Exercise 10

1. Find the co-ordinates of the images of the following points under reflection in the x- axis:
(i) (2, -5)
(ii) (-3/2, -1/2)
(iii) (-7, 0)
Solution:

The co-ordinates of the images of the points under reflection in the x-axis will be:
(i) Image of (2, -5) will be (2, 5)
(ii) Image of (-3/2, -1/2) will be (-3/2, 1/2)
(iii) Image of (-7, 0) will be (-7, 0)

2. Find the co-ordinates of the images of the following points under reflection in the y-axis:
(i) (2, -5)
(ii) (-3/2, 1/2)
(iii) (0, -7)
Solution:

The co-ordinates of the image of the points under reflection in the y-axis will be:
(i) Image of (2, -5) will be (-2, -5)
(ii) Image of (-3/2, 1/2) will be (3/2, 1/2)
(iii) Image of (0, -7) will be (0, -7)

3. Find the co-ordinates of the images of the following points under reflection in the origin:
(i) (2, -5)
(ii) (-3/2, -1/2)
(iii) (0, 0)
Solution:

The co-ordinate of the image of the points under reflection in the y-axis will be:
(i) Image of (2, -5) will be (-2, 5)
(ii) Image of (-3/2, -1/2) will be (3/2, 1/2)
(iii) Image of (0, 0) will be (0, 0)

4. The image of a point P under reflection in the x-axis is (5, -2). Write down the coordinates of P.
Solution:

Given that (5, -2) are the co-ordinates of the image of a point P under x-axis
Thus, the co-ordinates of P will be (5, 2).

5. A point P is reflected in the x-axis. Co-ordinates of its image are (8, -6).
(i) Find the co-ordinates of P.
(ii) Find the co-ordinates of the image of P under reflection in the y-axis.
Solution:

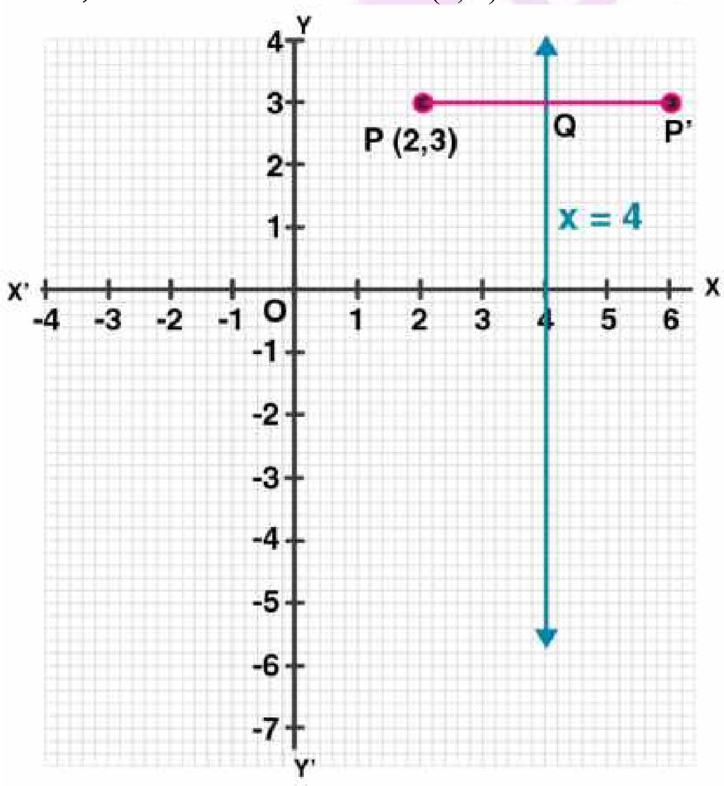
(i) The co-ordinates of image of P which is reflected in x-axis are (8, -6)
(ii) The co-ordinates of image of P under reflection in the y-axis will be (-8, 6)

6. A point P is reflected in the origin. Co-ordinates of its image are (2, -5). Find
(i) the co-ordinates of P.
(ii) the co-ordinates of the image of P in the x-axis.
Solution:

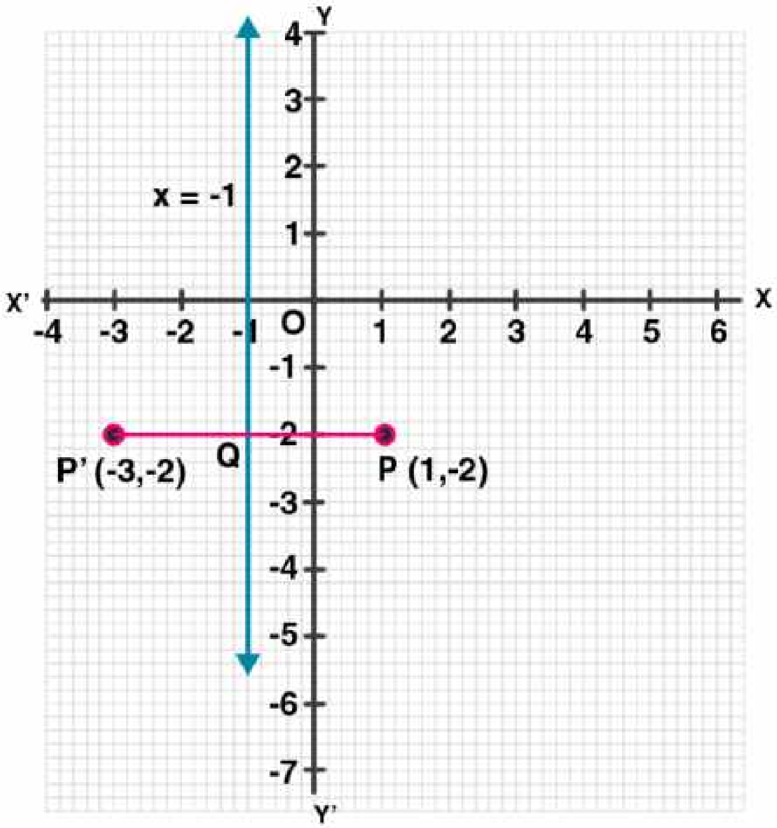
The co-ordinates of image of a point P which is reflected in origin are (2, -5), then
(i) Co-ordinates of P will be (-2, 5)
(ii) Co-ordinates of the image of P in the x-axis will be (-2, -5)

7. (i) The point P (2, 3) is reflected in the line x = 4 to the point P'. Find the co-ordinates of the point P'.
(ii) Find the image of the point P (1, -2) in the line x = -1.
Solution:

(i) The steps are:
(a) Draw axis XOX' and YOY' and take 1 cm = 1 unit
(b) Plot point P (2, 3) on it.
(c) Draw a line x = 4 which is parallel to y-axis.
(d) From P, draw a perpendicular on x = 4, which intersects x = 4 at Q.
(e) Produce PQ to P', such that QP' = QP.
Thus, P' is the reflection of P in the line x = 4
Hence, the co-ordinates of P' are (6, 3).

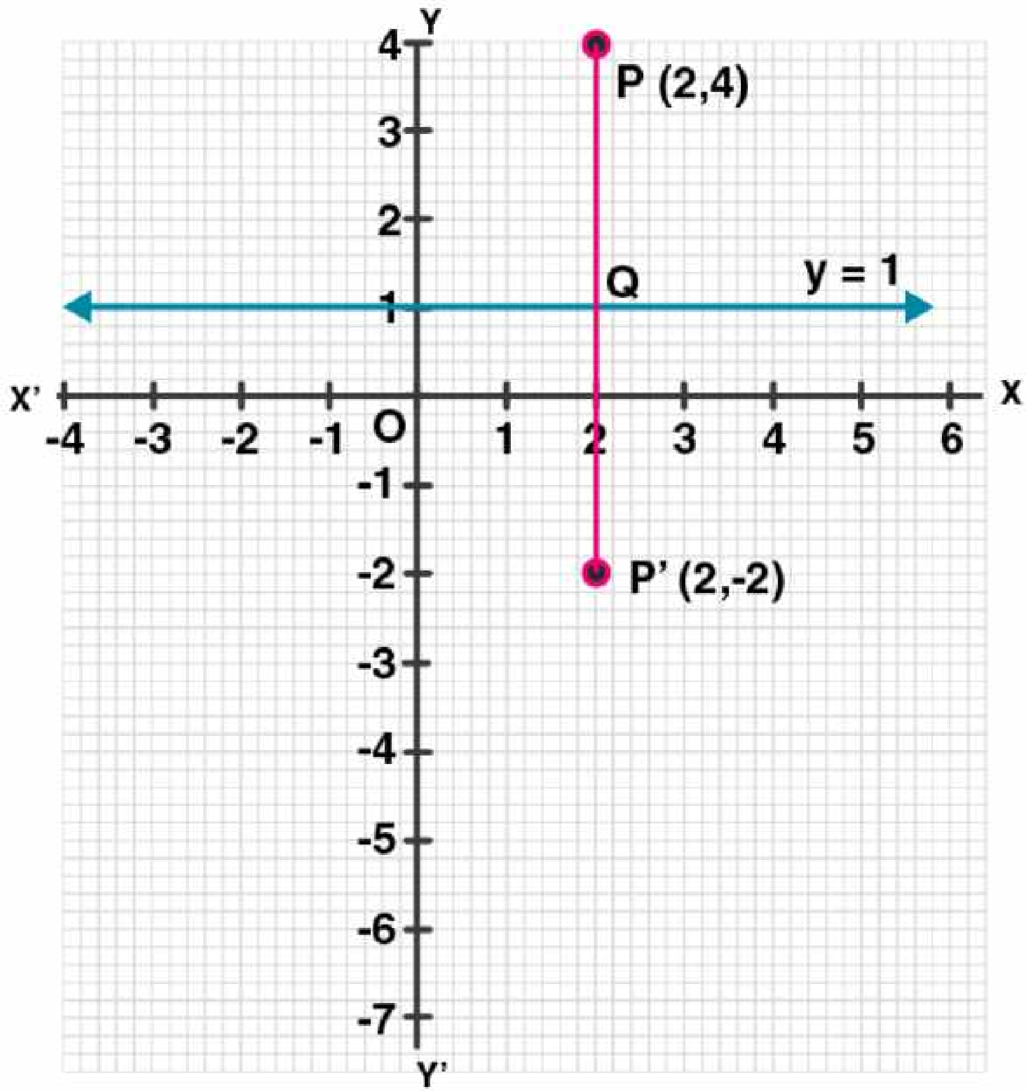


(ii) The steps are:
(a) Draw axis XOX’ and YOY’ and take 1 cm = 1 unit
(b) Plot point P (2, 3) on it.
(c) Draw a line x = 4 which is parallel to y-axis.
(d) From P, draw a perpendicular on x = 4, which intersects x = 4 at Q.
(e) Produce PQ to P’, such that QP’ = QP.
Thus, P’ is the reflection of P in the line x = 4
Hence, the co-ordinates of P’ are (6, 3)

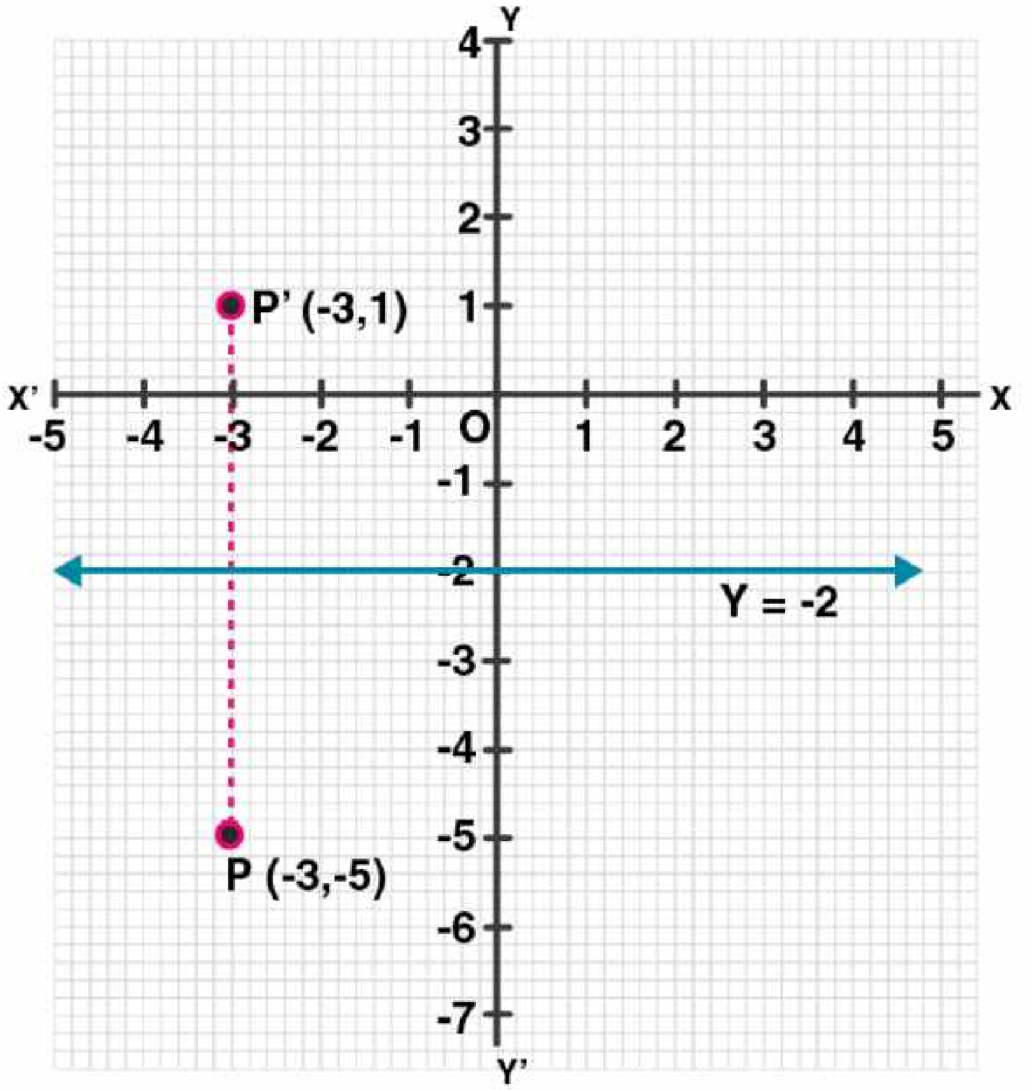


8. (i) The point P (2, 4) on reflection in the line y = 1 is mapped onto P’ Find the co-ordinates of P’.
(ii) Find the image of the point P (-3, -5) in the line y = -2.
Solution:

(i) The steps are:
(a) Draw axis XOX’ and YOY’ and take 1 cm = 1 unit.
(b) Plot point P (2, 4) on it.
(c) Draw a line y = 1, which is parallel to x-axis.
(d) From P, draw a perpendicular on y = 1 meeting it at Q.
(e) Produce PQ to P’ such that QP’ = PQ.
Therefore, P’ is the reflection of P whose co-ordinates are (2, -2).



(ii) The steps are:
(a) Draw axis XOX' and YOY' and take 1 cm = 1 unit.
(b) Plot point P (-3, -5) on it.
(c) Draw a line y = -2 which is parallel to the x-axis.
(d) From P, draw a perpendicular on y = -2 which meets it at Q.
(e) Produce PQ to P' such that QP' = PQ.
Therefore, P' is the image of P, whose co-ordinates are (-3, 1).



9. The point P (-4, -5) on reflection in y-axis is mapped on P'. The point P' on reflection in the origin is mapped on P". Find the co-ordinates of P' and P". Write down a single transformation that maps P onto P".
Solution:

Given, point P (-4, -5)
And, P' is the image of point P in y-axis
Thus, the co-ordinates of P' will be (4, -5)
Again,
P" is the image of P' under reflection in origin.
Thus, the co-ordinates of P'' will be (-4, 5).
The single transformation that maps P onto P" is the x-axis.

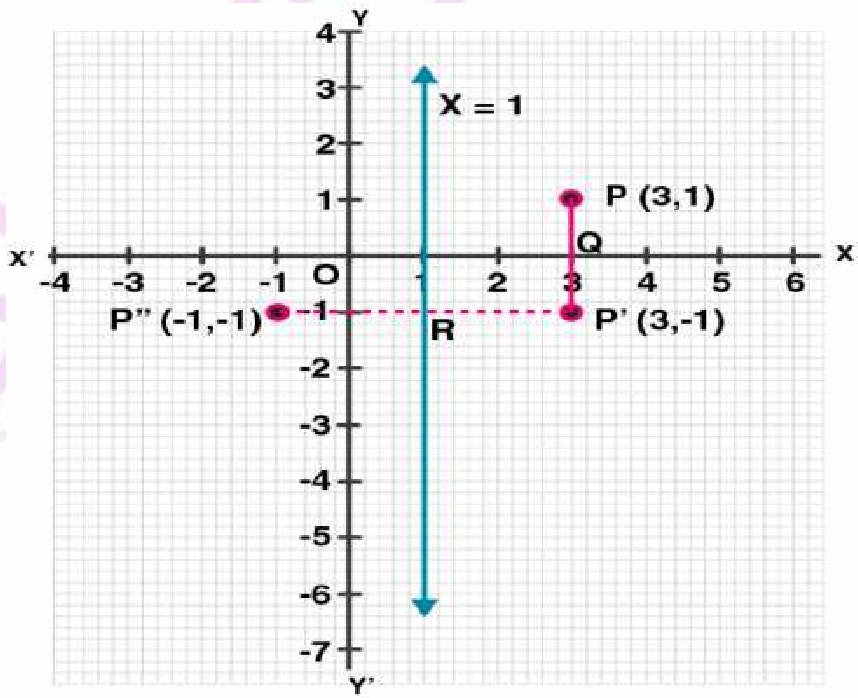
10. Write down the co-ordinates of the image of the point (3, -2) when:
(i) reflected in the x-axis

(ii) reflected in the y-axis
(iii) reflected in the x-axis followed by a reflection in the y-axis
(iv) reflected in the origin.
Solution:

The co-ordinates of the given point are (3, -2).
Now,
(i) Co-ordinates of the image reflected in x- axis will be (3, 2)
(ii) Co-ordinates of the image reflected in y- axis will be (-3, -2)
(iii) Co-ordinates of the point reflected in x- axis followed by reflection in the y-axis will be (-3, 2)
(iv) Co-ordinates of the point reflected in the origin will be (-3, 2).

11. Find the co-ordinates of the image of (3, 1) under reflection in x-axis followed by a reflection in the line x = 1.
Solution:

The steps are:
(i) Draw axis XOX' and YOY' taking 1 cm = 1 unit.
(ii) Plot a point P (3, 1).
(iii) Draw a line x = 1, which is parallel to y-axis.
(iv) From P, draw a perpendicular on x-axis meeting it at Q.
(v) Produce PQ to P' such that QP' = PQ, then
P' is the image of P is x-axis. Then co-ordinates of P' will be (3, -1)
(vi) From P', draw a perpendicular on x = 1 meeting it at R.
(vii) Produce P'R to P" such that RP" = P'R
Thus, P" is the image of P' in the line x = 1

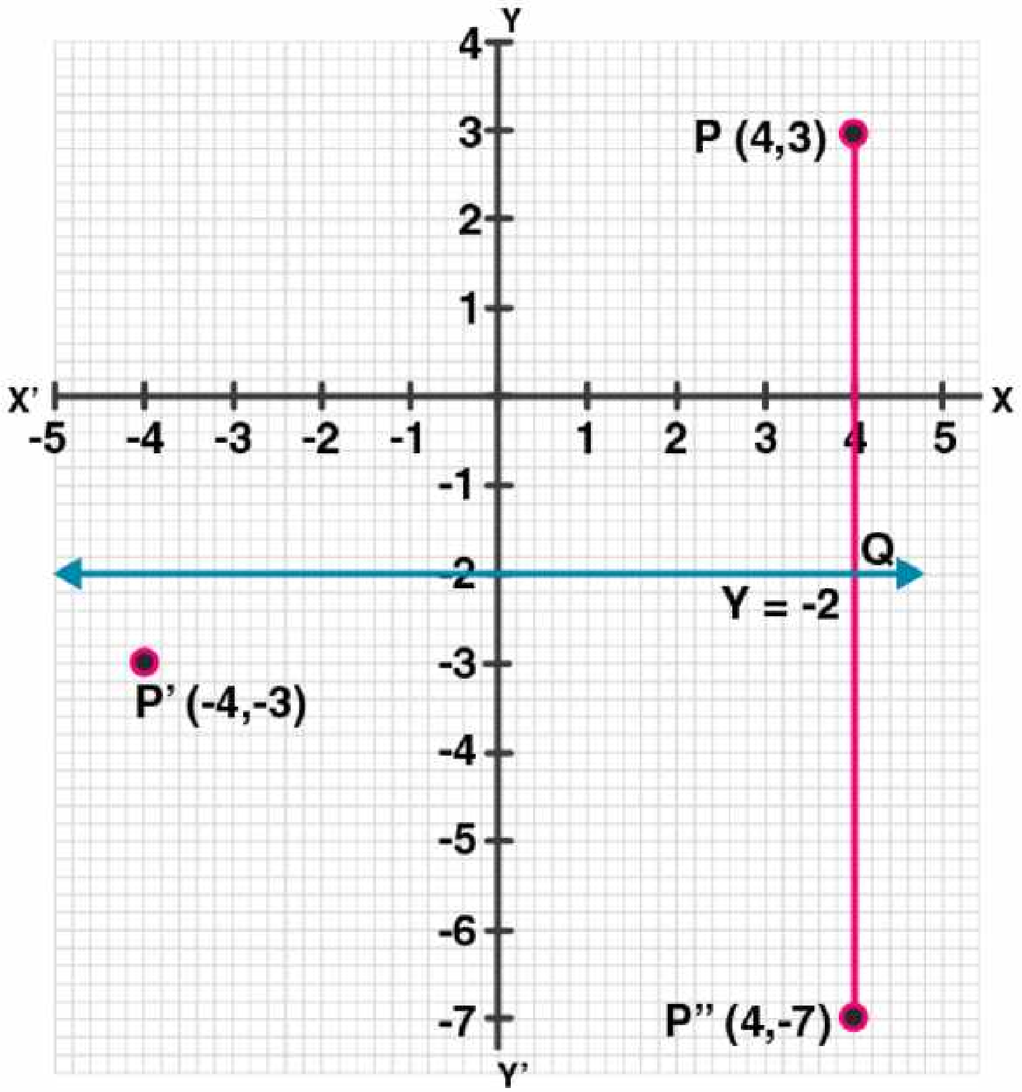


Hence, the co-ordinates of P" are (-1, -1)

12. If P' (-4, -3) is the image of a point P under reflection in the origin, find
(i) the co-ordinates of P.
(ii) the co-ordinates of the image of P under reflection in the line y = -2.
Solution:

(i) Given, reflection of P is P' (-4, -3) in the origin
Thus, the co-ordinates of P will be (4, 3)
Now,
Draw a line y = -2, which is parallel to x-axis
(ii) From P, draw a perpendicular on y = -2 meetings it at Q
Produce PQ to P" such that QP" = PQ

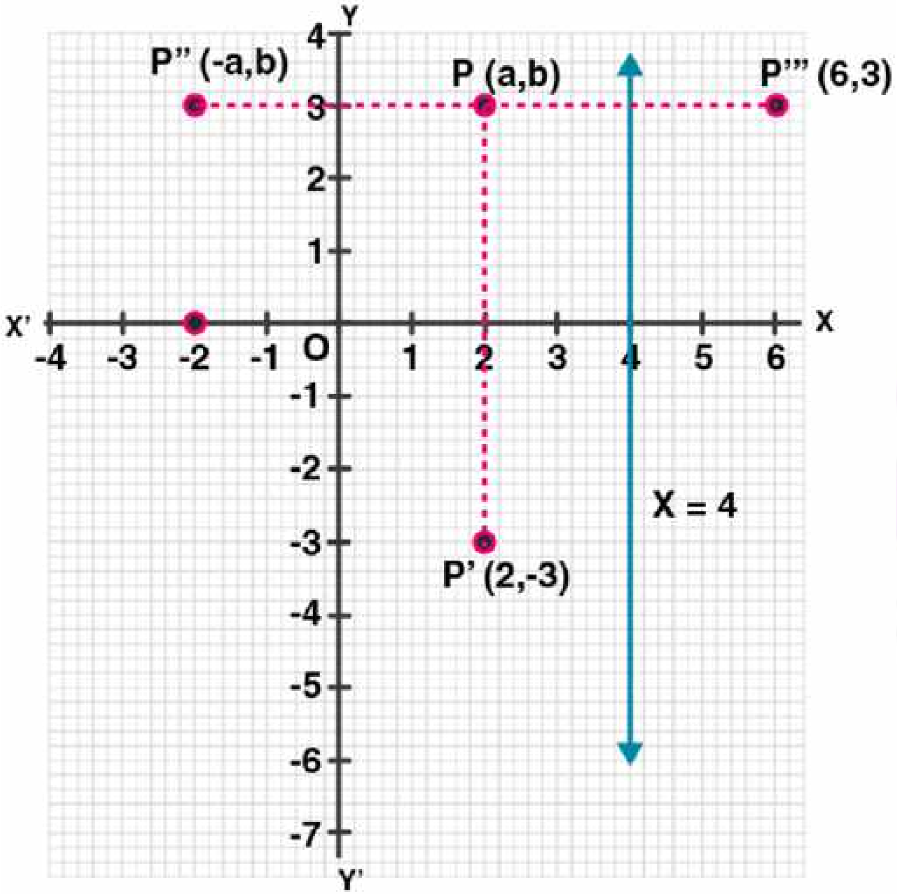
Thus, P" will the image of P in the line y = -2
Hence, the co-ordinates of P" will be (4, -7).



13. A point P (a, b) is reflected in the x-axis to P' (2, -3), write down the values of a and b. P" is the image of P, when reflected in the y-axis. Write down the co-ordinates of P". Find the co-ordinates of P", when P is reflected in the line parallel to y-axis such that x = 4.
Solution:

Given,
P' (2, -3) is the reflection of P (a, b) in the x-axis
Hence, the co-ordinates of P' will be (a, – b) but P' is (2, -3)
On comparing, we get a = 2, b = 3
Thus, the co-ordinates of P will be (2, 3)
And,
P" is the image of P when reflected in y-axis
Hence, the co-ordinate of P" will be (– 2, 3)

Now, draw a line x = 4, which is parallel to y-axis
As P''' is the image of P when it is reflected in the line x = 4,
So, P''' is its reflection.
Thus, the co-ordinates of P''' will be (6, 3).

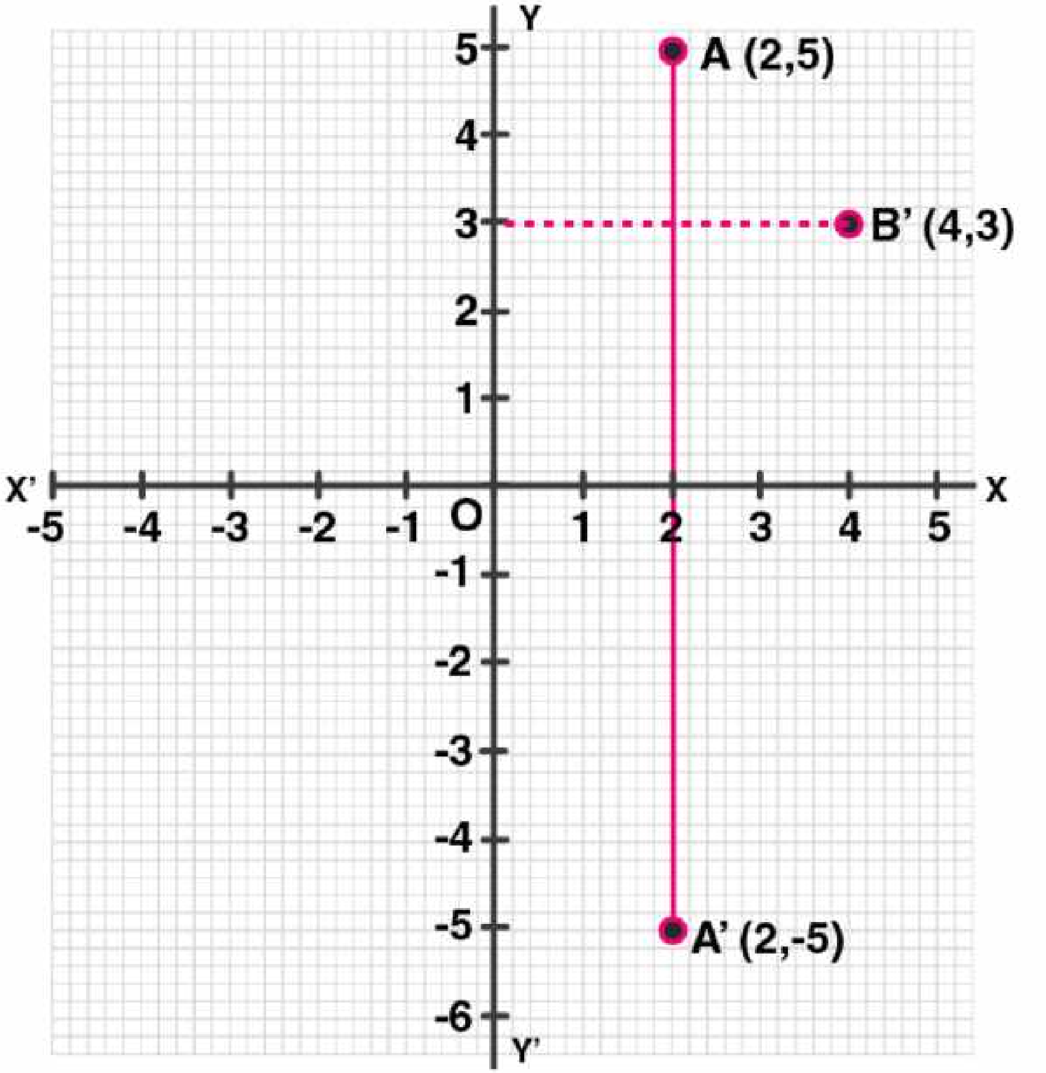


14. (i) Point P (a, b) is reflected in the x-axis to P' (5, -2). Write down the values of a and b.
(ii) P'' is the image of P when reflected in the y-axis. Write down the co-ordinates of P''.
(iii) Name a single transformation that maps P' to P''.
Solution:

(i) Image of P (a, b) reflected in the x-axis to P' (5, -2)
So, the co-ordinates of P will be (5, 2)
Hence, a = 5 and b = 2
(ii) P'' is the image of P when reflected in the y-axis
Thus, its co-ordinates will be (-5, -2).
(iii) The single transformation that maps P' to P'' is the origin.

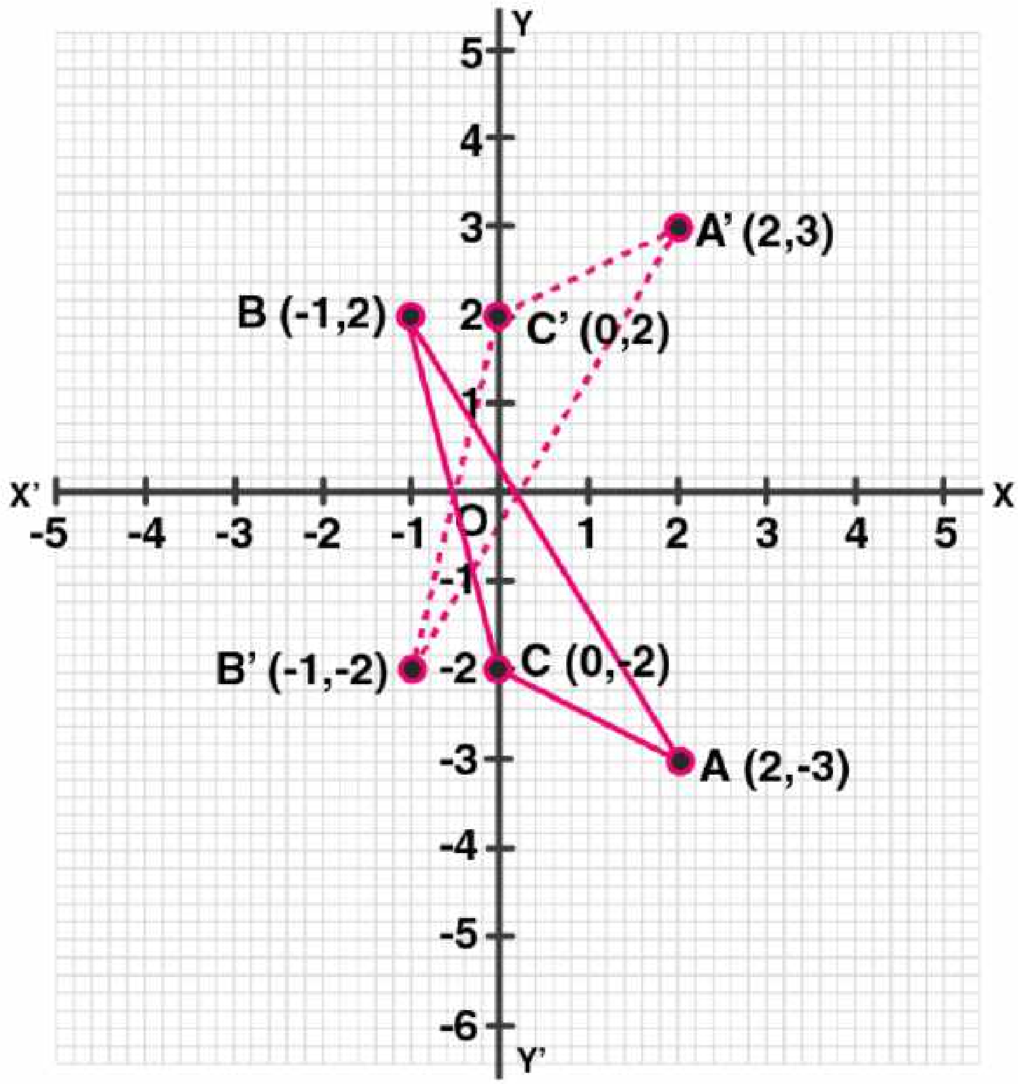
15. Points A and B have co-ordinates (2, 5) and (0, 3). Find
(i) the image A' of A under reflection in the x-axis.
(ii) the image B' of B under reflection in the line AA'.
Solution:

Given, co-ordinates of A are (2, 5) and of B are (0, 3)
(i) Co-ordinates of A’, the image of A reflected in the x-axis will be (2, -5)
(ii) Co-ordinates of B’, the image of B under reflection in the line AA’ will be (4, 3).



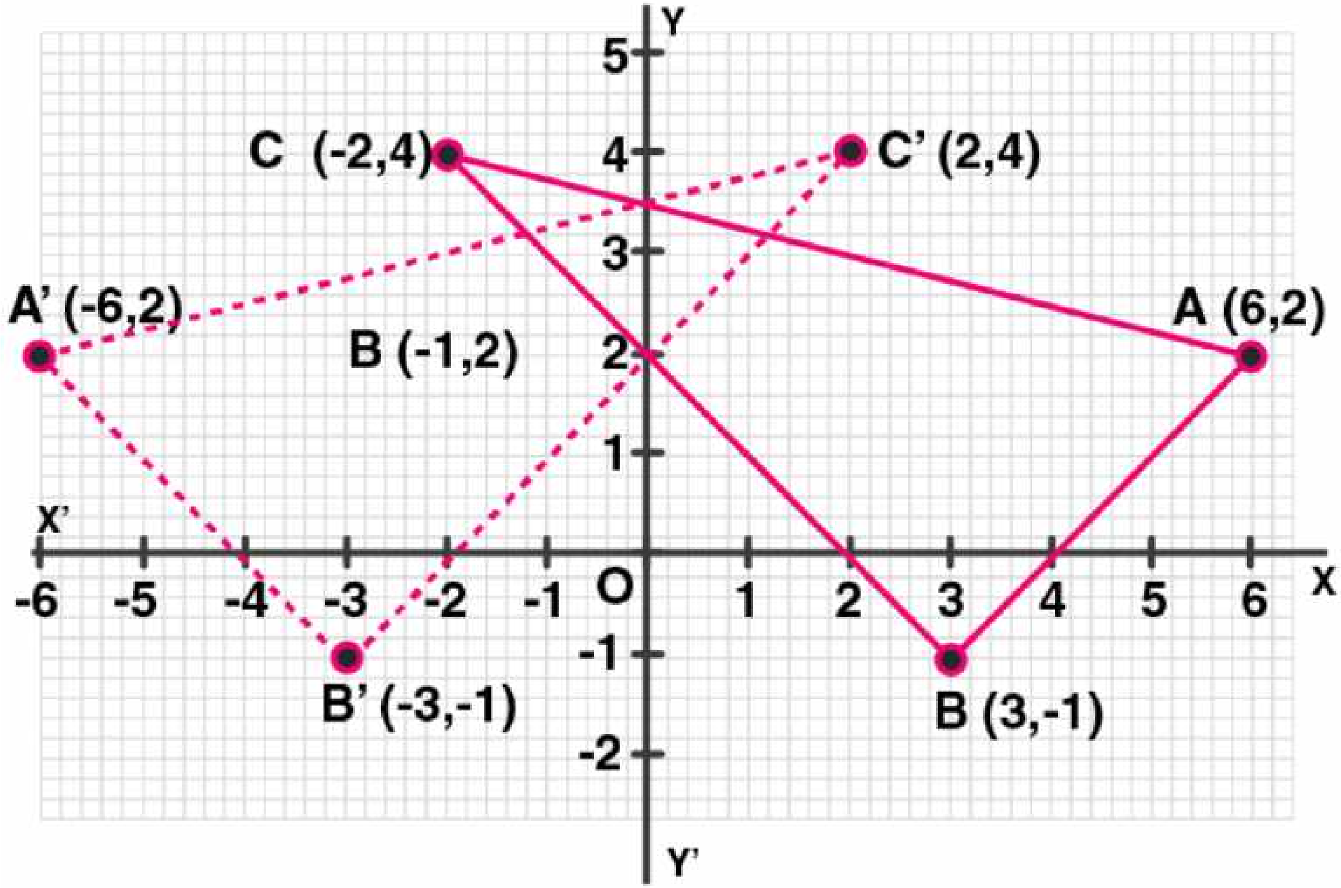
16. Plot the points A (2, -3), B (-1, 2) and C (0, -2) on the graph paper. Draw the triangle formed by reflecting these points in the x-axis. Are the two triangles congruent?
Solution:

The points A (2, -3), B (-1, 2) and C(0, -2) has been plotted on the graph paper as shown and are joined to form a triangle ABC.
Hence, the co-ordinates of the images of A, B and C reflected in x-axis will be A’ (2, 3), B’ (-1, -2), C’ (0, 2) respectively.
And, these are joined to from another Δ A’B’C’
Yes, these two triangles are congruent.



17. The points (6, 2), (3, -1) and (-2, 4) are the vertices of a right-angled triangle. Check whether it remains a right-angled triangle after reflection in the y-axis.
Solution:

Let A (6, 2), B (3, -1) and C (-2, 4) be the points of a right-angled triangle
Then,
The co-ordinates of the images of A, B, C reflected in y-axis will be:
A' (-6, 2), B' (-3, -1) and C' (2, 4).
Hence, by joining these points
We see that ΔA'B'C' is also a right-angled triangle.



18. The triangle ABC where A (1, 2), B (4, 8), C (6, 8) is reflected in the x-axis to triangle A' B' C'. The triangle A' B' C' is then reflected in.the origin to triangle A"B"C" Write down the co-ordinates of A", B", C". Write down a single transformation that maps ABC onto A" B" C".
Solution:

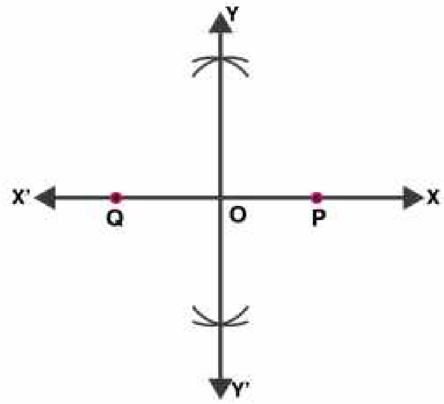
Given,
The co-ordinates of Δ ABC are A (1, 2) B (4, 8), C (6, 8)
These vertices are reflected in x- axis as A', B' and C'.
Hence, their co-ordinates are A' (1, -2), B' (4, -8) and C' (6, -8).
Now,
A', B' and C' are again reflected in origins to form an ΔA"B"C".
Hence, the co-ordinates will be A" (-1, 2), B" (-4, 8) and C" (-6, 8)
The single transformation that maps ABC onto A" B" C" is y-axis.

19. The image of a point P on reflection in a line l is point P'. Describe the location of the line l.
Solution:

The line will be the right bisector of the line segment joining P and P'.

20. Given two points P and Q, and that (1) the image of P on reflection in the y-axis is the point Q and (2) the midpoint of PQ is invariant on reflection in x-axis. Locate:

(i) the x-axis
(ii) the y-axis and
(iii) the origin.
Solution:



Given, Q is the image of P on reflection in y-axis and mid-point of PQ is invariant on reflection in x-axis

(i) x-axis will be the line joining the points P and Q.
(ii) The line perpendicular bisector of line segment PQ is the y-axis.
(iii) The origin will be the mid-point of line segment PQ.

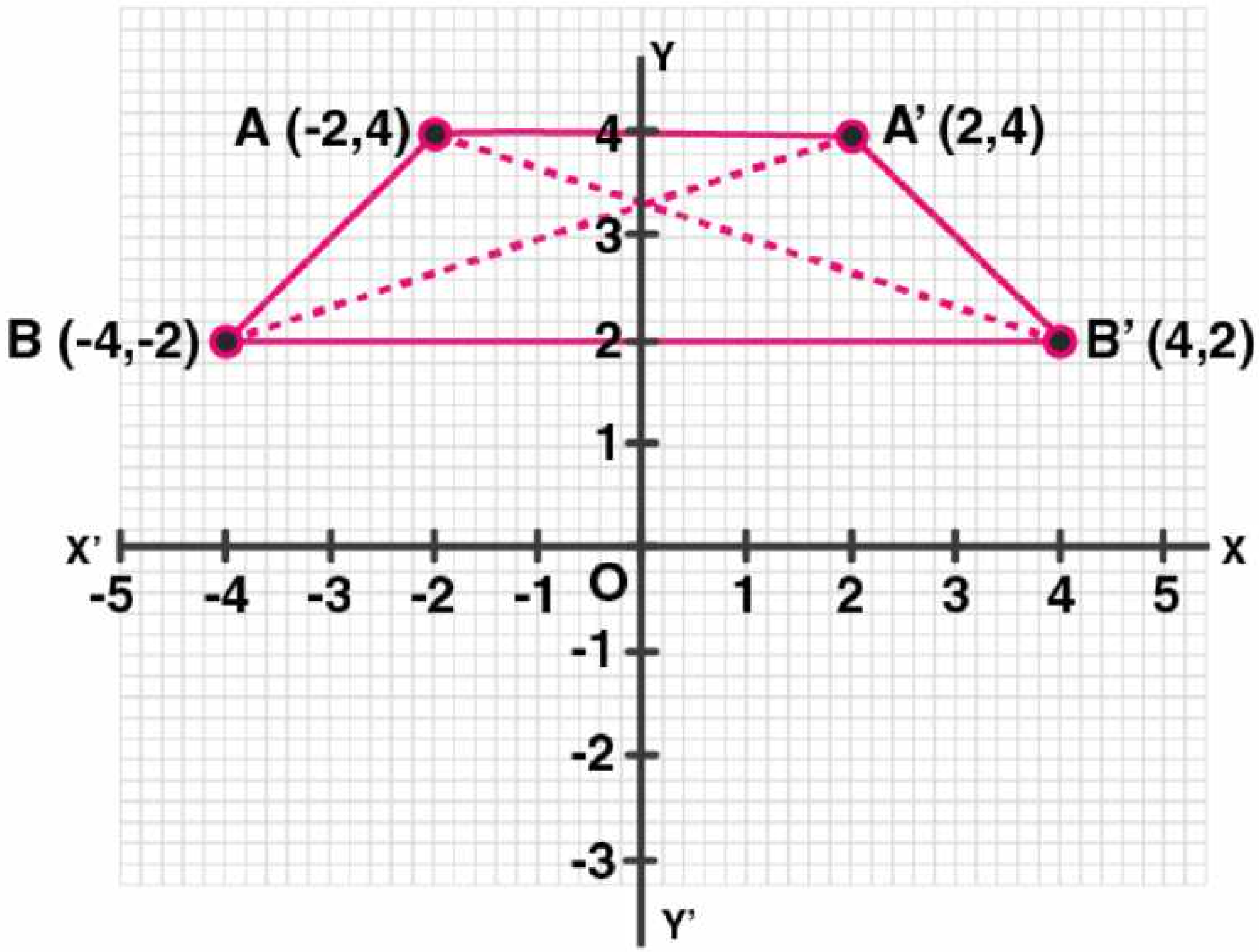
21. The point (-3, 0) on reflection in a line is mapped as (3, 0) and the point (2, -3) on reflection in the same line is mapped as (-2, -3).
(i) Name the mirror line.
(ii) Write the co-ordinates of the image of (-3, -4) in the mirror line.
Solution:

Given,
The point (-3,0) is the image of point (3, 0) and point (2, -3) is image of point (-2, -3) reflected on the same line.
(i) Clearly, it's seen that the mirror line will be y-axis.
(ii) The co-ordinates of the image of the point (-3, -4) reflected in the same line i.e. y-axis will be (3, -4).

22. A (-2, 4) and B (-4, 2) are reflected in the y-axis. If A' and B' are images of A and B respectively, find
(i) the co-ordinates of A' and B'.
(ii) Assign a special name to a quad. AA'B'B.
(iii) State whether AB' = BA'.
Solution:

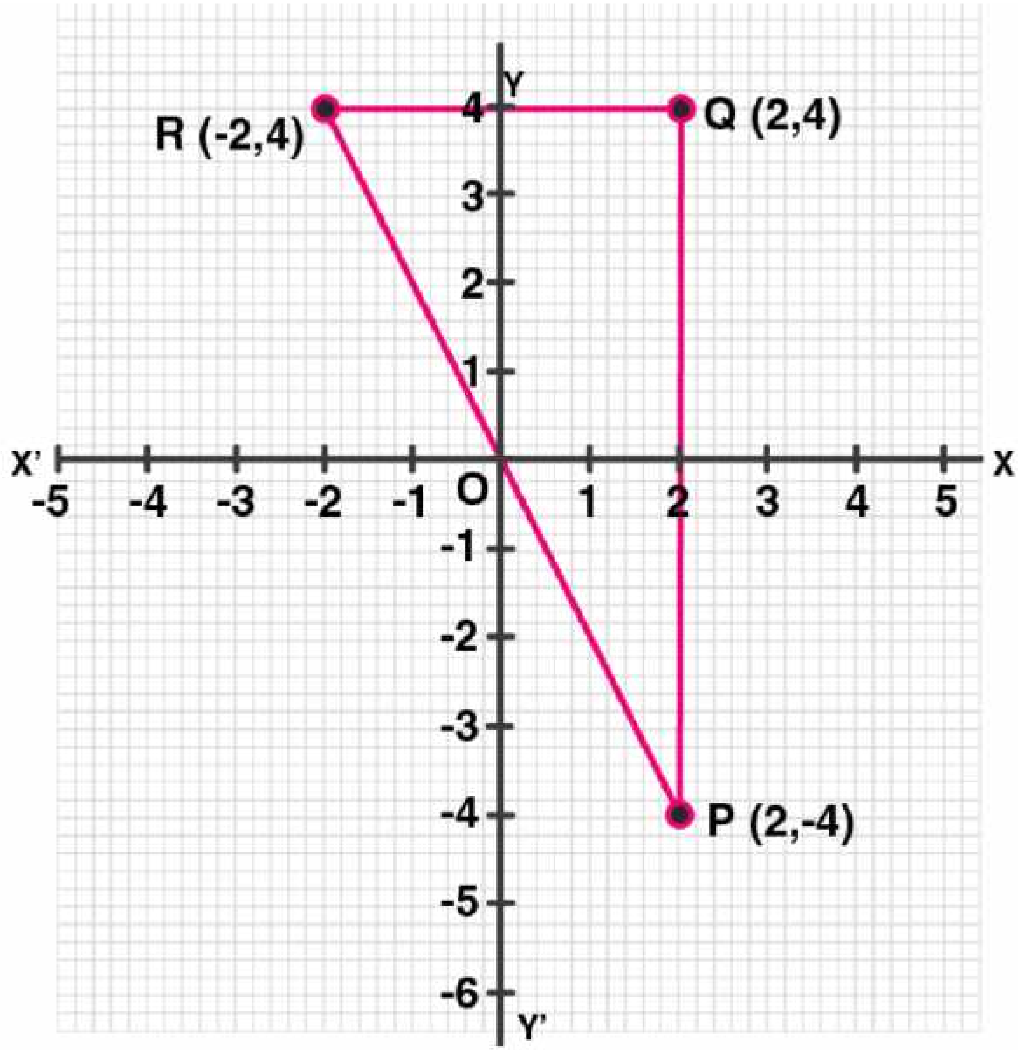
Given,
A (-2, 4) and B (-4, 2) are reflected in the y-axis as A' and B'.

(i) The co-ordinates of A' are (2, 4) and of B are (4, 2).
(ii) The quadrilateral AA'B'B is an isosceles trapezium.
(iii) Yes, it is found out that AB' = BA'



23. Use graph paper for this question.
(i) The point P (2, -4) is reflected about the line x = 0 to get the image Q. Find the co-ordinates of Q.
(ii) Point Q is reflected about the line y = 0 to get the image R. Find the co-ordinates of R.
(iii) Name the figure PQR.
(iv) Find the area of figure PQR.
Solution:

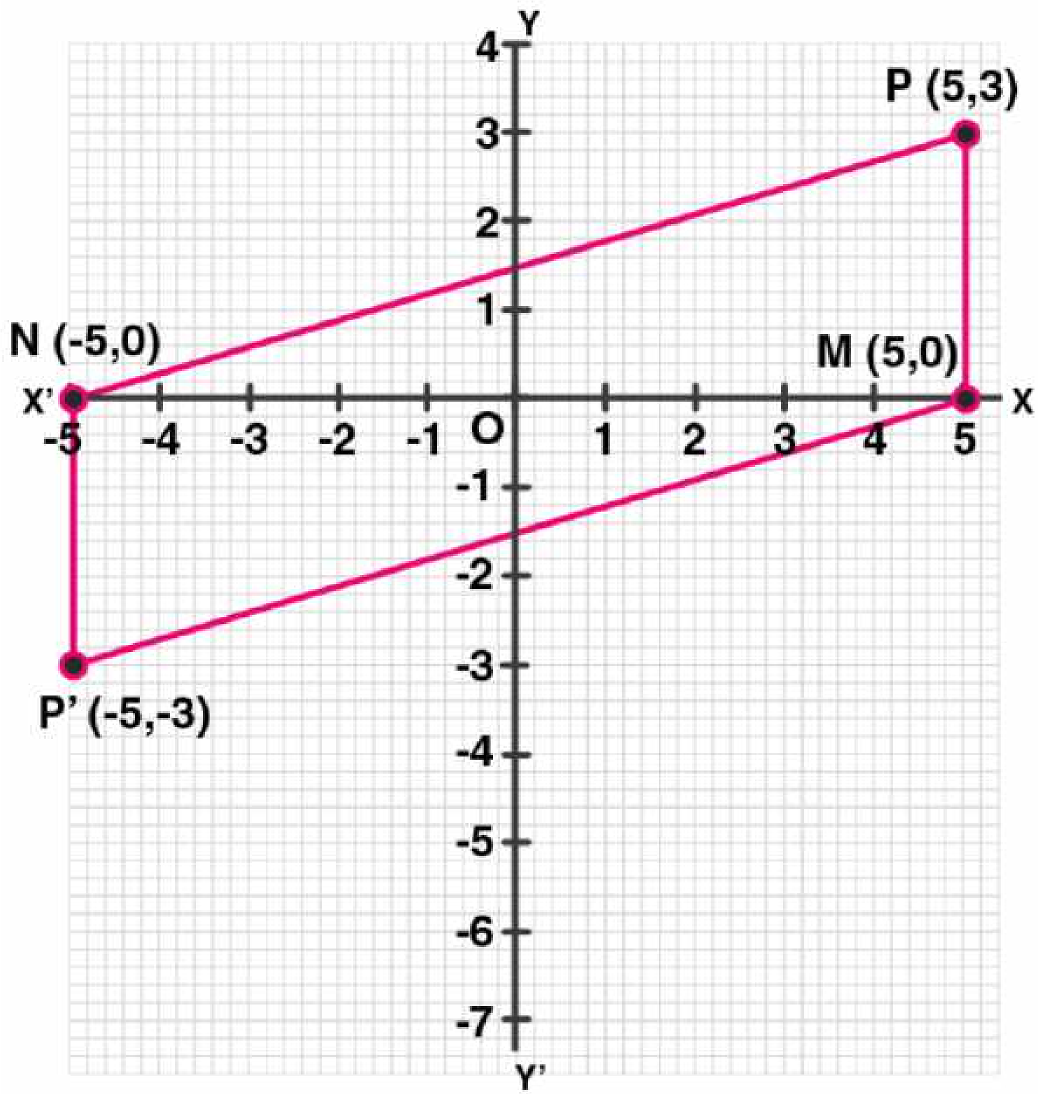
(i) As the point Q is the reflection of the point P (2, -4) in the line x = 0,
Thus, the co-ordinates of Q are (2, 4).
(ii) As R is the reflection of Q (2, 4) about the line y = 0,
Thus, the co-ordinates of R are (– 2, 4).
(iii) Figure PQR is the right-angled triangle PQR.
(iv) Area of Δ PQR = ½ x QR x PQ
= ½ x 4 x 8
= 16 sq. units.



24. Use graph paper for this question. The point P (5, 3) was reflected in the origin to get the image P'.
(i) Write down the co-ordinates of P'.
(ii) If M is the foot of the perpendicular from P to the x-axis, find the co-ordinates of M.
(iii) If N is the foot of the perpendicular from P' to the x-axis, find the co-ordinates of N.
(iv) Name the figure PMP'N.
(v) Find the area of the figure PMP'N. (2001)
Solution:

Given, P' is the image of point P (5, 3) reflected in the origin.
(i) Co-ordinates of P' will be (-5, -3).
(ii) M is the foot of the perpendicular from P to the x-axis.
Hence, the co-ordinates of M will be (5, 0)
(iii) N is the foot of the perpendicular from P' to x-axis.
Hence, the co-ordinates of N will be (-5, 0).
(iv) By joining the points, the figure PMP'N is a parallelogram.

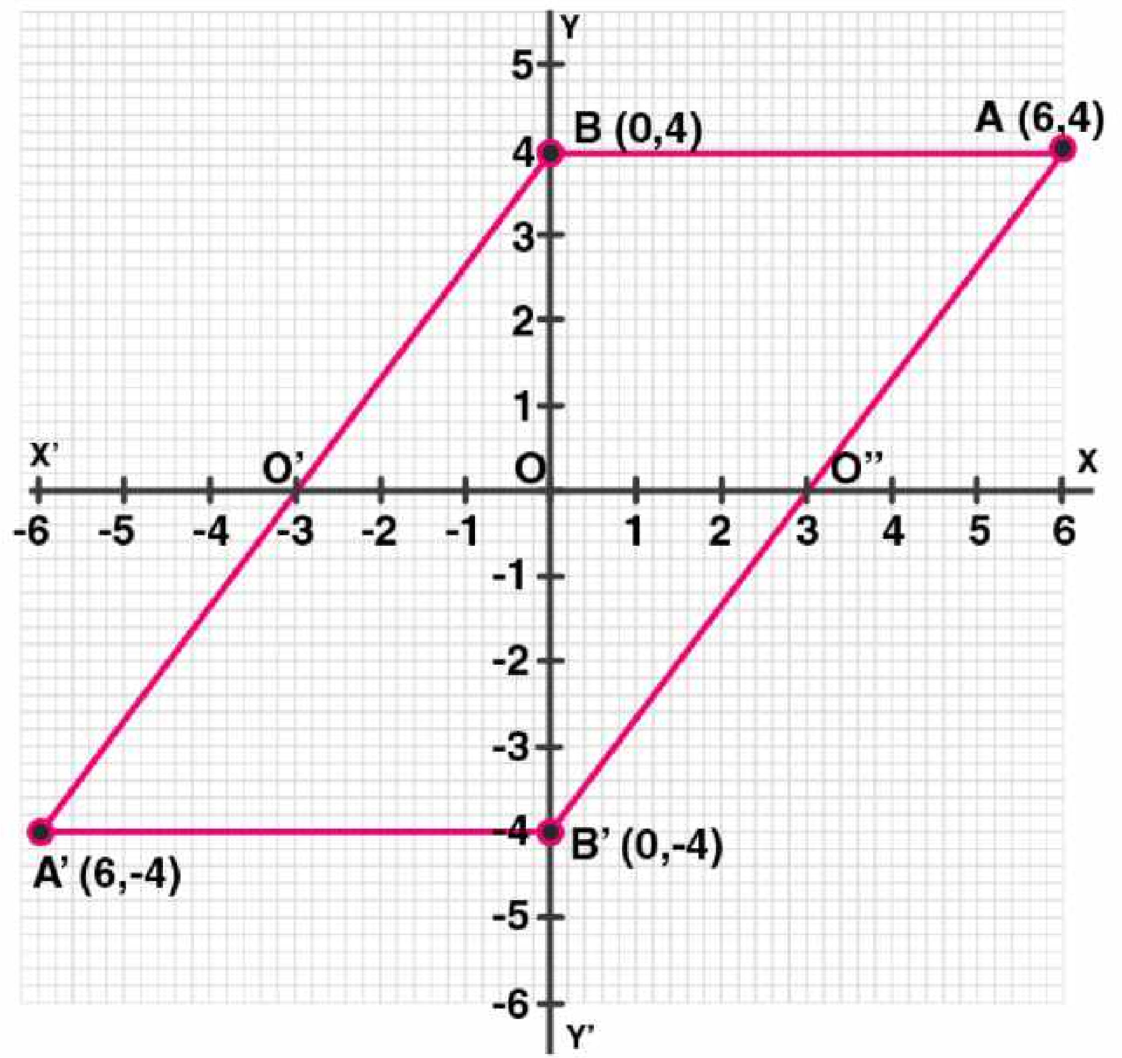
(v) Area of the parallelogram = 2 x area of Δ MPN
= 2 x ½ x MN x PM
= MN x PM = 10 x 3
= 30 sq. units.



25. Using a graph paper, plot the points A (6, 4) and B (0, 4).
(i) Reflect A and B in the origin to get the images A' and B'.
(ii) Write the co-ordinates of A' and B'.
(iii) State the geometrical name for the figure ABA'B'.
(iv) Find its perimeter.
Solution:

Points A (6, 4) and B (0, 4) are plotted on a graph paper.
(i) A and B are reflected in the origin to get images A' and B'
(ii) Hence,
The co-ordinates of A' are (-6, -4)
The co-ordinates of B' are (0, -4)

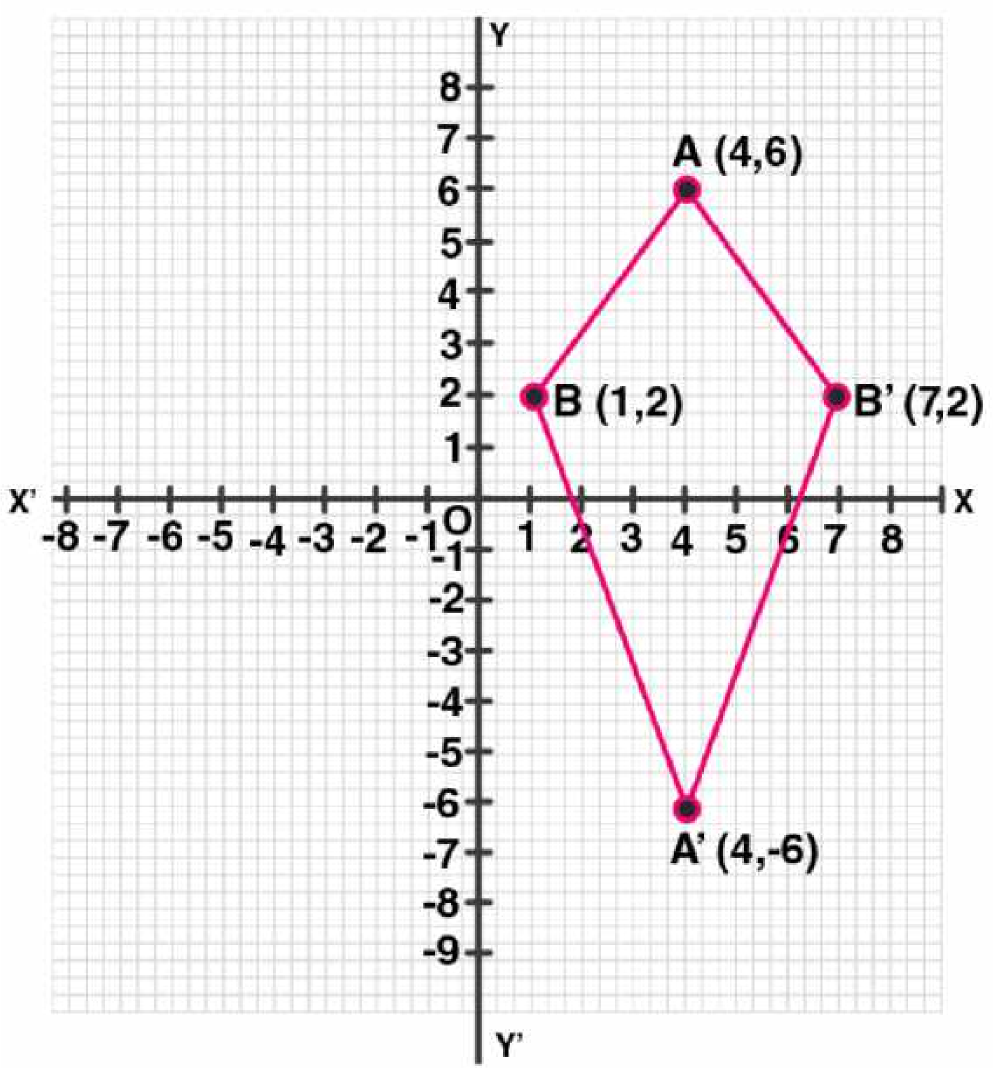
(iii) The geometrical name for ABA'B' is parallelogram
(iv) From the figure in graph paper, we see that
Length of AB = A'B' = 6 units
And, BB' = 8 units
In Δ ABB', by Pythagoras theorem
$(AB')^2 = AB^2 + (BB')^2$
$= 6^2 + 8^2$
$= 36 + 64 = 100$
$AB' = \sqrt{100} = 10$ units



Hence, the perimeter of ABA'B' = (6 + 10 + 6 + 10) = 32 units

26. Use graph paper to answer this question
(i) Plot the points A (4, 6) and B (1, 2).
(ii) If A' is the image of A when reflected in x-axis, write the co-ordinates of A'.
(iii) If B' is the image of B when B is reflected in the line AA', write the co-ordinates of B'.
(iv) Give the geometrical name for the figure ABA'B'.
Solution:

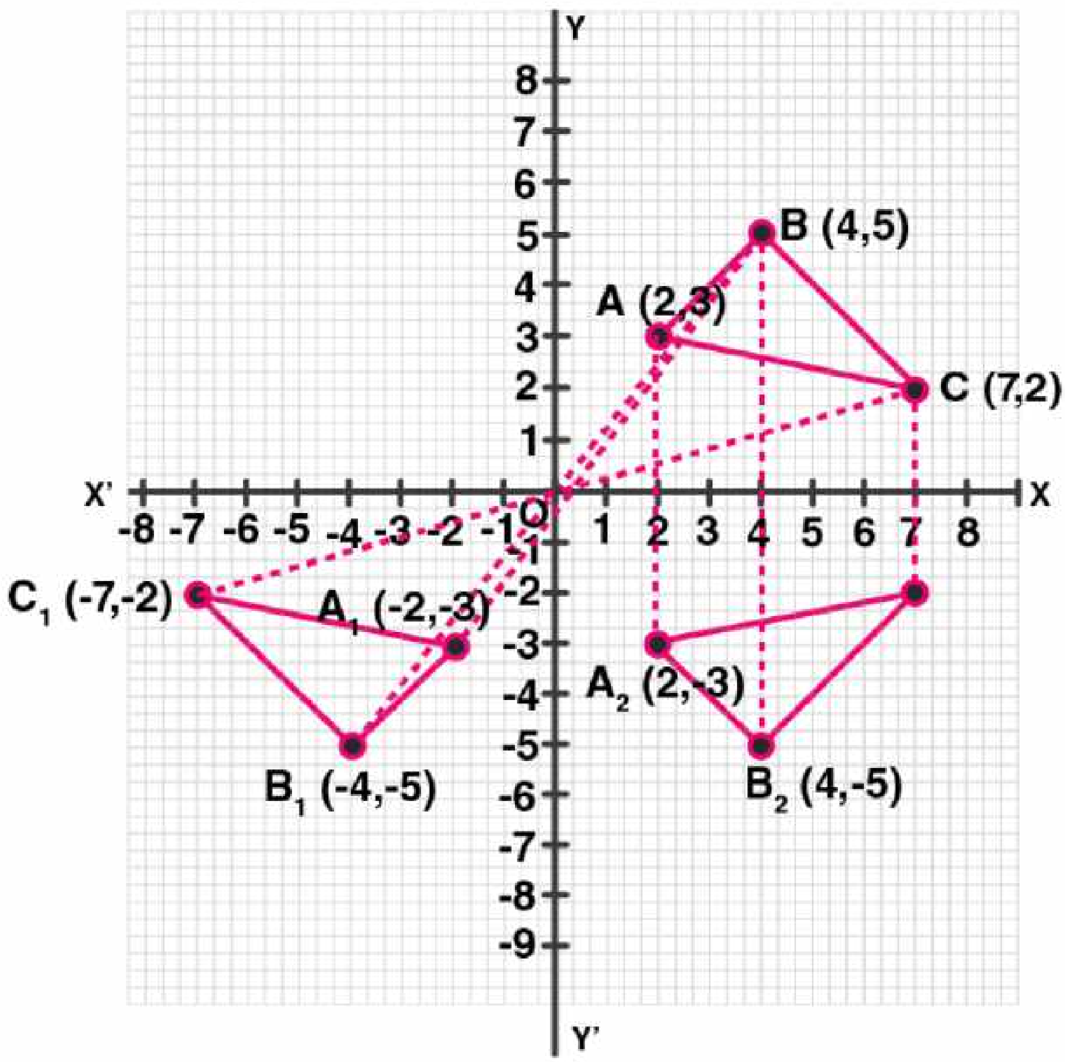
(i) Plotting the points A (4, 6) and B (1, 2) on the given graph.
(ii) The co-ordinates of the image of A when reflected in axis are A'(4, -6)
(iii) The co-ordinates of the image of B when reflected in the line AA' are B' = (7, 2)
(iv) It's seen that in the quadrilateral ABA'B', we have
AB = AB' and A'B = A'B'
Thus, ABA'B' is a kite.



27. The points A (2, 3), B (4, 5) and C (7, 2) are the vertices of ΔABC.
(i) Write down the co-ordinates of A_1, B_1, C_1 if Δ $A_1B_1C_1$ is the image of Δ ABC when reflected in the origin.
(ii) Write down the co-ordinates of A_2, B_2, C_2 if Δ $A_2B_2C_2$ is the image of Δ ABC when reflected in the x-axis.
(iii) Assign the special name to the quadrilateral BCC_2B_2 and find its area.
Solution:

Given, points A (2, 3), B (4, 5) and C (7, 2) are the vertices of Δ ABC.

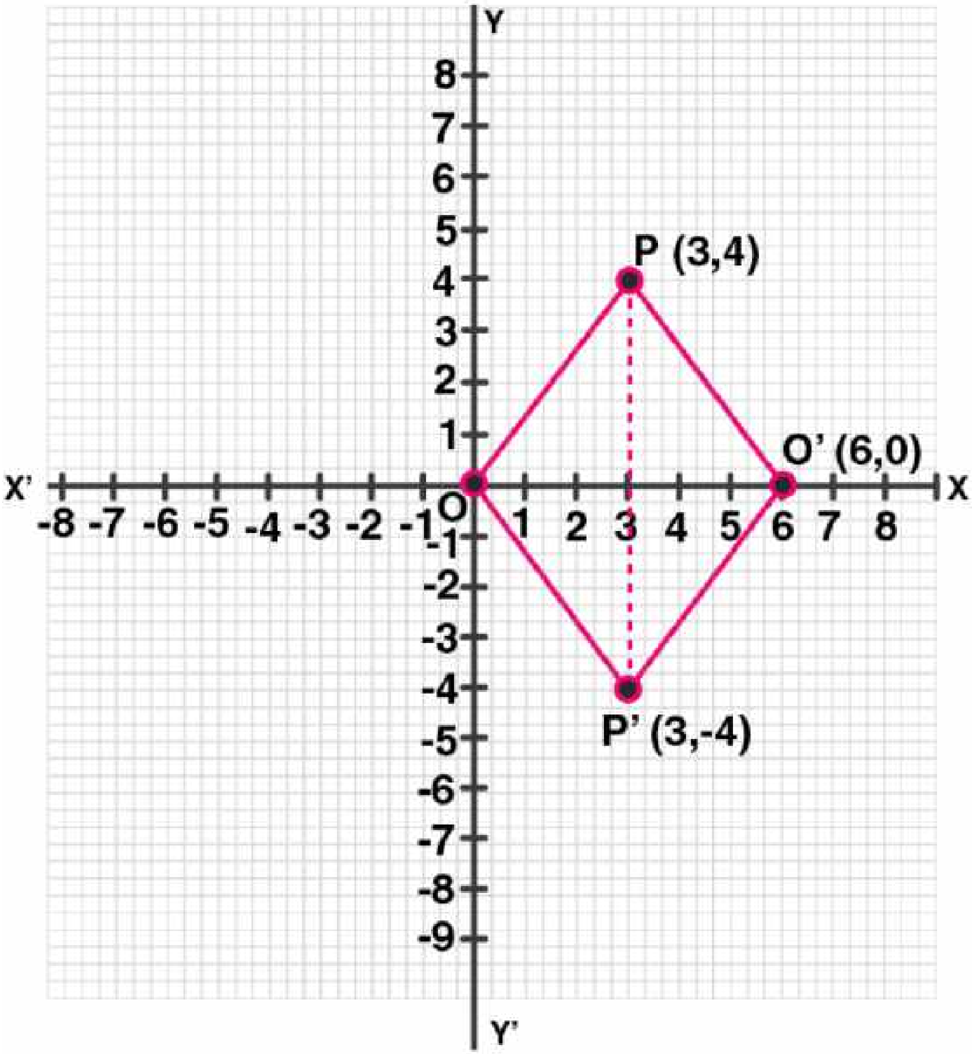
And A_1, B_1 and C_1 are the images of A, B and C reflected in the origin.
(i) Hence,
Co-ordinates of A_1 = (-2, -3)
Co-ordinates of B_1 (-4, -5) and
Co-ordinates of C_1 (-7, -2).
(ii) Now,
Co-ordinates of A_2, B_2 and C_2 the images of A, B and C when reflected in x-axis are:
A_2 (2, – 3), B_2 (4, – 5), C_2 (7, – 2)
(iii) The quadrilateral formed by joining the points, BCC_2B_2 is an isosceles trapezium and its area is
= ½ (BB_2 + CC_2) x 3
= ½ (10 + 4) x 3
= ½ x 14 x 3
= 21 sq. units



28. The point P (3, 4) is reflected to P' in the x-axis and O' is the image of O (origin) in the line PP'. Find:
(i) the co-ordinates of P' and O',

(ii) the length of segments PP' and OO'.
(iii) the perimeter of the quadrilateral POP'O'.
Solution:

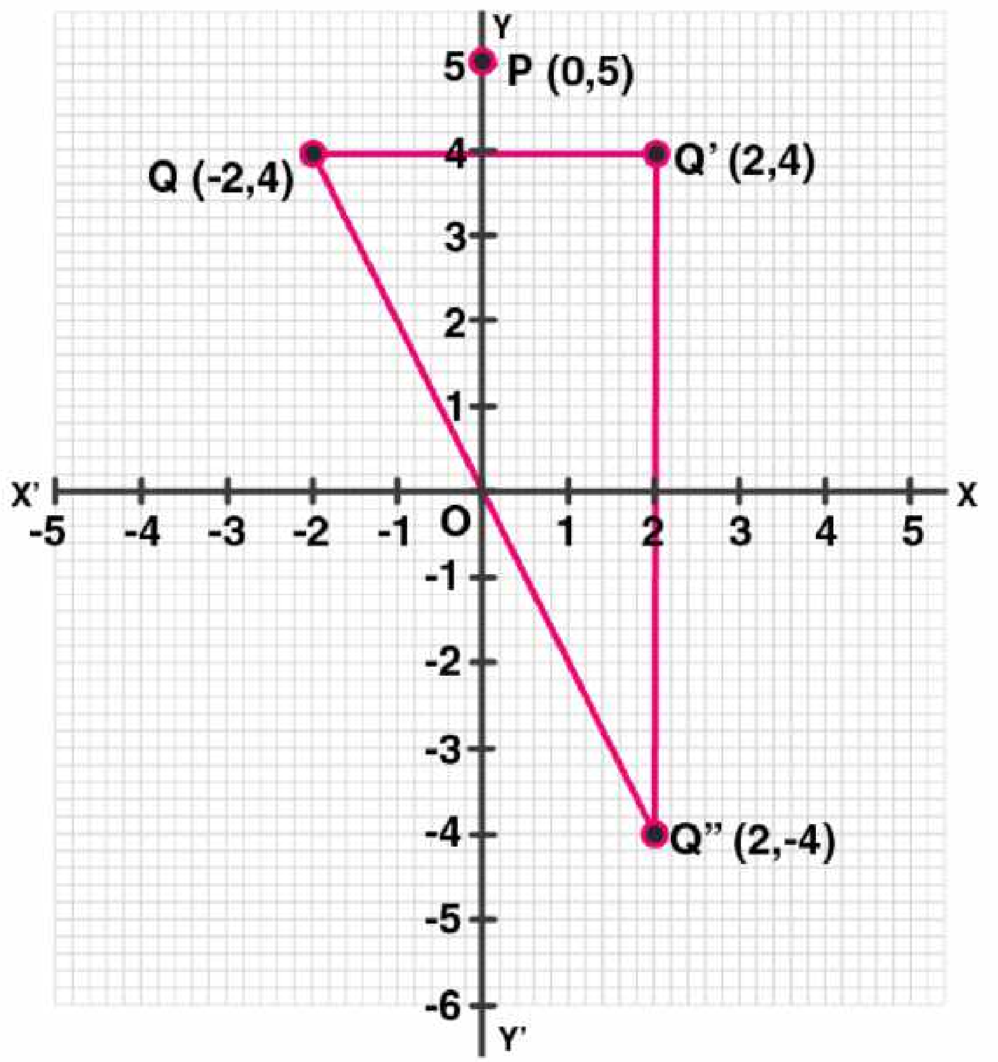
Given,
P' is the image of P (3, 4) reflected in x- axis and O' is the image of O the origin in the line P'P.
(i) Hence, co-ordinates of P' are (3, -4) and co-ordinates of O' reflected in PP' are (6, 0)
(ii) Length of PP' = 8 units and OO' = 6 units
(iii) Perimeter of POP'O' is (4 x OP) units.
Let Q be the point of intersection of diagonals OO' and PP'.
So, OQ = 3 units and OP = 4 units
Hence,
OP = $\sqrt{[(OQ)^2 + (PQ)^2]} = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$ units
Thus, the perimeter of POP'O' = 4 x 5 = 20 units



29. Use a graph paper for this question. (Take 10 small divisions = 1 unit on both axes). P and Q have co-ordinates (0, 5) and (-2, 4).
(i) P is invariant when reflected in an axis. Name the axis.

(ii) Find the image of Q on reflection in the axis found in (i).
(iii) (0, k) on reflection in the origin is invariant. Write the value of k.
(iv) Write the co-ordinates of the image of Q, obtained by reflecting it in the origin followed by a reflection in x-axis.
Solution:

Given, two points P (0, 5) and Q (-2, 4)
(i) As the abscissa of P is 0. It is invariant when is reflected in y-axis.
(ii) Let Q’ be the image of Q on reflection in y-axis.
Thus, the co-ordinate of Q’ will be (2, 4)
(iii) (0, k) on reflection in the origin is invariant.
So, the co-ordinates of image will be (0, 0)
Hence, k = 0
(iv) The reflection of Q in the origin is the point Q” and its co-ordinates will be (2, – 4) and reflection of Q” (2, – 4) in x-axis is (2, 4) which is the point Q’.



Chapter Test

1. The point P (4, – 7) on reflection in x-axis is mapped onto P'. Then P' on reflection in the y-axis is mapped onto P". Find the co-ordinates of P' and P". Write down a single transformation that maps P onto P".
Solution:

Given,
P' is the image of P (4, -7) reflected in x-axis
Thus, the co-ordinates of P' are (4, 7)
Again P" is the image of P' reflected in y-axis
Hence, the co-ordinates of P" are (-4, 7)
Therefore, single transformation that maps P and P" is in the origin.

2. The point P (a, b) is first reflected in the origin and then reflected in the y-axis to P'. If P' has co-ordinates (3, – 4), evaluate a, b
Solution:

The co-ordinates of image of P (a, b) reflected in origin are (-a, -b).
Again, the co-ordinates of P' which is image of the above point (-a, -b) reflected in the y-axis are (a, -b).
But the co-ordinates of P' are (3, -4)
Thus, a = 3 and -b = -4 ⇒ b = 4

3. A point P (a, b) becomes (– 2, c) after reflection in the x-axis, and P becomes (d, 5) after reflection in the origin. Find the values of a, b, c and d.
Solution:

Given, point P (a, b) and the image of P (a, b) after reflected in the x-axis be (a, -b)
But, it is given as (-2, c)
Thus, a = -2, c = -b
Next,
If P is reflected in the origin, then its co-ordinates will be (-a, -b)
But, it is given as (d, 5)
Thus,
-b = 5 ⇒ b = -5,
d = -a = -(-2) = 2,
c = -b = -(-5) = 5
Thus,
a = -2, b = -5, c = 5 and d = 2

4. A (4, – 1), B (0, 7) and C (– 2, 5) are the vertices of a triangle. Δ ABC is reflected in the y-axis and then reflected in the origin. Find the co-ordinates of the final images of the vertices.
Solution:

Given, A (4, -1), B (0, 7) and C (-2, 5) are the vertices of ΔABC.
ΔABC after reflecting in y-axis, the co-ordinates of points will be A' (-4, -1), B' (0, 7), C' (2, 5).
Again, when ΔA'B'C' reflecting in origin:

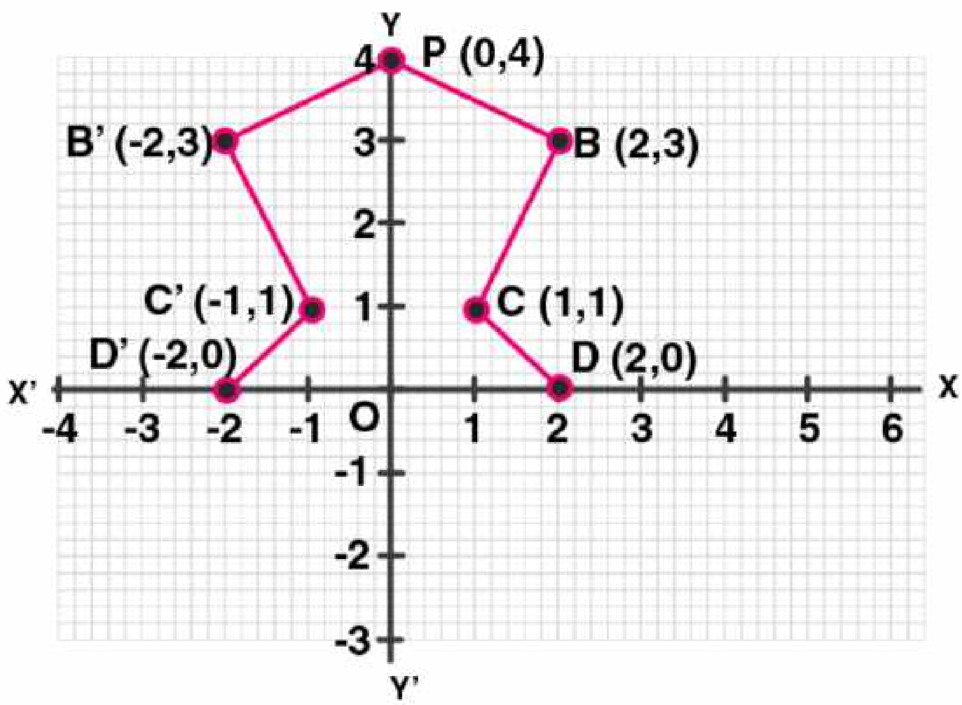
The co-ordinates of the images of the vertices will be A" (4, 1), B" (0, -7), C" (-2, -5).

5. The points A (4, – 11), B (5, 3), C (2, 15), and D (1, 1) are the vertices of a parallelogram. If the parallelogram is reflected in the y-axis and then in the origin, find the co-ordinates of the final images. Check whether it remains a parallelogram. Write down a single transformation that brings the above change.
Solution:

Given, points A (4, -11), B (5, 3), C (2, 15) and D (1, 1) are the vertices of a parallelogram.
After reflecting in y-axis, the images of these points will be
A' (-4, 11), B' (-5, 3), C (-2, 15) and D' (-1, 1).
Again, reflecting these points in origin, the image of these points will be A" (4, -11), B" (5, -3), C" (2, -15) and D" (0, -1).
Yes, the reflection of a single transformation is in the x-axis.

6. Use a graph paper for this question (take 2 cm = 1 unit on both x and y axes).
(i) Plot the following points: A (0, 4), B (2, 3), C (1, 1) and D (2, 0).
(ii) Reflect points B, C, D on y-axis and write down their coordinates. Name the images as B', C', D' respectively.
(iii) Join points A, B, C, D, D', C', B' and A in order, so as to form a closed figure. Write down the equation of line of symmetry of the figure formed.
Solution:

(i) On graph: A (0, 4), B (2, 3), C (1, 1) and D (2, 0)
(ii) Point after reflection on y-axis are B' = (-2, 3), C' = (-1, 1) and D' = (-2, 0)
(iii) The points A, B, C, D, D', C', B' and A in order to form a closed figure.
Hence, the equation of the line of symmetry is x = 0



7. The triangle OAB is reflected in the origin O to triangle OA'B'. A' and B' have coordinates (– 3, – 4) and (0, – 5) respectively.
(i) Find the co-ordinates of A and B.
(ii) Draw a diagram to represent the given information.
(iii) What kind of figure is the quadrilateral ABA'B'?
(iv) Find the coordinates of A", the reflection of A in the origin followed by reflection in the y-axis.
(v) Find the co-ordinates of B", the reflection of B in the x-axis followed by reflection in the origin.
Solution:

Given,
Δ OAB is reflected in the origin O to Δ OA'B',
And the co-ordinates of A' = (-3, -4) and B' = (0, -5).
(i) Hence, the co-ordinates of A will be (3, 4) and of B will be (0, 5).
(ii) The diagram representing the given information has been drawn here.
(iii) The figure in the diagram is a rectangle.
(iv) The co-ordinates of B', the reflection of B is the x-axis is (0, -5) and co-ordinates of B", the reflection in origin of the point (0, -5) will be (0, 5).
(v) The co-ordinates of the points, the reflection of A in the origin are (-3, -4) and coordinates of A", the reflected in y-axis of the point (-3, – 4) are (3, -4).