EXERCISE 9.1

1. For the following A.P.s, write the first term 'a' and the common difference 'd':

(i) 3, 1, – 1, – 3, …

Solution:-

From the question,

The first term a = 3

Then, difference d = 1 – 3 = - 2

- 1 – 1 = - 2

- 3 – (-1) = - 3 + 1 = -2

Therefore, common difference d = - 2

(ii) 1/3, 5/3, 9/3, 13/3, ….

Solution:-

From the question,

The first term a = 1/3

Then, difference d = 5/3 – 1/3 = (5 - 1)/3 = 4/3

9/3 – 5/3 = (9 - 5)/3 = 4/3

13/3 – 9/3 = (13 - 9)/3 = 4/3

Therefore, common difference d = 4/3

(iii) -3.2, -3, -2.8, -2.6, ….

Solution:-

From the question,

The first term a = -3.2

Then, difference d = -3 – (-3.2) = -3 + 3.2 = 0.2

-2.8 – (-3) = -2.8 + 3 = 0.2

-2.6 – (-2.8) = -2.6 + 2.8 = 0.2

Therefore, common difference d = 0.2

2. Write first four terms of the A.P., when the first term a and the common difference d are given as follows :

(i) a = 10, d = 10

Solution:-

From the question it is given that,

First term a = 10

Common difference d = 10

Then the first four terms are = 10 + 10 = 20

20 + 10 = 30
30 + 10 = 40
Therefore, first four terms are 10, 20, 30 and 40.

(ii) a = -2, d = 0
Solution:-
From the question it is given that,
First term a = -2
Common difference d = 0
Then the first four terms are = -2 + 0 = -2
-2 + 0 = -2
-2 + 0 = -2
Therefore, first four terms are -2, -2, -2 and -2.

(iii) a = 4, d = -3
Solution:-
From the question it is given that,
First term a = 4
Common difference d = -3
Then the first four terms are = 4 + (-3) = 4 - 3 = 1
1 + (-3) = 1 – 3 = - 2
-2 + (-3) = -2 – 3 = - 5
Therefore, first four terms are 4, 1, -2 and -5.

(iv) a = ½, d = -1/6
Solution:-
From the question it is given that,
First term a = ½
Common difference d = -1/6
Then the first four terms are = ½ + (-1/6) = ½ - 1/6 = (3 - 1)/6 = 2/6 = 1/3
1/3 + (-1/6) = 1/3 – 1/6 = (2 - 1)/6 = 1/6
1/6 + (-1/6) = 1/6 – 1/6 = 0
Therefore, first four terms are ½, 1/3, 1/6 and 0.

3. Which of the following lists of numbers form an A.P.? If they form an A.P., find the common difference d and write the next three terms:
(i) 4, 10, 16, 22, …

Solution:-
From the question it is given that,
First term a = 4
Then, difference d = 10 – 4 = 6
16 – 10 = 6
22 – 16 = 6
Therefore, common difference d = 6
Hence, the numbers are form A.P.

(ii) -2, 2, -2, 2, …
Solution:-
From the question it is given that,
First term a = -2
Then, difference d = -2 – 2 = - 4
-2 – 2 = -4
2 – (-2) = 2 + 2 = 4
Therefore, common difference d is not same in the given numbers.
Hence, the numbers are not form A.P.

(iii) 2, 4, 8, 16, …
Solution:-
From the question it is given that,
First term a = 2
Then, difference d = 4 – 2 = 2
8 – 4 = 4
16 - 8 = 8
Therefore, common difference d is not same in the given numbers.
Hence, the numbers are not form A.P.

(iv) 2, 5/2, 3, 7/2, …
Solution:-
From the question it is given that,
First term a = 2
Then, difference d = 5/2 – 2 = (5 - 4)/2 = ½
3 – 5/2 = (6 - 5)/2 = ½
7/2 – 3 = (7 - 6)/2 = ½
Therefore, common difference d = ½

Hence, the numbers are form A.P.

(v) – 10, -6, -2, 2, …

Solution:-

From the question it is given that,

First term a = -10

Then, difference d = -6 – (- 10) = - 6 + 10 = 4

-2 – (-6) = - 2 + 6 = 4

2 – (-2) = 2 + 2 = 4

Therefore, common difference d = 4

Hence, the numbers are form A.P.

(vi) $1^2, 3^2, 5^2, 7^2$, …

Solution:-

From the question it is given that,

First term a = 1^2 = 1

Then, difference d = $3^2 – 1^2$ = 9 - 1 = 8

$5^2 – 3^2$ = 25 - 9 = 16

$7^2 – 5^2$ = 49 - 25 = 24

Therefore, common difference d is not same in the given numbers.

Hence, the numbers are not form A.P.

(vii) 1, 3, 9, 27, …

Solution:-

From the question it is given that,

First term a = 1 = 1

Then, difference d = 3 – 1 = 2

9 - 3 = 6

27 – 9 = 18

Therefore, common difference d is not same in the given numbers.

Hence, the numbers are not form A.P.

(viii) √2, √8, √18, √32, …

Solution:-

Given numbers can be written as, √2, 2√2, 3√2, 4√2, …

From the question it is given that,

First term a = √2

Then, difference d = 2√2 - √2 = √2

3√2 - 2√2 = √2

4√2 - 3√2 = √2

Therefore, common difference d = √2

Hence, the numbers are form A.P.

(ix) a, 2a, 3a, 4a, …

From the question it is given that,

First term a = a

Then, difference d = 2a – a = a

3a – 2a = a

4a – 3a = a

Therefore, common difference d = a

Hence, the numbers are form A.P.

(x) a, 2a + 1, 3a + 2, 4a + 3, …

From the question it is given that,

First term a = a

Then, difference d = (2a + 1) – a = 2a + 1 – a = a + 1

(3a + 2) – (2a + 1) = 3a + 2 – 2a - 1 = a + 1

(4a + 3) – (3a + 2) = 4a + 3 – 3a – 2 = a + 1

Therefore, common difference d = a +1

Hence, the numbers are form A.P.

EXERCISE 9.2

1. Find the A.P. whose n^{th} term is 7 – 3K. Also find the 20th term.

Solution:-

From the question it is given that,

n^{th} term is 7 – 3k

So, $T_n = 7 - 3n$

Now, we start giving values, 1, 2, 3, … in the place of n, we get,

$T_1 = 7 - (3 \times 1) = 7 - 3 = 4$

$T_2 = 7 - (3 \times 2) = 7 - 6 = 1$

$T_3 = 7 - (3 \times 3) = 7 - 9 = -2$

$T_4 = 7 - (3 \times 4) = 7 - 12 = -5$

$T_{20} = 7 - (3 \times 20) = 7 - 60 = -53$

Therefore, A.P. is 4, 1, -2, -5, …

So, 20^{th} term is – 53

2. Find the indicated terms in each of following A.P.s:

(i) 1, 6, 11, 16, …; a_{20}

Solution:-

From the question,

The first term a = 1

Then, difference d = 6 – 1 = 5

11 – 6 = 5

16 – 11 = 5

Therefore, common difference d = 5

From the formula, $a_n = a + (n - 1)d$

So, $a_{20} = a + (20 - 1)d$

$= 1 + (20 - 1)5$

$= 1 + (19)5$

$= 1 + 95$

$= 96$

Therefore, $a_{20} = 96$

(ii) -4, -7, -10, -13, …, a_{25}, a_n

Solution:-

From the question,

The first term a = -4

Then, difference d = -7 – (-4) = - 7 + 4 = -3
-10 – (-7) = -10 + 7 = -3
-13 – (-10) = -13 + 10 = -3
Therefore, common difference d = -3
From the formula, $a_n = a + (n - 1)d$
So, $a_{25} = a + (25 - 1)d$
$= -4 + (25 - 1)(-3)$
$= -4 + (24)-3$
$= - 4 - 72$
$= -76$
Therefore, $a_{25} = -76$
Now, $a_n = a + (n - 1)d$
$a_n = -4 + (n - 1)-3$
$= -4 – 3n + 3$
$= -1 – 3n$

3. Find the n^{th} term and the 12^{th} term of the list of numbers: 5, 2, – 1, – 4, …
Solution:-
From the question,
The first term a = 5
Then, difference d = 2 – 5 = - 3
-1 – 3 = -3
- 4 – (-1) = -4 + 1 = -3
Therefore, common difference d = -3
From the formula, $a_n = a + (n - 1)d$
$T_n = a + (n - 1)d$
$= 5 + (n - 1)-3$
$= 5 – 3n + 3$
$= 8 – 3n$
So, $T_{12} = a + (12 - 1)d$
$= 5 + (12 - 1)(-3)$
$= 5 + (11)-3$
$= 5 - 33$
$= - 28$

4. Find the 8^{th} term of the A.P. whose first term is 7 and common difference is 3.
Solution:-

From the question it is given that,
The first term a = 7
Then, common difference d = 3
$T_n = a + (n - 1)d$
So, $T_8 = a + (8 - 1)d$
$= 7 + (8 - 1)3$
$= 7 + (7)3$
$= 7 + 21$
$= 28$

5.
(i) If the common difference of an A.P. is – 3 and the 18th term is – 5, then find its first term.
Solution:-
From the question it is given that,
The 18th term = -5
Then, common difference d = -3
$T_n = a + (n - 1)d$
So, $T_{18} = a + (18 - 1)d$
$-5 = a + (18 - 1)(-3)$
$-5 = a + (17)(-3)$
$-5 = a - 51$
$a = 51 – 5$
$a = 46$
Therefore, first term a = 46

(ii) If the first term of an A.P. is – 18 and its 10th term is zero, then find its common difference.
Solution:-
From the question it is given that,
The 10th term = 0
Then, first term a = -18
$T_n = a + (n - 1)d$
So, $T_{10} = a + (10 - 1)d$
$0 = -18 + (10 - 1)d$
$0 = -18 + 9d$
$9d = 18$

$d = 18/9$

$d = 2$

Therefore, common difference d = 2

6. Which term of the A.P.

(i) 3, 8, 13, 18, … is 78?

Solution:-

Let us assume 78 as n^{th} term.

From the question,

The first term a = 3

Then, difference d = 8 – 3 = 5

13 – 8 = 5

18 – 13 = 5

Therefore, common difference d = 5

$T_n = a + (n - 1)d$

So, $78 = a + (n - 1)d$

$78 = 3 + (n - 1)5$

$78 = 3 + 5n - 5$

$78 = -2 + 5n$

$5n = 78 + 2$

$5n = 80$

$n = 80/5$

$n = 16$

Therefore, 78 is 16^{th} term.

(ii) 7, 13, 19, … is 205 ?

Solution:-

Let us assume 205 as n^{th} term.

From the question,

The first term a = 7

Then, difference d = 13 – 7 = 6

19 – 13 = 6

Therefore, common difference d = 6

$T_n = a + (n - 1)d$

So, $205 = a + (n - 1)d$

$205 = 7 + (n - 1)6$

$205 = 7 + 6n - 6$

$205 = 1 + 6n$
$6n = 205 - 1$
$6n = 204$
$n = 204/6$
$n = 34$

Therefore, 205 is 34th term.

(iii) 18, 15½, 13, … is – 47 ?

Solution:-

Convert mixed fraction into improper fraction = 15½ = 31/2

Let us assume -47 as n^{th} term.

From the question,

The first term a = 18

Then, difference $d = 31/2 – 18 = (31 – 36)/2 = -5/2$

$13 – 31/2 = (26 - 31)/2 = -5/2$

Therefore, common difference d = -5/2

$T_n = a + (n - 1)d$

So, $-47 = a + (n - 1)d$

$-47 = 18 + (n - 1)(-5/2)$
$-47 = 18 – 5/2n + 5/2$
$-47 – 18 = -5/2n + 5/2$
$-65 = -5/2n + 5/2$
$-65 – 5/2 = - 5/2n$
$(-130 - 5)/2 = -5/2n$
$-135/2 = -5/2n$
$n = (-135/2) \times (-2/5)$
$n = -135/-5$
$n = 27$

Therefore, -47 is 27th term.

7.

(i) Check whether – 150 is a term of the A.P. 11, 8, 5, 2, …

Solution:-

From the question it is given that,

The first term a = 11

Then, difference $d = 8 – 11 = -3$

$5 – 8 = -3$

2 – 5 = -3
Then, common difference d = - 3
Let us assume -150 as n^{th} term,
$T_n = a + (n - 1)d$
So, -150 = 11 + (n - 1)(-3)
-150 = 11 – 3n + 3
-150 = 14 - 3n
3n = 150 + 14
3n = 164
n = 164/3
n = $54\frac{2}{3}$
Therefore, – 150 is not a term of the A.P. 11, 8, 5, 2, …

(ii) Find whether 55 is a term of the A.P. 7, 10, 13, … or not. If yes, find which term is it.
Solution:-
From the question it is given that,
The first term a = 7
Then, difference d = 10 – 7 = 3
13 – 10 = 3
Then, common difference d = 3
Let us assume 55 as n^{th} term,
$T_n = a + (n - 1)d$
So, 55 = 7 + (n - 1)3
55 = 7 + 3n - 3
55 = 4 + 3n
3n = 55 - 4
3n = 51
n = 51/3
n = 17
Therefore, 55 is 17^{th} term of the A.P. 7, 10, 13, …

(iii) Is 0 a term of the A.P. 31, 28, 25,…? Justify your answer.
Solution:-
From the question it is given that,
The first term a = 31
Then, difference d = 28 – 31 = -3
25 – 28 = -3

Then, common difference d = - 3
Let us assume 0 as n^{th} term,
$T_n = a + (n - 1)d$
So, 0 = 31 + (n - 1)(-3)
0 = 31 – 3n + 3
0 = 34 - 3n
3n = 34
n = 34/3
n = $11\frac{1}{3}$
Therefore, 0 is not a term of the A.P. 31, 28, 25, …

8.
(i) Find the 20th term from the last term of the A.P. 3, 8, 13, …, 253.
Solution:-
Let us assume 253 as n^{th} term.
From the question,
The first term a = 3
Then, difference d = 8 – 3 = 5
13 – 8 = 5
Therefore, common difference d = 5
$T_n = a + (n - 1)d$
So, 253 = a + (n - 1)d
253 = 3 + (n - 1)5
253 = 3 + 5n - 5
253 = -2 + 5n
5n = 253 + 2
5n = 255
n = 255/5
n = 51
Therefore, 253 is 51^{th} term.
Now, assume 'P' be the 20^{th} term from the last.
Then, P = L – (n - 1)d
= 253 – (20 - 1) 5
= 253 – (19) 5
= 253 – 95
P = 158
Therefore, 158 is the 20^{th} term from the last.

(ii) Find the 12^{th} from the end of the A.P. – 2, – 4, – 6, …, – 100.

Solution:-

Let us assume -100 as n^{th} term.

From the question,

The first term a = -2

Then, difference d = - 4 – (-2) = - 4 + 2 = -2

-6 – (-4) = -6 + 4 = - 2

Therefore, common difference d = -2

$T_n = a + (n - 1)d$

So, - 100 = a + (n - 1)d

- 100 = -2 + (n - 1)(-2)

- 100 = -2 - 2n + 2

- 100 = -2n

n = -100/-2

n = 50

Therefore, -100 is 50^{th} term.

Now, assume 'P' be the 12^{th} term from the last.

Then, P = L – (n - 1)d

= -100 – (12 - 1) (-2)

= -100 – (11) (-2)

= -100 + 22

P = - 78

Therefore, -78 is the 12^{th} term from the last of the A.P. – 2, – 4, – 6, …

9. Find the sum of the two middle most terms of the A.P.

-4/3, -1, -2/3, …, $4\frac{1}{3}$

Solution:-

From the question,

Last term (n^{th}) = $4\frac{1}{3}$ = 13/3

First term a = -4/3

Then, difference d = -1 – (-4/3) = -1 + 4/3 = (-3 + 4)/3 = 1/3

= -2/3 – (-1) = -2/3 + 1 = (-2 + 3)/3 = 1/3

Therefore, common difference d = 1/3

We know that,

$T_n = a + (n - 1)d$

So, $13/3 = -4/3 + (n - 1)(1/3)$

$13/3 + 4/3 = 1/3n - 1/3$

$13/3 + 4/3 + 1/3 = 1/3n$

$(13 + 4 + 1)/3 = 1/3n$

$18/3 = 1/3n$

$6 = 1/3n$

$n = 6 \times 3$

$n = 18$

So, middle term is 18/2 and (18/2) + 1 = 9^{th} and 10^{th} term

Then, $a_9 + a_{10} = a + 8d + a + 9d$

$= 2a + 17d$

Now substitute the value of 'a' and 'd'.

$= 2(-4/3) + 17(1/3)$

$= -8/3 + 17/3$

$= (-8 + 17)/3$

$= 9/3$

$= 3$

Therefore, the sum of the two middle most terms of the A.P is 3.

10. Which term of the A.P. 53, 48, 43,… is the first negative term ?

Solution:-

From the question,

The first term a = 53

Then, difference d = 48 – 53 = -5

= 43 – 48 = -5

Therefore, common difference d = -5

$T_n = a + (n - 1)d$

$= 53 + (n - 1)(-5)$

$= 53 - 5n + 5$

$= 58 - 5n$

$5n = 58$

$n = 58/5$

$n = 11.6 \approx 12$

Therefore, 12^{th} term is the first negative term of the A.P. 53, 48, 43,…

11. Determine the A.P. whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

Solution:-

From the question it is given that,

$T_5 = 19$

$T_8 – T_{13} = 20$

We know that, $T_n = a + (n - 1)d$

So, $T_5 = a + 4d = 19$ … [equation (i)]

$T_{13} – T_8 = (a + 12d) – (a + 7d)$ … [equation (ii)]

$20 = a + 12d – a – 7d$

$20 = 5d$

$d = 20/5$

$d = 4$

Now, substitute value of d in equation (i) we get,

Then, $T_5 = a + 4d$

$19 = a + 4(4)$

$a = 19 – 16$

$a = 3$

Therefore, A.P. is 3 + 4 = 7, 7 + 4 = 11, 11 + 4 = 15

Hence, the four term of A.P. is 3, 7, 11, 15, …

12. Determine the A.P. whose third term is 16 and the 7th term exceeds the 5th term by 12.

Solution:-

From the question it is given that,

$T_3 = 16$

The 7th term exceeds the 5th term by 12 = $T_7 – T_5 = 12$

We know that, $T_n = a + (n - 1)d$

So, $T_3 = a + 2d = 16$ … [equation (i)]

$T_7 – T_5 = (a + 6d) – (a + 4d) = 12$ … [equation (ii)]

$12 = a + 6d – a – 4d$

$12 = 2d$

$d = 12/2$

$d = 6$

Now, substitute value of d in equation (i) we get,

Then, $T_3 = a + 2d$

$16 = a + 2(6)$

$a = 16 – 12$

$a = 4$

Therefore, A.P. is 4 + 6 = 10, 10 + 6 = 16, 16 + 6 = 22
Hence, the four term of A.P. is 4, 10, 16, 22, ….

13. Find the 20th term of the A.P. whose 7th term is 24 less than the 11th term, first term being 12.
Solution:-
From the question it is given that,
First term a = 12
7th term is 24 less than the 11th term = $T_{11} – T_7 = 24$
$T_{11} – T_7 = (a + 10d) – (a + 6d) = 24$
24 = a + 10d – a – 6d
24 = 4d
d = 24/4
d = 6
Now, $T_{20} = a + 19d$
Substitute the values of a and d,
$T_{20} = 12 + 19(6)$
$T_{20} = 12 + 114$
$T_{20} = 126$

14. Find the 31st term of an A.P. whose 11th term is 38 and 6th term is 73.
Solution:-
From the question it is given that,
$T_{11} = 38$
$T_6 = 73$
Let us assume 'a' be the first term and 'd' be the common difference,
So, $T_{11} = a + 10d = 38$ equation (i)
$T_6 = a + 5d = 73$ equation (ii)
Subtracting both equation (i) and equation (i),
(a + 10d) – (a + 5d) = 73 – 38
a + 10d – a – 5d = 35
5d = 35
d = 35/5
d = 7
now, substitute the value of d in equation (i) to find out a, we get
a + 10d = 38
a + 10(7) = 38

a + 70 = 38
a = 38 – 70
a = -32
Therefore, T_{31} = a + 30d
= -32 + 30(7)
= -32 + 210
= 178

15. If the seventh term of an A.P. is 1/9 and its ninth term is 1/7, find its 63rd term.
Solution:-
From the question it is given that,
T_9 = 1/7
T_7 = 1/9
Let us assume 'a' be the first term and 'd' be the common difference,
So, T_9 = a + 8d = 1/7 equation (i)
T_7 = a + 6d = 1/9 equation (ii)
Subtracting both equation (i) and equation (i),
(a + 6d) – (a + 8d) = 1/9 – 1/7
a + 6d – a – 8d = (7 - 9)/63
-2d = -2/63
d = (-2/63) × (-1/2)
d = 1/63
now, substitute the value of d in equation (ii) to find out a, we get
a + 6(1/63) = 1/9
a = 1/9 – 6/63
a = (7 - 6)/63
a = 1/63
Therefore, T_{63} = a + 62d
= 1/63 + 62(1/63)
= 1/63 + 62/63
= (1 + 62)/63
= 63/63
= 1

16.
(i) The 15th term of an A.P. is 3 more than twice its 7th term. If the 10th term of the A.P. is 41, find its nth term.

Solution:-

From the question it I s given that,

$T_{10} = 41$

$T_{10} = a + 9d = 41$ … [equation (i)]

$T_{15} = a + 14d = 2T_7 + 3$

$= a + 14d = 2(a + 6d) + 3$

$= a + 14d = 2a + 12 d + 3$

$-3 = 2a – a + 12d – 14d$

$a – 2d = -3$ … [equation (ii)]

Now, subtracting equation (ii) from (i), we get,

$(a + 9d) – (a – 2d) = 41 – (-3)$

$a + 9d – a + 2d = 41 + 3$

$11d = 44$

$d = 44/11$

$d = 4$

Then, substitute the value of d is equation (i) to find a,

$a + 9(4) = 41$

$a + 36 = 41$

$a = 41 – 36$

$a = 5$

Therefore, n^{th} term $= T_n = a + (n - 1)d$

$= 5 + (n - 1)4$

$= 5 + 4n – 4$

$= 1 + 4n$

(ii) The sum of 5^{th} and 7^{th} terms of an A.P. is 52 and the 10^{th} term is 46. Find the A.P.

Solution:-

From the question it is given that,

$a_5 + a_7 = 52$

$(a + 4d) + (a + 6d) = 52$

$a + 4d + a + 6d = 52$

$2a + 10d = 52$

Divide both the side by 2 we get,

$a + 5d = 26$ … equation (i)

Given, $a_{10} = a + 9d = 46$

$a + 9d = 46$ … equation (ii)

Now subtracting equation (i) from equation (ii),

$(a + 9d) – (a + 5d) = 46 – 26$
$a + 9d – a – 5d = 20$
$4d = 20$
$d = 20/4$
$d = 5$
Substitute the value of d in equation (i) to find out a,
$a + 5d = 26$
$a + 5(5) = 26$
$a + 25 = 26$
$a = 26 – 25$
$a = 1$
Then, $a_2 = a + d$
$= 1 + 5 = 6$
$a_3 = a_2 + d$
$= 6 + 5$
$= 11$
$a_4 = a_3 + d$
$= 11 + 5$
$= 16$
Therefore, 1, 6, 11, 16,… are A.P.

17. If 8th term of an A.P. is zero, prove that its 38th term is triple of its 18th term.
Solution:-
Froom the question it is given that,
$T_8 = 0$
We have to prove that, 38th term is triple of its 18th term = $T_{38} = 3T_{18}$
$T_8 = a + 7d = 0$
$T_8 = a = -7d$
$T_{38} = a + 37d$
$= -7d + 37d$
$= 30d$
Take, $T_{18} = a + 17d$
Substitute the value of a and d,
$T_{18} = -7d + 17d$
$T_{18} = 10d$
By comparing results of T_{38} and T_{18}, 38th term is triple of its 18th term.

18. Which term of the A.P. 3, 10, 17,… will be 84 more than its 13th term?
Solution:-
From the question it is given that,
First term a = 3
Common difference d = 10 – 3 = 7
Then, $T_{13} = a + 12d$
$= 3 + 12(7)$
$= 3 + 84$
$= 87$
Let us assume that, n^{th} term is 84 more than its 13^{th} term
So, $T_n = 84 + 87$
$= 171$
We know that, $T_n = a + (n - 1)d = 171$
$3 + (n - 1)7 = 171$
$3 + 7n – 7 = 171$
$7n – 4 = 171$
$7n = 171 + 4$
$7n = 175$
$n = 175/7$
$n = 25$

19. If the nth terms of the two A.P.s 9, 7, 5, … and 24, 21, 18, … are the same, find the value of n. Also, find that term.
Solution:-
First take, A.P. 9, 7, 5, ….
From the above A.P.
First term a = 9
Then, common difference d = 7 – 9 = -2
We know that, $T_n = a + (n - 1)d$
$= 9 + (n - 1)(-2)$
$= 9 – 2n + 2$
$= 11 – 2n$
Now, consider A.P. 24, 21, 18, …
From the above A.P.
First term $a_1 = 24$
Then, common difference d = 21 – 24 = - 3
We know that, $T_n = a + (n - 1)d$

= 24 + (n - 1) (-3)
= 24 – 3n + 3
= 27 – 3n

Froom the question it is given that, n^{th} term of both A.P. is same,
So, 11- 2n = 27 – 3n
-2n + 3n = 27 – 11
n = 16
Then, T_{16} = a + (n - 1)d
= 9 + 15 (-2)
= 9 – 30
= - 21

20.

(i) How many two digit numbers are divisible by 3 ?

Solution:-

The two digits numbers divisible by 3 are, 12, 15, 18, 21, 24,…..,99.
The above numbers are A.P.
So, first number a = 12
Common difference d = 15 – 12 = 3
Then, last number is 99
We know that, T_n (last number) = a + (n - 1)d
99 = 12 + (n - 1)3
99 = 12 + 3n – 3
99 = 9 + 3n
99 – 9 = 3n
3n = 90
n = 90/3
n = 30
Therefore, 30 two digits number are divisible by 3.

(ii) Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Solution:-

The natural numbers which are divisible by both 2 and 5 are 110, 120, 130, 140, ….,999
The above numbers are A.P.
So, first number a = 110
Common difference d = 120 – 110 = 10

Then, last number is 999

We know that, T_n (last number) = a + (n - 1)d

999 = 110 + (n - 1)10

999 = 110 + 10n – 10

999 = 100 + 10n

999 – 100 = 10n

10n = 888

n = 888/10

n = 88

The number of natural numbers which are divisible by both 2 and 5 are 88.

(iii) How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

Solution:-

The numbers which are lie between 10 and 300, when divisible by 4 leave a remainder 3 are 11, 15, 19, 23, 27,….299

The above numbers are A.P.

So, first number a = 11

Common difference d = 15 – 11 = 4

Then, last number is 299

We know that, T_n (last number) = a + (n - 1)d

299 = 11 + (n - 1)4

299 = 11 + 4n – 4

299 = 7 + 4n

299 – 7 = 4n

4n = 292

n = 292/4

n = 73

The total which are lie between 10 and 300, when divisible by 4 leave a remainder 3 are 73.

21. If the numbers n – 2, 4n – 1 and 5n + 2 are in A.P., find the value of n.

Solution:-

From the question it is given that, n – 2, 4n – 1 and 5n + 2 are in A.P.

Multiplying by 2 to 4n – 1 then it becomes = 8n – 2

So, 8n – 2 = n – 2 + 5n + 2

8n – 2 = 6n

$8n - 6n = 2$
$2n = 2$
$n = 2/2$
$n = 1$

22. The sum of three numbers in A.P. is 3 and their product is – 35. Find the numbers.
Solution:-
From the question it is given that,
The sum of three numbers in A.P. = 3
Given, Their product = -35
Let us assume the 3 numbers which are in A.P. are, a – d, a, a + d
Now adding 3 numbers = a – d + a + a + d = 3
$3a = 3$
$a = 3/3$
$a = 1$
From the question, product of 3 numbers is – 35
So, $(a - d) \times (a) \times (a + d) = - 35$
$(1 - d) \times (1) \times (1 + d) = - 35$
$1^2 - d^2 = - 35$
$d^2 = 35 + 1$
$d^2 = 36$
$d = \sqrt{36}$
$d = \pm 6$
Therefore, the numbers are $(a - d) = 1 - 6 = - 5$
$a = 1$
$(a + d) = 1 + 6 = 7$
If $d = - 6$
The numbers are $(a - d) = 1 - (-6) = 1 + 6 = 7$
$a = 1$
$(a + d) = 1 + (-6) = 1 - 6 = -5$
Therefore, the numbers -5, 1, 7,… and 7, 1, -5,… are in A.P.

23. The sum of three numbers in A.P. is 30 and the ratio of first number to the third number is 3 : 7. Find the numbers.
Solution:-
From the question it is given that, sum of three numbers in A.P. = 30
The ratio of first number to the third number is 3: 7

Let us assume the 3 numbers which are in A.P. are, a – d, a, a + d

Now adding 3 numbers = a – d + a + a + d = 30

3a = 30

a = 30/3

a = 10

Given ratio 3 : 7 = a – d : a + d

3/7 = (a - d)/(a + d)

(a - d)7 = 3(a + d)

7a – 7d = 3a + 3d

7a – 3a = 7d + 3d

4a = 10d

4(10) = 10d

40 = 10d

d = 40/10

d = 4

Therefore, the numbers are a – d = 10 – 4 = 6

a = 10

a + d = 10 + 4 = 14

So, 6, 10, 14, … are in A.P.

24. The sum of the first three terms of an A.P.is 33. If the product of the first and the third terms exceeds the second term by 29, find the A.P.

Solution:-

From the question it is given that, sum of the first three terms of an A.P. is 33.

Let us assume the 3 numbers which are in A.P. are, a – d, a, a + d

Now adding 3 numbers = a – d + a + a + d = 33

3a = 33

a = 33/3

a = 11

Given, the product of the first and the third terms exceeds the second term by 29.

(a - d) (a + d) = a + 29

$a^2 – d^2 = 11 + 29$

$11^2 – d^2 = 40$

$121 – 40 = d^2$

$d^2 = 81$

d =√81

d = ±9

If d = 9
Therefore, the numbers are (a - d) = 11 – 9 = 2
a = 11
(a + d) = 11 + 9 = 20
If d = - 9
The numbers are (a - d) = 1 – (-9) = 11 + 9 = 20
a = 11
(a + d) = 11 + (-9) = 11 – 9 = 2
Therefore, the numbers 2, 11, 20,… and 20, 11, 2,… are in A.P.

25. Justify whether it is true to say that the following are the n^{th} terms of an A.P.
(i) 2n – 3
(ii) n^2 + 1
Solution:-
(i) 2n - 3
From the question it is given that,
n^{th} term is 2n – 3
So, T_n = 2n – 3
Now, we start giving values, 1, 2, 3, … in the place of n, we get,
(2 × 1) – 3 = 2 – 3 = - 1
(2 × 2) – 3 = 4 – 3 = 1
(2 × 3) – 3 = 6 – 3 = 3
(2 × 4) – 3 = 8 – 3 = 5
From the above results, -1, 1, 3, 5, …. Are in A.P.
So, first term a = - 1
Common difference d = 1 – (-1) = 1 + 1 = 2
= 3 – 2 = 2

(ii) n^2 + 1
From the question it is given that,
n^{th} term is n^2 + 1
So, $T_n = n^2 + 1$
Now, we start giving values, 1, 2, 3, … in the place of n, we get,
$1^2 + 1 = 1 + 1 = 2$
$2^2 + 1 = 4 + 1 = 5$
$3^2 + 1 = 9 + 1 = 10$
$4^2 + 1 = 16 + 1 = 17$

From the above results, 2, 5, 10, 17, ….
So, first term a = 2
Common difference d = 5 – 2 = 3
= 10 – 5 = 5
The common difference is not same.
Therefore, 2, 5, 10, 17,… are not in A.P.

EXERCISE 9.3

1. Find the sum of the following A.P.s :

(i) 2, 7, 12, … to 10 terms

Solution:-

From the question,

First term a = 2

Then, d = 7 – 2 = 5

12 – 7 = 5

So, common difference d = 5

n = 10

S_{10} = n/2(2a + (n - 1)d)

= 10/2 ((2 × 2) + (10 - 1)5)

= 5(4 + 45)

= 5(49)

= 245

(ii) 1/15, 1/12, 1/10, … to 11 terms

Solution:-

From the question,

First term a = 1/15

Then, d = 1/12 – 1/15

= (5 - 4)/60

= 1/60

So, common difference d = 1/60

n = 11

S_{11} = 11/2(2a + (n - 1)d)

= 11/2 ((2 × (1/15)) + (11 - 1)(1/60))

= 11/2 ((2/15) + (10/60))

= 11/2 (2/15 + 1/6)

= 11/2 (4 + 5)/30

= 11/2 (9/30)

= 11/2(3/10)

= 33/20

2. How many terms of the A.P. 27, 24, 21, …, should be taken so that their sum is zero?

Solution:-

From the question,

The first term a = 27

Difference d = 24 – 27 = - 3

= 21 – 24 = - 3

So, common difference d = -3

$S_n = 0$

Let us assume n be there in A.P.

So, $S_n = (n/2)(2a + (n - 1)d)$

$0 = n/2 ((2 \times 27) + (n - 1)(-3))$

$0 = n/2(54 – 3n + 3)$

$0 = n/2(57 – 3n)$

$0 \times (2/n) = 57 – 3n$

$0 = 57 – 3n$

$3n = 57$

$n = 57/3$

$n = 19$

3. Find the sums given below :

(i) 34 + 32 + 30 + … + 10

Solution:-

From the question,

First term a = 34,

Difference d = 32 – 34 = -2

So, common difference d = - 2

Last term $T_n = 10$

We know that, $T_n = a + (n - 1)d$

$10 = 34 + (n - 1)(-2)$

$-24 = - 2(n - 1)$

$-24 = - 2n + 2$

$2n = 24 + 2$

$2n = 26$

$n = 26/2$

$n = 13$

$S_n = n/2(a + 1)$

$= 13/2 (34 + 10)$

$= 13/2 (44)$

$= 13 (22)$

$= 286$

(ii) – 5 + (– 8) + (– 11) + … + (– 230)

Solution:-

From the question,

First term a = -5,

Difference d = -8 – (-5) = -8 + 5 = -3

So, common difference d = - 3

Last term T_n = -230

We know that, $T_n = a + (n - 1)d$

$-230 = -5 + (n - 1)(-3)$

$-230 = - 5 – 3n + 3$

$-230 = - 2 – 3n$

$3n = 230 - 2$

$3n = 228$

$n = 228/3$

$n = 76$

Therefore, $S_n = n/2 (a + l)$

$= 76/2 (-5 + (-230))$

$= 38 (-5 - 230)$

$= 38 (235)$

$= - 8930$

4.

In an A.P. (with usual notations) :

(i) given a = 5, d = 3, a_n = 50, find n and S_n

Solution:-

From the question,

First term a = 5

Then common difference d = 3

a_n = 50,

We know that, $a_n = a + (n - 1)d$

$50 = 5 + (n - 1)3$

$50 = 5 + 3n – 3$

$50 = 2 + 3n$

$3n = 50 – 2$

$3n = 48$

$n = 48/3$

$n = 16$

So, $S_n = (n/2)(2a + (n - 1)d)$

$= (16/2)((2 \times 5) + (16 - 1) \times 3)$

$= 8(10 + 45)$

$= 8(55)$

$= 440$

(ii) given $a = 7$, $a_{13} = 35$, find d and S_{13}

Solution:-

From the question,

First term $a = 7$

$a_{13} = 35$,

We know that, $a_n = a + (n - 1)d$

$35 = 7 + (13 - 1)d$

$35 = 7 + 13d – d$

$35 = 7 + 12d$

$12d = 35 – 7$

$12d = 28$

$d = 28/12$ … [divide by 4]

$d = 7/3$

So, $S_{13} = (n/2)(2a + (n - 1)d)$

$= (13/2)((2 \times 7) + ((13 - 1) \times (7/3))$

$= (13/2)((14 + (12 \times 7/3))$

$= (13/2)(14 + 28)$

$= (13/2)(42)$

$= 13 \times 21$

$= 273$

(iii) given $d = 5$, $S_9 = 75$, find a and a_9.

Solution:-

From the question it is given that,

Common difference $d = 5$

$S_9 = 75$

We know that, $a_n = a + (n - 1)d$

$a_9 = a + (9 - 1)5$

$a_9 = a + 45 – 5$

$a_9 = a + 40$ … [equation (i)]

Then, $S_9 = (n/2) (2a + (n - 1)d)$

$75 = (9/2) (2a + (9 - 1)5)$

$75 = (9/2) (2a + (8)5)$

$(75 \times 2)/9 = 2a + 40$

$150/9 = 2a + 40$

$2a = 150/9 – 40$

$2a = 50/3 – 40$

$2a = (50 - 120)/3$

$2a = -70/3$

$a = -70/(3 \times 2)$

$a = - 35/3$

Now, substitute the value of a in equation (i),

$a_9 = a + 40$

$= -35/3 + 40$

$= (-35 + 120)/3$

$= 85/3$

(iv) given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d

Solution:-

From the question it is give that,

First term $a = 8$,

$a_n = 62$ and $S_n = 210$

We know that, $a_n = a + (n - 1)d$

$62 = 8 + (n - 1)d$

$(n - 1)d = 62 – 8$

$(n - 1)d = 54$ … [equation (i)]

Then, $S_n = (n/2) (2a + (n - 1)d)$

$210 = (n/2) ((2 \times 8) + 54)$ … [from equation (i) $(n - 1)d = 54$]

$210 = (n/2) (16 + 54)$

$420 = n(70)$

$n = 420/70$

$n = 6$

Now, substitute the value of n in equation (i),

$(n - 1)d = 54$

$(6 - 1)d = 54$

$5d = 54$

$d = 54/5$

Therefore, d = 54/5 and n = 6

(v) given a = 3, n = 8, S = 192, find d.
Solution:-
From the question it is given that,
First term a = 3
n = 8
S = 192
We know that, $S_n = (n/2)(2a + (n - 1)d)$
$192 = (8/2)((2 \times 3) + (8 - 1)d)$
$192 = 4(6 + 7d)$
$192/4 = 6 + 7d$
$48 = 6 + 7d$
$48 - 6 = 7d$
$42 = 7d$
$d = 42/7$
$d = 6$
Therefore, common difference d is 6.

5.
(i) The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.
Solution:-
From the question it is give that,
First term a = 5
Last term = 45
Then, sum = 400
We know that, last term = a + (n – 1)d
$45 = 5 + (n - 1)d$
$(n - 1)d = 45 - 5$
$(n - 1)d = 40$ … [equation (i)]
So, $S_n = (n/2)(2a + (n - 1)d)$
$400 = (n/2)((2 \times 5) + 40)$ … [from equation (i) (n - 1)d = 40]
$800 = n(10 + 40)$
$800 = 50n$
$n = 800/50$
$n = 16$

(ii) The sum of first 15 terms of an A.P. is 750 and its first term is 15. Find its 20th term.
Solution:-
From the question it is give that,
First term a = 15
Therefore, sum of first n terms of an A.P. is given by,
$S_n = (n/2) (2a + (n - 1)d)$
$S_{15} = (15/2)(2a + (15 - 1)d)$
$750 = (15/2) (2a + 14d)$
$(750 \times 2)/15 = 2a + 14d$
$100 = 2a + 14d$
Dividing both the side by 2 we get,
$50 = a + 7d$
Now, substitute the value a,
$50 = 15 + 7d$
$7d = 50 – 15$
$7d = 35$
$d = 35/7$
$d = 5$
So, 20th term $a_{20} = a + 19d$
$= 15 + 19(5)$
$= 15 + 95$
$= 110$

6. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?
Solution:-
From the question it is give that,
First term a = 17
Last term (l) = 350
Common difference d = 9
We know that, $l = T_n = a + (n - 1)d$
$350 = 17 + (n - 1) \times 9$
$350 – 17 = 9n – 9$
$333 + 9 = 9n$
$342 = 9n$
$n = 342/9$

n = 38

So, $S_n = (n/2)(2a + (n - 1)d)$

$= (38/2)((2 \times 17) + (38 - 1)d)$

$= 19(34 + (37 \times 9))$

$= 19(34 + 333)$

$= 19 \times 367$

$= 6973$

Therefore, n = 38 and $S_n = 6973$

7. Solve for x : 1 + 4 + 7 + 10 + … + x = 287.

Solution:-

From the question,

First term a = 1

Difference d = 4 – 1 = 3

n = x

x = a = (n - 1)d

x – 1 = (n - 1)d

$S_n = (n/2)(2a + (n - 1)d)$

$287 = (n/2)((2 \times 1) + (n - 1)3)$

$= n(2 + 3n - 3)$

$574 = n(2 + 3n - 3)$

$574 = 2n + 3n^2 – 3n$

$574 = - n + 3n^2$

$3n^2 – n – 574 = 0$

$3n^2 – 42n + 41 – 574 = 0$

$3n(n - 14) + 41(n - 14) = 0$

$(n - 14)(3n + 41) = 0$

If n – 14 = 0

n = 14

or 3n + 41 = 0

3n = -41

n = -41/3

We have to take positive number so n = 14

Then, = a + (n - 1)d

= 1 + (14 - 1) 3

= 1 + (13)3

= 1 + 39

= 40
Therefore, x = 40

8.

(i) How many terms of the A.P. 25, 22, 19, … are needed to give the sum 116 ? Also find the last term.

Solution:-

From the question it is given that,
First term a = 25
Common difference d = 22 – 25 = - 3
Sum = 116
$S_n = (n/2)(2a + (n - 1)d)$
116 = (n/2) (2a + (n - 1)d)
By cross multiplication,
232 = n ((2 × 25) + (n - 1) (-3))
232 = n (50 – 3n + 3)
232 = n (53 – 3n)
$232 = 53n - 3n^2$
$3n^2 - 53n + 232 = 0$
$3n^2 - 24n - 29n + 232 = 0$
3n (n - 8) – 29 (n - 8) = 0
(n - 8) (3n - 29) = 0
If n – 8 = 0
n = 8
or 3n - 29 = 0
3n = 29
n = 29/3
not possible to take fraction,
So, n = 8
Then, T = a + (n - 1)d
= 25 +(8 - 1) (-3)
= 25 + 7 (-3)
= 25 – 21
= 4

(ii) How many terms of the A.P. 24, 21, 18, … must be taken so that the sum is 78 ? Explain the double answer.

Solution:-

From the question it is given that,

First term a = 24

Common difference d = 21 – 24 = - 3

Sum = 78

$S_n = (n/2)(2a + (n - 1)d)$

$78 = (n/2)(2a + (n - 1)d)$

By cross multiplication,

$156 = n((2 \times 24) + (n - 1)(-3))$

$156 = n(48 - 3n + 3)$

$156 = n(51 - 3n)$

$156 = 51n - 3n^2$

$3n^2 - 51n + 156 = 0$

$3n^2 - 12n - 39n + 156 = 0$

$3n(n - 4) - 39(n - 4) = 0$

$(n - 4)(3n - 39) = 0$

If $n - 4 = 0$

$n = 4$

or $3n - 39 = 0$

$3n = 39$

$n = 39/3$

$n = 13$

now we have to consider both values

So, n = 4

Then, $T = a + (n - 1)d$

$= 24 + (4 - 1)(-3)$

$= 24 + 3(-3)$

$= 24 - 9$

$= 15$

n = 13

Then, $T = a + (n - 1)d$

$= 24 + (13 - 1)(-3)$

$= 24 + 12(-3)$

$= 24 - 36$

$= -12$

So, $(12 + 9 + 6 + 3 + 0 + (-3) + (-6) + (-9) + (-12)) = 0$

Hence, the sum of 5^{th} term to 13^{th} term = 0

9. Find the sum of first 22 terms, of an A.P. in which d = 7 and a_{22} is 149.

Solution:-

From the question it is given that,

Common difference d = 7

$a_{22} = 149$

n = 22

we know that,

$a_{22} = (n - 1)d$

149 = a + (22 - 1)7

149 = a + (22)7

149 = a + 147

a = 149 – 147

a = 2

So, $S_{22} = (n/2)(2a + (n - 1)d)$

$= (22/2)((2 \times 2) + (22 - 1)7)$

$= 11(4 + (21)7)$

$= 11(4 + 147)$

$= 11(151)$

$= 1661$

10.

(i) Find the sum of first 51 terms of the A.P. whose second and third terms are 14 and 18 respectively.

Solution:-

From the question it is given that,

$T_2 = 14$, $T_3 = 18$

So, common difference $d = T_3 - T_2$

$= 18 - 14$

$= 4$

Where, $a = T_1 = 14 - 4 = 10$

n = 51

We know that,

$S_{51} = (n/2)(2a + (n - 1)d)$

$= (51/2)((2 \times 10) + (51 - 1)4)$

$= (51/2)(20 + (50 \times 4))$

$= (51/2)(20 + 200)$

= (51/2) × 220
= 5610

(ii) If the third term of an A.P. is 1 and 6^{th} term is – 11, find the sum of its first 32 terms.

Solution:-

From the question it is given that,

$T_3 = 1$, $T_6 = -11$ and n = 32

We know that,

$T_3 = a + 2d = 1$ … [equation (i)]

$T_6 = a + 5d = -11$ … [equation (ii)]

Now, subtracting equation (ii) from equation (i), we get,

(a + 2d) – (a + 5d) = 1 – (-11)

a + 2d – a – 5d = 1 + 11

-3d = 12

d = -12/3

d = -4

Now, substitute value of d in equation (i),

a + 2d = 1

a + 2(-4) = 1

a – 8 = 1

a = 8 + 1

a = 9

S_{32} = (n/2) (2a + (n - 1)d)
= (32/2) (2(9) + (32 - 1)(-4))
= 16 (18 + (31)(-4))
= 16 (18 - 124)
= 16 (-106)
= - 1696

Therefore, the sum of its first 32 terms is – 1696.

11. If the sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

Solution:-

From the question it is given that,

$S_6 = 36$

$S_{16} = 256$

We know that,
$S_n = (n/2)(2a + (n - 1)d)$
$S_6 = (6/2)(2a + (6 - 1)d) = 36$
$3(2a + 5d) = 36$
Divide both the side by 3,
$2a + 5d = 12$ … [equation (i)]
Now, $S_{16} = (16/2)(2a + (16 - 1)d) = 256$
$8(2a + 15d) = 256$
Divide both the side by 8,
$2a + 15d = 32$ … [equation (ii)]
Then, subtract equation (ii) from equation (i) we get,
$(2a + 5d) – (2a + 15d) = 12 – 32$
$2a + 5d – 2a – 15d = -20$
$-10d = -20$
$d = -20/-10$
$d = 2$
substitute the value of d in equation (i) to find a,
$2a + 5d = 12$
$2a + 5(2) = 12$
$2a + 10 = 12$
$2a = 12 – 10$
$2a = 2$
$a = 2/2$
$a = 1$
So, $S_{10} = (n/2)(2a + (n - 1)d)$
$= (10/2)((2 \times 1) + (10 - 1)2)$
$= 5(2 + 18)$
$= 5(20)$
$= 100$
Therefore, the sum of first 10 terms is 100.

12. Show that a_1, a_2, a_3, … form an A.P. where a_n is defined as $a_n = 3 + 4n$. Also find the sum of first 15 terms.
Solution:-
From the question it is given that,
n^{th} term is $3 + 4n$
So, $a_n = 3 + 4n$

Now, we start giving values, 1, 2, 3, … in the place of n, we get,

$a_1 = 3 + (4 \times 1) = 3 + 4 = 7$

$a_2 = 3 + (4 \times 2) = 3 + 8 = 11$

$a_3 = 3 + (4 \times 3) = 3 + 12 = 15$

$a_4 = 3 + (4 \times 4) = 3 + 16 = 19$

So, The numbers are 7, 11, 15, 19, ….

Then, first term a = 7, common difference d = 11 – 7 = 4

We know that,

$S_{15} = (n/2)(2a + (n - 1)d)$

$= (15/2)((2 \times 7) + (15 - 1) \times 4)$

$= (15/2)(14 + (14 \times 4))$

$= (15/2)(14 + 56)$

$= (15/2) \times 70$

$= 525$

Therefore, the sum of first 15 terms is 525.

13.

(i) If $a_n = 3 – 4n$, show that $a_1, a_2, a_3, \ldots$ form an A.P. Also find S_{20}.

Solution:-

From the question it is given that,

n^{th} term is 3 + 4n

So, $a_n = 3 - 4n$

Now, we start giving values, 1, 2, 3, … in the place of n, we get,

$a_1 = 3 - (4 \times 1) = 3 - 4 = - 1$

$a_2 = 3 - (4 \times 2) = 3 - 8 = -5$

$a_3 = 3 - (4 \times 3) = 3 - 12 = -9$

$a_4 = 3 - (4 \times 4) = 3 - 16 = - 13$

So, The numbers are -1, -5, -9, -13, ….

Then, first term a = -1, common difference d = -5 – (-1) = -5 + 1 = -4

We know that,

$S_{20} = (n/2)(2a + (n - 1)d)$

$= (20/2)((2 \times (-1)) + (20 - 1) \times (-4))$

$= 10(-2 + (19 \times (-4)))$

$= 10(-2 - 76)$

$= 10(-78)$

$= -780$

Therefore, the S_{20} is -780.

(ii) Find the common difference of an A.P. whose first term is 5 and the sum of first four terms is half the sum of next four terms.

Solution:-

From the question it is given that,

First term a = 5

And also it is given that, the sum of first four terms is half the sum of next four terms,

$a_1 + a_2 + a_3 + a_4 = ½ (a_5 + a_6 + a_7 + a_8)$

then,

a + (a + d) + (a + 2d) + (a + 3d) = ½ ((a + 4d) + (a + 5d) + (a + 6d) + (a + 7d))

a + a + d + a + 2d + a + 3d = ½ (a + 4d + a + 5d + a + 6d + a + 7d)

4a + 6d = ½ (4a + 22d)

By cross multiplication,

2(4a + 6d) = (4a + 22d)

2 ((4 × 5) + 6d) = ((4 × 5) + 22d) … [given a = 5]

2(20 + 6d) = (20 + 22d)

40 + 12d = 20 + 22d

40 – 20 = 22d – 12d

20 = 10d

d = 20/10

d = 2

Therefore, the common difference d is 2.

EXERCISE 9.4

1. Can 0 be a term of a geometric progression?

Solution:-

No, 0 is not a term of geometric progression.

2.

(i) Find the next term of the list of numbers 1/6, 1/3, 2/3, …

Solution:-

From the question,

First term a = 1/6

Then, r = (1/3) ÷ (1/6)

r = (1/3) × (6/1)

r = 6/3

r = 2

Therefore, next term = 2/3 × 2 = 4/3

(ii) Find the next term of the list of numbers 3/16, -3/8, ¾, -3/2,…

Solution:-

From the question,

First term a = 3/16

Then, r = (-3/8) ÷ (3/16)

r = (-3/8) × (16/3)

r = (-3 × 16)/(8 × 3)

r = (-1 × 2)/ (1 × 1)

r = -2

Therefore, next term = -3/2 × (-2) = 6/2 = 3

(iii) Find the 15th term of the series √3 + 1/√3 + 1/3√3 + …

Solution:-

From the question,

First term a = √3

Then, r = (1/√3) ÷ (√3)

r = (1/√3) × (1/√3)

r = (1 × 1)/(√3 × √3)

r = $1/(\sqrt{3})^2$

r = 1/3

So, $a_{15} = ar^{n-1}$

$= \sqrt{3}(1/3)^{15-1}$
$= \sqrt{3}(1/3)^{14}$
$= \sqrt{3} \times (1/3^{14})$
Therefore, $a_{15} = \sqrt{3} \times (1/3^{14})$

(iv) Find the n^{th} term of the list of numbers $1/\sqrt{2}$, -2, $4\sqrt{2}$, - 16,…
Solution:-
From the question it is given that,
First term $a = 1/\sqrt{2}$
Then, $r = -2 \div (1/\sqrt{2})$
$r = (-2/1) \times (\sqrt{2}/1)$
$r = (-2 \times \sqrt{2})/(1 \times 1)$
$r = -2\sqrt{2}$
So, $a_n = ar^{n-1}$
$= (1/\sqrt{2})(-2\sqrt{2})^{n-1}$
$= (1/\sqrt{2}) \times (-1)^{n-1} \times [(\sqrt{2})^2 \times \sqrt{2}]^{n-1}$
$= (-1)^{n-1} \times 1/\sqrt{2} \times [(\sqrt{2})^3]^{n-1}$
$= (-1)^{n-1} \times 1/\sqrt{2} \times (\sqrt{2})^{3n-3}$
$= (-1)^{n-1} (\sqrt{2})^{3n-3-1}$
$= (-1)^{n-1} (\sqrt{2})^{3n-4}$
$= (-1)^{n-1} \times 2^{(3n-4)/2}$
Therefore, $a_n = (-1)^{n-1} \times 2^{(3n-4)/2}$

(v) Find the 10^{th} and n^{th} terms of the list of numbers 5, 25, 125, …
Solution:-
From the question it is given that,
First term a = 5,
Then, $r = (25) \div (5)$
$r = (25) \times (1/5)$
$r = 5$
So, $a_{10} = ar^{n-1}$
$= 5 \times (5)^{10-1}$
$= 5 \times 5^9$
$= 5^{9+1}$ … [by $a^m \times a^n = a^{m+n}$]
$= 5^{10}$
Therefore, $a_n = ar^{n-1}$
$= 5 \times 5^{n-1}$

$= 5^{n-1+1}$

$= 5^n$

(vi) Find the 6^{th} and the n^{th} terms of the list of numbers 3/2, ¾, 3/8,…

Solution:-

From the question it is given that,

First term a = 3/2,

Then, r = (3/4) ÷ (3/2)

r = (3/4) × (2/3)

r = (3 × 2)/(4 × 3)

r = (1 × 1)/(2 × 1)

r = ½

So, $a_n = ar^{n-1}$

$= (3/2) \times (1/2)^{n-1}$

$= 3 \times ½ \times (½)^{n-1}$

$= 3 \times (½)^{n-1+1}$

$= 3 \times (½)^n$

$= 3/2^n$

Therefore, $a_6 = 3/2^n$

$= 3/2^6$

$= 3/64$

(vii) Find the 6th term from the end of the list of numbers 3, – 6, 12, – 24, …, 12288.

Solution:-

From the question it is given that,

Last term = 12288

First term a = 3,

Then, r = (-6) ÷ (3)

r = (-6) × (1/3)

r = (-6 × 1)/(1 × 3)

r = (-2 × 1)/(1 × 1)

r = -2

Then, 6^{th} term from the end,

$a_6 = l \times (1/r)^{n-1}$

$= 12288 \times (1/-2)^{6-1}$

$= 12288 \times (1/-2^5)$

$= 12288/-32$

= - 384

3. Which term of the G.P.

(i) 2, 2√2, 4, … is 128?

Solution:-

From the question it is given that,

Last term = 128

First term a = 2,

Then, r = (2√2) ÷ (2)

r = (2√2)/2

r = √2

Then, $a_n = ar^{n-1}$

So, $128 = 2(\sqrt{2})^{n-1}$

$2^7 = 2(\sqrt{2})^{n-1}$

$2^7/2 = (\sqrt{2})^{n-1}$

$2^{7-1} = (\sqrt{2})^{n-1}$

$2^6 = (\sqrt{2})^{n-1}$

$(\sqrt{2})^{n-1} = (\sqrt{2})^{12}$

Now, comparing the powers

n – 1 = 12

n = 12 + 1

n = 13

Therefore, 128 is the 13th term.

(ii) 1, 1/3, 1/9, … is 1/243

Solution:-

From the question it is given that,

Last term (a_n) = 1/243

First term a = 1,

Then, r = (1/3) ÷ (1)

r = (1/3) × (1/1)

r = 1/3

Then, $a_n = ar^{n-1}$

$1/243 = 1 \times (1/3)^{n-1}$

$(1/3)^5 = (1/3)^{n-1}$

By comparing both left hand side and right hand side,

5 = n – 1

$n = 5 + 1$
$n = 6$
Therefore, 1/243 is 6^{th} term.

(iii) 1/3, 1/9, 1/27, … is 1/19683?
Solution:-
From the question it is given that,
Last term (a_n) = 1/19683
First term a = 1/3
Then, r = (1/9) ÷ (1/3)
r = (1/9) × (3/1)
r = 1/3
Then, $a_n = ar^{n-1}$
$1/19683 = (1/3) \times (1/3)^{n-1}$
$(1/3)^9 = (1/3)^{n-1+1}$
$(1/3)^9 = (1/3)^n$
By comparing both left hand side and right hand side,
$9 = n$
$n = 9$
Therefore, 1/19683 is 9^{th} term.

4. Which term of the G.P. 3, - 3√3, 9, -9√3, … is 729?
Solution:-
From the question it is given that,
Last term (a_n) = 729
First term a = 3
Then, r = (-3√3) ÷ 3
r = (-3√3/3)
r = -√3
Then, $a_n = ar^{n-1}$
$729 = (3) \times (-\sqrt{3})^{n-1}$
$729/3 = (-\sqrt{3})^{n-1}$
$243 = (-\sqrt{3})^{n-1}$
$(-\sqrt{3})^{10} = (-\sqrt{3})^{n-1}$
By comparing both left hand side and right hand side,
$10 = n - 1$
$n = 10 + 1$

n = 11
Therefore, 729 is 11th term.

5. Determine the 12th term of a G.P. whose 8th term is 192 and common ratio is 2.
Solution:-
From the question it is given that,
$a_8 = 192$ and $r = 2$
Then, by the formula $a_n = ar^{n-1}$
$a_8 = ar^{8-1}$
$192 = a(2)^{8-1}$
$192 = a(2)^7$
$a = 192/2^7$
$a = 192/128$
$a = 3/2$
Now, $a_{12} = (3/2)(2)^{12-1}$
$= (3/2) \times (2)^{11}$
$= (3/2) \times 2048$
$= 3072$
$a_8 = 3072$

6. In a GP., the third term is 24 and 6th term is 192. Find the 10th term.
Solution:-
From the question it is given that,
$a_3 = 24$
$a_6 = 192$
Then, by the formula $a_n = ar^{n-1}$
$a_6 = ar^{6-1}$
$192 = ar^{6-1}$
$192 = ar^5$ … [equation (i)]
Now, $a_3 = ar^{n-1}$
$24 = ar^{3-1}$
$24 = ar^2$ … [equation (ii)]
By dividing equation (i) by equation (ii)
$ar^5/ar^2 = 192/24$
$r^{5-2} = 8$
$r^3 = 8$
$r^3 = 2^3$

$r = 2$
Now, substitute the value r in equation (i),
$192 = ar^5$
$192 = a\ (2)^5$
$a = 192/32$
$a = 6$
So, $a_{10} = ar^{10-1}$
$= ar^9$
$= 6(2)^9$
$= 6\ (512)$
$= 3072$

7. Find the number of terms of a G.P. whose first term is ¾, common ratio is 2 and the last term is 384.
Solution:-
From the question it is given that,
First term of G.P. a = ¾
Common ratio (r) = 2
Last term = 384
Then, by the formula $a_n = ar^{n-1}$
$384 = (3/4)\ (2)^{n-1}$
$(384 \times 4)/3 = (2)^{n-1}$
$(1536)/3 = (2)^{n-1}$
$512 = 2^{n-1}$
$2^9 = 2^{n-1}$
By comparing both left hand side and right hand side,
$9 = n - 1$
$n = 9 + 1$
$n = 10$
The number of terms of a G.P. is 10.

8. Find the value of x such that,
(i) -2/7, x, -7/2 are three consecutive terms of a G.P.
Solution:-
From the question,
$x^2 = -2/7 \times -7/2$
$x^2 = 1$

$x = \pm 1$
Therefore, $x = 1$ or $x = -1$

(ii) x + 9, x – 6 and 4 are three consecutive terms of a G.P.
Solution:-
From the question,
$(x - 6)^2 = (x + 9) \times 4$
$x^2 - 12x + 36 = 4x + 36$
$x^2 - 12x - 4x + 36 - 36 = 0$
$x^2 - 16x = 0$
$x(x - 16) = 0$
Either let us take $x - 16 = 16$
Or $x = 0$
So, $x = 0, 16$

(iii) x, x + 3, x + 9 are first three terms of a G.P. Find the value of x.
Solution:-
From the question,
$(x + 3)^2 = x(x + 9)$
$x^2 + 6x + 9 = x^2 + 9x$
$9 = 9x - 6x$
$9 = 3x$
$X = 9/3$
$X = 3$

9. If the fourth, seventh and tenth terms of a G.P. are x, y, z respectively, prove that x, y, z are in G.P.
Solution:-
From the question it is given that,
$a_4 = x$
$a_7 = y$
$a_{10} = z$
Now we have to prove that, x, y, z are in G.P.
Then, by the formula $a_n = ar^{n-1}$
$a_4 = ar^{4-1}$
$a_4 = a^3$
$a_4 = x$

So, $a_7 = a^{7-1}$

$a_7 = a^6$

$a_7 = y$

$a_{10} = a^{10-1}$

$a_{10} = a^9$

$a_{10} = z$

x, y, z are in G.P. then,

$y^2 = xz$

Then, $xz = ar^3 \times ar^9$

$= a^2r^{3+9}$

$= a^2r^{12}$

$y^2 = (ar^6)^2$

$y^2 = a^2r^{12}$

By comparing left hand side and right hand side

LHS = RHS

Therefore, x, y and z are in G.P.

10. The 5th, 8th and 11th terms of a G.P. are p, q and s respectively. Show that $q^2 = ps$.

Solution:-

From the question it is given that,

$a_5 = p$

$a_8 = q$

$a_{11} = s$

Now we have to prove that, $q^2 = ps$

Then, by the formula $a_n = ar^{n-1}$

$a_5 = ar^{5-1}$

$a_5 = a^4$

$a_5 = p$

So, $a_8 = a^{8-1}$

$a_8 = a^7$

$a_8 = q$

$a_{11} = a^{11-1}$

$a_{11} = a^{10}$

$a_{11} = s$

p, q, s are in G.P. then,

$q^2 = (ar^7)^2$

$= ar^{14}$

Then, $px = ar^4 \times ar^{10}$
$= a^2r^{4+10}$
$= a^2r^{14}$
Therefore, $q^2 = ps$

11. If a, b, c are in G.P., then show that a^2, b^2, c^2 are also in G.P.
Solution:-
From the question it is given that,
a, b, c are in G.P.
We have to show that a^2, b^2, c^2 are also in G.P
Then,
$b^2 = ac$ … equation (i)
Therefore, a^2, b^2, c^2 will be in G.P.
if $(b^2)^2 = a^2 \times c^2$
$(ac)^2 = a^2c^2$ … [from the equation (i)]
$a^2c^2 = a^2c^2$
Therefore, it is proved that a^2, b^2, c^2 are also in G.P.

12. If a, b, c are in A.P., then show that $3^a, 3^b, 3^c$ are in G.P.
Solution:-
From the question it is given that,
a, b and c are in A.P.
So, $2b = a + c$
We have to show that $3^a, 3^b, 3^c$ are also in G.P.
If $(3^b)^2 = 3^a \times 3^c$
$3^{2b} = 3^{a+c}$
Now, comparing the results we get,
$2b = a + c$
Therefore, $3^a, 3^b, 3^c$ are in G.P

13. If a, b, c are in A.P., then show that $10^{ax+10}, 10^{bx+10}, 10^{cx+10}, x \neq 0$, are in G.P.
Solution:-
From the question it is given that,
a, b and c are in A.P.
So, $2b = a + c$
We have to show that $10^{ax+10}, 10^{bx+10}, 10^{cx+10}, x \neq 0$, are also in G.P.
$2b = a + c$

If $(10^{bx + 10})^2 = (10^{ax + 10}) \times (10^{cx + 10})$
$(10^{2bx + 20}) = 10^{ax + 10 + cx + 10}$
$(10^{2bx + 20}) = 10^{ax + cx + 20}$
By comparing left hand side and right hand side we get,
$2bx + 20 = ax + cx + 20$
$2bx = ax + cx$
$2b = a + c$
Therefore, $10^{ax + 10}$, $10^{bx + 10}$, $10^{cx + 10}$ are in G.P.

14. If a, a^2+ 2 and a^3 + 10 are in G.P., then find the values(s) of a.
Solution:-
From the question,
$(a^2 + 2)^2 = a(a^3 + 10)$
$a^4 + 4 = a^4 + 10a$
$4a^2 – 10a + 4 = 0$
$2a^2 – 5a + 2 = 0$
$2a^2 – a – 4a + 2 = 0$
$a(2a - 1) – 2(2a - 1) = 0$
$(2a - 1) (a - 2) = 0$
Then, $2a – 1 = 0$
$a = ½$
$a – 2 = 0$
$a = 2$
Therefore, a = 2 or a= ½

15. The first and the second terms of a GP. are x^{-4} and x^m . If its 8^{th} term is x^{52}, then find the value of m.
Solution:-
From the question it is given that,
First term of G.P. $a_1 = x^{-4}$
Second term of G.P. $a_2 = x^m$
Eighth term of G.P $a_8 = x^{52}$
Then, $r = a_2/a_1$
$= x^m/x^{-4}$
$= x^{m – (-4)}$ … [by $a^m/a^n = a^{m - n}$]
$= x^{m + 4}$
$a_8 = ar^{8 – 1}$

$a_8 = ar^7$

$x^{52} = x^{-4} \times r^7$

$= x^{-4} \times x^{7(m + 4)}$

$= x^{-4 + 7m + 28}$

$X^{52} = x^{7m + 24}$

By comparing left hand side and right hand side we get,

52 = 7m + 24

7m = 52 – 24

7m = 28

m = 28/7

m = 4

Therefore, the value of m is 4.

16. Find the geometric progression whose 4th term is 54 and the 7th term is 1458.

Solution:-

From the question it is given that,

4th term $a_4 = 54$

7th term $a_7 = 1458$

$ar^3 = 54$

$ar^6 = 1458$

Now dividing we get,

$ar^6/ar^3 = (1458/54)$

$r^{6-3} = 27$

$r^3 = 3^3$

r = 3

Then, $ar^3 = 54$

a × 27 = 54

a = 54/27

a = 2

Therefore G.P. is 2, 6, 18, 54, …

17. The fourth term of a G.P. is the square of its second term and the first term is – 3. Determine its seventh term.

Solution:-

From the question it is given that,

$a_1 = -3$

a_n is square of a_2 i.e. $a_n = (a_2)^2$

$a_n = ar^{n-1}$
$a_4 = ar^{4-1}$
$= ar^3$
$a_3 = ar^{3-1}$
$a_3 = ar^2$
So, $ar^3 = ar^2$
$ar^3 = a^2r^2$
$r^3/r^2 = a^2/a$
$r^{3-2} = a^{2-1}$
$r = a$
Therefore, $a_7 = ar^{7-1}$
$a_7 = ar^6$
$= (-3)(-3)^6$
$= -3^{1+6}$
$= -3^7$
$a_7 = -2187$

18. The sum of first three terms of a G.P. is 39/10 and their product is 1. Find the common ratio and the terms.
Solution:-
From the question it is give that,
The sum of first three terms of a G.P. is 39/10
The product of first three terms of a G.P. is 1
Let us assume that a be the first term and 'r' be the common ratio,
And also assume that, three terms of the G.P. is a/r, a, ar,
The sum of three terms = $(a/r) + a + ar = 39/10$
Take out 'a' as common then, we get
$a(1/r + 1 + r) = 39/10$ … [equation (i)]
Now, product of three terms = $(a/r) \times a \times ar = 1$
$a^3r/r = 1$
$a^3 = 1$
$a^3 = 1^3$
$a = 1$
Substitute the value of 'a' in equation (i),
$1(1/r + 1 + r) = 39/10$
$(1 + r + r^2)/r = 39/10$
By cross multiplication we get,

$10(1 + r + r^2)/r = 39r$

$10 + 10r + 10r^2 = 39r$

Transposing 39r from right hand side to left hand side it becomes – 39r,

$10 + 10r + 10r^2 – 39r = 0$

$10r^2 – 29r + 10 = 0$

$10r^2 – 25r – 4r + 10 = 0$

$5r(2r - 5) – 2(2r - 5) = 0$

$(2r - 5) (5r - 2) = 0$

So, $2r – 5 = 0$

$r = 5/2$

$5r – 2 = 0$

$r = 2/5$

Therefore, r = 5/2 or 2/5

Then the terms if r = 5/2 are, 1, 5/2, 25/4, …

The terms if r = 2/5 are, 1, 2/5, 4/25, …

19. Three numbers are in A.P. and their sum is 15. If 1, 4 and 19 are added to these numbers respectively, the resulting numbers are in G.P. Find the numbers.

Solution:-

From the question it is give that,

The sum of first three terms of a A.P. is 15

Let us assume three numbers are a – d, a, a + d.

The sum of three terms = a – d + a + a + d = 15

$a = 15/3$

$a = 5$

Then, adding 1, 4, 19 in the terms

The numbers become, a – d + 1, a + 4, a + d + 19

Therefore, $b^2 = ac$

$(a + 4)^2 = (a – d + 1) (a + d + 19)$

Simplify the above terms,

$a^2 + 8a + 16 = a^2 + ad + 19a – ad – a^2 – 19d + a + d + 19$

$a^2 + 8a + 16 = a^2 – d^2 – 18d + 20a + 19$

$8a + 16 = 20a – 18d – d^2 + 19$

$8a + 16 – 20a + 18d + d^2 – 19 = 0$

$d^2 + 18d – 12a – 3 = 0$

$d^2 + 18d – (12 \times 5) – 3 = 0$

$d^2 + 18d – 60 – 3= 0$

$d^2 + 18d - 63 = 0$
$d^2 + 21d - 3d - 63 = 0$
$d(d + 21) - 3(d + 21) = 0$
$(d + 21)(d - 3) = 0$
So, $d + 21 = 0$
$d = -21$
$d - 3 = 0$
$d = 3$
Then the terms if $d = 3$ and $a = 5$,
Then G.P. $5 - 3 = 2, 5, 5 + 3 = 8$
The terms if $d = -21$ are $5 - (-21) = 5 + 21 = 26, 5, 5 - 21 = -16$

20. Three numbers form an increasing G.P. If the middle term is doubled, then the new numbers are in A.P. Find the common ratio of the G.P.
Solution:-
From the question it is given that,
Three numbers form an increasing G.P.
Let us assume the three numbers a/r, a, are
Then, double the middle term we get,
a/r, 2a, ar will be in A.P.
So, $2(2a) = a/r + ar$
$4a = a(1/r + r)$
$4 = 1/r + r$
By cross multiplication,
$4r = 1 + r^2$
$r^2 - 4r + 1 = 0$
$r = -b \pm \sqrt{(b^2 - 4ac)}/2a$
$= -(-4) \pm \sqrt{((-4)^2 - 4 \times 1 \times 1)}/(2 \times 1)$
$= 4 \pm \sqrt{(16 - 4)}/2$
$= 4 \pm \sqrt{12}/2$
$= 4 \pm 2\sqrt{3}/2$
$= 2 \pm \sqrt{3}$
Therefore, the common ratio of the G.P. is $2 \pm \sqrt{3}$.

21. Three numbers whose sum is 70 are in GP. If each of the extremes is multiplied by 4 and the mean by 5, the numbers will be in A.P. Find the numbers.
Solution:-

From the question it is given that,
Three numbers are in G.P. whose sum is 70.
Let us assume the three number be a/r, a, ar
Then, sum = (a/r) + a + ar = 70
Take out a as common,
a((1/r) + 1 + r) = 70 … [equation (i)]
Now, multiplying the extremes by 4 and mean by 5,
Then, (a/r) × 4 = 4a/r
(a × 5) = 5a
(ar × 4) = 4ar
4a/r, 5a, 4ar
Therefore, these are in A.P.
So, 2(5a) = (4a/r) + 4ar
10a = 4a(1/r) + r
Divide both the side by 2 we get,
(10/2)a = (4/2)a (1/r) + r
5a = 2a((1/r) + r)
$5r = 2 + 2r^2$
$2r^2 – 5r + 2 = 0$
$2r^2 – r – 4r + 2 = 0$
r(2r - 1) – 2(2r - 1) = 0
(2r - 1) (r - 2) = 0
So, 2r – 1 = 0
2r = 1
r = ½
r – 2 = 0
r = 2
Now substitute the value r in equation (i),
a((1/2) + 1 + 2) = 70
a ((½) + 3) = 70
a ((1 + 6)/2) = 70
a (7/2) = 70
a = 70 × (2/7)
a = 10 × 2
a = 20
Then,
r = 2, a = 20

= (a/r), a, ar
= (20/2), 20, (20 × 2)
= 10, 20, 40
Then,
r = ½, a = 20
= (a/r), a, ar
= (20/½), 20, (20 × ½)
= (20 × 2), 20, 10
= 40, 20, 10

22.
(i) If a, b, c are in A.P. as well in G.P., prove that a = b = c.
Solution:-
From the question it is given that,
a, b, c are in A.P. as well in G.P.
We have to prove that, a = b = c.
a, b, c are in A.P.
2b = a + c
b = (a + c)/2 … [equation (i)]
Now, a, b, c are in G.P.
b^2 = ac … [equation (ii)]
Now substitute the value of 'b' in equation (ii),
$((a + c)/2)^2 = ac$
$(a + c)^2/4 = ac$
$(a + c)^2 = 4ac$
$(a + c)^2 – 4ac = 0$
Then, $(a - c)^2 = 0$
(a - c) = 0
a = c … [equation (iii)]
From the equation (i), 2b = a + c
Substitute the value of a
Then, 2b = a + a
2b = 2a
Therefore, b = a … [equation (iv)]
By comparing equation (iii) and equation (iv),
a = b = c

(ii) If a, b, c are in A.P as well as in G.P., then find the value of $a^{b-c} + b^{c-a} + c^{a-b}$

Solution:-

From the question it is given that,

a, b, c are in A.P.

So, $2b = a + c$

Now, a, b, c are in G.P

$b^2 = ac$

from question 22(i) $a = b = c$,

Given, $a^{b-c} + b^{c-a} + c^{a-b}$

Therefore, $b - c = 0$, $c - a = 0$ and $a - b = 0$

So, $a^0 + b^0 + c^0$

We know that, $x^0 = 1$

$1 + 1 + 1$

$= 3$

23. The terms of a G.P. with first term a and common ratio r are squared. Prove that resulting numbers form a G.P. Find its first term, common ratio and the n^{th} term.

Solution:-

From the question it is given that,

First term of G.P = a

Common ratio = r

Then the terms of G.P. is a, ar, ar^2.

By squaring the terms of G.P. we get,

a^2, a^2r^2, a^2r^4

We know that, $b^2 = 4ac$

$(a^2r^2)^2 = a^2 \times a^2r^4$

$a^4r^4 = a^4r^4$

Therefore, the first term is a^2

Common ratio is r^2

Then, n^{th} term will be

$a_n = ar^{n-1}$

$a_n = a^2(r^{n-1})^2$

$a_n = a^2r^{2n-2}$

24. Show that the products of the corresponding terms of two G.P.'s a, ar, ar^2, …, ar^{n-1} and A, AR, AR^2, …, AR^{n-1} form a G.P. and find the common ratio.

Solution:-

From the question it is given that,
The corresponding terms of two G.P.'s a, ar, ar^2, …, ar^{n-1} and A, AR, AR^2, …, AR^{n-1}
We have to show that, the products of the corresponding terms of two G.P.'s form a G.P.
Consider first and second term,
So, ratio = second term/third term = arAR/aA
= rR
Then, Consider second and third term,
So, ratio = third term/second term = ar^2AR^2/arAR
= rR
By comparing the both the results the common ratio is rR.

25.
(i) If a, b, c are in G.P. show that 1/a, 1/b, 1/c are also in G.P.
Solution:-
From the question it is given that,
a, b, c are in G.P.
We know that, $b^2 = ac$
We have to show that, 1/a, 1/b, 1/c are also in G.P.
$(1/b)^2 = (1/a) \times (1/c)$
$(1/b^2) = = (1/ac)$
By cross multiplication we get,
$ac = b^2$
Hence it is proved that, 1/a, 1/b, 1/c are in G.P.

(ii) If K is any positive real number and K^a, K^b, K^c are three consecutive terms of a G.P., prove that a, b, c are three consecutive terms of an A.P.
Solution:-
From the question it is given that,
K is any positive real number
K^a, K^b, K^c are three consecutive terms of a G.P.
We know that, $b^2 = ac$
$(K^b)^2 = k^a \times K^c$
$K^{2b} = k^{a + c}$ … [from $a^m \times a^n = a^{m + n}$]
By comparing left hand side and right hand side we get,
$2b = a + c$
Therefore, a, b, c are three consecutive terms of an A.P.

(iii) If p, q, r are in A.P., show that p^{th}, q^{th} and r^{th} terms of any G.P. are themselves in GP.

Solution:-

From the question it is given that,

p, q, r are in A.P.

So, 2p = p + r

We have to show that p^{th}, q^{th} and r^{th} terms of any G.P.

P^{th} term in G.P. = AR^{p-1}

Q^{th} term in G.P. = AR^{q-1}

R^{th} term in G.P. = AR^{r-1}

So, if $(AR^{q-1})^2 = AR^{p-1} \times AR^{r-1}$

$A^2R^{2q-2} = A^2R^{p-1+r-1}$

$A^2R^{2q-2} = A^2R^{p+r-2}$

$R^{2q-2} = R^{p+r-2}$

By comparing left hand side and right hand side we get,

2p – 2 = p + r – 2

2p = P + r

Therefore, p, q, r are in A.P

26.

If a, b, c are in GP., prove that the following are also in G.P.

(i) a^3, b^3, c^3

Solution:-

From the question it is given that,

a, b, c are in GP.

So, $b^2 = ac$

We have to prove that, a^3, b^3, c^3 are in G.P.

Then, $(b^3)^2 = a^3 \times c^3$

It can be written as, $(b^2)^3 = (a \times c)^3$

$b^2 = ac$

Therefore, it is proved that a^3, b^3, c^3 are in G.P.

(ii) $a^2 + b^2$, ab + bc, $b^2 + c^2$.

Solution:-

From the question it is given that,

a, b, c are in GP.

So, $b^2 = ac$

We have to prove that, $a^2 + b^2$, $ab + bc$, $b^2 + c^2$. are in G.P.

Then, $(ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$

$a^2b^2 + b^2c^2 + 2ab^2c = a^2b^2 + a^2c^2 + b^2c^2 + b^4$

By transposing and simplification, we get,

$b^4 + a^2c^2 - 2ab^2c = 0$

$(b^2 - ac)^2 = 0$

$b^2 = ac$

Therefore, $a^2 + b^2$, $ab + bc$, $b^2 + c^2$ are in GP.

27. If a, b, c, d are in G.P., show that

(i) $a^2 + b^2$, $b^2 + c^2$, $c^2 + d^2$ are in G.P.

Solution:-

From the question it is given that

a, b, c, d are in G.P

So, $bc = ad$ … [equation (i)]

$b^2 = ac$ … [equation (ii)]

$c^2 = bd$ … [equation (iii)]

We have to show that, $a^2 + b^2$, $b^2 + c^2$, $c^2 + d^2$ are in G.P.

Then, $(b^2 + c^2)^2 = (a^2 + b^2)(c^2 + d^2)$

Consider the LHS $= (b^2 + c^2)^2$

$= b^4 + c^4 + 2b^2c^2$

From the equation (ii) and equation (iii),

$= a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$

$= c^2(a^2 + b^2) + d^2(a^2 + b^2)$

$= (a^2 + b^2) + (c^2 + d^2)$

Now consider the RHS $= (a^2 + b^2)(c^2 + d^2)$

By comparing the LHS and RHS

LHS = RHS

Hence it is proved that, $a^2 + b^2$, $b^2 + c^2$, $c^2 + d^2$ are in G.P.

(ii) $(b - c)^2 + (c - a)^2 + (d - b)^2 = (a - d)^2$.

Solution:-

From the question it is given that

a, b, c, d are in G.P

We have to prove that, $(b - c)^2 + (c - a)^2 + (d - b)^2 = (a - d)^2$.

Consider the LHS $= (b - c)^2 + (c - a)^2 + (d - b)^2$

We know that, the first, second and third terms of G.P. generally a, ar, ar^2

So, LHS = $(ar - ar^2)^2 + (ar^2 - a)^2 + (ar^3 - ar)^2$

$= a^2r^2(1 - r)^2 + a^2(r^2 - 1)^2 + a^2r^2(r^2 - 1)^2$

By taking out a^2 as common we get,

$= a^2[r^2(1 - r^2 - 2ar) + r^4 - 2r^2 + 1 + r^2(r^4 - 2r^2 + 1)]$

$= a^2[r^2 - r^4 - 2ar^3 + r^4 - 2r^2 + 1 + r^6 - 2r^4 + r^2]$

$= a^2(r^6 - 2r^3 + 1)$

Now, consider the RHS = $(a - d)^2$

$= (a - ar^3)^2$

$= a^2 (1 - r^3)^2$

$= a^2(1 + r^6 - 2r^3)$

$= a^2 (r^6 - 2r^3 + 1)$

By comparing the LHS and RHS

LHS = RHS

Hence it is proved that, $(b - c)^2 + (c - a)^2 + (d - b)^2 = (a - d)^2$.

EXERCISE 9.5

1. Find the sum of:

(i) 20 terms of the series 2 + 6 + 18 + …

Solution:-

From the question,

First term a = 2,

Common ratio r = 6/2 = 3

Number of terms n = 20

So, $S_{20} = a(r^n - 1)/r – 1$

$= 2(3^{20} - 1)/3 – 1$

$= 2(3^{20} - 1)/2$

$= 3^{20} – 1$

Therefore, $S_{20} = 3^{20} - 1$

(ii) 10 terms of series 1 + √3 + 3 + …

Solution:-

From the question,

First term a = 1,

Common ratio r = √3/1 = √3

Number of terms n = 10

So, $S_{10} = a(r^n - 1)/r – 1$

$= 1((\sqrt{3})^{10} - 1)/ \sqrt{3} – 1$

Multiplying (√3 + 1) for both numerator and denominator we get,

$= ((\sqrt{3}^{10} - 1) (\sqrt{3} + 1))/ ((\sqrt{3} – 1) (\sqrt{3} + 1)$

$= (3^5 - 1) (\sqrt{3} + 1))/3 – 1$ … [by rationalizing the denominator]

$= ((243 - 1)(\sqrt{3} + 1))/2$

$= 242(\sqrt{3} + 1)/2$

$= 121(\sqrt{3} + 1)$

Therefore, $S_{10} = 121(\sqrt{3} + 1$

(iii) 6 terms of the GP 1, -2/3, 4/9, …

Solution:-

From the question,

First term a = 1,

Common ratio r = -2/3 × 1= -2/3

Number of terms n = 6

So, $S_6 = a(r^n - 1)/r – 1$

$= 1[1 - (-2/3)^6]/(1 + (2/3))$
$= (3/5) (1 – (-2^6/3^6))$
$= (3/5) (1 – (64/729))$
$= (3/5) ((729 - 64)/729)$
$= 3/5 \times (665/729)$
$= 133/243$

(iv) 5 terms and n terms of the series 1 + 2/3 + 4/9 + …
Solution:-
From the question,
First term a = 1,
Common ratio r = 2/3 × 1= -/3
Number of terms n = 5
So, $S_n = a(1 - r^n)/1 – r$
$= 1[1 – (2/3)^n]/(1 – 2/3)$
$S_n = 3[1 – (2/3)^n]$
Then, $S_5 = 3[1 – (2/3)^5]$
$= 3[1 – (32/243)]$
$= 3((243 – 32)/243)$
$= 211/81$

(v) n terms of the G.P. √7, √21, 3√7, …
Solution:-
From the question,
First term a = √7,
Common ratio r = √21/√7 = √3
Number of terms n = 10
So, $S_n= a(r^n - 1)/r – 1$
$= \sqrt{7}((\sqrt{3})^n - 1)/ \sqrt{3} – 1$
Multiplying (√3 + 1) for both numerator and denominator we get,
$= \sqrt{7}((\sqrt{3}^n - 1) (\sqrt{3} + 1))/ ((\sqrt{3} – 1) (\sqrt{3} + 1)$
$= [\sqrt{7}((\sqrt{3})^n – 1) (\sqrt{3} + 1)]/((\sqrt{3})^2 – 1^2)$ … [by rationalizing the denominator]
$= (\sqrt{7}[(\sqrt{3})^n - 1] (\sqrt{3} + 1))/3 - 1$
Therefore, $S_n = \sqrt{7}/2 [(\sqrt{3})^n - 1] (\sqrt{3} + 1)$

(vi) n terms of the G.P. 1, -a, a^2, $-a^3$, … (a ≠ -1)
Solution:-

From the question,
First term a = 1,
Common ratio r = -a/1 = -a
So, $S_n = a(1 - r^n)/1 - r$
$= 1[1 - (-a)^n]/(1 - (-a))$
$= (1 - (-a)^n)/(1 + a)$

(vii) n terms of the G.P. $x^3, x^5, x^7, \ldots$ ($x \neq -1$)
Solution:-
From the question,
First term $a = x^3$,
Common ratio $r = x^5/x^3 = x^{5-3} = x^2$
So, $S_n = a(1 - r^n)/1 - r$
$= x^3[(1 - (x^2)^n]/(1 - x^2)$ if r < 1
$= x^3(1 - x^{2n})/1 - x^2$
And also $S_n = a(r^n - 1)/(1 - r)$
$= x^3[(x^2)^n - 1]/x^2 - 1$
$= x^3(x^{2n} - 1)/(x^2 - 1)$

2. Find the sum of the first 10 terms of the geometric series
$\sqrt{2} + \sqrt{6} + \sqrt{18} + \ldots$.
Solution:-
From the question it is given that,
$a = \sqrt{2}$
$r = \sqrt{3}$
We know that, $S_n = a(r^n - 1)/(r - 1)$
$S_{10} = \sqrt{2}[(\sqrt{3})^{10} - 1]/(\sqrt{3} - 1)$
$= (\sqrt{2}/(\sqrt{3} - 1))\,[(3)^5 - 1]$
$= (\sqrt{2}/(\sqrt{3} - 1))\,[243 - 1]$
$= (\sqrt{2}/(\sqrt{3} - 1)) \times 242$
$= (\sqrt{2}\,(\sqrt{3} + 1)\,242)/[(\sqrt{3} - 1)(\sqrt{3} + 1)]$
Rationalizing the denominator, we get,
$= 242(\sqrt{6} + \sqrt{2})/(3 - 1)$
$= 242(\sqrt{6} + \sqrt{2})/2$
$= 121(\sqrt{6} + \sqrt{2})$

3. Find the sum of the series 81 – 27 + 9 … - 1/27

Solution:-

From the question it is given that,

First term a = 81

r = -27/81

= -1/3

Last term l = -1/27

$S_n = (a - lr)/(l - r)$

= [81 + ((1/27) × (-1/3)]/[1 + (1/3)]

= [(81 – (1/81))]/(4/3)

= (6561 - 1)/[81 × (4/3)]

= (6560 × 3)/(81 × 4)

= 1640/27

4. The n^{th} term of a G.P. is 128 and the sum of its n terms is 255. If its common ratio is 2, then find its first term.

Solution:-

From the question it is given that,

The n^{th} term of a G.P. $T_n = 128$

The sum of its n terms $S_n = 255$

Common ratio r = 2

We know that, $T_n = ar^{n-1}$

$128 = a2^{n-1}$

$a = 128/2^{n-1}$ … [equation (i)]

Also we know that, $S_n = a(r^n - 1)/(r - 1)$

$255 = a(2^n - 1)/(2 - 1)$

By cross multiplication we get,

$255 = a(2^n - 1)$

$a = 255/(2^n - 1)$ … [equation (ii)]

Now, consider both the equation(i) and equation (ii)

$255/(2^n - 1) = 128/(2^{n-1})$

By cross multiplication we get,

$255 \times 2^{n-1} = 128(2^n - 1)$

$255 \times 2^{n-1} = 128 \times 2^n - 128$

$(255 \times 2^n)/2 = 128 \times 2^n - 128$

$255 \times 2^n = 256 \times 2^n - 256$

$256 \times 2^n - 255 \times 2^n = 256$

By simplification,
$2^n = 256$
$2^n = 2^8$
By comparing both LHS and RHS, we get,
Then, $128 = a2^7$
$128 = a \times 128$
$a = 128/128$
$a = 1$
Therefore, the value of a is 1.

5. If the sum of first six terms of any G.P. is equal to 9 times the sum of the first three terms, then find the common ratio of the G.P.
Solution:-
From the question it is given that,
the sum of first six terms of any G.P. is equal to 9 times the sum of the first three terms,
$S_6 = 9S_3$
We know that,
$S_n = a(r^n - 1)/(r - 1)$
$S_6 = a(r^6 - 1)/(r - 1)$
$S_3 = a(r^3 - 1)/(r - 1)$
Now,
$a(r^6 - 1)/(r - 1) = 9 \times a(r^3 - 1)/(r - 1)$
By simplification we get,
$r^6 – 1 = 9(r^3 - 1)$
$(r^6 - 1)/(r^3 - 1) = 9$
$[(r^3 + 1) (r^3 - 1)]/(r^3 - 1) = 9$
$r^3 + 1 = 9$
$r^3 = 9 – 1$
$r^3 = 8$
$r^3 = 2^3$
$r = 2$
Therefore, common ratio r = 2

6.
(i) How many terms of the G.P. 3, 3^2, 3^3, … are needed to give the sum 120?
Solution:-

From the question it is given that,
Terms of the G.P. 3, 3^2, 3^3, …
Sum of the terms = 120
The first term a = 3
r = $3^2/3$
= 9/3
= 3
We know that, $S_n = a(r^n - 1)/r – 1 = 120$
$3(3^n - 1)/(3 - 1) = 120$
$3(3^n - 1)/2 = 120$
By cross multiplication we get,
$3^n – 1 = (120 \times 2)/3$
$3^n – 1 = 240/3$
$3^n – 1 = 80$
$3^n = 80 + 1$
$3^n = 81$
$3^n = 3^4$
Therefore, n = 4

(ii) How many terms of the G.P. 1, 4, 16, … must be taken to have their sum equal to 341?
Solution:-
From the question it is given that,
Terms of the G.P. 1, 4, 16, …
Sum of the terms = 341
The first term a = 1
r = 4/1
= 4
We know that, $S_n = a(r^n - 1)/r – 1 = 341$
$1(4^n - 1)/(4 - 1) = 341$
$1(4^n - 1)/3 = 341$
By cross multiplication we get,
$4^n – 1 = (341 \times 3)$
$4^n – 1 = 1023$
$4^n = 1023 + 1$
$4^n = 1024$

4	1024
4	256
4	64
4	16
	4

$4^n = 4^5$

Therefore, n = 5

7. How many terms of the GP. 1, √2 > 2, 2 √2, … are required to give a sum of 1023(√2 + 1)?

Solution:-

From the question it is given that,

Terms of the G.P. 1, √2 > 2, 2 √2, …

Sum of the terms = 1023(√2 + 1)

The first term a = 1

r = √2/1 = √2

We know that, $S_n = a(r^n - 1)/r – 1 = 1023(\sqrt{2} + 1)$

$1[(\sqrt{2}^n - 1)]/(\sqrt{2} - 1) = 1023(\sqrt{2} + 1)$

$(\sqrt{2}^n - 1) = 1023(\sqrt{2} + 1)(\sqrt{2} - 1)$

$(\sqrt{2})^n – 1 = 1023[(\sqrt{2})^2 – 1^2]$

$(\sqrt{2})^n – 1 = 1023(2 – 1)$

$(\sqrt{2})^n – 1 = 1023(1)$

$(\sqrt{2})^n – 1 = 1023$

$(\sqrt{2})^n = 1023 + 1$

$(\sqrt{2})^n = 1024$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

$(\sqrt{2})^n = 2^{10}$

$(\sqrt{2})^n = (\sqrt{2})^{20}$

n = 20

8. How many terms of the 2/9 – 1/3 + ½ + … will make the sum 55/72?

Solution:-

From the question it is given that,

Terms of G.P. is 2/9 – 1/3 + ½ + …

Sum of the terms = 55/72

The first term a = 2/9

r = -1/3 ÷ 2/9 = (-1/3) × (9/2) = - 3/2

We know that, $S_n = a(r^n - 1)/r - 1 = 55/72$

$[(2/9)(1 - (-3/2)^n)]/(1 + (3/2)) = 55/72$

$1 - (-3/2)^n = (55/72) \times (5/2) \times (9/2)$

$(1 - (-1))^n (3/2)^n = 275/32$

$1 + 1(3/2)^n = 275/32$

$(3/2)^n = 275/32 - 1$

$(3/2)^n = (275 - 32)/32$

$(3/2)^n = 243/32$

$(3/2)^n = (3/2)^5$

Therefore, n = 5

9. The 2nd and 5th terms of a geometric series are -½ and sum 1/16 respectively. Find the sum of the series up to 8 terms.

Solution:-

From the question it is given that,

$a_2 = -½$

$a_5 = 1/16$

We know that, $a_2 = ar^{n-1}$

$= ar^{2-1}$

$a_2 = ar = -½$ … [equation (i)]

$a_5 = ar^{5-1}$

$= ar^4$

$a_5 = ar^4 = 1/16$ … [equation (ii)]

Now, dividing equation (ii) by (i) we get,

$r^3 = 1/16 \div (-½)$

$= (1/16) \times (-2)$

$= -1/8$

$r^3 = (-1/2)^3$

So, r = -½

$ar = -½$

$a \times (-½) = -½$

$a = -½ \times (-2/1)$

$a = 1$

Therefore, $a = 1$ and $r = -½$

Then, $S_8 = a(1 – r^n)/(1 - r)$

$= 1[1 - (-1/2)^8]/(1 + ½)$

$= [1 – (1/256)]/(3/2)$

$= (255/256) \times (2/3)$

$= (510/768)$

$= 85/128$

10. The first term of G.P. is 27 and 8th term is 1/81. Find the sum of its first 10 terms.

Solution:-

From the question it is given that,

First term $a = 27$

8th term $a_8 = 1/81$

Then, $a_n = ar^{n – 1}$

$a_8 = ar^{8 – 1} = 1/81$

$a_8 = ar^7 = 1/81$

$ar^7 = 1/81$

$27r^7 = 1/81$

$r^7 = 1/(81 \times 27)$

$r^7 = 1/2187$

$r^7 = 1/(3^7)$

$r = 1/3$

So, $S_{10} = a(1 – r^n)/(1 - r)$

$= 27[1 - (1/3)^{10}]/(1 – 1/3)$

$= 27[1 – (1/3^{10})]/((3 – 1)/3)$

$= ((27 \times 3)/2) [1 – 1/3^{10}]$

$= (81/2) [1 – 1/3^{10}]$

11. Find the first term of the G.P. whose common ratio is 3, last term is 486 and the sum of whose terms is 728.

Solution:-

From the question it is given that,

Common ratio $r = 3$

Last term = 486

Sum of the terms = 728

We know that, $S_n = a(r^n - 1)/(r - 1)$

$= a(3^n - 1)/(3 – 1) = 728$

$a(3^n - 1)/2 = 728$

$a(3^n - 1) = 728 \times 2$

$a(3^n - 1) = 1456$ … [equation (i)]

Then, last term = ar^{n-1}

$486 = a \times 3^{n-1}$

$486 = a(3^n/3)$

$486 \times 3 = a3^n$

$1458 = a3^n$ … [equation (ii)]

Consider equation (i), $a(3^n - 1) = 1456$

$a3^n – a = 1456$

Substitute the value of $a3^n$ in equation (i),

$1458 – a = 1456$

$a = 1458 – 1456$

$a = 2$

Therefore, the first term a is 2.

12. In a G.P. the first term is 7, the last term is 448, and the sum is 889. Find the common ratio.

Solution:-

From the question it is given that,

First term a is = 7

Then, last term is = 448

Sum = 889

We know that, last term = ar^{n-1}

$7r^{n-1} = 448$

$r^{n-1} = 448/7$

$r^{n-1} = 64$ … [equation (i)]

So, sum = $a(r^n - 1)/(r – 1) = 889$

$7(r^n - 1)/(r - 1) = 889$

$(r^n - 1)/(r - 1) = 889/7$

$(r^n - 1)/(r - 1) = 127$ … [equation (ii)]

Consider the equation (i),

$r^n/r = 64$

$r^n = 64r$

Now substitute the value of r^n in equation (ii),

$(64r – 1)/(r - 1) = 127$

$64r – 1 = 127r – 127$

$127r – 64r = -1 + 127$

$63r = 126$

$r = 126/63$

$r = 2$

Therefore, common ratio = 2

13. Find the third term of a G.P. whose common ratio is 3 and the sum of whose first seven terms is 2186.

Solution:-

From the question it is given that,

Common ratio r = 3

$S_7 = 2186$

We know that, $S_n = a(r^n - 1)/(r - 1)$

$S_7 = a(3^n - 1)/(3 - 1)$

$2186 = a(3^n - 1)/2$

By cross multiplication,

$(2186 \times 2) = a(3^7 - 1)$

$(4372) = a(2187 - 1)$

$4372 = a2186$

$a = 4372/2186$

$a = 2$

Then, $a_3 = ar^{3-1}$

$= ar^2$

$= 2 \times 3^2$

$= 2 \times 9$

$a_3 = 18$

14. If the first term of a G.P. is 5 and the sum of first three terms is 31/5, find the common ratio.

Solution:-

From the question it is given that,

First term of a G.P. is a = 5

The sum of first three terms is $S_3 = 31/5$

We know that, $S_n = a(r^n - 1)/(r - 1)$

$S_3 = a(r^3 - 1)/(r - 1)$

$31/5 = 5(r^3 - 1)/(r - 1)$

$31/(5 \times 5) = (r^3 - 1)/(r - 1)$

$31/25 = (r^3 - 1)/(r - 1)$

$(r - 1)(r^2 + r + 1)/(r - 1) = 31/25$

$r^2 + r + 1 = 31/25$

By cross multiplication we get,

$25(r^2 + r + 1) = 31$

$25r^2 + 25r + 25 = 31$

Transposing 31 from right hand side to left hand side it becomes – 31,

$25r^2 + 25r + 25 - 31 =$

$25r^2 + 25r - 6 = 0$

$25r^2 + 30r - 5r - 6 = 0$

$5r(5r + 6) - 1(5r + 6) = 0$

$(5r - 1)(5r + 6) = 0$

Take $5r - 1 = 0$

$r = 1/5$

or $5r + 6 = 0$

$r = -6/5$

Therefore, common ratio $r = 1/5$ or $-6/5$.

15. The sum of first three terms of a GP. is to the sum of first six terms as 125 : 152. Find the common ratio of the GP.

Solution:-

Froom the question it is given that,

Ratio of the sum of first three terms to the sum of first six terms $S_3 \div S_6 = 125 : 152$

We know that, $S_n = a(r^3 - 1)/(r - 1)$

$S_3 : S_6 = 125 : 152$

$[a(r^3 - 1)/(r - 1)] : [a(r^6 - 1)/(r - 1)] = 125 : 152$

$(r^3 - 1) : (r^6 - 1) = 125 : 152$

$(r^3 - 1) : (r^3 + 1)(r^3 - 1) = 125 : 152$

$(r^3 - 1)/[(r^3 + 1)(r^3 - 1)] = 125/152$

$1/(r^3 + 1) = 125/152$

By cross multiplication,

$(1 \times 152) = (r^3 + 1) \times 125$

$152 = 125r^3 + 125$

$125r^3 = 152 – 125$
$125r^3 = 27$
$r^3 = 27/125$
$r^3 = (3/5)^3$
$r = 3/5$
Therefore, common ratio $r = 3/5$

16. $\sum_{n=1}^{50} (2^n - 1)$
Solution:-
From the question it is given that,
n = 1, 2, 3, 4, …., 50
Then, $S_n = (2^1 – 1) + (2^2 - 1) + (2^3 - 1) + (2^4 - 1) … 2^{50} - 1$
$S_n = (2^1 + 2^2 + 2^3 + 2^4 … 2^{50}) – 1 \times 50$
$S_n = 2 + 4 + 8 + 16 … 2^{50} – 50$
We know that, $S_n = [a(a^n - 1)/(r - 1)] – 50$
$= [2(2^{50} - 1)/(2 - 1)] – 50$
$= (2 \times 2^{50}) – 2 – 50$
$= 2^{51} – 52$

17. Sum the series $x(x + y) + x^2(x^2 + y^2) + x^3 (x^3 + y^3)$ … to n terms.
Solution:-
From the question it is given that, $x(x + y) + x^2(x^2 + y^2) + x^3 (x^3 + y^3)$ …
Then, $S_n = x^2 + xy + x^4 + x^2y^2 + x^6 + x^3y^3 + …$ n terms
By separating the terms,
$S_n = x^2 + x^4 + x^6 + …$ n terms G.P. … (1)
$S_n = xy + x^2y^2 + x^3y^3 + …$ … (2)
In G.P. (1) first term $a = x^2$, $r = x^4/x^2 = x^{4-2} = x^2$
In G.P. (2) first term $a = xy$, $r = x^2y^2/xy = x^{2-1}y^{2-1} = xy$
$S_n = a(r^n - 1)/(r - 1)$
$S_n = [(x^2((x^2)^n – 1))/(x^2 - 1)] + [(xy((xy)^n – 1))/(xy - 1)]$
$S_n = [x^2(x^{2n} - 1)/(x^2 - 1)] + [xy((xy)^n - 1)/(xy - 1)]$

18. Find the sum of the series $1 + (1 + x) + (1 + x + x^2) + …$ to n terms, $x \neq 1$.
Solution:-
From the question it is given that,
$1 + (1 + x) + (1 + x + x^2) + …$ to n terms

Now, multiply and divide by (1 - x) we get,

= [(1 - x)/(1 - x)] + [((1 - x)(1 + x))/(1 - x)] + [(1 - x) (1 + x + x^2)/(1 - x)] + …

By taking common we get,

= 1/(1 - x) [(1 - x) + $(1 + x)^2$ + (1 + x^3) + ….]

= 1/(1 - x) [1 – x + 1 + x^2 + 1 + x^3 + …]

= 1/(1 - x) [(1 + 1 + 1 + … n terms) – (x + x^2 + x^3+ … n terms)]

We know that, $S_n = a(1 – r^n)/(1 - r)$

= (1/(1 - x)) [n – (x(1 – x^n)/(1 - x))]

= (1/(1 - x)) [(n(1 - x) – x(1- x^n))/(1- x)]

= (1/(1 – x^2)) [n(1 - x) – x(1 – x^n)]

19. Find the sum of the following series to n terms:

(i) 7 + 77 + 777 + …

Solution:-

Consider the given numbers 7 + 77 + 777 + … n terms

Take out 7 as common we get,

= 7 (1 + 11 + 111 + … n terms)

= 7/9 (9 + 99 + 999 + … n terms)

= 7/9 ((10 - 1) + (100 - 1) + (1000 - 1) + … n terms)

= 7/9 (10 + 100 + 100 + … n terms – (1 + 1 + 1 + …. n terms))

We know that, $S_n = a(r^n - 1)/(r - 1)$

First term a = 10

Common ratio r = 10

= 7/9 [(10(10^n - 1)/(10 - 1)) - n]

= 7/9 [(((10 × 10^n) - 10)/9) - n]

= 7/81 [10^{n+1} – 10 – 9n]

= 7/81 [10^{n+1} – 9n - 10]

(ii) 8 + 88 + 888 + …

Solution:-

Consider the given numbers 8 + 88 + 888 + … n terms

Take out 8 as common we get,

= 8(1 + 11 + 111 + … n terms)

= 8/9 (9 + 99 + 999 + … n terms)

= 8/9 ((10 - 1) + (100 - 1) + (1000 - 1) + … n terms)

= 8/9 (10 + 100 + 100 + … n terms – (1 + 1 + 1 + …. n terms))

We know that, $S_n = a(r^n - 1)/(r - 1)$

First term a = 10
Common ratio r = 10

= 8/9 [(10(10^n - 1)/(10 - 1)) - n]
= 8/9 [(((10 × 10^n) - 10)/9) - n]
= 8/81 [10^{n+1} – 10 – 9n]
= 8/81 [10^{n+1} – 9n - 10]

(iii) 0.5 + 0.55 + 0.555 + …

Solution:-

Consider the given numbers 0.5 + 0.55 + 0.555 + … n terms
Take out 5 as common we get,

= 5(0.1 + 0.11 + 0.111 + … n terms)
= 5/9 (0.9 + 0.99 + 0.999 + … n terms)
= 5/9 ((1 – 0.1) + (1 – 0.01) + (1 – 0.001) + … n terms)
= 5/9 (1 + 1 + 1 + … n terms – (0.1 + 0.01 + 0.001 + …. n terms))

We know that, S_n = a(1 - r^n)/(1 - r)

= 5/9 [n - (0.1(1 – $(-0.1)^n$)/(1 – 0.1))]
= 5/9 [n – ((1/9) (1 – (1/10^n)))]
= 5/81 [9n – 1 + (1/10^n)]

CHAPTER - TEST

1. Write the first four terms of the A.P. when its first term is – 5 and the common difference is – 3.

Solution:-

From the question it is given that,

First term a = - 5

Common difference d = -3

Then the first four terms are = - 5 + (-3) = -5 - 3 = - 8

-8 + (-3) = - 8 - 3 = -11

- 11 + (-3) = - 11 - 3 = - 14

Therefore, first four terms are -5, -8, -11 and -14.

2. Verify that each of the following lists of numbers is an A.P., and the write its next three terms:

(i) 0, ¼, ½, ¾, …

Solution:-

From the question it is given that,

First term a = 0

Common difference = ¼ - 0 = ¼

So, next three numbers are ¾ + ¼ = 4/4 = 1

1 + ¼ = (4 + 1)/4 = 5/4

5/4 + ¼ = 6/4 = 3/2

Therefore, the next three term are 1, 5/4 and 3/2.

(ii) 5, 14/3, 13/3, 4, …

Solution:-

From the question it is given that,

First term a = 5

Common difference = 14/3 - 5 = (14 - 15)/3 = -1/3

So, next three numbers are 4 + (-1/3) = (12 - 1)/3 = 11/3

11/3 + (-1/3) = (11 - 1)/3 = 10/3

10/3 + (-1/3) = (10 - 1)/3 = 9/3 = 3

Therefore, the next three term are 11/3, 10/3 and 3.

3. The n^{th} term of an A.P. is 6n + 2. Find the common difference.

Solution:-

From the question it is given that,
n^{th} term is 6n + 2
So, $T_n = 6n + 2$
Now, we start giving values, 1, 2, 3, … in the place of n, we get,
$T_1 = (6 \times 1) + 2 = 6 + 2 = 8$
$T_2 = (6 \times 2) + 2 = 12 + 2 = 14$
$T_3 = (6 \times 3) + 2 = 18 + 2 = 20$
$T_4 = (6 \times 4) + 2 = 24 + 2 = 26$
Therefore, A.P. is 8, 14, 20, 26, …
So, common difference d = 14 – 8 = 6

4. Show that the list of numbers 9, 12, 15, 18, … form an A.P. Find its 16^{th} term and the n^{th}.
Solution:-
From the question,
The first term a = 9
Then, difference d = 12 – 9 = 3
15 – 12 = 3
18 – 15 = 3
Therefore, common difference d = 3
From the formula, $a_n = a + (n - 1)d$
$T_n = a + (n - 1)d$
$= 9 + (n - 1)3$
$= 9 + 3n - 3$
$= 6 + 3n$
So, $T_{16} = a + (n - 1)d$
$= 9 + (16 - 1)3$
$= 9 + (15)(3)$
$= 9 + 45$
$= 54$

5. Find the 6^{th} term from the end of the A.P. 17, 14, 11, …, – 40.
Solution:-
From the question it is given that,
First term a = 17
Common difference = 14 – 17 = - 3
Last term l = - 40

L = a + (n - 1)d
-40 = 17 + (n - 1)(-3)
-40 – 17 = -3n + 3
- 57 - 3 = -3n
n = -60/-3
n = 20
Therefore, 6^{th} term form the end = l – (n - 1)d
= - 40 – (6 - 1)(-3)
= - 40 – (5)(-3)
= - 40 + 15
= - 25

6. If the 8^{th} term of an A.P. is 31 and the 15^{th} term is 16 more than its 11^{th} term, then find the A.P.
Solution:-
From the question it is given that,
a_8 = 31
a_{15} = the 15^{th} term is 16 more than its 11^{th} term = a_{11} + 16
we know that, a_n = a + (n - 1)d
So, a_8 = a + 7d = 31 … [equation (i)]
a_{15} = a + 14d = a + 10d + 16
14d – 10d = 16
4d = 16
d = 16/4
d = 4
Now substitute the value of d in equation (i) we get,
a + (7 × 4) = 31
a + 28 = 31
a = 31 – 28
a = 3
So, 3 + 4 = 7, 7 + 4 = 11, 11 + 4 = 15
Therefore, A.P. is 3, 7, 11, 15, …

7. The 17^{th} term of an A.P. is 5 more than twice its 8^{th} term. If the 11^{th} term of the A.P. is 43, then find the w^{th} term.
Solution:-
From the question it is given that,

a_{17} = 5 more than twice its 8^{th} term = $2a_8 + 5$
$a_{11} = 43$
$a_n = ?$
We know that, $a_{11} = a + 10d = 43$ … [equation (i)]
$a_{17} = 2a_8 + 5$
$a + 16d = 2(a + 7d) + 5$
$a + 16d = 2a + 14d + 5$
$2a – a = 16d – 14d – 5$
$a = 2d – 5$ … [equation (ii)]
Now substitute the value of a in equation (i) we get,
$2d – 5 + 10d = 43$
$12d = 43 + 5$
$12d = 48$
$d = 48/12$
$d = 4$
To find out the value of a substitute the value of d in equation (i)
$a + (10 \times 4) = 43$
$a + 40 = 43$
$a = 43 – 40$
$a = 3$
Then, $a_n = a + (n - 1)d$
$= 3 + 4(n - 1)$
$= 3 + 4n – 4$
$= 4n – 1$

8. The 19^{th} term of an A.P. is equal to three times its 6^{th} term. If its 9^{th} term is 19, find the A.P.
Solution:-
From the question it is given that,
a_{19} = 19^{th} term of an A.P. is equal to three times its 6^{th} term = $3a_6$
$a_9 = 19$
As we know, $a_n = a + (n - 1)d$
$a_9 = a + 8d = 19$ … [equation (i)]
Then, $a_{19} = 3(a + 5d)$
$a + 18d = 3a + 15d$
$3a – a = 18d – 15d$
$2a = 3d$

$a = (3/2)d$

Now substitute the value of a in equation (i) we get,

$(3/2)d + 8d = 19$

$(3d + 16d)/2 = 19$

$(19/2)d = 19$

$d = (19 \times 2)/19$

$d = 2$

To find out the value of a substitute the value of d in equation (i)

$a + 8d = 19$

$a + (8 \times 2) = 19$

$a + 16 = 19$

$a = 19 – 16$

$a = 3$

Therefore, A.P. is 3, 5, 7, 9, …

9. If the 3rd and the 9th terms of an A.P. are 4 and – 8 respectively, then which term of this A.P. is zero?

Solution:-

From the question it is given that,

$a_3 = 4$

$a_9 = - 8$

We know that, $a_3 = a + 2d = 4$ … [equation (i)]

$a_9 = a + 8d = -8$ … [equation (ii)]

Now, subtracting equation (i) from equation (ii)

$(a + 8d) – (a + 2d) = -8 – 4$

$a + 8d – a – 2d = -12$

$6d = -12$

$d = -12/6$

$d = -2$

To find out the value of a substitute the value of d in equation (i)

$a + 2d = 4$

$a + (2 \times (-2)) = 4$

$a – 4 = 4$

$a = 4 + 4$

$a = 8$

let us assume n^{th} term be zero, then

$a + (n - 1)d = 0$

8 + (n - 1)(-2) = 0
-2n + 2 = -8
-2n = -8 – 2
-2n = -10
n = -10/-2
n = 5
Therefore, 0 will be the fifth term.

10. Which term of the list of numbers 5, 2, – 1, – 4, … is – 55?
Solution:-
From the question it is given that,
First term a = 5
n^{th} term = -55
Common difference d = 2 – 5 = - 3
We know that, $a_n = a + (n - 1)d$
- 55 = 5 + (n - 1)(-3)
-55 – 5 = - 3n + 3
-60 – 3 = -3n
-63 = -3n
n = -63/-3
n = 21
Therefore, -55 is the 21^{st} term.

11. The 24^{th} term of an A.P. is twice its 10^{th} term. Show that its 72^{nd} term is four times its 15^{th} term.
Solution:-
From the question it is given that,
The 24^{th} term of an A.P. is twice its 10^{th} term = $a_{24} = 2a_{10}$
We have to show that, 72^{nd} term is four times its 15^{th} term = $a_{72} = 4a_{15}$
We know that, $a_{24} = a + 23d = 2a_{10}$
a + 23d = 2(a + 9d)
a + 23d = 2a + 18d
2a - a = 23d – 18d
a = 5d … [equation (i)]
$a_{72} = 4a_{15}$
a + 71d = 4(a + 14d)
Substitute the value of a we get,

$5d + 71d = 4(5d + 14d)$
$76d = 4(19d)$
Therefore, it is proved that 72^{nd} term is four times its 15^{th} term.

12. Which term of the list of numbers 20, 19¼, 18½, 17¾, … is the first negative term?
Solution:-
From the question it is given that,
First term a = 20
Common difference $d = 19\frac{1}{4} - 20 = 77/4 - 20 = (77 - 80)/4 = -\frac{3}{4}$
We know that, $a_n = a + (n - 1)d$
$a_n = 20 + (n - 1)(-\frac{3}{4})$
$a_n = 20 - \frac{3}{4}n + \frac{3}{4}$
$a_n = 20 + \frac{3}{4} - \frac{3}{4}n$
$a_n = (80 + 3)/4 - \frac{3}{4}n$
$a_n = 83/4 - \frac{3}{4}n < 0$
$83/4 < \frac{3}{4}n$
$83 < 3n$
$83/3 < n$
$28 < n$
Therefore, 28^{th} is the first negative term.

13. If the p^{th} term of an A.P. is q and the q^{th} term is p, show that its n^{th} term is (p + q – n)
Solution:-
From the question it is given that,
p^{th} term = q
q^{th} term = p
We have to show that, n^{th} term is $(p + q - n)$
We know that, $a_n = a + (n - 1)d$
So, p^{th} term $= a + (p - 1)d = q$ … [equation (i)]
q^{th} term $= a + (q - 1)d = p$ … [equation (ii)]
Now subtracting equation (ii) from equation (i), we get
$q - p = (a + (p - 1)d) - (a + (q - 1)d)$
$q - p = (a + pd - d) - (a + qd - d)$
$q - p = a + pd - d - a - qd + d$
$q - p = pd - qd$
$q - p = d(p - q)$

d = (q – p)/(p - q)
d = -(p - q)/(p - q)
d = - 1
Substitute the value of d in equation (i), we get
a + (p -1)(-1) = q
a – p + 1= q
a = q + p – 1
Then, n^{th} term = a + (n - 1)d
= (p + q - 1) + (n - 1) (- 1)
= (p + q - 1) - n + 1
= p + q – 1 – n + 1
= p + q – n

14. How many three digit numbers are divisible by 9?
Solution:-
The three digits numbers which are divisible by 9 are 108, 117, 126, …, 999
Then, first term a = 108
Common difference = 9
Last term = 999
We know that, l = a_n = a + (n - 1)d
999 = 108 + (n - 1)9
999 – 108 = 9n – 9
891 + 9 = 9n
900 = 9n
n = 900/9
n = 100
Therefore, there are 100 three digits numbers.

15. The sum of three numbers in A.P. is – 3 and the product is 8. Find the numbers.
Solution:-
From the question it is given that,
The sum of three numbers in A.P. = – 3
The product of three numbers in A.P. = 8
Let us assume the 3 numbers which are in A.P. are, a – d, a, a + d
Now adding 3 numbers = a – d + a + a + d = - 3
3a = -3
a = -3/3

$a = -1$

From the question, product of 3 numbers is – 35

So, $(a - d) \times (a) \times (a + d) = 8$

$a(a^2 – d^2) = 8$

$-1 ((-1)^2 – d^2 = 8$

$1 - d^2 = 8/-1$

$1 - d^2 = -8$

$d^2 = 8 + 1$

$d^2= 9$

$d = \sqrt{9}$

$d = \pm 3$

Therefore, the numbers are if $d = 3$ $(a - d) = - 1 – 3 = - 4$

$a = -1$

$(a + d) = - 1 + 3 = 2$

If $d = - 6$

The numbers are $(a - d) = - 1 – (-3) = - 1 + 3 = 2$

$a = -1$

$(a + d) = -1 + (-3) = -1 – 3 = -4$

Therefore, the numbers -4, -1, 2,… and 2, -1, -4,… are in A.P.

16. The angles of a quadrilateral are in A.P. If the greatest angle is double of the smallest angle, find all the four angles.

Solution:-

From the question it is given that,

The angles of a quadrilateral are in A.P.

Greatest angle is double of the smallest angle

Let us assume the greatest angle of the quadrilateral is $a + 3d$,

Then, the other angles are $a + d$, $a – d$, $a – 3d$

So, $a – 3d$ is the smallest

Therefore, $a + 3d = 2(a – 3d)$

$a + 3d = 2a – 6d$

$6d + 3d = 2a – a$

$9d = a$ … [equation (i)]

We know that the sum of all angles of quadrilateral is 360^o.

$a – 3d + a – d + a + d + a+ 3d = 360^o$

$4a = 360^o$

$a = 360/4$

$a = 90^o$
Now, substitute the value of a in equation (i) we get,
$9d = 90$
$d = 90/9$
$d = 10$
Substitute the value of a and d in assumed angles,
Greatest angle = $a + 3d = 90 + (3 \times 10) = 90 + 30 = 120^o$
Then, other angles are = $a + d = 90^o + 10^o = 100^o$
$a - d = 90^o - 10^o = 80^o$
$a - 3d = 90^o - (3 \times 10) = 90 - 30 = 60^o$
Therefore, the angles of quadrilateral are 120^o, 100^o, 80^o and 60^o.

17. The n^{th} term of an A.P. cannot be $n^2 + n + 1$. Justify your answer.
Solution:-
From the question it is given that,
The n^{th} term of an A.P. cannot be $n^2 + n + 1$.
Now, we start giving values, 1, 2, 3, … in the place of n, we get,
$a_1 = 1^2 + 1 + 1 = 1 + 2 = 3$
$a_2 = 2^2 + 2 + 1 = 4 + 3 = 7$
$a_3 = 3^2 + 3 + 1 = 9 + 4 = 13$
$a_4 = 4^2 + 4 + 1 = 16 + 5 = 21$
Then, difference $d = a_2 - a_1 = 7 - 3 = 4$
$d = a_3 - a_2 = 13 - 7 = 6$
$d = a_4 - a_3 = 21 - 13 = 8$
Therefore, common difference d is not same in the numbers.
Hence, the numbers are not form A.P.
So, $a_n \neq n^2 + n + 1$

18. Find the sum of first 20 terms of an A.P. whose n^{th} term is 15 – 4n.
Solution:-
From the question it is given that,
n^{th} term is $15 - 4n$
So, $a_n = 15 - 4n$
Now, we start giving values, 1, 2, 3, … in the place of n, we get,
$a_1 = 15 - (4 \times 1) = 15 - 4 = 11$
$a_2 = 15 - (4 \times 2) = 15 - 8 = 7$
$a_3 = 15 - (4 \times 3) = 15 - 12 = 3$

$a_4 = 15 – (4 × 4) = 15 – 16 = - 1$
Then, $a_{20} = 15 – (4 × 20) = 15 – 80 = -65$
So, 11, 7, 3, -1, … -65 are in A.P.
Therefore, first term a = 11
Common difference = -4
n = 20
$S_{20} = (n/2) [2a + (n - 1)d]$
$= (20/2) [(2 × 11) + (20 - 1)(-4)]$
$= 10 [22 – (19) (-4)]$
$= 10 [22 – 76]$
$= 10(-54)$
$= -540$
Therefore, the sum of first 20 terms of an A.P. is -540.

19. Find the sum : 18 + 15½ + 13 + … + (-49½)
Solution:-
From the question it is given that,
First term a = 18
Common difference d = 15½ - 18
$= 31/2 – 18$
$= (31 – 36)/2$
$= -5/2$
Last term = -49½ = -99/2
We know that, $a_n = a + (n - 1)d$
$-99/2 = 18 + (n - 1)(-5/2)$
$(-99/2) – (18/1) = (n - 1)(-5/2)$
$(-99 - 36)/2 = (-5/2)(n - 1)$
$(-135/2) = (-5/2) (n - 1)$
$(-135/2) × (-2/5) = n – 1$
$-135/-5 = n – 1$
$27 = n – 1$
$n = 27 + 1$
$n = 28$
Then, $S_n = (n/2) [2a + (n - 1)d]$
$S_{28} = (28/2) [(2 × 18) + (28 - 1)(-5/2)]$
$S_{28} = 14[36 + (27 × (-5/2))]$
$S_{28} = 14[36 – (135/2)]$

S_{28} = 14 [(72 - 135)/2]
S_{28} = 14 (-63/2)
S_{28} = - 441

20.
(i) How many terms of the A.P. – 6, (-11/2), – 5, … make the sum – 25?
Solution:-
From the question it is given that,
Terms of the A.P. is -6, (-11/2) – 5, …
The first term a = -6
Common difference d = (-11/2) – (-6)
= (-11/2) + 6
= (-11 + 12)/2
= ½
The terms are make the sum – 25
Then, S_n = (n/2)(2a + (n – 1)d)
-25 = (n/2) [(2 × (-6)) + (n - 1) (½)]
(-25 × 2) = n [-12 + ½n – ½]
-50 = n [(-25/2) + (½n)]
½n^2 – (25/2)n + 50 = 0
n^2 – 25n + 100 = 0
n^2 – 5n – 20n + 100 = 0
n(n - 5) – 20(n - 5) = 0
(n - 5) (n - 20) = 0
So, n – 5 = 0
n = 5
or n – 20 = 0
n = 20
Therefore, number of terms are 5 or 20.

(ii) Solve the equation 2 + 5 + 8 + … + x = 155
Solution:-
From the question it is given that,
First term a = 2
Last term = x
Common difference d = 5 – 2 = 3
Then, sum of the terms = 155

$L = a + (n - 1)d$
$x = 2 + (n - 1)3$
$x = 2 + 3n – 3$
$x = 3n – 1$ … [equation (i)]
We know that, $S_n = (n/2)\ [2a + (n - 1)d]$
$155 = (n/2)\ [(2 \times 2) + (n - 1) \times 3]$
$155 \times 2 = n[4 + 3n - 3]$
$310 = n(3n + 1)$
$310 = 3n^2 + n$
$3n^2 + n – 310 = 0$
$3n^2 – 30n + 31n – 310 = 0$
$3n(n - 10) + 31(n - 10) = 0$
$(n - 10)\ (3n + 31) = 0$
So, $n – 10 = 0$
$n = 10$
or $3n + 31 = 0$
$n = -31/3$
negative is not possible.
Therefore, $n = 10$
Now, substitute the value of n in equation (i),
$x = 3n – 1$
$= (3 \times 10) – 1$
$= 30 – 1$
$= 29$

21. If the third term of an A.P. is 5 and the ratio of its 6^{th} term to the 10^{th} term is 7 : 13, then find the sum of first 20 terms of this A.P.
Solution:-
From the question it is given that,
The third term of an A.P. $a_3 = 5$
The ratio of its 6^{th} term to the 10^{th} term $a_6 : a_{10} = 7 : 13$
We know that, $a_n = a + (n - 1)d$
$a_3 = a + (3 - 1)d = 5$
$= a + 2d = 5$ … [equation (i)]
Then, $a_6/a_{10} = 7/13$
$(a + 5d)/(a + 9d) = 7/13$
By cross multiplication we get,

13(a + 5d) = 7(a + 9d)
13a + 75d = 7a + 63d
13a – 7a + 65d – 63d = 0
6a + 2d = 0
Divide by 2 on both side we get,
3a + d = 0
d = -3a … [equation (ii)]
Substitute the value of d in equation (i),
a + 2(-3a) = 5
a – 6a = 5
-5a = 5
a = -5/5
a = -1
Now substitute the value of a in equation (ii),
d = -3(-1)
d = 3
Then, sum of first 20 terms,
= (n/2) [2a + (n - 1)d]
= (20/2)[(2 × (-3)) + (2- - 1)3]
= 10[-2 + 57]
= 10 × 55
= 550

22. In an A.P., the first term is 2 and the last term is 29. If the sum of the terms is 155, then find the common difference of the A.P.
Solution:-
From the question it is given that,
First term a = 2
Last term = 29
The sum of terms = 155
We know that, last term = a_n = a + (n - 1)d
29 = 2 + (n - 1)d
29 – 2 = d(n - 1)
27 = d(n - 1) … (i)
Then, S_n = (n/2)[2a + (n - 1)d]
155 = (n/2)[(2 ×2) + 27]
155 = (n/2)[4 + 27]

$155 = (31/2)n$
$n = (155 \times 2)/31$
$n = 10$
$d(n - 1) = 27$
$d(10 - 1) = 27$
$d(9) = 27$
$d = 27/9$
$d = 3$

23. The sum of first 14 terms of an A.P. is 1505 and its first term is 10. Find its 25th term.

Solution:-

From the question it is given that,
First term a = 10
The sum of first 14 terms of an A.P. = 1505
25th term = ?
We know that, $S_n = (n/2)\ [2a + (n - 1)d]$
$S_{14} = (n/2)\ [2a + (n - 1)d]$
$1505 = (14/2)\ [(2 \times 10) + (14 - 1)d]$
$1505 = 7[20 + 13d]$
$1505/7 = 20 + 13d$
$215 = 20 + 13d$
$13d = 215 – 20$
$13d = 195$
$d = 195/13$
$d = 15$
Then, $a_n = a + (n - 1)d$
$a_{25} = 10 + (25 - 1)(15)$
$= 10 + (24)15$
$= 10 + 360$
$= 370$

24. The sum of first n term of an A.P. is 3n² + 4n. Find the 25th term of this A.P.

Solution:-

From the question it is given that,
The sum of first n term of an A.P. is $3n^2 + 4n$
$S_n = 3n^2 + 4n$

So, $S_{n-1} = 3(n - 1)^2 + 4(n - 1)$
$= 3(n^2 – 2n + 1) + 4(n - 1)$
$= 3n^2 – 6n + 3 + 4n – 4$
$= 3n^2 – 2n – 1$
Now, $a_n = S_n – S_{n-1}$
$= (3n^2 + 4n) – (3n^2 – 2n - 1)$
$= 3n^2 + 4n – 3n^2 + 2n + 1$
$= 6n + 1$
Therefore, $a_{25} = 6(25) + 1$
$= 150 + 1$
$= 151$

25. In an A.P., the sum of first 10 terms is – 150 and the sum of next 10 terms is – 550. Find the A.P.
Solution:-
From the question it is given that,
The sum of first 10 terms = - 150
The sum of next 10 terms = - 550
A.P = ?
We know that, $S_n = (n/2) [2a + (n - 1)d]$
$S_{10} = (n/2) [2a + (10 - 1)d]$
$-150 = (10/2) [2a + 9d]$
$-150 = 5[2a + 9d]$
$- 150 = 10a + 45d$ … [equation (i)]
Then, $S_{20} = S_{10} + S_{10}$
$= - 150 – 550$
$= -700$
$S_{20} = (20/2) [2a + 19d]$
$-700 = 10(2a + 19d)$
$-700 = 20a + 190d$ … [equation (ii)]
Now, multiplying equation (i) by 2 we get,
$20a + 90d = - 300$ … [equation (iii)]
Subtract equation (iii) from equation (ii),
$(20a + 190d) – (20a + 90d) = -700 – (-300)$
$20a + 190d – 20a – 90d = -700 + 300$
$100d = -400$
$d = -400/100$

d = -4
Substitute the value of d in equation (i) we get,
10a + 45(-4) = -150
10a – 180 = - 150
10a = -150 + 180
10a = 30
a = 30/10
a = 3
$a_2 = 3 + (-4) = 3 – 4 = -1$
$a_3 = -1 + (-4) = -1 – 4 = -5$
$a_3 = -5 + (-4) = -5 – 4 = -9$
Therefore, A.P. is 3, -1, -5, -9, …

26. The sum of first m terms of an A.P. is $4m^2 – m$. If its n^{th} term is 107, find the value of n. Also find the 21^{st} term of this A.P.
Solution:-
From the question it is given that,
$a_n = 107$
The sum of first m terms of an A.P. $S_m = 4m^2 – m$
n^{th} term S_n is $= 4n^2 – n$
Then, $S_{n-1} = 4(n - 1)^2 – (n - 1)$
$= 4(n^2 – 2n + 1) – n + 1$
$= 4n^2 – 8n + 4 – n + 1$
$= 4n^2 – 9n + 5$
Therefore, $a_n = S_n – S_{n-1}$
$107 = 4n^2 – n – 4n^2 + 9n – 5$
$107 = 8n – 5$
$107 + 5 = 8n$
$112 = 8n$
$n = 112/8$
$n = 14$
$a_n = 8n – 5$
$a_{21} = (8 \times 14) – 5$
$a_{21} = 168 – 5$
$a_{21} = 163$

27. Find the geometric progression whose 4^{th} term is 54 and 7^{th} term is 1458.

Solution:-
From the question it is given that,
The geometric progression whose 4th term $a_4 = 54$
The geometric progression whose 7th term $a_7 = 1458$
We know that, $a_n = ar^{n-1}$
$a_4 = ar^{4-1}$
$a_4 = ar^3 = 54$
$a_7 = ar^6 = 1458$
By dividing both we get,
$ar^6/ar^3 = 1458/54$
$r^{6-3} = 27$
$r^3 = 3^3$
$r = 3$
To find out a, consider $ar^3 = 54$
$a(3)^3 = 54$
$a = 54/27$
$a = 2$
Therefore, $a = 2$, $r = 3$
So, G.P. is 2, 6, 18, 54,…

28. The fourth term of a G.P. is the square of its second term and the first term is – 3. Find its 7th term.
Solution:-
From the question it is given that,
The fourth term of a G.P. is the square of its second term = $a_4 = (a_2)^2$
The first term $a_1 = -3$
We know that, $a_n = ar^{n-1}$
$a_4 = ar^{4-1}$
$a_4 = ar^3$
$a_2 = ar$
Now, $ar^3 = (ar)^2$
$ar^3 = a^2r^2$
$r^3/r^2 = a^2/a$
$r^{3-2} = a^{2-1}$ … [from $a^m/a^n = a^{m-n}$]
$r = a$
$a_1 = -3$
$a_7 = ar^{7-1}$

$a_7 = ar^6$

$= -3 \times (-3)^6$

$= -3 \times 729$

$= -2187$

Therefore, the 7th term $a_7 = -2187$

29. If the 4th, 10th and 16th terms of a G.P. are x, y and z respectively, prove that x, y and z are in G.P.

From the question it is given that,

$a_4 = x$

$a_{10} = y$

$a_{16} = z$

Now, we have to show that x, y and z are in G.P.

We know that,

$a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$a_4 = ar^3 = x$

$a_{10} = ar^9 = y$

$a_{16} = ar^{15} = z$

x, y, z are in G.P.

If $y^2 = xy$

Substitute the value of x and y,

$y^2 = (ar^9)^2$

$y^2 = a^2r^{18}$

Then, $xz = ar^3 \times ar^{15}$

$= a^{1+1} r^{3+15}$ … [from $a^m \times a^n = a^{m+n}$]

$= a^2r^{18}$

So, $y^2 = xy$

Therefore, it is proved that x, y, z are in G.P.

30. How many terms of the G.P. 3, 3/2, ¾ are needed to give the sum 3069/512?

Solution:-

From the question it is given that,

Sum of the terms $S_n = 3069/512$

First term a = 3

Common ratio r = (3/2)/3

= (3/2) × (1/3)

$= ½$

We know that, $S_n = a(1 – r^n)/(1 - r)$

$(3069/512) = 3[1 – (½)^n]/ (1 – ½)$

$(3069/512) = (2 \times 3) [1 – (½)^n]$

$1 – (½)^n = 3069/(512 \times 6)$

$1 – (½)^n = 1023/1024$

$(½)^n = 1 – (1023/1024)$

$(½)^n = (1024 - 1023)/1024$

$(½)^n = 1/1024$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

$(½)^n = (½)^{10}$

By comparing both LHS and RHS,

$n = 10$

Therefore, there are 10 terms are in the G.P.

31. Find the sum of first n terms of the series: 3 + 33 + 333 + …

Solution:-

Consider the given numbers 3 + 33 + 333 + … n terms

Take out 3 as common we get,

$= 3 (1 + 11 + 111 + … \text{n terms})$

$= 3/9 (9 + 99 + 999 + … \text{n terms})$

$= 3/9 ((10 - 1) + (100 - 1) + (1000 - 1) + … \text{n terms})$

$= 3/9 (10 + 100 + 100 + … \text{n terms} – (1 + 1 + 1 + …. \text{n terms}))$

We know that, $S_n = a(r^n - 1)/(r - 1)$

First term $a = 10$

Common ratio $r = 10$

$= 3/9 [(10(10^n - 1)/(10 - 1)) - n]$

$= 3/9 [(((10 \times 10^n) - 10)/9) - n]$

$= 3/81 [10^{n + 1} – 10 – 9n]$

$= 1/27\ [10^{n+1} - 9n - 10]$

32. Find the sum of the series 7 + 7.7 + 7.77 + 7.777 + … to 50 terms.

Solution:-

Consider the given numbers 7 + 7.7 + 7.77 + 7.777 + … to 50 terms

Take out 7 as common we get,

= 7(1 + 1.1 + 1.11 + 1.111 + … 50 terms)

= 7/9 (9 + 9.9 + 9.99 + 9.999 + … 50 terms)

= 7/9 ((10 - 1) + (10 – 0.1) + (10 – 0.01) + (10 – 0.001) + … 50 terms)

= 7/9 (10 + 10 + 10 + 10 + … n terms – (0.1 + 0.01 + 0.001 + …. 50 terms))

We know that, $S_n = a(1 - r^n)/(1 - r)$

$= 7/9\ [500 - (1(1 - (0.1)^{50})/(1 - 0.1))]$

$= 7/9\ [500 - ((10/9)\ (1 - (1/10^{50})))]$

$= 7/81\ [4500 - 10 + 10^{-49}]$

$= 7/81[4490 + 10^{-49}]$