Date of Exam: 7th January 2020 (Shift 1) Time: 9:30 am- 12:30 pm Subject: Physics

1. A polarizer-analyzer set is adjusted such that the intensity of light coming out of the analyzer is just 10 % of the original intensity. Assuming that the polarizer-analyzer set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero is

b.

d.

 71.6°

 18.4^{0}

a. 45°

c. 90°

Solution:(d)

Intensity after polarisation through polaroid = $I_o cos^2 \phi$ So, $0.1I_o = I_o cos^2 \phi$ $\Rightarrow cos \phi = \sqrt{0.1}$ $\Rightarrow cos \phi = 0.316$

Since, $\cos \phi < \cos 45^{\circ}$ therefore, $\phi > 45^{\circ}$ If the light is passing at 90° from the plane of polaroid, than its intensity will be zero.

Then, $\theta = 90^{\circ} - \phi$ therefore, θ will be less than 45° . So, the only option matching is option d which is 18.4°

2. Which of the following gives reversible operation?





Since, there is only one input hence the operation is reversible.

3. A 60 HP electric motor lifts an elevator with a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (Given 1 HP = 746 W, $g = 10 m/s^2$)

b.

 $2.0 \ m/s$

d. 1.9 m/s

a. 1.5 m/s

c.
$$1.7 m/s$$

Solution:(d) Friction will oppose the motion

> Net force = 2000 kg + 4000 = 24000 N Power of lift = 60 HP Power =Force × Velocity $v = \frac{P}{-} = \frac{60 \times 746}{F24000}$ v = 1.86 m/s= 1.9 m/s

- 4. A long solenoid of radius R carries a time (t) dependent current $I(t) = I_0 t(1 t)$. A ring of radius 2R is placed coaxially near its middle. During the time instant $0 \le t \le 1$, the induced current (I_R) and the induced EMF (V_R) in the ring changes as:
 - a. Direction of I_R remains unchanged and V_R is maximum at t = 0.5
 - b. Direction of I_R remains unchanged and V_R is zero at t = 0.25
 - c. At t = 0.5 direction of I_R reverses and V_R is zero
 - d. At t = 0.25 direction of I_R reverses and V_R is maximum

Solution:(c)



Field due to solenoid near the middle $= \mu_o NI$

Flux, $\phi = BA$ where $(A = \pi(R)^2)$ $= \mu_o N I_o t (1 - t) \pi R^2$ $E = -\frac{d\phi}{dt}$ [By Lenz's law] $E = -\frac{d}{dt} (\mu_o N I_o t (1 - t) \pi R^2)$ $E = -\mu_o N I_o \pi R^2 \frac{d}{dt} [t(1 - t)]$ $E = -\pi \mu_o I_o N R^2 (1 - 2t)$

Current will change its direction when EMF will be zero

 $\implies (1 - 2t) = 0$ So, $t = 0.5 \ sec$

5. Two moles of an ideal gas with $\frac{C_p}{C_v} = 5/3$ are mixed with 3 moles of another ideal gas with $\frac{C_p}{C_v} = 4/3$. The value of $\frac{C_p}{C_v}$ for the mixture is

a.1.47b.1.42c.1.45d.1.50

Solution:(b)

For first gas having $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$ Using formula $C_p = \frac{R\gamma}{\gamma - 1}$ $C_v = \frac{R}{\gamma - 1}$





- 6. Consider a circular coil of wire carrying current *I*, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i . The magnetic flux through the area of the circular coil area is given by ϕ_0 . Which of the following option is correct?
 - a. $\phi_i = -\phi_o$ b. $\phi_i > \phi_o$ c. $\phi_i < \phi_o$ d. $\phi_i = \phi_o$

Solution:(a)



As magnetic field line of ring will form close loop therefore all outgoing from circular hole passing through the infinite plate.

- $\therefore \phi_1 = -\phi_0$ (because the magnetic field lines going inside is equal to the magnetic field lines coming out.)
- 7. The current (i_1) (in A) flowing through 1 Ω resistor in the following circuit is
 - a. 0.40 *A* b. 0.20 *A*
 - c. 0.25 A d. 0.5 A

Solution:(b)





8. Two infinite planes each with uniform surface charge density $+\sigma C/m^2$ are kept in such a way that the angle between them is 30° . The electric field in the region shown between them is given by:



d.
$$\frac{\sigma}{2\epsilon_0}\left[(1+\frac{\sqrt{3}}{2})\hat{y}+\frac{1}{2}\hat{x}\right]$$

Solution:(a)

Field due to single plate $= \frac{\sigma}{2\epsilon_o} = \left[\vec{E_1}\right] = \left[\vec{E_2}\right]$ Net electric field $\vec{E} = \vec{E_1} + \vec{E_2}$ $= \frac{\sigma}{2\epsilon_0} \cos 30^0(-\hat{j}) + \frac{\sigma}{2\epsilon_0} \sin 30^0(-\hat{i}) + \frac{\sigma}{2\epsilon_0}(\hat{j})$ $= \frac{\sigma}{2\epsilon_0} \left(1 - \frac{\sqrt{3}}{2}\right)(\hat{j}) - \frac{\sigma}{4\epsilon_0}(\hat{i})$ $= \frac{\sigma}{2\epsilon_0} \left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{y} - \frac{1}{2}\hat{x}\right]$

9. If the magnetic field in a plane electromagnetic wave is given by $\vec{B} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\hat{j} T$ then what will be expression for electric field?

a.
$$\vec{E} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{i} V/m$$
 b. $\vec{E} = 3 \times 10^{-8} \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{j} V/m$
c. $\vec{E} = 60 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{k} V/m$ d. $\vec{E} = 9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t) \hat{k} V/m$

Solution:(d)

We know that,

$$\begin{vmatrix} E_0 \\ B_0 \end{vmatrix} = c$$

$$B_0 = 3 \times 10^{-8}$$

$$\implies E_0 = B_0 \times c = 3 \times 10^{-8} \times 3 \times 10^8$$

$$= 9 N/C$$

$$\therefore E = E_o \sin(\omega t - kx + \phi)\hat{k} = 9 \sin(\omega t - kx + \phi)\hat{k}$$

$$\vec{E} = 9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\hat{k} V/m$$

- 10. The time period of revolution of electron in its ground state orbit in a hydrogen atom is $1.6 \times 10^{-16}s$. The frequency of revolution of the electron in its first excited state (in s^{-1}) is :
 - a. 6.2×10^{15} b. 1.6×10^{14} c. 7.8×10^{14} d. 5.6×10^{12}

Solution:(c)

Time period is proportional to $\frac{n^3}{Z^2}$.

Let T_1 be the time period in ground state and T_2 be the time period in it's first excited state.

$$T_1 = \frac{n^3}{2^2}$$

(Where, n = excitation level and 2 is atomic no.)

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3$$

$$T_1 = 1.6 \times 10^{-16} s$$

So,

Given,

$$\frac{1.6 \times 10^{-16}}{T2} = \left(\frac{1}{2}\right)^3$$
$$T_2 = 12.8 \times 10^{-16} s$$

Frequency is given by $f = \frac{1}{T}$

11. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence will be

 $f = \frac{1}{12.8} \times 10^{16} \ Hz$

 $f = 7.8128 \times 10^{14} Hz$

a.
$$L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$$

c. $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

b.
$$L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$$

d. $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

Solution:(d)

For damped oscillator by Newton's second law

$$-kx - bv = ma$$

$$kx + bv + ma = 0$$

$$kx + b\frac{dx}{dt} + m\frac{d^2x}{dt^2} = 0$$
For LCR circuit by KVL
$$-IR - L\frac{dI}{dt} - \frac{q}{c} = 0$$

$$\implies IR + L\frac{dI}{dt} + \frac{q}{c} = 0$$

$$\implies \frac{q}{c} + R\frac{dq}{dt} + L\frac{d^2q}{dt^2} = 0$$
By comparing
$$R \implies b$$

 $c \implies \frac{1}{k}$

 $m \implies L$



12. Visible light of wavelength $6000 \times 10^{-8} cm$ falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minima is at 60° from the central maxima. If the first minimum is produced at θ_1 , then θ_1 is close to

b.

d.

 30°

 25°

a. 20°

c. 45°

Solution:(d)

For single slit diffraction experiment:

Angle of minima are given by

$$\sin \theta_n = \frac{n\lambda}{d} \quad (\sin \theta_n \neq \theta_n \text{ as } \theta \text{ is large })$$

$$\sin \theta_2 = \sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{2\lambda}{d} = \frac{2 \times 6000 \times 10^{-10}}{d} \tag{1}$$

$$\sin \theta_1 = \frac{\lambda}{d} = \frac{6000 \times 10^{-10}}{d} \tag{2}$$

Dividing (1) and (2)

$$\implies \frac{\sqrt{3}}{2sin\theta_1} = 2 \implies sin\theta_1 = \frac{\sqrt{3}}{4} = 0.43$$

As, the value is coming less than 30° the only available option are 20° and 25° but by using approximation we get $\theta_1 = 25^{\circ}$

13. The radius of gyration of a uniform rod of length l about an axis passing through a point l/4 away from the center of the rod, and perpendicular to it, is



Solution:(a)



Moment of inertia of rod about axis perpendicular to it passing through its centre is given by

$$I = \frac{Ml^2}{12} + M\left(\frac{l}{4}\right)^2$$
$$= \frac{3Ml^2 + 4Ml^2}{48}$$
$$= \frac{7Ml^2}{48}$$

Now, comparing with $I = Mk^2$ where k is the radius of gyration

$$k = \sqrt{\frac{7l^2}{48}}$$
$$k = l\sqrt{\frac{7}{48}}$$

14. A satellite of mass m is launched vertically upward with an initial speed u from the surface of the earth. After it reaches height R(R = radius of earth), it ejects a rocket of mass m/10 so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G = gravitational constant; M is the mass of earth)

a.
$$5m\left[u^2 - \frac{119}{200}\frac{GM}{R}\right]$$

b. $\frac{m}{20}\left[u - \sqrt{\frac{2GM}{3R}}\right]^2$
c. $\frac{3m}{8}\left[u + \sqrt{\frac{5GM}{6R}}\right]^2$
d. $\frac{m}{20}\left[u^2 + \frac{113}{200}\frac{GM}{R}\right]$

Solution:(a) As we know,

 $T.E_{ground} = T.E_R$ $\frac{1}{2}mu^2 + \left(\frac{-GMm}{R}\right) = \frac{1}{2}mv^2 + \left(\frac{-GMm}{2R}\right)$ $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \left(\frac{-GMm}{2R}\right)$ $v^2 = u^2 + \left(\frac{-GM}{R}\right)$ $\implies v = \sqrt{u^2 + \left(\frac{-GM}{R}\right)}$

The rocket splits at height R. Since, sepration of rocket is impulsive therefore conservation of momentum in both radial and tangential direction can be applied.

$$\frac{m}{10}V_T = \frac{9m}{10}\sqrt{\frac{GM}{2R}}$$
$$\frac{m}{10}V_r = m\sqrt{u^2 - \frac{GM}{R}}$$

(1)







- 15. Three point particles of mass 1 kg, 1.5 kg and 2.5 kg are placed at three corners of a right triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The centre of mass of the system is at the point:
 - a. 0.9 cm right and 2.0 cm above 1 kg mass
 - b. 2.0 cm right and 0.9 cm above 1 kg mass
 - c. 1.5 cm right and 1.2 cm above 1 kg mass
 - d. 0.6 cm right and 2.0 cm above 1 kg mass

Solution: (a)



Taking 1 kg as the origin

$$\begin{aligned} x_{com} &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \\ x_{com} &= \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5} \\ x_{com} &= 0.9 \\ y_{com} &= \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \\ y_{com} &= \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5} \\ y_{com} &= 2 \end{aligned}$$

Centre of mass is at (0.9, 2)

16. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece should be close to:

a.	22 mm	b.	2 mm
c.	12 mm	d.	33 mm

Solution:(a)

Magnification of compound microscope for least distance of distinct vision setting(strained eye)

$$M = \frac{L}{f_0} \left(1 + \frac{D}{f_e} \right)$$

where L is the tube length

 f_0 is the focal length of objective

D is the least distance of distinct vision = 25 cm

i.e. $375 = \frac{150 \times 10^{-3}}{5 \times 10^{-3}} \left(1 + \frac{25 \times 10^{-2}}{f_e} \right)$ i.e. $12.5 = 1 + \frac{25 \times 10^{-2}}{f_e}$ i.e. $\frac{25 \times 10^{-2}}{f_e} = 11.5$ $\therefore f_e \approx 21.7 \times 10^{-3} \text{m} = 22 \text{ mm}$

17. Speed of transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm²) is 90 m/s. If the Young's modulus of wire is $16 \times 10^{11} Nm^{-2}$, the extension of wire over its natural length is

b.

d.

- a. 0.03 mm
- **c.** 0.04 *mm*

Solution:(a) Given, $M = 6 \ grams = 6 \times 10^{-3} \ kg$

$$\begin{split} L &= 60 \ cm = 0.6 \ m \\ A &= 1 \ mm^2 = 1 \times 10^{-6} \ m^2 \\ \text{Using the relation, } v^2 &= \frac{T}{\mu} \\ &\Rightarrow T = \mu v^2 = V^2 \times \frac{M}{L} \\ \text{As Young's modulus, } Y &= \frac{stress}{strain} \\ strain &= \frac{stress}{Y} = \frac{T}{AY} \\ strain &= \frac{\Delta L}{L} = \frac{V^2 \frac{M}{L}}{AY} = V^2 \frac{M}{AYL} \\ &\Rightarrow \Delta L = \frac{V^2 M}{AY} \\ \Delta L &= \frac{8100 \times 6 \times 10^{-3}}{1 \times 10^{-6} \times 16 \times 10^{11}} \\ \Delta L &= 0.03 \ mm \end{split}$$



 $0.02 \ mm$ $0.01 \ mm$

18. 1 liter of dry air at STP expands adiabatically to a volume of 3 litres. If $\gamma = 1.4$, the work done by air is $(3^{1.4} = 4.655)$ (take air to be an ideal gas)

b.

d.

90.5 J

60.7 J

- a. 48 J
- c. 100.8 J

Solution:(b) Given, $P_1 = 1 atm$, $T_1 = 273 K$ (At STP) In adiabatic process,

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$
$$P_2 = P_1 \left[\frac{V_1}{V_2}\right]^{\gamma}$$
$$P_2 = 1 \times \left[\frac{1}{3}\right]^{1.4}$$
$$P_2 = 0.2164 \ atm$$

Work done in adiabatic process is given by

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$
$$W = \frac{1 \times 1 - 3 \times .2164}{0.4} \times 101.325$$

Since, 1 atm = 101.325 kPa and $1 Liter = 10^{-3} m^3$

 $W=89.87\;J$

19. A bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from the rest, the bob starts falling vertically. When it has covered a distance h, the angular speed of the wheel will be:

a.
$$r\sqrt{\frac{3}{4gh}}$$

b. $\frac{1}{r}\sqrt{\frac{4gh}{3}}$
c. $r\sqrt{\frac{3}{2gh}}$
d. $\frac{1}{r}\sqrt{\frac{2gh}{3}}$

Solution:(b) By energy conservation,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow gh = \frac{v^2}{2} + \frac{\omega^2 r^2}{4}$$
(1)

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Since the rope is inextensible and also it is not slipping,

$$\therefore v = r\omega$$

from eq. (1) and (2)
$$gh = \frac{\omega^2 r^2}{2} + \frac{\omega^2 r^2}{4}$$
$$\Rightarrow gh = \frac{3}{4}r^2\omega^2$$
$$\Rightarrow \omega^2 = \frac{4gh}{3r^2}$$
$$\Rightarrow \omega = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

20. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant varies as $k(x) = k(1 + \alpha x)$, where 'x' is the distance measured from one of the plates. If ($\alpha d \ll 1$), the total capacitance of the system is best given by the expression:



Solution:(b)

Given, $k(x) = k (1 + \alpha x)$ $dC = \frac{A\varepsilon_0 k}{dx}$ Since all capacitance are in series, we can apply

 $\frac{1}{Ceq} = \int \frac{1}{dC} = \int_0^d \frac{dx}{k(1+\alpha x)\epsilon_0 A}$ $\frac{1}{Ceq} = \left[\frac{\ln\left(1+\alpha x\right)}{k\epsilon_0 A\alpha}\right]_0^d$

(2)

On putting the limits from 0 to d

$$=\frac{\ln\left(1+\alpha d\right)}{k\epsilon_0 A\alpha}$$

Using expression $\ln (1 + x) = x - \frac{x^2}{2} + \dots$ And putting $x = \alpha d$ where, x approaches to 0.

$$\frac{1}{C} = \frac{d}{k\epsilon_0 A d\alpha} \left[\alpha d - \frac{(\alpha d)^2}{2} \right]$$
$$\frac{1}{C} = \frac{d}{k\epsilon_0 A} \left[1 - \frac{\alpha d}{2} \right]$$
$$C = \frac{k\epsilon_0 A}{d} \left[1 + \frac{\alpha d}{2} \right]$$

21. A non- isotropic solid metal cube has coefficient of linear expansion as $5 \times 10^{-5}/^{\circ}C$ along the x-axis and $5 \times 10^{-6}/^{\circ}C$ along y-axis and z-axis. If the coefficient of volumetric expansion of the solid is $C \times 10^{-6}/^{\circ}C$ then the value of C is _____

Solution:(60) We know that, V = xyz

$$\frac{\Delta v}{v} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$\frac{1}{T} \frac{\Delta v}{v} = \frac{1}{T} \frac{\Delta x}{x} + \frac{1}{T} \frac{\Delta y}{y} + \frac{1}{T} \frac{\Delta z}{z}$$

$$y = \alpha_n + \alpha_y + \alpha_z$$

$$y = 50 \times 10^{-6} ^{\circ}C + 5 \times 10^{-6} ^{\circ}C + 5 \times 10^{-6} ^{\circ}C$$

$$y = 60 \times 10^{-6} ^{\circ}C$$

$$\therefore C = 60$$

22. A loop ABCDEFA of straight edges has six corner points A(0,0,0), B(5,0,0), C(5,5,0), D(0,5,0), E(0,5,5), F(0,0,5). The magnetic field in this region is $\vec{B} = (3\hat{i} + 4\hat{k}) T$. The quantity of flux through the loop ABCDEFA (in *Wb*) is _____

Solution:(175)



As we know, magnetic flux = $\vec{B} \cdot \vec{A}$





 $\implies (B_x + B_z) \cdot (A_x + A_z)$ $\implies (3\hat{i} + 4\hat{k}) \cdot (25\hat{i} + 25\hat{k})$ $\implies (75 + 100) Wb$ $\implies 175 Wb$

23. A carnot engine operates between two reservoirs of temperature 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy in(J) delivered by the engine to the low temperature reservoir, in a cycle, is

Solution: (600 J)

$$\begin{split} \eta &= 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = \frac{2}{3} \\ \text{Given, } W &= 1200 \ J \\ \text{From conservation of energy} \\ Q_1 - Q_2 &= W \\ \eta &= \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1} \implies Q_1 = 1800 \text{ J} \\ \Rightarrow Q_2 &= Q_1 - W = 600 \text{ J} \end{split}$$

24. A particle of mass 1 kg slides down a frictionless track (AOC) starting from rest at a point A(height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaches its highest point P(height 1 m) the kinetic energy of the particle (in J) is:(Figure drawn is schematic and not to scale; take $g = 10 m/s^2$)_____

Solution:(10)



As the particle starts from rest the total energy at point $A = mgh = T.E_A$ (where h = 2 m) After reaching point P



 $T.E_c = K.E. + mgh$ By conservation of energy $T.E_A = T.E_p$ $\implies K.E. = mgh = 10 J$

25. A beam of electromagnetic radiation of intensity $6.4 \times 10^{-5} W/cm^2$ is comprised of wavelength, $\lambda = 310 nm$. It falls normally on a metal (work function $\phi = 2 eV$) of surface area $1 cm^2$. If one in 10^3 photons ejects an electron, total number of electrons ejected in 1s is 10^x (hc = 1240 eV - nm, $1 eV = 1.6 \times 10^{-19} J$), then x is

Solution:(11)

 $P = \text{Intensity} \times \text{Area}$ $= 6.4 \times 10^{-5} \text{W} - \text{cm}^{-2} \times 1 \text{ cm}^{2}$ $= 6.4 \times 10^{-5} \text{ W}$

For photoelectric effect to take place, energy should be greater than work function Now,

$$E = \frac{1240}{310} = 4 \ eV > 2 \ eV$$

Therefore, photoelectric effect takes place Here n is the number of photons emitted.

$$\begin{aligned} n\times E &= I\times A\\ \Longrightarrow \ n = \frac{IA}{E} = \frac{6.4\times 10^{-5}}{6.4\times 10^{-19}} = 10^{14} \end{aligned}$$

Where, n is number of incident photon

Since, 1 out of every 1000 photons are successfull in ejecting 1 photoelectron Therefore, the number of photoelectrons emitted is

$$=\frac{10^{14}}{10^3}$$
$$\therefore x = 11$$