Exercise Solutions

Question 1: A flint glass prism and a crown glass prism are to be combined in such a way that the deviation of the mean ray is zero. The refractive index of flint and crown glasses for the mean ray are 1.620 and 1.518 respectively. If the refracting angle of the flint prism is 6.0°, what would be the refracting angle of the crown prism?

Solution:
Refractive index of flint glass = μ_f = 1.620
Refracting angle of flint prism = A_f = 6.0°
Refractive index of crown glass = μ_c =1.518

Now,
For zero net deviation of mean ray:
$(\mu_c - 1)A_c = (\mu_f - 1)A_f$

=> $A_c = [(\mu_f - 1)/(\mu_c - 1)]A_f$

= [1.620-1]/[1.518-1] x 6°

= 7.2°

Question 2: A certain material has refractive indices 1.56, 1.60 and 1.68 for red, yellow and violet light respectively. A Calculate the dispersive power. B Find the angular dispersion produced by a thin prism of angle 6^0 made of this material.

Solution:

(a) Dispersive power = $\omega = [\mu_v - \mu_r]/[\mu_y - 1]$
Where μ_r = Refractive index of red light = 1.56
μ_y = Refractive index of yellow light = 1.60 and
μ_v = Refractive index of violet light = 1.68

=> ω = 0.2

(b) Angular dispersion = $\delta = [\mu_v - \mu_r]A$
Here, Refracting angle of prism = A = 6.0°

=> $\delta = (1.68 - 1.56)6^0 = 7.2^o$

Question 3: The focal lengths of a convex lens for red, yellow and violet rays are 100 cm, 98 cm and 96 cm respectively. Find the dispersive power of the material of the lens.

Solution:
The focal length of a lens is given by

$1/f = (\mu - 1)[1/R_1 - R_2]$

$$(\mu - 1) = \frac{1}{f}\frac{1}{(\frac{1}{R_1} - \frac{1}{R_2})}$$

Let $k = 1/[\ 1/R_1 - 1/R_2]$

Then, $\mu - 1 = k/f$

So, $\mu_r - 1 = k/100$ ...(1)
$\mu_v - 1 = k/98$ ....(2) and
$\mu_v - 1 = k/96$ .....(3)

Where, μ_r = Refractive index for the red color
μ_v = Refractive index for the violet color
μ_y = Refractive index for the yellow color

Now,
Dispersive power = $\omega = [\mu_v - \mu_r]/[\ \mu_y - 1]$

$= [(\mu_v - \mu_r) - (\mu_r - 1)\ ]/[\ \mu_y - 1]$

$= [k/96 - k/100]/[k/98]$

=> $\omega = 0.0408$

Question 4: The refractive index of a material changes by 0.014 as the color of the light changes from red to violet. A rectangular slab of height 2.00 cm made of this material is placed on a newspaper. When viewed normally in yellow light, the letters appear 1.32 cm below the top surface of the slab. Calculate the dispersive power of the material.

Solution:

$\mu_v - \mu_r = 0.014$ [given]

μ_y = [Real depth]/[Apparent depth] = 2/1.30 = 1.515

Dispersive power = $\omega = [\mu_v - \mu_r]/[\mu_y - 1]$

= 0.014/[1.515 -1]

= 0.027

Question 5: A thin prism is made of material having refractive indices 1.61 and 1.65 for red and violet light. The dispersive power of the material is 0.07. It is found that a beam of yellow light passing through the prism suffers a minimum deviation of 4.0° in favourable conditions. Calculate the angle of the prism.

Solution:

Dispersive power = $\omega = [\mu_v - \mu_r]/[\mu_y - 1]$

Here, $\mu_v = 1.65$, $\mu_r = 1.61$, $\omega = 0.07$ and $\delta_y = 4^o$

=> $0.07 = [1.65 – 1.61]/[\mu_y - 1]$

=> $\mu_y – 1 = 0.04/0.07 = 4/7$

Again, $\delta = (\mu - 1)A$

=> $A = \delta_y/[\mu_y – 1] = 4/(4/7) = 7^o$

Question 6: The minimum deviations suffered by red, yellow and violet beams passing through and equilateral transparent prism are 38.4°, 38.7° and 39.2° respectively. Calculate the dispersive power of the medium.

Solution:
Minimum Deviations by Red = δ_r = 38.4^0
Minimum Deviations by Yellow = δ_y = 38.7^0
Minimum Deviations by Violet = δ_v = 39.2^0

Dispersive power = $\omega = [\mu_v - \mu_r]/[\mu_y - 1]$

We know, $\delta = (\mu - 1)A$

$\omega = [\mu_v - \mu_r]/[\mu_y - 1] = [(\mu_v - 1) - (\mu_r - 1)]/[\mu_y - 1]$

$= [(\delta_v/A) - (\delta_r/A)]/[(\delta_v/A)]$

$= [\delta_v - \delta_r]/\delta_v$

= [39.2-38.4]/38.7

= 0.0204

Question 7: Two prisms of identical geometrical shape are combined with their refracting angles oppositely directed. The materials of the prisms have refractive indices 1.52 and 1.62 for violet light. A violet ray is deviated by 1.0° when passes symmetrically through this combination. What is the angle of the prisms?

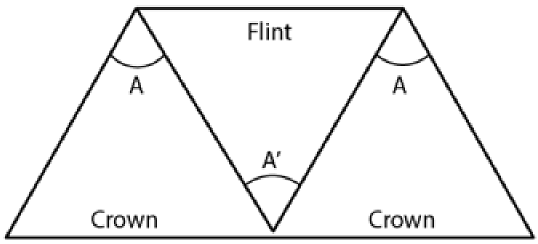
Solution: Two prisms of identical geometrical shape are combined.
Let A = Angle of the prisms.

$\delta = (\mu_{v1} - 1)A - (\mu_{v2} - 1)A$

Where, Refractive index of violet light through first prism = μ_{v1} = 1.52
Refractive index of violet light from second prism = μ_{v2} = 1.62
Deviation of violet light through prism = $\delta = 1^0$

Or, $\delta = (\mu_{v1} - \mu_{v2})A$
$\Rightarrow A = \delta/(\mu_{v1} - \mu_{v2})$
On substituting the values, we get
$A = 10^o$

Question 8: Three thin prisms are combined as shown in figure (below). The refractive indices of the crown glass for red, yellow and violet rays are μ_r, μ_y and μ_v respectively and those for the flint glass are μ'_r, μ'_y and μ'_v respectively. Find the ratio A'/A for which (a) there is no net angular dispersion, and (b) there is no net deviation in the yellow ray.



Solution:

Total deviation for yellow ray produced by the prism combination:

$\delta_v - \delta_r = \delta_{cy} - \delta_{fy} + \delta_{cy} = 2\delta_{cy} - \delta_{fy} = 2(\mu_{cy} - 1)A - (\mu_{fy} - 1)A'$

Similarly, angular dispersion produced by the combination:

$\delta_v - \delta_r = 2(\mu_{vc} - 1)A - (\mu_{vf} - 1)A'$

(a) for net angular dispersion to be zero, $\delta v - \delta r = 0$

$\Rightarrow 2(\mu_{vc} - 1) A - (\mu_{vf} - 1) A' = 0$

$\Rightarrow 2(\mu_{vc} - 1) A = (\mu_{vf} - 1) A'$

$\Rightarrow A'/A = [2(\mu_{vc} - 1)]/[(\mu_{vf} - 1)]$

(b) For net deviation in the yellow ray to be zero, $\delta y = 0$

$2(\mu_{cy} - 1) A - (\mu_{fy} - 1) A' = 0$

$\Rightarrow A'/A = [2(\mu_{cy} - 1)]/[\mu_{fy} - 1]$

Question 9: A thin prism of crown glass ($\mu_r = 1.515$, $\mu_v = 1.525$) and a thin prism of flint glass (($\mu_r = 1.612$, $\mu_v = 1.632$) are placed in contact with each other. Their refracting angles are 5.0° each and are similarly directed. Calculate the angular dispersion produced by the combination.

Solution:
A thin prism of crown glass (μ_r = 1.515, μ_v = 1.525) and a thin prism of flint glass ((μ_r = 1.612, μ_v = 1.632) are placed in contact with each other.

Since, they are similarly directed, the total deviation produced

$\delta = \delta_c + \delta_f = (\mu_c - 1)A + (\mu_f - 1)A$

$= (\mu_c + \mu_f - 2)A$

So, angular dispersion of the combination:

$\delta_v - \delta_r = (\mu_{cv} + \mu_{fv} - 2)A - (\mu_{cr} + \mu_{fr})A$

$= (\mu_{cv} + \mu_{fv} - \mu_{cr} - \mu_{fr})A$

$= (1.525 + 1.632 - 1.515 - 1.612)5$

$= 0.15^o$

Question 10: A thin prism of angle 6.0°, ω = 0.07 and μ_y = 1.50 is combined with another thin prism having ω = 0.08 and μ_y = 1.60. The combination produces no deviation in the mean ray.
(a) Find the angle of the second prim.
(b) Find the net angular dispersion produced by the combination when a beam of white light passes through it.
(c) If the prisms are similarly directed, what will be the deviation in the mean ray?
(d) Find the angular dispersion in the situation described in C.

Solution:

Here, For first prism:
$A_1 = 60$, $\omega_1 = 0.07$, $\mu_1 = 1.50$

For Second Prism
$A_2 = ?$, $\omega_2 = 0.08$ and $\mu_2 = 1.60$

The combination produces no deviation in the mean ray.

(a) $\delta = (\mu_2 - 1)A_2 - (\mu_1 - 1)A_1 = 0$

$\Rightarrow (1.6 – 1)A_2 – (1.5 -1)6 = 0$
$\Rightarrow A_2 = 5^o$

(b) When a beam of white light passes through it,
Net angular dispersion = $(\mu_2 -1)\omega_2A_2 - (\mu_1 -1)\omega_1A_1$

= $(1.60 - 1)5^o - (1.60 - 1)5^o$

= 0.03^o

(c) If the prisms are similarly directed.

$\delta = (\mu_2 - 1)A_2 - (\mu_1 - 1)A_1 = 0$

= $(1.60 -1)5^o – (1.50 – 1)6^o$
$\Rightarrow \delta = 6^o$

(d) Similarly, if the prisms are similarly directed,
Net angular dispersion :
$\mu_v - \mu_r = (\mu_2 - 1)\omega_2A_2 - (\mu_1 - 1)\omega_1A_1$

= $(1.6 – 1)\text{x } 0.08 \text{ x } 5 – (1.50-1)\text{x}0.07\text{x}6$

= 0.45^o

Question 11: The refractive index of a material M1 changes by 0.014 and that of another material M2 changes by 0.024 as the color of the light is changed from red to violet. Two thin prisms, one made of M1(A=5.3°) and the other made of M2(A=3.7°) are combined with their refracting angles oppositely directed.
(a) Find the angular dispersion produced by the combination.
(b) The prisms are now combined with their refracting angles similarly directed. Find the angular dispersion produced by the combination.

Solution:

Refractive index of a material M1 changes = $(\mu_{v1} - \mu_{r1}) = 0.014$

Refractive index of a material M2 changes = $(\mu_{v2} - \mu_{r2}) = 0.024$

Prism angle of a material $M_1 = A_1 = 5.3^0$

Prism angle of a material $M_2 = A_2 = 3.7^0$

(a) When the prisms are oppositely directed,

Angular dispersion = $\delta = (\mu_{v2} - \mu_{r2})A_2 - (\mu_{v1} - \mu_{r1})A_1$

=> $\delta = (0.024) x 3.7^0 - (0.024) x 5.3^o = 0.0146^o$

(b) When they are similarly directed,

Angular dispersion = $\delta = (\mu_{v2} - \mu_{r2})A_2 + (\mu_{v1} - \mu_{r1})A_1$

=> $\delta = (0.024) x 3.7^0 + (0.024) x 5.3^o = 0.0163^o$