½ ½ 1
	(OR) $U=\sin^2 x\Rightarrow\frac{du}{dx}=2.\sin x.\cos x$ $V=e^{\cos x}\Rightarrow\frac{dv}{dx}=e^{\cos x}.-\sin x$ $\frac{du}{dv}=\frac{2\sin x.\cos x}{e^{\cos x}.(-\sin x)}=-2\cos ec.e^{-\cos x}$	½ ½ 1
24	Put $2x=t\ \Rightarrow\ 2dx=dt$ If $x=1\ \Rightarrow\ t=2$ $x=2\ \Rightarrow\ t=4$ $I=\frac{1}{2}\int_2^4 e^t\left(\frac{2}{t}-\frac{2}{t^2}\right)dt$ $=\int_2^4\left(\frac{1}{t}-\frac{1}{t^2}\right)et.dt$	½ ½ ½

	$=\left[e^t.\frac{1}{t}\right]_2^4 \qquad \left[\because \int\left[f(x)+f'(x)\right]e^x=e^x.f(x)+c\right]$ $=\frac{e^4}{4}-\frac{e^2}{2}=\frac{e^2\left(e^2-2\right)}{4}$	½
25	Let $I={}_0^1\int x(1-x)^n.dx$ $I={}_0^1\int(1-x)\left[1-(1-x)\right]^n.dx \quad \left[\because {}_0^a\int f(x)dx={}_0^a\int f(a-x).dx\right]$ $={}_0^1\int(1-x).x^n.dx$ $={}_0^1\int\left(x^n-x^{n+1}\right)dx=\left[\frac{x^{n+1}}{1n+1}-\frac{x^{n+2}}{n+2}\right]_0^1$ $=\left(\frac{1}{n+1}-\frac{1}{n+2}\right)-(0-0)=\frac{1}{(n+1)(n+2)}$	1 ½ ½
26	$P(A)=0.3$ $P(B)=0.6$ $P(A'\cap B')=P(A\cup B)'$ $=1-P(A\cup B)$ $=1-\left[P(A)+P(B)-P(A).P(B)\right]$ $=1-\left[0.3+0.6-(0.3)(0.6)\right]$ $=1-0.72=0.28$	1 1
27	$\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ $\Rightarrow \sin^{-1}(1-x)=\frac{\pi}{2}+2\sin^{-1}x$ $\Rightarrow 1-x=\cos\left[\cos^{-1}\left(1-2x^2\right)\right]$ $\Rightarrow 1-x=1-2x^2 \qquad \Rightarrow 2x^2-x=0$ $\Rightarrow x=0,\frac{1}{2}$	1 1 1

	But $\Rightarrow x=\frac{1}{2}$ does not satisfy the equation So $x=0$	1
28	$y=(\log x)^{x}+x^{\log x}$ Let $u=(\log x)^{x}$ and $v=x^{\log x}$ Differentiating the above $w.r.t.\ x,$ we get $\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \longrightarrow (i)$ Now, $u=(\log x)^{x}$ $\log u=x.\log(\log x)\Rightarrow \frac{1}{u}.\frac{du}{dx}=\frac{1}{\log x}+\log(\log x)$ $\frac{du}{dx}=(\log x)^{x}\left[\frac{1}{\log x}+\log(\log x)\right]\longrightarrow (ii)$ $v=x^{\log x}$ $\log v=(\log x)^{2}$ $\frac{1}{v}.\frac{dv}{dx}=2\log x.\frac{1}{x}$ $\frac{dv}{dx}=x^{\log x}\left[\frac{2\log x}{x}\right]\longrightarrow (3)$ $\frac{dy}{dx}=(\log x)^{x}\left[\frac{1}{\log x}+\log(\log x)\right]+x^{\log x}\left[\frac{2\log x}{x}\right]$	1 1 1 1
29	$\frac{dy}{dx}=\frac{y\sin\left(\frac{y}{x}\right)-x}{x.\sin(y/x)} \longrightarrow (i)$ Given differential equation is Homogeneous differential equation. Let $y=vx \Rightarrow \frac{dy}{dx}=v+x.\frac{dv}{dx} \longrightarrow (ii)$ Substitute (ii) in (i) $v+x.\frac{dv}{dx}=\frac{vx.\sin v-x}{x.\sin v}$ $x.\frac{dv}{dx}=\frac{v\sin v-1}{\sin v}-v$	1 1

	$x.\frac{dv}{dx}=\frac{-1}{\sin v}-v$ $x.\frac{dv}{dx}=\frac{-1}{\sin v}$ $-\int \sin v.dv=+\int \frac{dx}{x}$ $+\cos v=+\log x+\log c$ $\cos \frac{y}{x}=\log\lvert cx\rvert$ $x=1$ when $y=\frac{\pi}{2}$ $\Rightarrow \cos\ \pi/2=\log c$ $\Rightarrow c=1$ $\cos\left(\frac{y}{x}\right)=\log\lvert x\rvert$	1 1
30	Let $\overrightarrow{AB}=i+2j+3k$ And $\overrightarrow{BC}=2i+4j-5k$ $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$ $=(i+2j+3k)+(2i+4j-5k)$ $=3i+6j-2k$ Unit vector parallel to AC $=\frac{3i+6j-2k}{\sqrt{9+36+4}}=\frac{1}{7}(3i+6j-2k)$ Unit vector parallel to $BD=\frac{AB-BC}{\lvert AB-BC\rvert}=\frac{-i-2j+8k}{\sqrt{1+4+64}}$ $=\frac{1}{\sqrt{69}}(-i-2j+8k)$ 	1 1 1 1
	(OR) $\overrightarrow{AB}=(2-1)i+(-1-2)j+(4-3)k=i-3j+k$ $\overrightarrow{AC}=(4-1)i+(5-2)j+(-1-3)k=3i+3j+4k$	½ ½

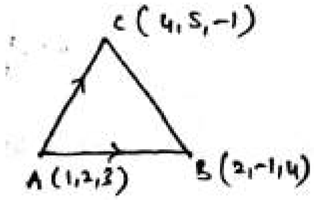
Now $\vec{AB}\times\vec{AC}=\begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$ 1

$$=i(12-3)-j(-4-3)+k(3+9)$$

$$=9i+7j+12k$$

$|AB\times AC|=\sqrt{9^2+7^2+12^2}=\sqrt{274}$ 1

Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{1}{2}\sqrt{274}$ sq. units. 1



31 Let the company manufacture x souvenirs of type A and y souvenirs of type B, clearly $x\geq 0, y\geq 0$. We make the following table from the given date.

	Novelty Souvenirs		Requirement (in mins)
	Type A (x)	Type B (y)	
Cutting	5	8	200
Assembling	10	8	240
Profit in Rs	100	120	

1

Since the time available for cutting is 3 hours 20 minutes and for assembling is 4 hours, we have the constraints

$$5x+8y\leq 200$$

$$10x+8y\leq 240$$ 1

Total profit (z) earned is $z=100x+120y$

Hence the mathematical formulation of the problem is maximize

$$z=100x+120y\longrightarrow(i)$$

Subject to the constraints

$$5x+8y\leq 200\longrightarrow(ii)$$

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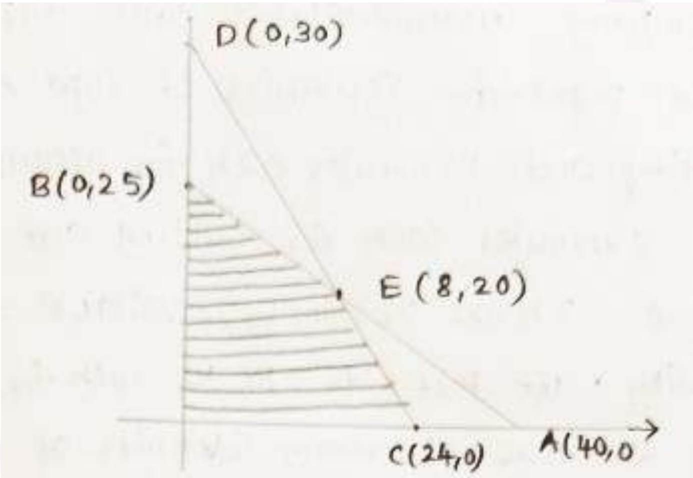
$5x+4y\le 120 \longrightarrow (iii)$

$x, y \ge 0$

Let us evaluate z at the corner points $0(0,0), C(24,0), E(8,20)$ and $B(0,25)$

Corner Point	$z=100x+120y$
$(0,0)$	0
$(24,0)$	2400
$(8,20)$	3200
$(0,25)$	3000



We find that the maximum value of z is 3200 at $E(8,20)$ Hence the company should manufacture 25 souvenirs of type B to realize maximum profit and maximum profit is Rs.3200.

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Rotten apples = 3
Good apples = 7
Total apples = 10

Probability of rotten apples $= \frac{3}{10} = 0.3$

Probability of good apple $= \frac{7}{10} = 0.7$

Three apples are chosen.
0 rotten apples = $3C_0\ (0.3)^0\ (0.7)^3$ = 0.343
1 rotten apples = $3C_1\ (0.3)^1\ (0.7)^2$ = 3(0.49) = 1.47

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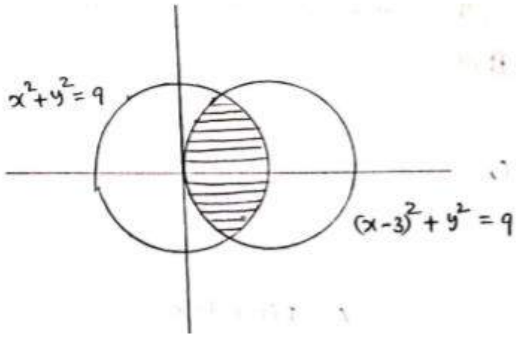
	2 rotten apples = $3C_2$ $(0.3)^2$ $(0.7)^1$ = 0.189 3 rotten apples = $3C_3$ $(0.3)^3$ $(0.7)^0$ = 0.027 Mean = 0 × (0.343) + 1(1.47) + 2(0.189) + 3(0.027) = 0 + 1.47 + 0.378 + 0.081 = 1.929	2 1
33	Required line is passing through (1, 1, 1) i.e $\frac{x-1}{a}=\frac{y-1}{b}=\frac{z-1}{c} \quad \to 1$ Given lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4} \quad \to 2$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \quad \to 3$ $\vec{b}_1 = a_i + bj + ck$ $\vec{b}_2 = i + 2j + 4k$ $b_3 = 2i + 3j + 4k$ $b_1 b_2 = 0$ and $b_1.b_3 = 0$ ➔ a + 2b + 4c = 0 ------ 4 And 2a + 3b + 4c = 0 ------ 5 $\frac{a}{8-12}=\frac{b}{8-4}=\frac{c}{3-4}$ $\frac{a}{-4}=\frac{b}{4}=\frac{c}{-1}=\lambda$ a = -4λ , b = 4λ, c = -λ put the values of a, b, c in equation 1 $\frac{x-1}{-4\lambda}=\frac{y-1}{4\lambda}=\frac{z-1}{-\lambda} \Rightarrow \frac{x-1}{-4}=\frac{y-1}{4}=\frac{z-1}{-1}$ $\vec{r}$ = (i + j + k) + λ (-4i+ 4j - k) Given two lines are (-2i + 3j - k) + λ (i + 2j + 4k) and (i + 2j + 3k) + 4 (2i + 3j + 4k) Angle between them is $\cos\theta = \left\|\frac{b_1 \cdot b_2}{\|b_1\| \cdot \|b_2\|}\right\|$ $\cos\theta = \frac{(i+2j+4k)\cdot(2i+3j+4k)}{\sqrt{21}\cdot\sqrt{29}}$ $\cos\theta = \left\|\frac{2+6+16}{\sqrt{21}\cdot\sqrt{29}}\right\| = \left\|\frac{24}{\sqrt{21}\cdot\sqrt{29}}\right\|$ $\theta = \cos^{-1}\left(\frac{24}{\sqrt{609}}\right)$	 1 1 1 1 1 1
34	The two circles are $x^2 + y^2 = 9$ ➔ $y^2 = 9 - x^2$ And $(x-3)^2 + y2 = 9$ ➔ $y^2 = 9 - (x-3)^2$ ➔ $9 - x^2 = 9 - (x-3)^2$ $9 - x^2 = 9 - x^2 + 6x - 9$	1

	x = 3/2  Area of shaded region = $2\left[\int_0^{3/2}\sqrt{9-(x-3)^2}\cdot dx+\int_{3/2}^{3}\sqrt{9-x^2}\cdot dx\right]$ $=2\left[\frac{x-3}{2}\sqrt{9-(x-3)^2}+\frac{9}{2}\sin^{-1}\frac{x-3}{3}\right]_0^{3/2}+2\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_{3/2}^{3}$ $=2\left[\frac{-9\sqrt{3}}{4}-\frac{6\pi}{4}+\frac{18\pi}{4}\right]$ $=6\pi-\frac{-9\sqrt{3}}{2}$ sq.units	1 2 1 1
35	P = ax + by P = $ax+b\frac{c^2}{x}$ $[\because xy=c^2]$ $\frac{dp}{dx}=a+\frac{bc^2}{-x^2}$ $a+\frac{bc^2}{-x^2}=0 \Rightarrow x=\sqrt{\frac{b}{a}}\cdot c$ $\frac{d^2P}{dx^2}=0+\frac{2bc^2}{x^3}=\frac{2bc^2}{\left(c\sqrt{b/a}\right)^3}=+ve$ $P_{\min}=a\sqrt{\frac{b}{a}}\cdot c+\frac{bc^2}{c}\cdot\sqrt{\frac{a}{b}}$ $=c\sqrt{ab}+c\sqrt{ab}=2c\sqrt{ab}$	1 1 1 1 1 1
36	t_p = a = A.R $^{p-1}$ A = First term t_q = b = A.R $^{q-1}$ R = common ratio t_r = c = A.R $^{r-1}$ ➔ log a = log A + (p - 1) log R log b = log A + (q – 1) log R log c = log A + (r – 1) log R $\begin{vmatrix} \log A+(P-1)\log R & p & 1 \\ \log A+(q-1)\log R & q & 1 \\ \log a+(r-1)\log R & r & 1 \end{vmatrix}$	1 1 1

	$= \begin{vmatrix} \log A & p & 1 \\ \log A & q & 1 \\ \log A & r & 1 \end{vmatrix} + \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$	1
	$= \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log r \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$	1
	$= 0 + \log R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$ $c_1 \rightarrow c_1 + c_3$ $= \log R \begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix} = \log R(0) = 0$	1