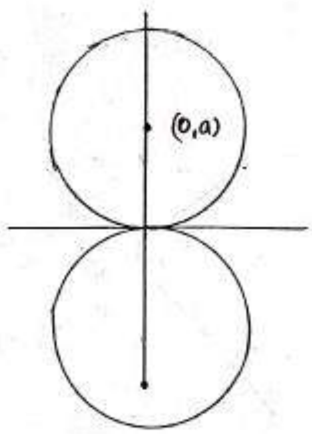
CBSE Class 12 Maths Question Paper 2020 Set 3 Solution

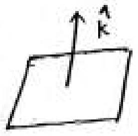
CLASS XII

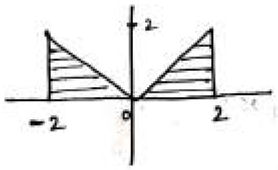
MATHS SET – III : 65/3/3

S.NO	SOLUTION	MARK
1	**(B)** $p\sqrt{3}=1 \quad \Rightarrow \quad p=\frac{1}{\sqrt{3}}$	1
2	**(B)** $\tan\left(\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{3}{4}\right)$ $=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{4}\right)$ $=\tan\left(2\tan^{-1}\frac{3}{4}\right)$ $=\tan\left(\tan^{-1}\frac{2\left(3/4\right)}{1-\left(3/4\right)^2}\right)=\frac{\left(6/4\right)}{\frac{16-9}{16}}=\frac{24}{7}$	1
3	**(B)** $z=3x-4y$ at $(0,0)\Rightarrow z=0$ at $(0,8)\Rightarrow z=-32$ at $(5,0)\Rightarrow z=15$ at $(4,10)\Rightarrow z=-28$ Minimum $=-32$	1
4	**(D)** $f(x)=\lvert x\rvert+x=\begin{cases}2x & , x\geq 0\\ 0 & , x<0\end{cases}$ $g(x)=\lvert x\rvert-x=\begin{cases}0 & , x\geq 0\\ -2x & , x<0\end{cases}$ $f[g(x)]=\lvert x\rvert-x=\begin{cases}2:g(x) & , g(x)\geq 0\\ 0 & , g(x)<0\end{cases}$ $f[g(x)]=-4x \quad , \quad x<0$	1
5	**(B)** $\int\frac{1}{\log x}.dx$ Let $\log x=t \quad \Rightarrow \quad \frac{1}{x}.dx=dt$ $\int\frac{1}{\log x}.dx=\int\frac{dt}{t}=\log t+c=\log\lvert\log x\rvert+c$	1

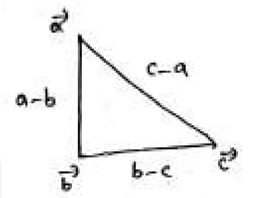
MATHS SET – III : 65/3/3

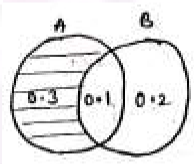
6	**(A)** $x^2+(y-a)^2=a^2$ $x^2+y^2-2ay+a^2=a^2$ $x^2+y^2=2ay$ $2x+2y.\frac{dy}{dx}=2a.\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{2x}{2a-2y}$ Order $=1$ 	1
7	**(A)** $A=\begin{bmatrix}-2&0&0\\0&-2&0\\0&0&-2\end{bmatrix}$ $\|A\|=-2(4-0)=-8$ $\|adj\ A\|=\|A\|^{3-1}=\|A\|^2=(-8)^2=64$	1
8	**(B)** Image of $(2,-1,4)$ in the YZ-plane is $(-2,-1,4)$	1
9	**(A)** $y=-x^3+3x^2+12x-5$ $\frac{dy}{dx}=-3x^2+6x+12$ $=-3(x^2-2x-4)$ $=-3((x-1)^2-5)$ $\frac{dy}{dx}=15-3(x-1)^2$	1

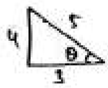
	Maximum value $=15$	
10	**(A)** $\vec{r}.\hat{k}=0$ 	1
11	Area of parallelogram $=\frac{1}{2}\lvert d_1\times d_2\rvert=\frac{1}{2}\times 2\times 3=3$	1
	(OR) $\left(2\hat{i}-\lambda\hat{j}+\hat{k}\right).\left(\hat{i}+2\hat{j}-\hat{k}\right)=0\Rightarrow 2-2\lambda-1=0\Rightarrow\lambda=\frac{1}{2}$	1
12	$\frac{4c_1\times 3c_1\times 2c_1}{9c_3}=\frac{2}{7}$	1
13	$f\left(x\right)=\lvert x+3\rvert-1$ Minimum value $=-1$	1
14	$y=\tan^{-1}x+\cot^{-1}x$ $\frac{dy}{dx}=\frac{1}{1+x^2}-\frac{1}{1+x^2}=0$	1
	(OR) $y=\tan^{-1}x+\cot^{-1}x$ $y=\pi/2$ $\frac{dy}{dx}=0$	1
	(OR) $\cos\left(xy\right)=k\quad\Rightarrow-\sin\left(xy\right).\left(x\frac{dy}{dx}+y\right)=0$ $\Rightarrow-\sin\left(x.y\right).x\frac{dy}{dx}=y.\sin\left(xy\right)$ $\Rightarrow\frac{dy}{dx}=\frac{-y\sin\left(xy\right)}{x\sin\left(xy\right)}=\frac{-y}{x}$	1
15	$RHL=\cos\pi=-1$ $LHL=\lambda\pi$ $\Rightarrow\lambda\pi=-1\quad\Rightarrow\lambda=-1/\pi$	1
16	$\int_{-2}^{2}\lvert x\rvert\,dx$	½

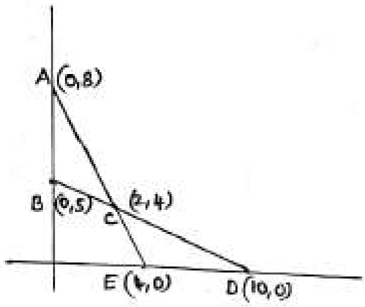
	Area $=\left(\frac{1}{2}\times 2\times 2\right)+\left(\frac{1}{2}\times 2\times 2\right)$ $=4$ sq. units  	½
	(OR) $\int \frac{dx}{9+4x^2}=\frac{1}{4}\int \frac{dx}{9/4+x^2}=\frac{1}{4}.\frac{2}{3}\tan^{-1}\left(\frac{2x}{3}\right)$ $\int \frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\left(x/a\right)+c=\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)$	½ ½
17	$f(x)=7-4x-x^2$ $f'(x)=-4-2x$ $f'(x)>0$ $-4-2x>0 \quad \Rightarrow -4>2x \quad \Rightarrow x<-2$	½ ½
18	$y=\sin^2\sqrt{x}$ $\frac{dy}{dx}=2\sin^2\sqrt{x}.\cos\sqrt{x}.\frac{1}{2\sqrt{x}}$ $\frac{dy}{dx}=\frac{\sin\sqrt{x}.\cos\sqrt{x}}{\sqrt{x}}$	1
19	$a_{ij}=\left\|(i)^2-j\right\|$ $a_{11}=1-1=0 \qquad a_{21}=4-1=3$ $a_{12}=\|1-2\|=1 \qquad a_{22}=4-2=2$ $\therefore \quad A=\begin{bmatrix}0 & 1\\ 3 & 2\end{bmatrix}$	½ ½
20	Black die – 5 Red die – 5, 6 Probability $=\frac{2}{6}=\frac{1}{3}$	½ ½

MATHS SET – III : 65/3/3

21	$\lvert a+b\rvert = \lvert a-b\rvert$ $a^2+b^2+2(ab) = a^2+b^2-2(ab)$ $ab = 0$ $\therefore$ a and b are perpendicular	1 1
	(OR) $a-b = -\hat{i} - 8\hat{j}$ $\lvert a-b\rvert \sqrt{1+64} = \sqrt{65}$ $b-c = -2\hat{i}+\hat{j}-\hat{k}$ $\lvert b-c\rvert = \sqrt{4+1+4} = \sqrt{6}$ $c-a = 3\hat{i}+7\hat{j}+\hat{k}$ $\lvert c-a\rvert = \sqrt{9+49+1} = \sqrt{59}$ $\lvert a-b\rvert^2 = \lvert b-a^1 + \lvert c-a\rvert^2$ $\therefore$ $\vec{a}, \vec{b}, \vec{c}$ are sides of Right angled $\Delta le.$  	½ ½ 1
22	$A\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix} = \begin{bmatrix}3 & 4\\ -1 & 6\end{bmatrix}$ Let $A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$ $\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix} = \begin{bmatrix}3 & 4\\ -1 & 6\end{bmatrix}$ $\begin{bmatrix}a-b & 2a\\ c-d & 2c\end{bmatrix} = \begin{bmatrix}3 & 4\\ -1 & 6\end{bmatrix}$ $a=2 \quad a-b=3 \quad \Rightarrow b=-1$ $c=3 \quad c-d=-1 \quad \Rightarrow d=4$ $\therefore$ $A = \begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$	½ ½ 1

23	$y=\tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right]$ Let $x=a\sin\theta \quad \Rightarrow \frac{x}{a}=\sin\theta \quad \Rightarrow \theta\sin^{-1}\left(\frac{x}{a}\right)$ $y=\tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$ $y=\tan^{-1}(\tan\theta)$ $y=\theta \quad \Rightarrow y=\sin^{-1}\left(\frac{x}{a}\right)$ $\Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}.\frac{1}{a}=\frac{1}{\sqrt{a^2-x^2}}$	½ ½ 1
24	$P(A)=0.4$ $P(B)=0.3$ $P(A\cup B)=0.6$ $P(B'\cap A)=0.3$ 	1 1
25	$\sin^{-1}4x+\sin^{-1}(3x)=\frac{-\pi}{2}$ $\sin^{-1}4x+\frac{\pi}{2}-\cos^{-1}(3x)=\frac{-\pi}{2}$ $\sin^{-1}4x+\frac{-\pi}{2}-\frac{\pi}{2}+\cos^{-1}(3x)$ $\sin^{-1}(4x)+-\pi+\cos^{-1}(3x)$ $\sin^{-1}(4x)+-\left[\pi-\cos^{-1}3x\right]$ $\sin^{-1}(4x)+-\cos^{-1}(-3x)$ $\sin^{-1}(-4x)+\cos^{-1}(-3x)$ Let $\sin^{-1}(-4x)=\theta \qquad \cos^{-1}(-3x)=\theta$	½ ½ ½

	$-4x=\sin\theta$ $\qquad -3x=\cos\theta$ $\frac{\sin\theta}{\cos\theta}=\frac{4}{3}$ $\quad \Rightarrow \tan\theta = 4/3$ $\quad -4x = 4/5$ $x=\frac{-1}{5}$ 	½
	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ $=\tan^{-1}\left(\frac{\cos^2 x/2-\sin^2 x/2}{1-2\sin x/2 . \cos x/2}\right)$ $=\tan^{-1}\left(\frac{\left(\cos x/2+\sin x/2\right)\left(\cos x/2-\sin x/2\right)}{\left(\cos x/2-\sin x/2\right)^2}\right)$ $=\tan^{-1}\left(\frac{\cos x/2+\sin x/2}{\cos x/2-\sin x/2}\right)$ $=\tan^{-1}\left(\frac{1+\tan x/2}{1-\tan x/2}\right)$ $=\tan^{-1}\left[\tan\left(\pi/4+x/2\right)\right]$ $=\frac{\pi}{4}+\frac{x}{2}$	½ ½ ½ ½
26	On ZX plane y = 0 Dr's of the line $\rightarrow$ 6, -3, 18 Eqn of the line $\rightarrow \frac{x+1}{6}=\frac{y-1}{-3}=\frac{z+8}{18}=\lambda$ $x=6\lambda-1, y=-3\lambda+1, z=18\lambda-8$ $y=0 \quad \Rightarrow -3\lambda+1=0 \Rightarrow \lambda = 1/3$ $\therefore$ The point $=(1,0,-2)$	½ ½ 1
27	$2x+y=8\rightarrow(0,8),(4,0)$	1

	$2x+y>8 \rightarrow$ away from origin $x+2y=10 \rightarrow (0,5),(10,0)$ $x+2y>10 \rightarrow$ away from origin $z=5x+7y$ at $(0,8) \rightarrow z=56$ at $(2,4) \rightarrow z=38$ at $(10,0) \rightarrow z=50$ Minimum value = 38 at $c(2,4)$  	1 1 1
28	$\sin \pi x \begin{cases} <0 & -1<x<0 \\ >0 & 0<x<1 \\ <0 & 1<x<3/2 \end{cases}$ $x\sin \pi x \begin{cases} >0 & -1<x<0 \\ >0 & 0<x<1 \\ <0 & 1<x<3/2 \end{cases}$ $x\sin \pi x \begin{cases} >0 \ , & -1<x<1 \\ <0 \ , & 1<x<3/2 \end{cases}$ $I=2\int_0^1 x\sin \pi x.dx+\int_1^{3/2} -x\sin \pi y.dx$ $\int x\sin \pi x.dx=\frac{-x\cos \pi x}{\pi}+\int \frac{\cos \pi x}{\pi}.dx$ $=\frac{-x\cos \pi x}{\pi}+\int \frac{\sin \pi x}{\pi^2}$	1 1

<table>
<tr><td></td><td>$$I=2\left[\frac{-x\cos\pi x}{\pi}+\frac{\sin\pi x}{\pi^2}\right]_0^1-\left[\frac{-x\cos\pi x}{\pi}+\frac{\sin\pi x}{\pi^2}\right]_1^{3/2}$$ $$2\left[\left(\frac{-\cos\pi}{\pi}+\frac{\sin\pi}{\pi^2}\right)-(0+0)\right]-\left[\left(\frac{-3}{2\pi}.\cos\frac{3\pi}{2}+\frac{1}{\pi^2}\sin\frac{3\pi}{2}\right)-\left(\frac{-\cos\pi}{\pi}+\frac{\sin\pi}{\pi^2}\right)\right]$$ $$=\frac{-2}{\pi}\cos\pi-\frac{1}{\pi^2}\sin\frac{3\pi}{2}-\frac{\cos\pi}{\pi}$$ $$=\frac{-3}{\pi}(-1)-\frac{1}{\pi^2}(-1)=\frac{3}{\pi}+\frac{1}{\pi^2}$$</td><td>1
1</td></tr>
<tr><td>29</td><td><table><tr><td></td><td>Head</td><td>Tail</td></tr><tr><td>Biased</td><td>0.6</td><td>0.4</td></tr><tr><td>Unbiased</td><td>0.5</td><td>0.5</td></tr></table></td><td>2</td></tr>
<tr><td></td><td>$$P\left(\frac{U}{T}\right)=\frac{\frac{1}{2}\times 0.5}{\frac{1}{2}\times 0.4+\frac{1}{2}\times 0.5}=\frac{1/4}{\frac{1}{5}+\frac{1}{4}}=\frac{1/4}{\frac{9}{20}}=\frac{1}{4}\times\frac{20}{9}=\frac{5}{9}$$</td><td>2</td></tr>
<tr><td>30</td><td>$$\frac{dy}{dx}+y\sec x=\tan x$$ $$P=\sec x,\qquad Q=\tan x$$ $$IF=e^{\int P.dx}=e^{\int \sec x.dx}=e^{\log\left|\sec x+\tan x\right|}=\sec x+\tan x$$ $$y.IF=\int Q.IF+C$$ $$y(\sec x+\tan x)=\int\tan x(\sec x+\tan x)+C$$ $$y(\sec x+\tan x)=\int\tan x\sec x+\tan^2 x.dx+C$$ $$y(\sec x+\tan x)=\sec x+\int(\sec^2 x-1)dx+C$$ $$y(\sec x+\tan x)=\sec x+\tan x-x+C$$ at $y=1$ and $x=\pi/4$ $$1\left(\sec\pi/4+\tan\pi/4\right)=\sec\pi/4+\tan\pi/4-\pi/4+C$$ $$\sqrt{2}+1=\sqrt{2}+1-\pi/4+C\qquad\Rightarrow C=\pi/4$$ $$y(\sec x+\tan x)=\sec x+\tan x-x+\pi/4$$</td><td>1
1
1
1</td></tr>
</table>

<table>
<tr><td>31</td><td>

$$f(x)=\frac{x}{1+|x|}$$

$$|x|=\begin{cases} x \quad , \quad x\ge 0 \\ -x \quad , \quad x<0 \end{cases}$$

$$f(x)=\begin{cases} \frac{x}{1+x} \quad , \quad x\ge 0 \\ \underline{x} \quad , \quad x<0 \end{cases}$$

<u>**one-one:**</u>

For $x\ge 0$	For $x<0$
$f(x_1)=f(x_2)$	$f(x_1)=f(x_2)$
$\frac{x_1}{1+x_1}=\frac{x_2}{1+x_2}$	$\frac{x_1}{1-x_1}=\frac{x_2}{1-x_2}$
$x_1+x_1x_2=x_2+x_1x_2$	$x_1-x_1x_2=x_2-x_1x_2$
$x_1=x_2$	$x_1=x_2$

Hence $f(x_1)=f(x_2)\Rightarrow x_1x_2$

$\therefore$ f is one-one

<u>**onto:**</u>

For $x\ge 0$	For $x<0$
Let $f(x)=y$	Let $f(x)=y$
$y=\frac{x}{1+x}$	$y=\frac{x}{1-x}$
$y+xy=x$	$y-xy=x$
$y=x(1-y)$	$y=x(1+y)$
$x=\frac{y}{1-y}$	$x=\frac{y}{1+y}$

$\therefore$ f is onto.

Hence f is both one-one and onto.

</td><td>1

1

1

1</td></tr>
<tr><td></td><td>(OR)</td><td></td></tr>
</table>

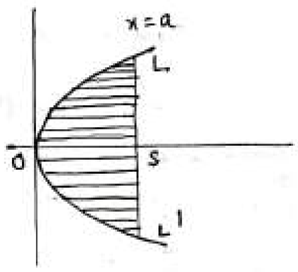
CLASS XII

MATHS SET – III : 65/3/3

32	$y = x^3(\cos x)^x + \sin^{-1}\sqrt{x}$ Let $u = (\cos x)^x \quad \Rightarrow \log u = x.\log(\cos x)$ $\Rightarrow \frac{1}{4}.\frac{du}{dx} = x\frac{1}{\cos x}(-\sin x) + \log(\cos x)$ $\Rightarrow \frac{du}{dx} = (\cos x)^x\left[\log(\cos x) - x\tan x\right]$ Now, $y = x^3(\cos x)^x + \sin^{-1}\sqrt{x}$ $\frac{dy}{dx} = x^3(\cos x)^x\left[\log(\cos x) - \tan x\right] + 3x^2(\cos x)^x + \frac{1}{\sqrt{1-x}}.\frac{1}{2\sqrt{x}}$	1 1 1 1
33	$9y^2 = x^3 \qquad \rightarrow (i)$ $18y.\frac{dy}{dx} = 3x^2$ Given $m = \pm 1$ $\frac{-6y}{x^2} = \pm 1$ $\frac{-6y}{x^2} = 1$ or $\frac{-6y}{x^2} = -1$ $x^2 = -6y$ or $x^2 = 6y$ Substitute the above in (i) $9\left(\frac{x^4}{36}\right) = x^3 \quad \Rightarrow \quad x = 0$ or 4 If $x = 4 \quad \Rightarrow y = \pm\frac{8}{3}$ Equation of normal $\Rightarrow y - y_1 = \frac{-dx}{dy}(x - x_1)$ $\Rightarrow y - 8/3 = \frac{-6\left(\frac{8}{3}\right)}{16}(x-4)$ $\Rightarrow \frac{3y-8}{3} = -x + 4$ $\Rightarrow 3y - 8 = -3x + 12$ $\Rightarrow 3x + 3y = 20$	1 1 1 1 1 1

CLASS XII

MATHS SET – III : 65/3/3

34	$\frac{x-2}{1}=\frac{y-2}{3}=\frac{2-3}{1}=\lambda$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}=\mu$ $x=\lambda+2 \quad x=\mu+2$ $y=3\lambda+2 \quad y=4\mu+3$ $z=\lambda+3 \quad z=2\mu+4$ $\lambda+2=\mu+2 \Rightarrow \lambda=\mu$ $3\lambda+2=4\mu+3 \Rightarrow \lambda=\mu=-1$ $\lambda+3=2\mu+4 \Rightarrow 2=2$ $\therefore$ The lines are intersect at $(1,-1,2)$ Equation of plane is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1 & m_1 & n_1 \\ x_2 & m_2 & n_2 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} x-2 & y-2 & z-3 \\ 1 & 3 & 1 \\ 1 & 4 & 2 \end{vmatrix}=0$ $\Rightarrow 2x-y+z=5$	1 1 1 1 1 1
35	For parabola $y^2=4ax$ Latus rectum is $x=a$ Area = Area OLSL' = 2 × Area OSL $=2\int_0^a y.dx$ $=4\sqrt{a}\int_0^a x^{1/2}.dx$ $=\frac{8}{3}\sqrt{a}\left[x^{3/2}\right]_0^a=\frac{8}{3}a^2$ 	1 1 2 1 1
	(OR)	
36	$x-y+2z=7$ $2x-y+3z=12$	

	$3x+2y-z=5$	
	$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix}$	1
	$\lvert A\rvert=1(1-6)+1(-2-9)+2(4+3)$	
	$=-5-11+14=-2$	1
	$adj\ A=\begin{bmatrix} -5 & 11 & 7 \\ 3 & -7 & -5 \\ -1 & 1 & 1 \end{bmatrix}^{-1}=\begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$	1
	$A^{-1}=\dfrac{adj\ A}{\lvert A\rvert}=\dfrac{-1}{2}\begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$	1
	$x=A^{-1}.B=\dfrac{-1}{2}\begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix}$	1
	$=\dfrac{-1}{2}\begin{bmatrix} -35+36-5 \\ 77-84+5 \\ 49-60+5 \end{bmatrix}=\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$	
	$\therefore\quad x=2, y=1, 2=3.$	1