# Exercise Solutions

**Question 1:** A body slipping on a rough horizontal plane moves with a deceleration of 4.0 $m/s^2$. What is the coefficient of kinetic friction between the block and the plane?

**Solution:**
Given:
The deceleration of body due to friction (a) = 4.0 $m/s^2$

Let m be the mass of the body and mg N be the weight of the body.

Frictional force (f)= ma

=> f = 4m N

Coefficient of kinetic friction = 4m/mg = 4/g = 0.4

Therefore, μ= 0.4

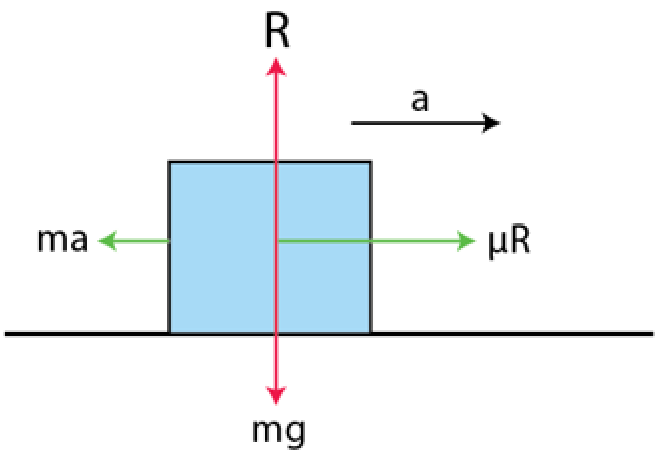
The co-efficient of kinetic friction between the block and the plane is 0.4.

**Question 2:** A block is projected along a rough horizontal road with speed of 10 m/s. if the coefficient of kinetic friction is 0.10, how far will it travel before coming to rest?

**Solution:**
Given,
Coefficient of kinetic friction = μ= 0.10
Initial velocity of the body = u = 10m/s

Let m be the mass of the body and mg N be the weight of the body

Normal force on the body N = mg




Frictional force = f= μN = 0.10 mg

Deacceleration due to kinetic friction =a =force/mass

= (0.10 mg)/m = 0.10g= 0.98 m/s$^2$

Final velocity due to deacceleration v= 0 m/s

Using equation, $V^2 = u^2 + 2as$, we have

$0 = 10^2 + 2(0.98)s$

=> s = 51 m
It will travel 51m before coming to rest.

**Question 3:** A block of mass m is kept on a horizontal table. If the static friction coefficient is μ, find the frictional force acting on the block.

**Solution:**
As the block is kept on horizonal surface and it is at rest, the frictional force will be zero. When the force is applied on the body to move it, the frictional force will act in opposite direction to oppose the motion. Hence, when body is at rest, frictional force will be zero.

**Question 4:** A block slides down an inclined surface of inclination 30$^o$ with the horizontal Starting from rest it covers 8 m in the first two seconds. Find the coefficient of kinetic friction between the two.

**Solution:**

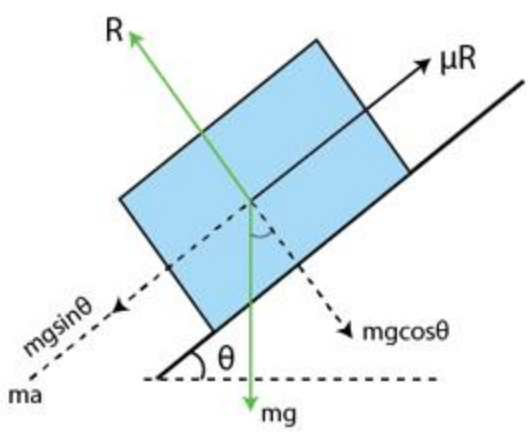
Angle of inclination= $30^o$
Time taken (t)= 2 sec
Distance travelled (s) = 8m
Initial velocity of the body= 0 m/s
Let "a" be the acceleration of the body.

Using equation, $s = ut + (1/2) at^2$

=> $8 = 0 + 1/2\, a\, (2^2)$

=> $a = 4\ m/s^2$

Now, the net force on the body = F = m.a = m (4)= 4m N




Let us consider "f" frictional force experienced by body while moving

So, the net force on the body:

= mg sin30°- f
= (1/2) mg - f

On equating the above two equations, we get

(1/2) mg - f = 4m

=> f = 0.9 m

Normal force on the body= mg cos30° = √3/2 (9.8)m= 8.48m
Coefficient of kinetic energy= 0.9m/8.48m = 0.11
Therefore, the coefficient of kinetic friction between the surfaces is 0.11,

**Question 5:** Suppose the block of the previous problem is pushed down the incline with a force of 4 N. How far will the block move in the first two seconds after starting from rest? The mass of the block is 4 kg.

**Solution:**
Mass of the block = m = 4kg
When no external force was applied, net force is "mgsin30° - f"

Here, external force F = 0.9m

Total net force along the inclination = mg sin30° - f + 4 N

Substituting the values,
=> 4× 9.8 × 1/2 – (0.9 × 4)+ 4 N

= 20 N

Hence, the acceleration of the body = a =force/mass = 20/4 = 5 $m/s^2$

The distance travelled in time 2 secs after starting from rest, s = ut + (1/2) $at^2$

=> 0 + 1/2(5)($2^2$) = 10 m
The block will move 10 m.

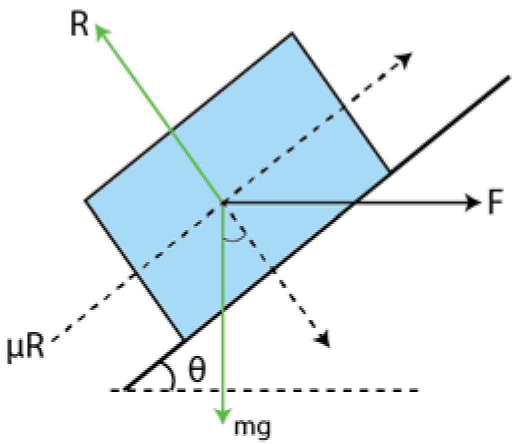
**Question 6:** A body of mass 2 kg is lying on a rough inclined plane of inclination 30°. Find the magnitude of the force parallel to the incline needed to make the block move (a) up the incline (b) down the incline. Coefficient of static friction = 0.2.

**Solution:**

mass of the body = m = 2kg

Angle of inclination = θ = 30°

Coefficient of static friction = μ = 0.2




(a)

To make the block move up the incline, the force should be equal and opposite to the net force acting down the incline i.e. μR + 2g sin 30°

= 0.2 x 9.8√3 + 2 x 9.8 x 1/2

= 13N

(b)

Net force acting down the incline

F = 2g sin 30° - μR

= 2 x 9.8 x 1/2 - 3.39

= 6.41 N

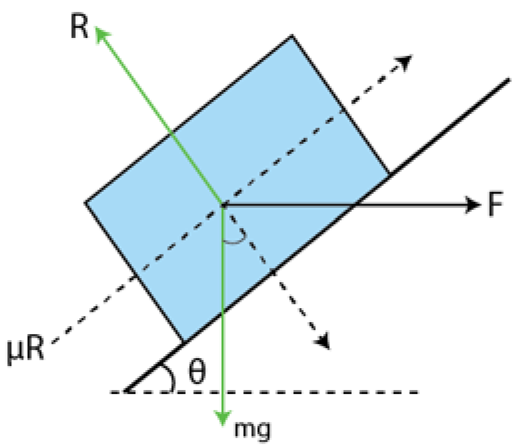
Due to force, F = 6.41 N, body will move in the incline with acceleration.

So, no external force is required. Therefore, force required is zero.

**Question 7:** Repeat part (a) of problem 6 if the push is applied horizontally and not parallel to the incline.

**Solution:**

mass of the body =m= 2kg
Angle of inclination =θ = $30^o$
Coefficient of static friction =μ = 0.2
and g = 10 $m/s^2$




from free body diagram,

R - mg cosθ - F sinθ = 0

or R = mg cosθ + F sinθ

and mg sin θ - μR - F cos θ = 0

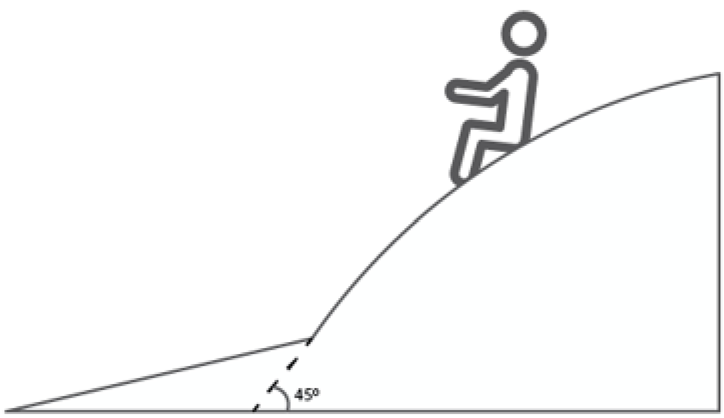
Using value of R,
mg sin θ - μ(mg cosθ + F sinθ ) - F cos θ = 0

$$\Rightarrow F = \frac{mg\sin\theta + \mu mg\cos\theta}{\mu\sin\theta - \cos\theta}$$

$$\Rightarrow F = \frac{2x10x\left(\frac{1}{2}\right) + 0.2x2x10x\left(\frac{\sqrt{3}}{2}\right)}{0.2x\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)}$$

$$\Rightarrow F = 17.5\ N$$

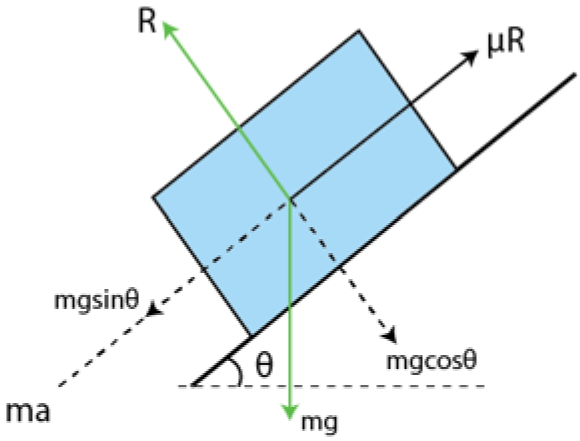
**Question 8:** In a children-park an inclined plane is constructed with an angle of incline $45^o$ in the middle part (figure below). Find the acceleration of a boy eliding on it if the friction coefficient between the cloth of the boy and the incline is 0.6 and g = 10 $m/s^2$.




**Solution:**

mass of the boy= m

Angle of inclination= θ = $45^o$
Coefficient of friction = μ = 0.6




R = mg cos(45°) = 0

=> R = mg/√2
Now, The force due to which boy is sliding down is

F = mg sin(45°) - μR

=> F = mg sin(45°) - μmg cos(45°)

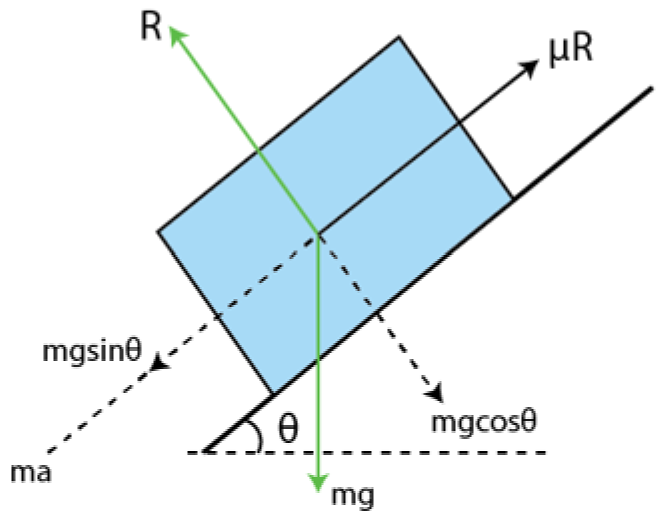
=> F = m(10)(1/√2) - 0.6 m (10)(1/√2)
=> F = (2√2)m

Thus, Acceleration of the boy = a= force/mass = (2√2)m/m = (2√2) m/$s^2$

**Question 9:** A body starts slipping down an incline and move, half meter in half second. How long will it take to move the next half meter?

**Solution:**

Let us consider "m" be the mass of the boy
Angle of inclination = θ
Acceleration of the body = a




Now,
R = mg cos θ ...(1)

and ma = mg sin θ - μR ...(2)

Using (1) in (2), we have

$$a = \frac{mg(\sin\theta - \mu\cos\theta)}{m}$$
$$a = g(\sin\theta - \mu\cos\theta)$$

Case 1:
In first half meter, the distance covered= s= 0.5 m

Time taken= t= 0.5sec

Initial velocity =u= 0

Using equation, $s = ut+1/2\ at^2$

=> $(0.5) = 1/2\ (a)(1/2)^2$

=> $a = 4\ m/s^2$

case 2: For next half meter,

Velocity =u=2 m/s

Acceleration= a= 4 $m/s^2$

Distance =s= 0.5m

Using equation of motion, $s = ut + (1/2) at^2$

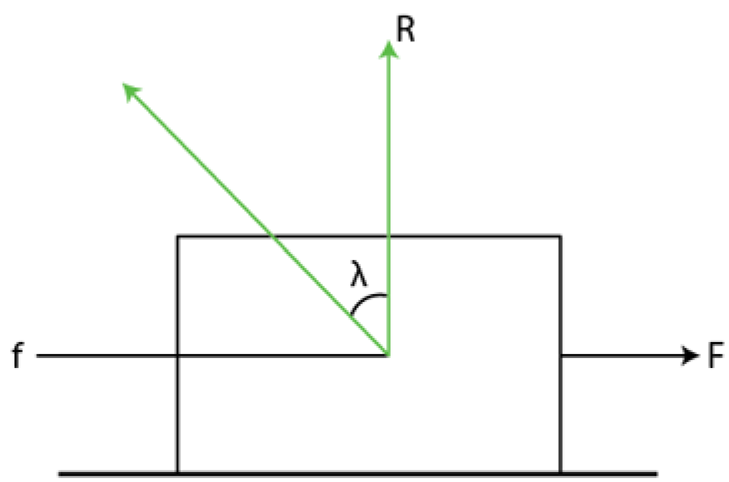
=> $(0.5) = 2t + (½) (4)(t^2)$

=> t = 0.21 sec.

**Question 10:** The angle between the resultant contact force and the normal force exerted by a body on the other is called the *angle of friction*. Show that, if λ be the angle of friction and μ the coefficient of static friction, $\lambda = \tan^{-1} \mu$

**Solution:**
Let the frictional force to be f, and F be the applied force and normal reaction be R

Now, coefficient of friction force = μ = tan λ = f/R



When the force applied on the body increases, the force of friction also increases. It increases up to limiting friction. Before reaching to the limiting friction,

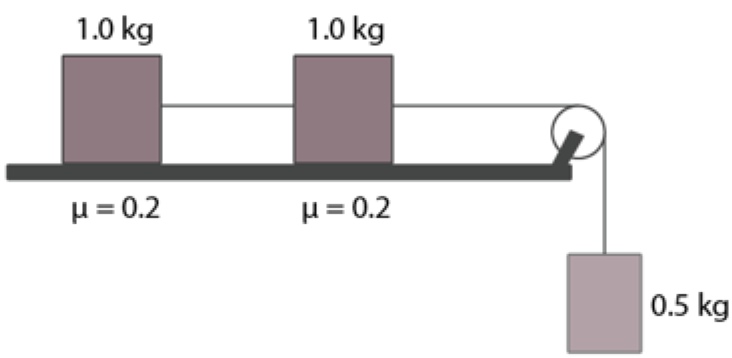
$$f < \mu R$$

$$\Rightarrow \tan\lambda = \frac{f}{R} \leq \frac{\mu R}{R}$$

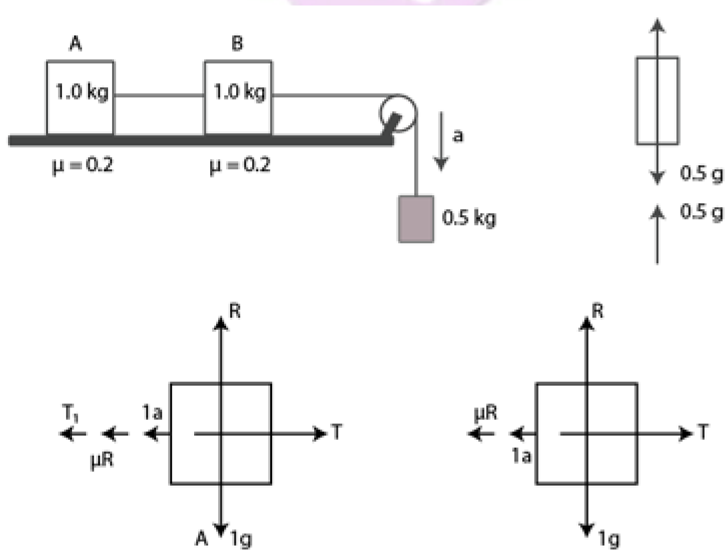
$$\Rightarrow \tan\lambda \leq \mu$$

$$\Rightarrow \lambda \leq \tan^{-1}\mu$$

**Question 11:** Consider the situation shown in figure (below). Calculate (a) the acceleration of the 1.0 kg blocks, (b) the tension in the string connecting the 1.0 kg block, and (c) the tension in the string attached to 0.50 kg.




**Solution:**



From the free body diagram,
$T + 0.5a - 0.5 g = 0$ ..(1)
$\mu R + a + T_1 - T = 0$ ...(2)
$\mu R + a = T_1$ ...(3)

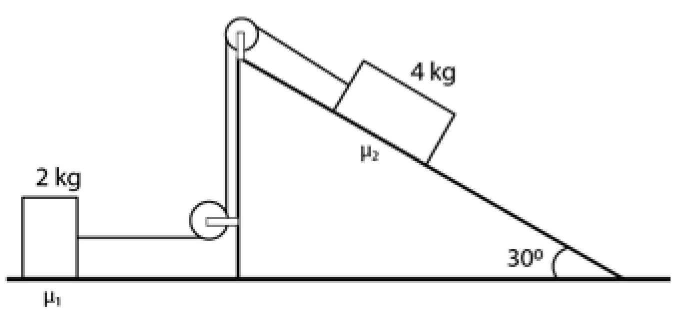
From equations (2) and (3
$T = 2T_1$

(2)=> $\mu R + a + T_1 - 2 T_1 = 0$

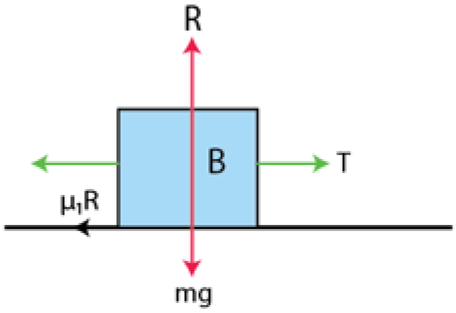
Or $T_1 = \mu R + a = 0.2 g + a$ ...(4)

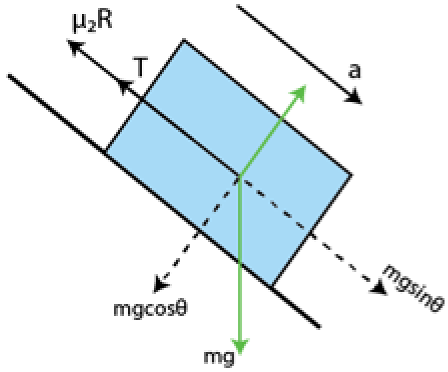
(1)=> $2T_1 + 0.5a - 0.5g = 0$
=> $T_1 = 0.25g - 0.25 a$ ...(5)
From equations (4) and (5)
$0.2g + a = 0.25g - 0.25a$
=> $a = 0.4 m/s^2$
Now,
(a) acceleration of 1 kg blocks each is $0.4 m/s^2$
(b) Tension $T_1 = 0.2g + a + 0.4 = 2.4N$
(c) Tension = $T = 0.5g - 0.5a = 4.8N$

**Question 12:** If the tension in the string in figure (below) is 16 N and the acceleration of each block is $0.5 m/s^2$, find the friction coefficients at the two contacts with the blocks.




**Solution:**




From first figure:
$\mu_1 R + ma = T$

$\mu_1 R + 2(0.5) = 16$ {Here $R = mg\cos\theta$}

$\Rightarrow \mu_1 (2g) = 15$

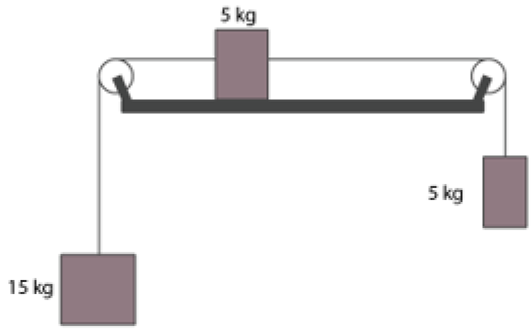
$\Rightarrow \mu_1 = 0.75$

From second figure:
$\mu_2 R + ma = F - mg \sin\theta$

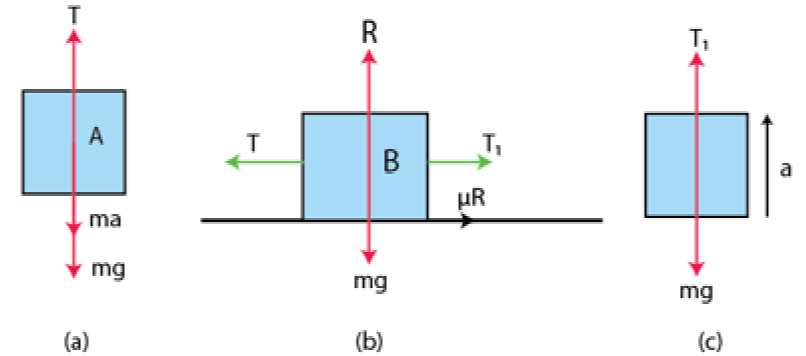
$\Rightarrow \mu_2 mg \cos\theta + 4(0.5) = 16 - 4g \sin 30°$

$\Rightarrow \mu_2 = 0.06$

**Question 13:** The friction coefficient between the table and the block shown in figure (below) is 0.2. Find the tensions in the two strings.




**Solution:**




Let us consider that the 15kg block is moving downward with the acceleration a.

Case 1: From figure (a)

$T + ma - mg = 0$

$T + 15a - 15g = 0$

$\Rightarrow T = 15g - 15a$……. (1)

Case 2: From the figure (c)

$T_1 - mg - ma = 0$

$T_1 - 5g - 5a = 0$

$\Rightarrow T_1 = 5g + 5a$ ………. (2)

Case 3: From figure (b)

$T = (T_1 + 5a + \mu R) = 0$

$\Rightarrow T - (5g + 5a + 5a + \mu R) = 0$ ……. (3)

(where $R = \mu g$)

From Equations (1) and (2),
$15g - 15a = 5g + 10a + 0.2\,(5g)$
$\Rightarrow 25a = 90$

$\Rightarrow a = 3.6\ m/s^2$

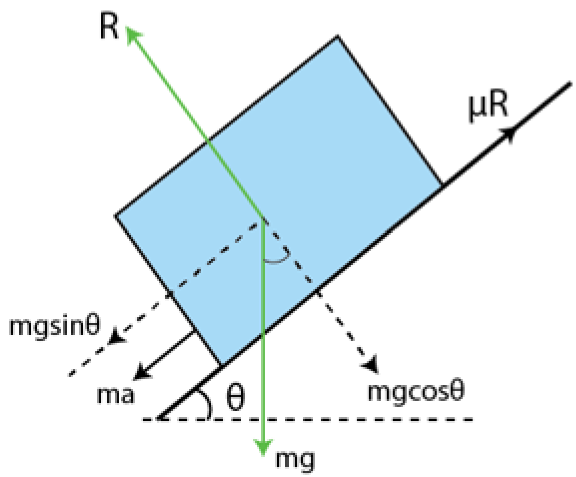
From Equation (3),
$T = 5 \times 10 + 10 \times 3.6 + 0.2 \times 5 \times 10$

T= 96 N in the left string.

From Equation (2),
$T_1 = 5g + 5a$
$= 5 \times 10 + 5 \times 36 = 50 + 18 T_1 = 68$ N in the right string.

**Question 14:** The friction coefficient between a road and the tyre of a vehicle is 4/3. Find the maximum incline the road may have so that once hard brakes are applied and the wheel starts skidding, the vehicle going down at a speed of 36 km/hr is stopped within 5 m.

**Solution:**




Let θ the maximum angle of incline.
distance travel s = 5 m,
Initial velocity of the vehicle= u = 36 km/h = 10 m/s
Final velocity of the vehicle= v = 0
and μ=4/3, g = 10 m/s$^2$

Now, $V^2 - u^2 = 2as$

r $a = -10\ m/s^2$

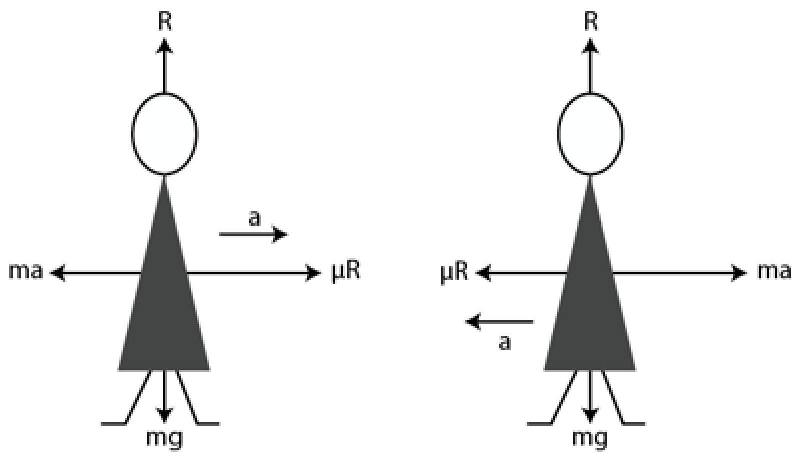
From the free body diagram
R = mg cosθ
Again, ma + mg sinθ = μR
=> ma + mg sinθ = μmg cosθ
=> a + g sinθ = μg cosθ
=> 10 +10 sinθ = 4/3×10 cosθ
=> 4 cosθ − 3 sin θ = 3
=> $4(1-\sin^2 \theta)^{1/2} = 3 + 3 \sin\theta$
=> $16 (1 - \sin^2 \theta) = 9 + 9 \sin^2 \theta + 18 \sin\theta$
=> $25 \sin^2 \theta + 18 \sin\theta - 7 = 0$

or sinθ = 0.28
or θ = 16°

Maximum incline is θ = 16°

**Question 15:** The friction coefficient between an athelete's shoes and the ground is 0.90. Suppose a superman wears these shoes and races for 50 m. There is no upper limit on his capacity of running at high speeds. (a) Find the minimum time that he will have to take in completing the 50 m starting from rest. (b) Suppose he takes exactly this minimum time to complete the 50 m, what minimum time will he take to stop?

**Solution:**




Superman has to move with maximum possible acceleration, to reach the given distance in minimum time,

Let us consider "a" be the maximum acceleration.
So, $ma - \mu R = 0$

=> $ma = \mu mg$
=> $a = \mu g = 0.9 \times 10 = 9\ m/s^2$

(a)
In this case,
initial velocity $= u = 0$,
$t = ?$
acceleration $= a = 9\ m/s^2$,
distance $s = 50$ m

From the equation of motion, $s = ut + (1/2)at^2$

$50 = 0 + 1/2 \times 9t^2$

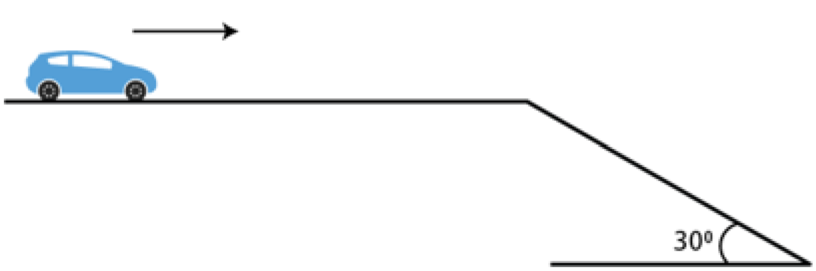
or t = 10/3 sec

(b)
After covering 50 m, the velocity of the athlete
v = u + at
=0+9×103 m/s

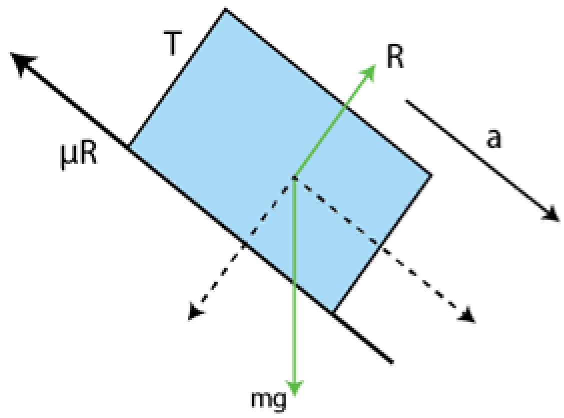
=30 m/s
He has to stop in minimum time. Hence, the deceleration,
$a = -9\ m/s^2$ (max)
R = mg
ma = μR (maximum frictional force)
ma = μmg
⇒ a = μg
$= 9\ m/s^2$ (deceleration)
$u_1 = 30$ m/s, v = 0
=> $t = (v_1 - u_1)/a$

or t = 10/3 sec.

**Question 16:** A car is going at a speed of 21.6 km/hr when it encounters a 12.8 m long slope of angle 30o (figure below). The friction coefficient between the road and the tyre is 1/2√3. Show that no matter how hard the driver applies the brakes; the car will reach the bottom with a speed greater than 36 km/hr. Take $g = 10\ m/s^2$.




**Solution:**




Hardest brake means maximum force of friction is produced between car's type and road.
maximum frictional force = μR

$R - mg \cos\theta = 0$
$\Rightarrow R = mg \cos\theta$ (i)
And $\mu R + ma - mg \sin\theta = 0$ (ii)
$\Rightarrow \mu\, mg \cos\theta + ma - mg \sin\theta = 0$

$\Rightarrow \mu g \cos\theta + a - 10(12) = 0$

$\Rightarrow a = 5 - \{1-(2\sqrt{3})\} \times 10(\sqrt{3}/2)$

or $a = -2.5\ m/s^2$

When brakes are applied, car will deaccelerate by $2.5\ m/s^2$

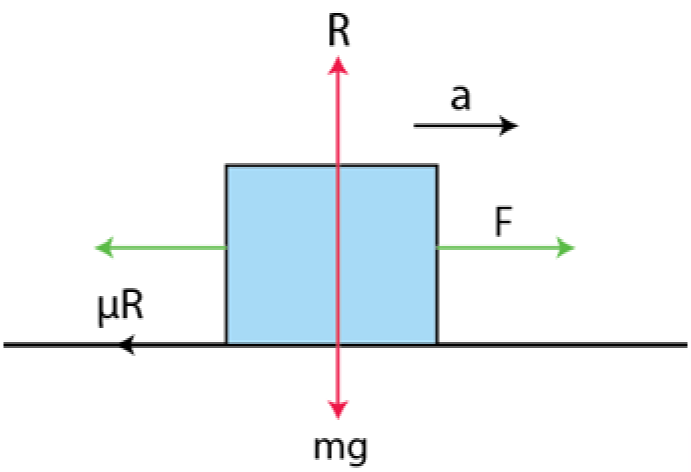
Distance =s= 12.8 m
initial velocity= u = 6 m/s
∴ Velocity at the end of incline

$v = \sqrt{(u^2+2as)} = \sqrt{(36+2(2.5)(12.8)} = 36$ km/h

Hence how hard the driver applies the breaks, car reaches the bottom with least velocity 36 km/h.

**Question 17:** A car starts from rest on a half kilometer long bridge. The coefficient of friction between the tyre and the road is 1.0. Show that one cannot drive through the bridge in less than 10 s.

**Solution:**




From diagram,
ma = μR
ma = μmg
a = μg = 1 × 10 = 10 m/s$^2$

To cross the bridge in minimum time, it must be at its maximum acceleration.

here, initial velocity u = 0, acceleration a = 10 m/s$^2$ and Distance s = 500 m,

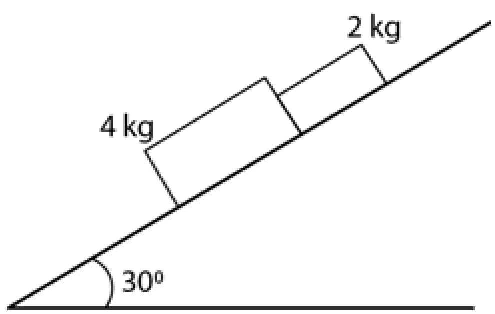
From equation $s = ut + 1/2\ at^2$

$500 = 1/2 \times 10 \times 10^2$

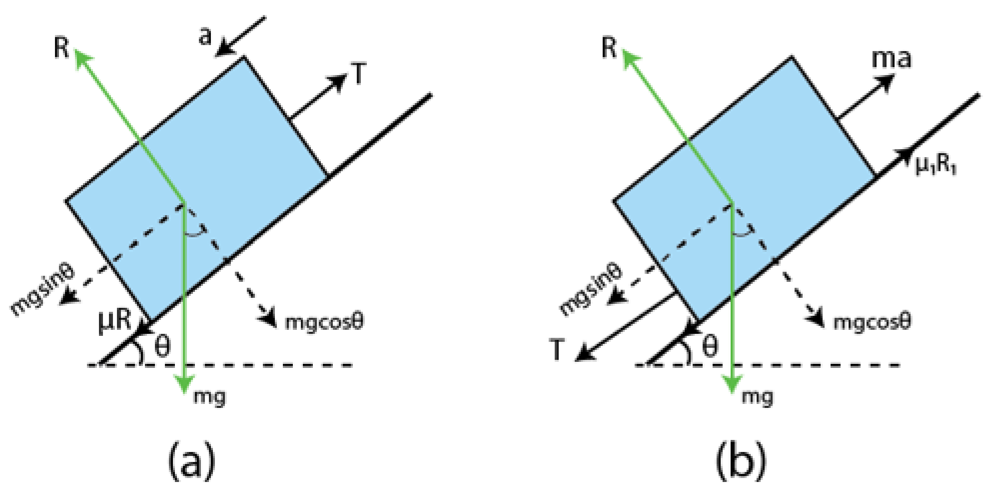
or t = 10 sec

car will take more than 10 sec to cross the bridge if the acceleration is less than 10 m/s$^2$.

**Question 18:** Figure (Below) shows two blocks in contact sliding down an inclined surface of inclination 30°. The friction coefficient between the block of mass 2.0 kg and the incline is and that between the block of mass 4.0 kg and the incline is $\mu_2$. Calculate the acceleration of the 2.0 kg block if (a) $\mu_1 = 0.20$ and $\mu_2 = 0.30$, (b) $\mu_1 = 0.30$ and $\mu_2 = 0.20$. Take $g = 10\ m/s^2$.




**Solution:**




Angle of inclination = $\theta = 30^o$

The free body diagram of the system is shown above

From the figure (a), mass = 4kg

$R = 4g \cos 30°$
=> $R = 20\sqrt{3}$ N ...(1)
and
$\mu_2 R + ma = T + mg \sin\theta$
$\mu_2 R + 4a = T + 4g \sin 30°$
$\Rightarrow 0.3\times (40) \cos 30° + 4a = T + 40 \sin 30°$ ....(2)

From the figure (a), mass = 2kg

$R_1 = 2g \cos 30°$
$=10/\sqrt{3}$ ..... (3)
$T + 2a - \mu_1 R_1 - 2g \sin 30° = 0$ .....(4)

From Equation (2),
$6\sqrt{3} + 4a - T - 20 = 0$

From Equation (4),

$T + 2a + 2\sqrt{3} - 10 = 0$

From equation (2) and (4)
$6\sqrt{3} + 6a - 30 + 2\sqrt{3} = 0$

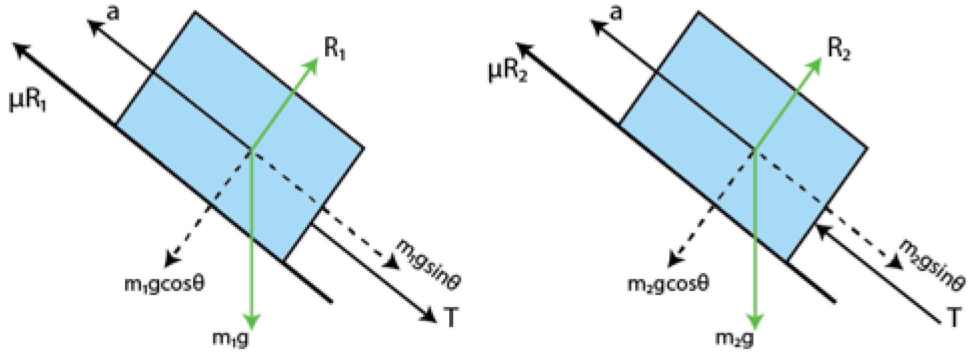
=> $6a = 30 - 8\sqrt{3}$

=> $a = 2.7$ m/s$^2$

(b) 4 kg block will move at a higher acceleration because the coefficient of friction is less than that of the 2 kg block. Therefore, the two blocks will move separately. By drawing the free body diagram of 2 kg mass, it can be shown that a = 2.4 m/s$^2$.

**Question 19:** Two masses $M_1$ and $M_2$ are connected by a light rod and the system is slipping down a rough incline of angle θ with the horizontal. The friction coefficient at both the contacts is μ. Find the acceleration of the system and the force by the rod on one of the blocks.

**Solution:**




(a) (b)

$R_1 = m_1g \cos \theta$

$T + m_1\sin \theta = m_1a + \mu R_1$

$T + m_1g \sin \theta = m_1a + \mu m_1g \cos \theta$ ..... (i)

From figure (b)
$R_2 = m_2g \cos \theta$
$T - m_2g \sin \theta = m_2a - \mu R_2$

$T - m_2g \sin \theta + m_2a + \mu m_2 g\cos \theta = 0$ ....(ii)

From Equations (i) and (ii),
$g \sin \theta(m_1 + m_2) - a(m_1 + m_2) - \mu g \cos \theta(m_1 + m_2) = 0$

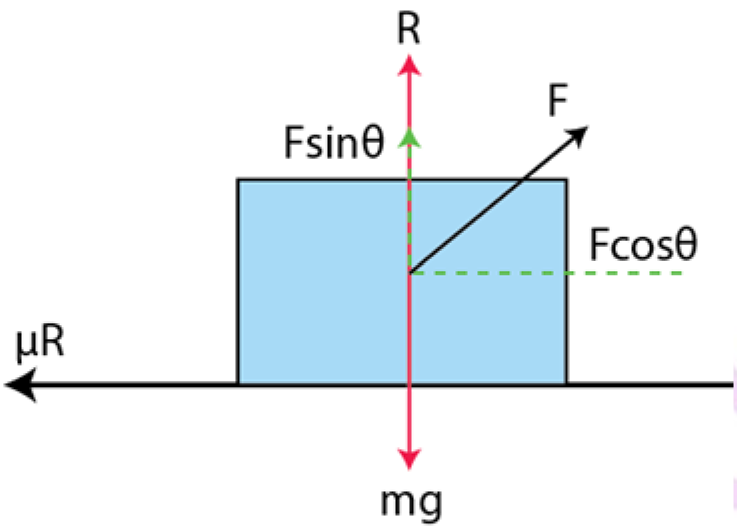
=> $a = g(\sin \theta - \mu \cos \theta)$
Hence, the acceleration of the system = $g(\sin \theta - \mu\cos \theta)$

The force exerted by the rod on one of the blocks is tension, T.
$T = m_1g \sin \theta + m_1a + \mu m_1g \cos \theta$
$T = - m_1g \sin \theta + m_1(g \sin \theta - \mu g \cos \theta) + \mu m_1g \cos \theta = 0$

=> T=0

**Question 20:** A block of mass M is kept on a rough horizontal surface. The coefficient of static friction between the block and the surface is μ. The block is to be pulled by applying a force to it. What minimum force is needed to slide the block? In which direction should this force act?

**Solution:**




From the free body diagram,
$R + F \sin\theta = mg$
$\Rightarrow R = -F \sin\theta + mg$ ...(1)
$\mu R = F \cos\theta$ .....(2)

From Equation (1),
$\mu(mg - F\sin\theta) - F\cos\theta = 0$
$\Rightarrow \mu mg = \mu F \sin\theta + F\cos\theta$

$$F = \frac{\mu mg}{\mu \sin\theta + \cos\theta}$$

F should be minimum, when $\mu \sin\theta + \cos\theta$ is maximum.
Again, $\mu \sin\theta + \cos\theta$ is maximum when its derivative is zero:

$d/d\theta\ (\mu \sin\theta + \cos\theta) = 0$

⇒ $\mu \cos\theta - \sin\theta = 0$
$\theta = \tan^{-1}\mu$
So,

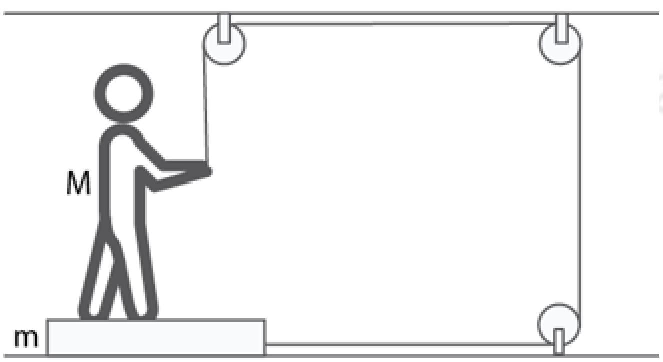
$$F = \frac{\mu mg}{\mu \sin\theta + \cos\theta}$$

$$F = \frac{\mu mg \cos\theta}{1 + \mu \tan\theta} = \frac{\mu mg \sec\theta}{1 + tan^2\theta}$$

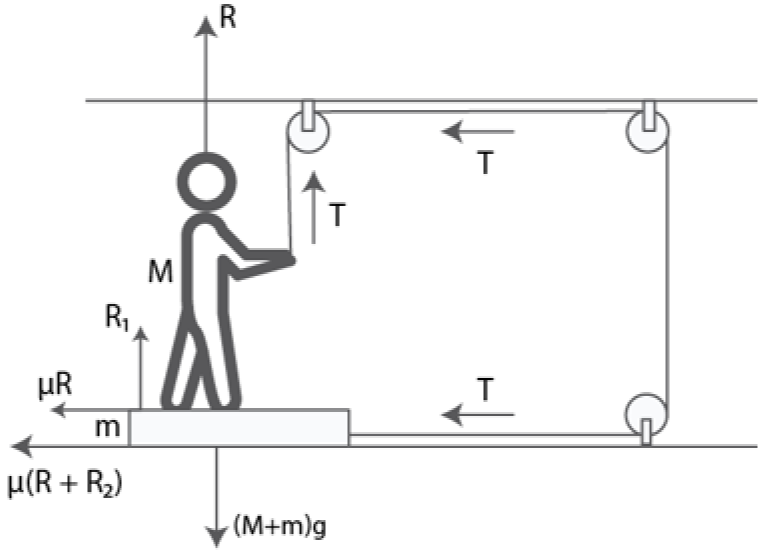
$$F = \frac{\mu mg}{\sec\theta} = \frac{\mu mg}{\sqrt{1 + tan^2\theta}} = \frac{\mu mg}{\sqrt{1 + \mu^2}}$$

Minimum force required is μmg/√(1+μ^2)

**Question 21:** The friction coefficient between the board and the floor shown in figure (below) is μ. Find the maximum force that the man can exert on the rope so that the board does not slip on the floor.




**Solution:**




$R + T = Mg$

$\Rightarrow R = Mg - T$ ....(1)

Also, $R_1 - R - mg = 0$

$\Rightarrow R_1 = R + mg$ .....(2)

And $T - \mu R_1 = 0$

From Equation (2),

$T - \mu(R + mg) = 0$

$\Rightarrow T - \mu R - \mu\, mg = 0$

$\Rightarrow T - \mu(Mg - T) - \mu mg = 0$

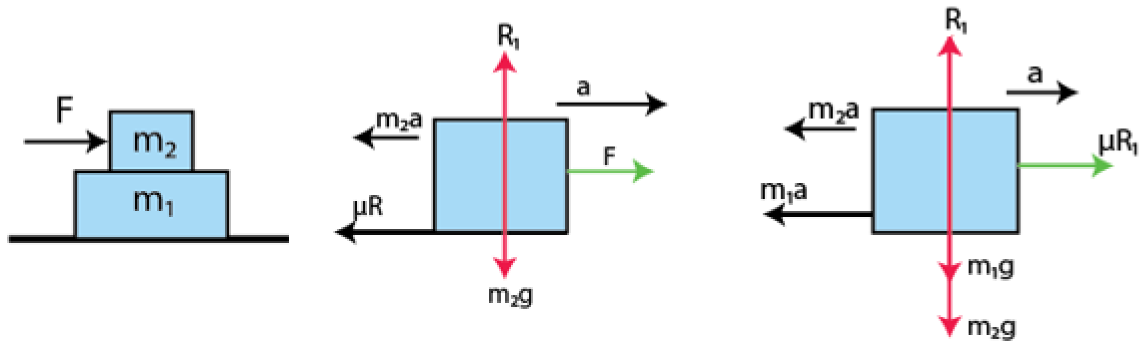
$T - \mu Mg + \mu t - \mu mg = 0$

$\Rightarrow T\,(1 + \mu) = \mu Mg + \mu mg$

=> $T = (\mu(M+m)g)/(1+\mu)$, which is the maximum force exerted by the man.

**Question 22:** A 2 kg block is placed over a 4 kg block and both are placed on a smooth horizontal surface. The coefficient of friction between the blocks is 0.20. Find the acceleration of the two blocks if a horizontal force of 12 N is applied to (a) the upper block, (b) the lower block. Take $g = 10\ m/s^2$.

**Solution:**




for the mass of 2 kg $m_2$

$R_1 - 2g = 0$
$\Rightarrow R_1 = 2 \times 10 = 20$
$2a + 0.2\ R_1 - 12 = 0$
$\Rightarrow 2a + 0.2\ (20) = 12$
$\Rightarrow 2a = 12 - 4$
$\Rightarrow a = 4\ m/s^2$

for 4kg block mass $m_1$

$4a - \mu R_1 = 0$
$\Rightarrow 4a = \mu R_1 = 0.2\ (20) = 4$
$\Rightarrow a = 1\ m/s^2$

The 2 kg block has acceleration 4 $m/s^2$ and the 4 kg block has acceleration 1 $m/s^2$.

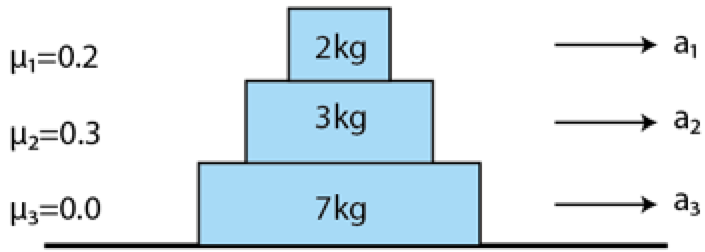
we can also write,

$R_1 = 2g = 20$
$Ma = \mu R_1 = 0$
$a = 0$

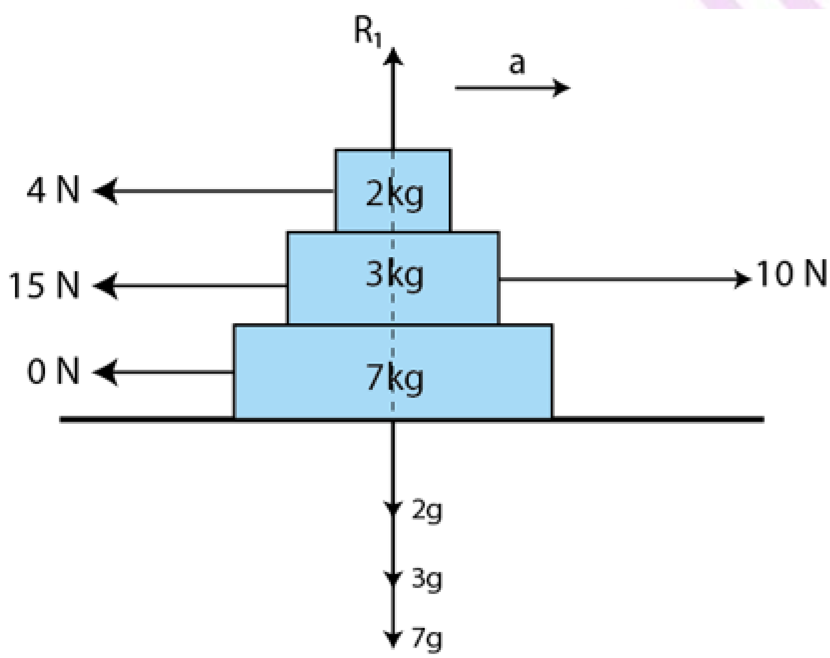
and
$Ma + \mu mg - F = 0$
$4a + 0.2 \times 2 \times 10 - 12 = 0$
$\Rightarrow 4a + 4 = 12$
$\Rightarrow 4a = 8$

$\Rightarrow a = 2\ m/s^2$

**Question 23:** Find the accelerations $a_1$, $a_2$, $a_3$ of the three blocks shown in figure (below) if a horizontal force of 10 N is applied on (a) 2 kg block, (b) 3 kg block, (c) 7 kg block. Take $g = 10\ m/s^2$.




**Solution:**




The coefficient of frictions are given as, $\mu_1 = 0.2$, $\mu_2 = 0.3$ and $\mu_3 = 0.4$

(a) When the 10 N force is applied to the 2 kg block, it experiences maximum frictional force.

Here, $\mu_1 R_1 = \mu_1 \times m_1 g$

$\mu_1 R_1 = \mu_1 \times 2g$

$= (0.2) \times 20$

= 4 N (From the 3 kg block)

The net force experienced by the 2 kg block = 10 – 4 = 6 N
Hence, $a_1$ = 6/2 = 3 m/s$^2$

In this case, the frictional force from the 2 kg block becomes the driving force (4N) and the maximum frictional force between the 3 kg and 7 kg blocks.
so, $\mu_2 R_2 = \mu_2 m_2 g$ = (0.3) × 5 kg = 15 N

3 kg block cannot move relative to the 7 kg block, because there is no friction from the floor.
So, $a_2 = a_3$ = 4/10 = 0.4 m/s$^2$

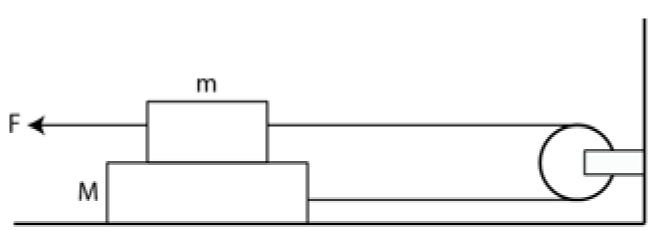
(b) When the 10 N force is applied to the 3 kg block, it experiences maximum frictional force of 19 N, from 2 kg and 7 kg block. As the floor is frictionless, all the three bodies will move together.

a = 10/12 = 5/6 m/s$^2$

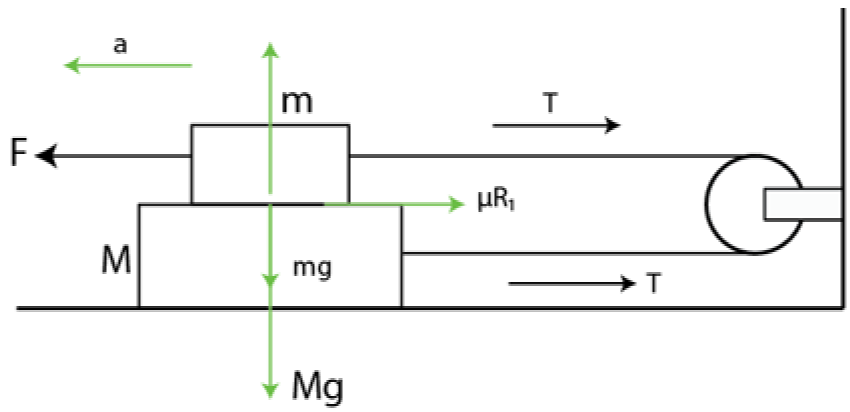
(c) when 10 N force is applied to the 7 kg block, all three blocks will move together with the same acceleration.

Hence, $a_1 = a_2 = a_3$ = 5/6 m/s$^2$

**Question 24:** The friction coefficient between the two blocks shown in figure (6-E9) is μ but the floor is smooth. (a) What maximum horizontal force F can be applied without disturbing the equilibrium of the system? (b) Suppose the horizontal force applied is double of that found in part (a). Find the accelerations of the two masses.




**Solution:**




(a) $R_1 = mg$ ...(i)
$F = \mu R_1 + T$ ...(ii)
$T - \mu R_1 = 0$ ..(iii)

From equations (i) and (ii)
$F - \mu mg = T$ ...(ii)

From equations (i) and (iii)
$T = \mu mg$

Putting $T = \mu mg$ in equation (ii)

$F = \mu mg + \mu mg = 2\mu mg$

(b)
From upper block of free body, we have
$2F - T - \mu mg = ma$

From lower block of free body, we have

$T = Ma + \mu mg$

Equating both the equations,
$2F - Ma - \mu mg - \mu mg = ma$

Putting $F = 2\mu mg$, we get

$2(2\mu mg) - 2\mu mg = a(M + m) \Rightarrow 4\mu mg - 2\mu mg = a(M + m)$

=> $a = 2\mu mg/(M+m)$ in opposite directions.

**Question 25:** Suppose the entire system of the previous question is kept inside an elevator which is coming down with an acceleration $a < g$. Repeat parts (a) and (b).

**Solution:**

Referring Question 24 image.

(a)
$R_1 + ma - mg = 0$
$\Rightarrow R_1 = m(g - a)$
$= mg - ma$

Now, $F - T - \mu R_1 = 0$ and
$T - \mu R_1 = 0$
$\Rightarrow F - [\mu (mg - ma)] - \mu(mg - ma) = 0$
$\Rightarrow F - \mu mg - \mu ma - \mu mg + \mu ma = 0$
$\Rightarrow F = 2\mu mg - 2\mu ma$
$= 2\mu m(g - a)$

(b) Let $a_1$ be acceleration of the blocks, then

$R_1 = mg - ma$ ....(i)
$2F - T - \mu R_1 = ma_1$ ...(ii)
Now, $T = \mu R_1 + Ma_1$
$= \mu mg - \mu ma + Ma_1$

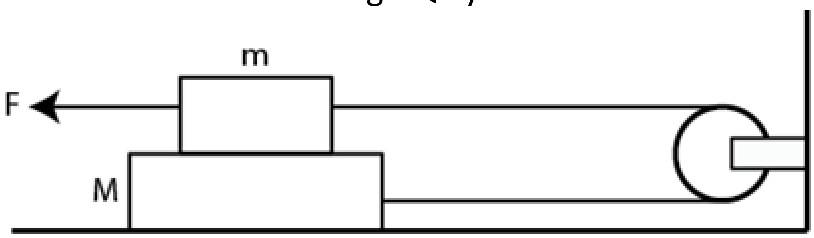
Substituting the value of F and T in equation (ii),
$2[2\mu m(g - a)] - (\mu mg - \mu ma + Ma_1) - \mu mg + \mu ma = ma_1$
$\Rightarrow 4\mu mg - 4\mu ma - 2\mu mg + 2\mu ma = ma_1 + Ma_1$

=>

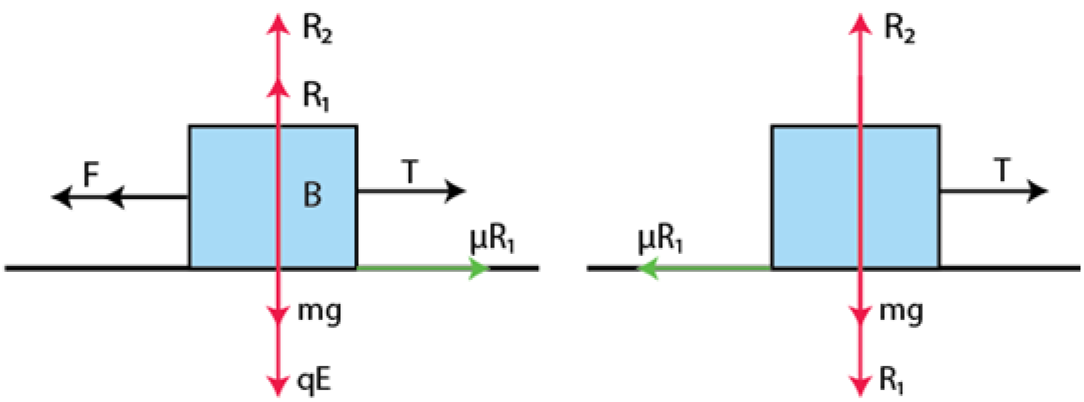
$$a_1 = \frac{2\mu m(g - a)}{m + M}$$

Both the blocks move with same acceleration $a_1$ but in opposite directions.

**Question 26:** Consider the situation shown in figure (below). Suppose a small electric field E exists in the space in the vertically upward direction and the upper block carries a positive charge Q On its top surface. The friction coefficient between the two blocks is μ but the floor is smooth. What maximum horizontal force F can be applied without disturbing the equilibrium?
[Hint: The force on a charge Q by the electric field E is F = QE in the direction of E.]




**Solution:**




$R_1$ is the normal reaction force
E is the small electric field
Q is the charge
From the figure (b)
$R_1 + qE = mg$

$\Rightarrow R_1 = mg - qE$ ...(1)

From figure (a)

$F - T - \mu R_1 = 0$

=> $F - T - \mu mg + \mu qE = 0$ ....(2)
[Using (1)]

Again from (1) and (2)

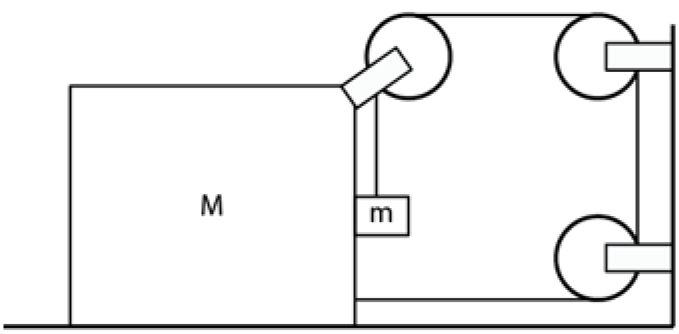
$T - \mu R_1 = 0$

=> $T - \mu R_1 = \mu(mg - qE) = \mu mg - \mu qE$
From (2),

$F - \mu mg + \mu qE - \mu mg + \mu qE = 0$

=> $F = 2\mu(mg - qE)$
Which is the maximum horizontal force applied.

**Question 27:** A block of mass m slips on a rough horizontal table under the action of a horizontal force applied to it. The coefficient of friction between the block and the table is μ. The table does not move on the floor. Find the total frictional force applied by the floor on the legs of the table. Do you need the friction coefficient between the table and the floor or the mass of the table?




**Solution:**
When the body is slipping from the surface, the maximum frictional force is acting on it.

$R = mg$
$F - \mu R = 0$
=> $F = \mu R = \mu mg$

As, table is at rest, the frictional force at table's legs will also be μR. Let the frictional force be f,

$f - \mu R = 0$
$\Rightarrow f = \mu R = \mu mg$
Therefore, the total frictional force on the table by the floor is μmg.

**Question 28:** Find the acceleration of the block of mass M in the situation of figure (6-E10). The coefficient of friction between the two blocks is $\mu_1$ and that between the bigger block and the ground is $\mu_2$.

**Solution:**
Let the acceleration of block of mass M is 'a' towards right. So, the block 'm' must go down with an acceleration '2a'. As the block 'm' is in contact with the block 'M', it will also have acceleration 'a' towards right.
$R_1 - ma = 0$
$\Rightarrow R_1 = ma$ ....(i)

Also,
$2ma + T - Mg + \mu_1 R_1 = 0$
$\Rightarrow T = Mg - (2 + \mu_1)\, ma$ .....(ii)

Also, $T + \mu_1 R_1 + Mg - R_2 = 0$
using (i), $R_2 = T + \mu_1\, ma + mg$

again using value of T from (2), we get
$R_2 = Mg + Ma - 2ma$ ...(iii)

Again, from figure we have
$T + T - R - Ma - \mu_2 R_2 = 0$
$\Rightarrow 2T - Ma - ma - \mu_2(Mg + mg - 2ma) = 0$

Substituting the values of $R_1$ and $R_2$ from (i) and (iii), we get:
$2T = (M + m)a + \mu_2\,(Mg + mg - 2ma)$ ….(iv)

From equations (ii) and (iv), we have:

=> $2mg – \mu_2 (M + m)g = a[M + m – 2\mu_2 m + 4m + 2\mu_1 m]$

Thus, acceleration of the block of mass M is

$$a = \frac{2m + \mu 2(M+m)g}{M+m(5+2(\mu 1-\mu 2))}$$

**Question 29:** A block of mass 2 kg is pushed against a rough vertical wall with a force of 40 N, coefficient of static friction being 05. Another horizontal force of 15 N, is applied on the block in a direction parallel to the wall. Will the block move? If yes, in which direction? If no, find the frictional force exerted by the wall on the block.

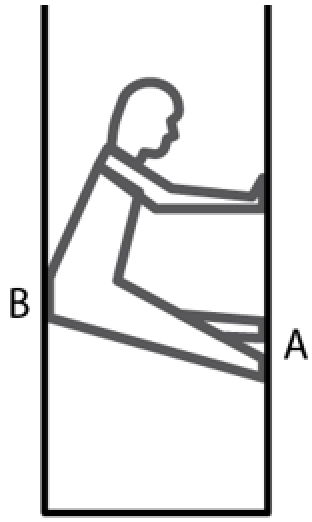
**Solution:**
Net force on the block = $(20)^2 + (15)^2 - 0.5 \times 40 = 5$ N

Therefore, $\tan \theta = 20/15 = 4/3$

or, $\theta = \tan^{-1} (4/3) = 53°$

The block will move at 53° angle with the 15 N force.

**Question 30:** A person (40 kg) is managing to be at rest between two vertical walls by pressing one wall A by his hands and feet and the other wall B by his back (figure 6-E11). Assume that the friction coefficient between his body and the walls is 0.8 and that limiting friction acts at all the contacts. (a) Show that the person pushes the two walls with equal force. (b) Find the normal force exerted by either wall on the person.
Take $g = 10$ m/s$^2$.




**Solution:**

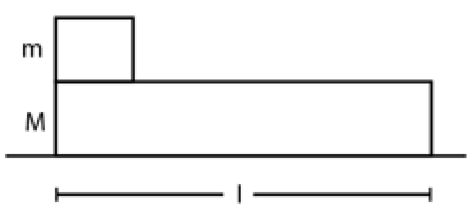
Mass of man = 50 kg and g = 10 $m/s^2$

Frictional force developed between hands, legs & back with the wall will be equal to the weight of the man. Man remains in equilibrium.
If man applies unequal forces on the wall, the reaction force will be different and he can't rest between the walls. Frictional force 2μR balance his body weight.

μR + μR = mg
=> 2μR = 40 × 10

=> R = 40 × 10/2 × 0.8 = 250 N, Which is normal force.

**Question 31:** Figure (6-E12) shows a small block of mass m kept at the left end of a larger block of mass M and length l. The system can slide on a horizontal road. The system is started towards right with an initial velocity u. The friction coefficient between the road and the bigger block is i and that between the block is μ/2. Find the time elapsed before the smaller blocks separates from the bigger block.



**Solution:**

Let masses m and M are having acceleration $a_1$ and $a_2$ respectively.

if $a_1 > a_2$ so, mass m can move on mass M.
Also, consider after time 't', the mass m separates from mass M
During this time, mass m covers the distance s,
$s = vt + 1/2\ a_1t^2$

and $s\_m = ut + 1/2\ a_2\ t^2$

For mass m to separate from mass M,

$vt + 1/2\ a_1t^2 = vt + 1/2\ a_2t^2$ .....(1)

$ma_1 + (1/2)\mu R = 0$

$ma_1 = -(1/2)\mu mg = (1/2)\mu m \times 10$

$a_1 = -5\mu$

Again, from figure, $Ma_2+\mu(M+m)\ g-(\mu/2)\ mg = 0$

$2Ma_2 = \mu mg - 2\mu mg - 2\mu mg$

=> $a_2 = (-\mu mg - 2\mu Mg)/2M$
Substituting the values of $a_1$ and $a_2$ in equation (1), we get:

$t = \sqrt{[(4ml)/(M+m)\mu g]}$