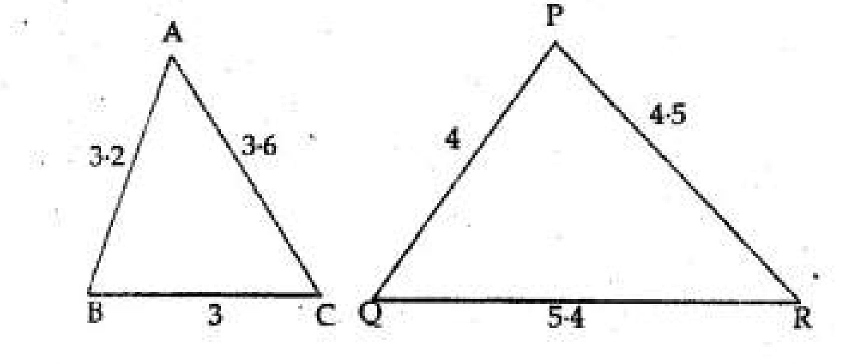
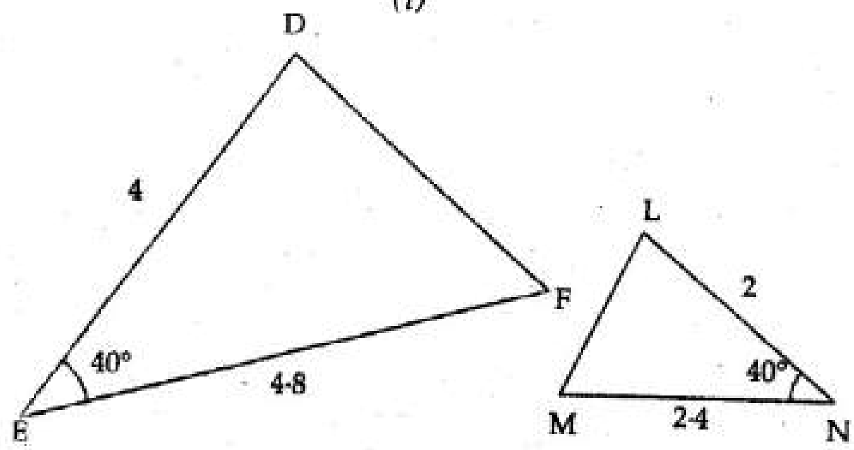
EXERCISE 13.1

1. State which pairs of triangles in the figure given below are similar. Write the similarity rule used and also write the pairs of similar triangles in symbolic form (all lengths of sides are in cm):



(i)



(ii)

Solution:-

(i) From the ΔABC and ΔPQR

AB/PQ = 3.2/4

= 32/40

Divide both numerator and denominator by 8 we get,

= 4/5

AC/PR = 3.6/4.5

= 36/45

Divide both numerator and denominator by 9 we get,

= 4/5

BC/QR = 3/5.4

= 30/54

Divide both numerator and denominator by 6 we get,

= 5/9

By comparing all the results, the side are not equal.

Therefore, the triangles are not equal.

(ii) From the ΔDEF and ΔLMN

∠E = ∠N = 40°

Then, DE/LN = 4/2

Divide both numerator and denominator by 2 we get,

= 2

EF/MN = 4.8/2.4

= 48/24

Divide both numerator and denominator by 24 we get,

= 2

Therefore, ΔDEF ~ ΔLMN

2. It is given that ΔDEF ~ ΔRPQ. Is it true to say that ∠D = ∠R and ∠F = ∠P ? Why?

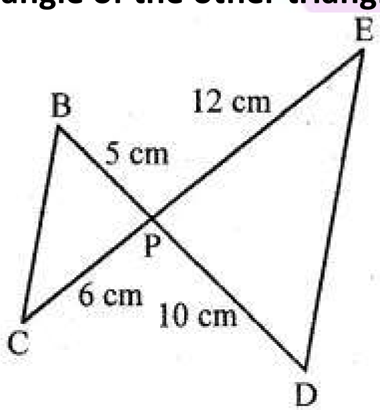
Solution:-

From the question is given that, ΔDEF ~ ΔRPQ

∠D = ∠R and ∠F = ∠Q not ∠P

No, ∠F ≠ ∠P

3. If in two right triangles, one of the acute angle of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles are similar? Why?



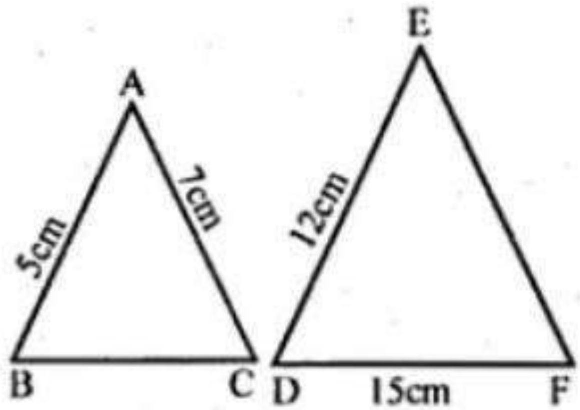
Solution:-

From the figure, two line segments are intersecting each other at P.

In ΔBCP and ΔDPE

5/10 = 6/12
Dividing LHS and RHS by 2 we get,
½ = ½
Therefore, ΔBCD ~ ΔDEP

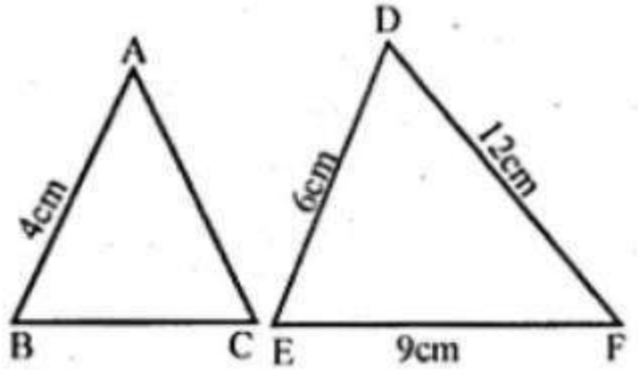
5. It is given that ΔABC ~ ΔEDF such that AB = 5 cm, AC = 7 cm, DF = 15 cm and DE = 12 cm.
Find the lengths of the remaining sides of the triangles.
Solution:-
As per the dimensions give in the questions,



From the question it is given that,
ΔDEF ~ ΔLMN
So, AB/ED = AC/EF = BC/DF
Consider AB/ED = AC/EF
5/12 = 7/EF
By cross multiplication,
EF = (7 × 12)/5
EF = 16.8 cm
Now, consider AB/ED = BC/DF
5/12 = BC/15
BC = (5 × 15)/12
BC = 75/12
BC = 6.25

6.
(a) If ΔABC ~ ΔDEF, AB = 4 cm, DE = 6 cm, EF = 9 cm and FD = 12 cm, then find the perimeter of ΔABC.
Solution:-

As per the dimensions give in the questions,



Now, we have to find out the perimeter of ΔABC

Let ΔABC ~ ΔDEF

So, AB/DE = AC/DF = BC/EF

Consider, AB/DE = AC/DE

4/6 = AC/12

By cross multiplication we get,

AC = (4 × 12)/6

AC = 48/6

AC = 8 cm

Then, consider AB/DE = BC/EF

4/6 = BC/9

BC = (4 × 9)/6

BC = 36/6

BC = 6 cm

Therefore, the perimeter of ΔABC = AB + BC + AC

= 4 + 6 + 8

= 18 cm

(b) If ΔABC ~ ΔPQR, Perimeter of ΔABC = 32 cm, perimeter of ΔPQR = 48 cm and PR = 6 cm, then find the length of AC.

Solution:-

From the question it is given that,

ΔABC ~ ΔPQR

Perimeter of ΔABC = 32 cm

Perimeter of ΔPQR = 48 cm

So, AB/PQ = AC/PR = BC/QR

Then, perimeter of ΔABC/perimeter of ΔPQR = AC/PR

32/48 = AC/6

AC = (32 × 6)/48

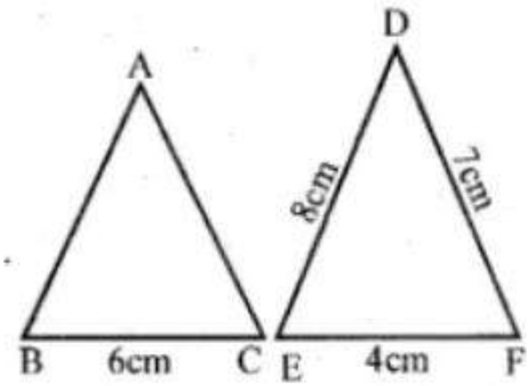
AC = 4

Therefore, the length of AC = 4 cm.

7. Calculate the other sides of a triangle whose shortest side is 6 cm and which is similar to a triangle whose sides are 4 cm, 7 cm and 8 cm.

Solution:-

Let us assume that, ΔABC ~ ΔDEF



ΔABC is BC = 6cm

ΔABC ~ ΔDEF

So, AB/DE = BC/EF = AC/DF

Consider AB/DE = BC/EF

AB/8 = 6/4

AB = (6 × 8)/4

AB = 48/4

AB = 12

Now, consider BC/EF = AC/DF

6/4 = AC/7

AC = (6 × 7)/4

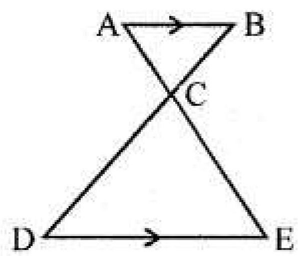
AC = 42/4

AC = 21/2

AC = 10.5 cm

8.

(a) In the figure given below, AB || DE, AC = 3 cm, CE = 7.5 cm and BD = 14 cm. Calculate CB and DC.



Solution:-

From the question it is given that,

AB||DE

AC = 3 cm

CE = 7.5 cm

BD = 14 cm

From the figure,

∠ACB = ∠DCE [because vertically opposite angles]

∠BAC = ∠CED [alternate angles]

Then, ΔABC ~ ΔCDE

So, AC/CE = BC/CD

3/7.5 = BC/CD

By cross multiplication we get,

7.5BC = 3CD

Let us assume BC = x and CD = 14 – x

7.5 × x = 3 × (14 - x)

7.5x = 42 – 3x

7.5x + 3x = 42

10.5x = 42

x = 42/10.5

x = 4

Therefore, BC = x = 4 cm

CD = 14 – x

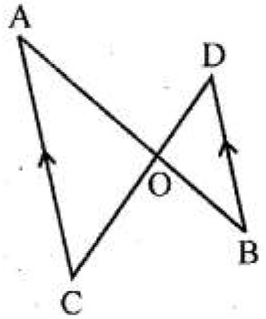
= 14 – 4

= 10 cm

(b) In the figure (2) given below, CA || BD, the lines AB and CD meet at G.

(i) Prove that ΔACO ~ ΔBDO.

(ii) If BD = 2.4 cm, OD = 4 cm, OB = 3.2 cm and AC = 3.6 cm, calculate OA and OC.



Solution:-

(i) We have to prove that, ΔACO ~ ΔBDO.

So, from the figure

Consider ΔACO and ΔBDO

Then,

∠AOC = ∠BOD [from vertically opposite angles]

∠A = ∠B

Therefore, ΔACO = ΔBDO

Given, BD = 2.4 cm, OD = 4 cm, OB = 3.2 cm, AC = 3.6 cm,

ΔACO ~ ΔBOD

So, AO/OB = CO/OD = AC/BD

Consider AC/BD = AO/OB

3.6/2.4 = AO/3.2

AO = (3.6 × 3.2)/2.4

AO = 4.8 cm

Now, consider AC/BD = CO/OD

3.6/2.4 = CO/4

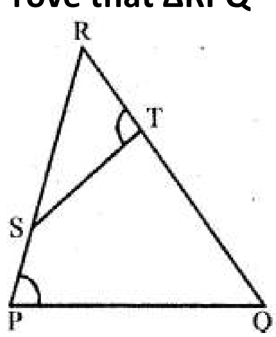
CO = (3.6 × 4)/2.4

CO = 6 cm

9. (a) In the figure

(i) given below, ∠P = ∠RTS.

Prove that ΔRPQ ~ ΔRTS.



Solution:-

From the given figure, ∠P = ∠RTS

So we have to prove that ΔRPQ ~ ΔRTS

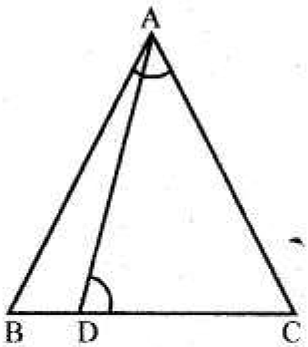
In ΔRPQ and ΔRTS

∠R = ∠R (common angle for both triangle)

∠P = ∠RTS (from the question)

ΔRPQ ~ ΔRTS

(b) In the figure (ii) given below, ∠ADC = ∠BAC. Prove that CA^2 = DC x BC



(ii)

Solution:-

From the figure, ∠ADC = ∠BAC

So, we have to prove that, CA^2 = DC x BC

In ΔABC and ΔADC

∠C = ∠C (common angle for both triangle)

∠BAC = ∠ADC (from the question)

ΔABC ~ ΔADC

Therefore, CA/DC = BC/CA

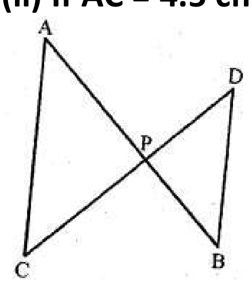
We know that, corresponding sides are proportional,

Therefore, $CA^2 = DC \times BC$

10. (a) In the figure (1) given below, AP = 2PB and CP = 2PD.

(i) Prove that ΔACP is similar to ΔBDP and AC || BD.

(ii) If AC = 4.5 cm, calculate the length of BD.



Solution:-

From the question it is give that,

AP = 2PB, CP = 2PD

(i) We have to prove that, ΔACP is similar to ΔBDP and AC || BD

AP = 2PB

AP/PB = 2/1

Then, CP = 2PD

CP/PD = 2/1

∠APC = ∠BPD [from vertically opposite angles]

So, ΔACP ~ ΔBDP

Therefore, ∠CAP = ∠PBD [alternate angles]

Hence, AC || BD

(ii) AP/PB = AC/BD = 2/1

AC = 2BD

2BD = 4.5 cm

BD = 4.5/2

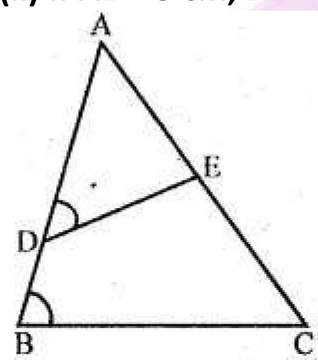
BD = 2.25 cm

(b) In the figure (2) given below,

∠ADE = ∠ACB.

(i) Prove that Δs ABC and AED are similar.

(ii) If AE = 3 cm, BD = 1 cm and AB = 6 cm, calculate AC.



Solution:-

From the given figure,

(i) ∠A = ∠A (common angle for both triangles)

∠ACB = ∠ADE [given]

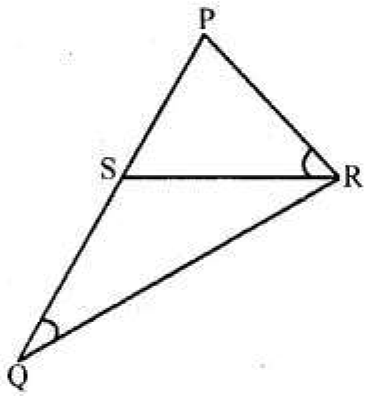
Therefore, ΔABC ~ ΔAED

(ii) from (i) proved that, ΔABC ~ ΔAED

So, BC/DE = AB/AE = AC/AD

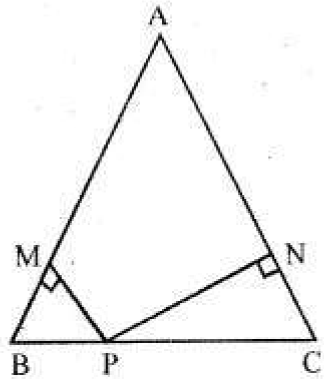
AD = AB – BD
= 6 – 1 = 5
Consider, AB/AE = AC/AD
6/3 = AC/5
AC = (6 × 5)/3
AC = 30/3
AC = 10 cm

(c) In the figure (3) given below, ∠PQR = ∠PRS. Prove that triangles PQR and PRS are similar. If PR = 8 cm, PS = 4 cm, calculate PQ.



Solution:-
From the figure,
∠P = ∠P (common angle for both triangles)
∠PQR = ∠PRS [from the question]
So, ΔPQR ~ ΔPRS
Then, PQ/PR = PR/PS = QR/SR
Consider PQ/PR = PR/PS
PQ/8 = 8/4
PQ = (8 × 8)/4
PQ = 64/4
PQ = 16 cm

11. In the given figure, ABC is a triangle in which AB = AC. P is a point on the side BC such that PM ⊥ AB and PN ⊥ AC. Prove that BM x NP = CN x MP.



Solution:-

From the question it is given that, ABC is a triangle in which AB = AC.

P is a point on the side BC such that PM ⊥ AB and PN ⊥ AC.

We have to prove that, BM x NP = CN x MP

Consider the ΔABC

AB = AC … [from the question]

∠B = ∠C … [angles opposite to equal sides]

Then, consider ΔBMP and ΔCNP

∠M = ∠N

Therefore, ΔBMP ~ ΔCNP

So, BM/CN = MP/NP

By cross multiplication we get,

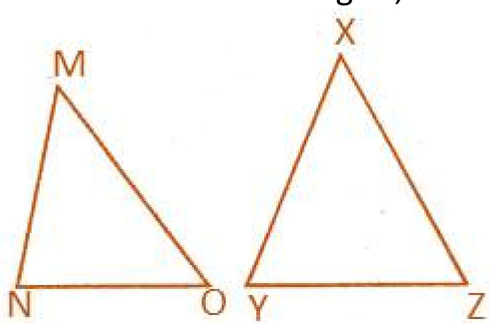
BM x NP = CN x MP

Hence it is proved.

12. Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

Solution:-

Consider the two triangles, ΔMNO and ΔXYZ



From the question it is given that, two triangles are similar triangles

So, ΔMNO ~ ΔXYZ

If two triangles are similar, the corresponding angles are equal and their corresponding sides are proportional.

MN/XY = NO/YZ = MO/XZ

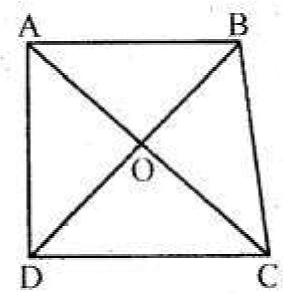
Perimeter of ΔMNO = MN + NO + MO

Perimeter of ΔXYZ = XY + YZ + XZ

Therefore, (MN/XY = NO/YZ = MO/XZ) = (MN/XY + NO/YZ + MO/XZ)

= Perimeter of ΔMNO/perimeter of ΔXYZ

13. In the adjoining figure, ABCD is a trapezium in which AB || DC. The diagonals AC and BD intersect at O. Prove that AO/OC = BO/OD



Using the above result, find the values of x if OA = 3x – 19, OB = x – 4, OC = x – 3 and OD = 4.

Solution:-

From the given figure, ABCD is a trapezium in which AB || DC,

The diagonals AC and BD intersect at O.

So we have to prove that, AO/OC = BO/OD

Consider the ΔAOB and ΔCOD,

∠AOB = ∠COD … [vertically opposite angles]

∠OAB = ∠OCD

Therefore, ΔAOB ~ ΔCOD

So, OA/OC = OB/OD

Now by using above result we have to find the value of x if OA = 3x – 19, OB = x – 4, OC = x – 3 and OD = 4.

OA/OC = OB/OD

(3x – 19)/(x – 3) = (x - 4)/4

By cross multiplication we get,

(x - 3) (x - 4) = 4(3x - 19)

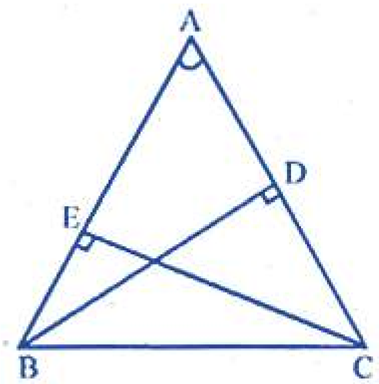
X^2 – 4x – 3x + 12 = 12x – 76

X^2 – 7x + 12 – 12x + 76 = 0

X^2 – 19x + 88 = 0

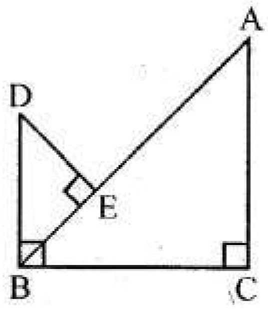
$X^2 - 8x - 11x + 88 = 0$
$X(x - 8) - 11(x - 8) = 0$
$(x - 8)(x - 11) = 0$
Take $x - 8 = 0$
$X = 8$
Then, $x - 11 = 0$
$X = 11$
Therefore, the value of x is 8 and 11.

14. In ΔABC, ∠A is acute. BD and CE are perpendicular on AC and AB respectively. Prove that AB x AE = AC x AD.
Solution:-
Consider the ΔABC,



So, we have to prove that, AB × AE = AC × AD
Now, consider the ΔADB and ΔAEC,
∠A = ∠A [common angle for both triangles]
∠ADB = ∠AEC [both angles are equal to 90^o]
ΔADB ~ ΔAEC
So, AB/AC = AD/AE
By cross multiplication we get,
AB × AE = AC × AD

15. In the given figure, DB ⊥ BC, DE ⊥ AB and AC ⊥ BC. Prove that BE/DE = AC/BC



Solution:-

From the figure, DB ⊥ BC, DE ⊥ AB and AC ⊥ BC

We have to prove that, BE/DE = AC/BC

Consider the ΔABC and ΔDEB,

∠C = 90°

∠A + ∠ABC = 90° [from the figure equation (i)]

Now in ΔDEB

∠DBE + ∠ABC = 90° [from the figure equation (ii)]

From equation (i), we get

∠A = ∠DBE

Then, in ΔABC and ΔDBE

∠C = ∠E [both angles are equal to 90°]

So, ΔABC ~ ΔDBE

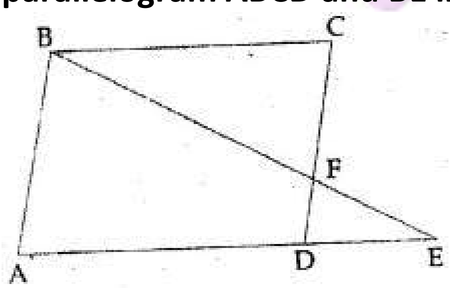
Therefore, AC/BE = BC/DE

By cross multiplication, we get

AC/BC = BE/DE

16.

(a) In the figure (1) given below, E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. show that ΔABE ~ ΔCFB.



Solution:-

From the figure, ABCD is a parallelogram,

Then, E is a point on AD and produced and BE intersects CD at F.

We have to prove that ΔABE ~ ΔCFB

Consider ΔABE and ΔCFB

∠A = ∠C [opposite angles of a parallelogram]

∠ABE = ∠BFC [alternate angles are equal]

ΔABE ~ ΔCFB

(b) In the figure (2) given below, PQRS is a parallelogram; PQ = 16 cm, QR = 10 cm. L is a point on PR such that RL : LP = 2 : 3. QL produced meets RS at M and PS produced at N.

(i) Prove that triangle RLQ is similar to triangle PLN. Hence, find PN.

Solution:-

From the question it is give that,

Consider the ΔRLQ and ΔPLN,

∠RLQ = ∠NLP [vertically opposite angles are equal]

∠RQL = ∠LNP [alternate angle are equal]

Therefore, ΔRLQ ~ ΔPLN

So, QR/PN = RL/LP = 2/3

QR/PN = 2/3

10/PN = 2/3

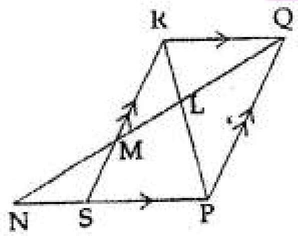
PN = (10 × 3)/2

PN = 30/2

PN = 15 cm

Therefore, PN = 15 cm

(ii) Name a triangle similar to triangle RLM. Evaluate RM.



Solution:-

From the figure,

Consider ΔRLM and ΔQLP

Then, ∠RLM = ∠QLP [vertically opposite angles are equal]

∠LRM = ∠LPQ [alternate angles are equal]

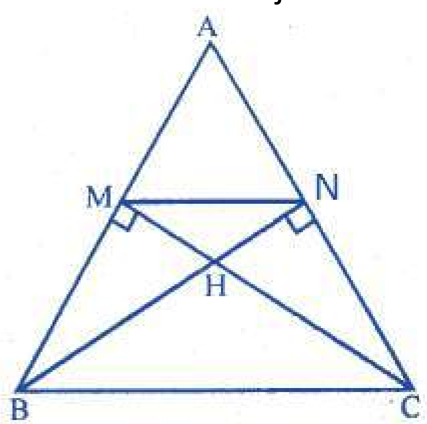
Therefore, ΔRLM ~ ΔQLP

Then, RM/PQ = RL/LP = 2/3

So, RM/16 = 2/3

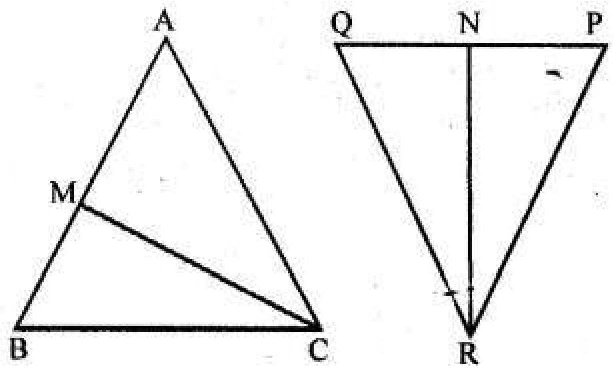
RM = (16 × 2)/3
RM = 32/3
RM = $10\frac{2}{3}$

17. The altitude BN and CM of ΔABC meet at H. Prove that
(i) CN × HM = BM × HN
(ii) HC/HB = √[(CN × HN)/(BM × HM)]
(iii) ΔMHN ~ ΔBHC
Solution:-
Consider the ΔABC,
Where, the altitude BN and CM of ΔABC meet at H.
and construction: join MN



(i) We have to prove that, CN × HM = BM × HN
In ΔBHM and ΔCHN
∠BHM = ∠CHN [because vertically opposite angles are equal]
∠M = ∠N [both angles are equal to 90°]
Therefore, ΔBHM ~ ΔCHN
So, HM/HN = BM/CN = HB/HC
Then, by cross multiplication we get
CN × HM = BM × HN
(ii) Now, HC/HB = √(HN × CN)/(HM × BM)
= √(CN × HN)/(BM × HM)
Because, M and N divide AB and AC in the same ratio.
(iii) Now consider ΔMHN and ΔBHC
∠MHN = ∠BHC [because vertically opposite angles are equal]
∠MNH = ∠HBC [because alternate angles are equal]
Therefore, ΔMHN ~ ΔBHC

18. In the given figure, CM and RN are respectively the medians of ΔABC and ΔPQR. If ΔABC ~ ΔPQR, prove that:
(i) ΔAMC ~ ΔPQR
(ii) CM/RN = AB/PQ
(iii) ΔCMB ~ ΔRNQ

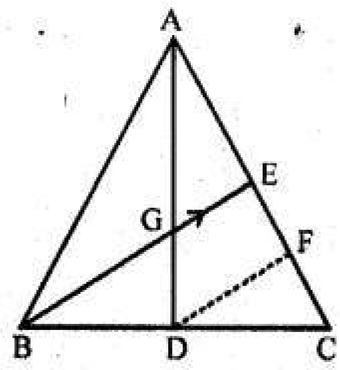


Solution:-
From the given figure it is given that, CM and RN are respectively the medians of ΔABC and ΔPQR.
(i) We have to prove that, ΔAMC ~ ΔPQR
Consider the ΔABC and ΔPQR
As ΔABC ~ ΔPQR
∠A = ∠P, ∠B = ∠Q and ∠C = ∠R
And also corresponding sides are proportional
AB/PQ = BC/QR = CA/RP
Then, consider the ΔAMC and ΔPNR,
∠A = ∠P
AC/PR = AM/PN
Because, AB/PQ = ½ AB/½PQ
AB/PQ = AM/PN
Therefore, ΔAMC ~ ΔPNR
(ii) From solution(i) CM/RN = AM/PN
CM/RN = 2AM/2PN
CM/RN = AB/PQ
(iii)Now consider the ΔCMB and ΔRNQ
∠B = ∠Q
BC/QP = BM/QN
Therefore, ΔCMB ~ ΔRNQ

19. In the adjoining figure, medians AD and BE of ΔABC meet at the point G, and DF is drawn parallel to BE. Prove that

(i) EF = FC

(ii) AG : GD = 2 : 1



Solution:-

From the figure it is given that, medians AD and BE of ΔABC meet at the point G, and DF is drawn parallel to BE.

(i) We have to prove that, EF = FC

From the figure, D is the midpoint of BC and also DF parallel to BE.

So, F is the midpoint of EC

Therefore, EF = FC

= ½ EC

EF = ½ AE

(ii) Now consider the ΔAGE and ΔADF

Then, (BG or GE) ||DF

Therefore, ΔAGE ~ ΔADF

So, AG/GD = AE/EF

AG/GD = 1/½

AG/GD = 1 × (2/1)

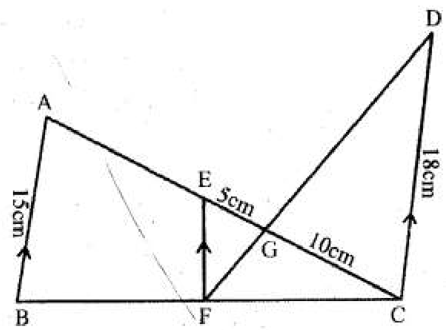
Therefore, AG: GD = 2: 1

20.

(a) In the figure given below, AB, EF and CD are parallel lines. Given that AB =15 cm, EG = 5 cm, GC = 10 cm and DC = 18 cm. Calculate

(i) EF

(ii) AC.



Solution:-

From the figure it is given that, AB, EF and CD are parallel lines.

(i) Consider the ΔEFG and ΔCGD

∠EGF = ∠CGD [Because vertically opposite angles are equal]

∠FEG = ∠GCD [alternate angles are equal]

Therefore, ΔEFG ~ ΔCGD

Then, EG/GC = EF/CD

5/10 = EF/18

EF = (5 × 18)/10

Therefore, EF = 9 cm

(ii) Now, consider the ΔABC and ΔEFC

EF ||AB

So, ΔABC ~ ΔEFC

Then, AC/EC = AB/EF

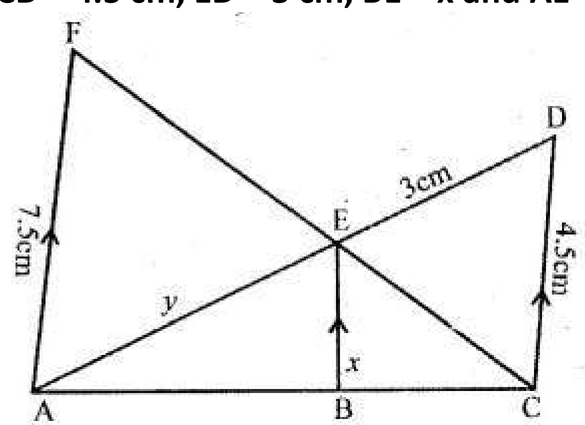
AC/(5 + 10) = 15/9

AC/15 = 15/9

AC = (15 × 15)/9

Therefore, AC = 25 cm

(b) In the figure given below, AF, BE and CD are parallel lines. Given that AF = 7.5 cm, CD = 4.5 cm, ED = 3 cm, BE = x and AE = y. Find the values of x and y.



Solution:-

From the figure, AF, BE and CD are parallel lines.

Consider the ΔAEF and ΔCED

∠AEF and ∠CED [because vertically opposite angles are equal]

∠F = ∠C [alternate angles are equal]

Therefore, ΔAEF ~ ΔCED

So, AF/CD = AE/ED

7.5/4.5 = y/3

By cross multiplication,

y = (7.5 × 3)/4.5

y = 5 cm

So, similarly in ΔACD, BE ||CD

Therefore, ΔABE ~ ΔACD

EB/CD = AE/AD

x/CD = y/y + 3

x/4.5 = 5/(5 + 3)

x/4.5 = 5/8

x = (4.5 × 5)/8

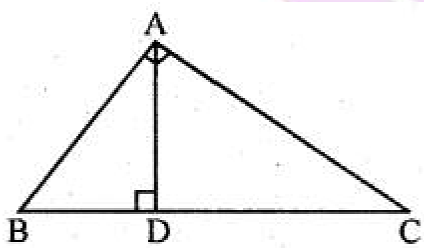
x = 22.5/8

x = 225/80

x = 45/16

x = $2\frac{13}{16}$

21. In the given figure, ∠A = 90° and AD ⊥ BC If BD = 2 cm and CD = 8 cm, find AD.



Solution:-

From the figure, consider ΔABC,

So, ∠A = 90°

And AD ⊥ BC

∠BAC = 90°

Then, ∠BAD + ∠DAC = 90° … [equation (i)]

Now, consider ΔADC

$\angle ADC = 90^o$

So, $\angle DCA + \angle DAC = 90^o$ … [equation (ii)]

From equation (i) and equation (ii)

We have,

$\angle BAD + \angle DAC = \angle DCA + \angle DAC$

$\angle BAD = \angle DCA$ … [equation (iii)]

So, from ΔBDA and ΔADC

$\angle BDA = \angle ADC$ … [both the angles are equal to 90^o]

$\angle BAD = \angle DCA$ … [from equation (iii)]

Therefore, $\Delta BDA \sim \Delta ADC$

BD/AD = AD/DC = AB/AC

Because, corresponding sides of similar triangles are proportional

BD/AD = AD/DC

By cross multiplication we get,

$AD^2 = BD \times CD$

$AD^2 = 2 \times 8 = 16$

$AD = \sqrt{16}$

AD = 4

22. A 15 metres high tower casts a shadow of 24 metres long at a certain time and at the same time, a telephone pole casts a shadow 16 metres long. Find the height of the telephone pole.

Solution:-

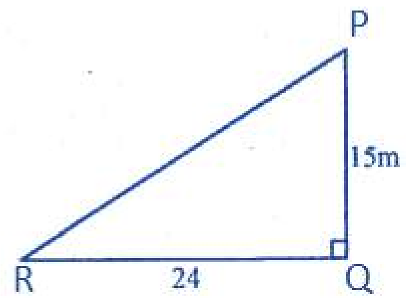
From the question it is given that,

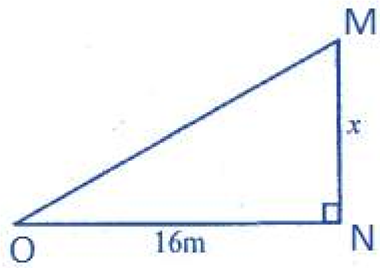
Height of a tower PQ = 15m

It's shadow QR = 24 m

Let us assume the height of a telephone pole MN = x

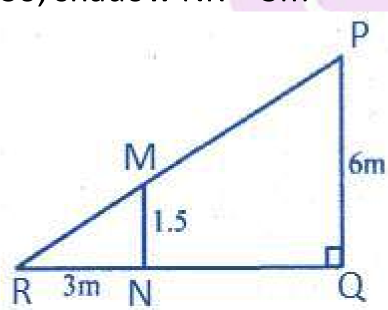
It's shadow NO = 16 m





Given, at the same time,
ΔPQR ~ ΔMNO
Therefore, PQ/MN = ON/RQ
15/x = 24/16
By cross multiplication we get,
x = (15 × 16)/24
x = 240/24
x = 10
Therefore, height of pole = 10 m.

23. A street light bulb is fixed on a pole 6 m above the level of street. If a woman of height casts a shadow of 3 m, find how far she is away from the base of the pole?
Solution:-
From the question it is given that,
Height of pole (PQ) = 6m
Height of a woman (MN) = 1.5m
So, shadow NR = 3m

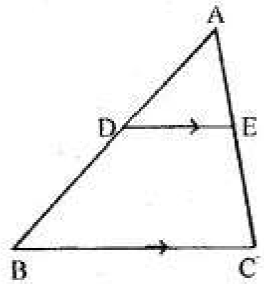


Therefore, pole and woman are standing in the same line
PM ||MR
ΔPRQ ~ ΔMNR
So, RQ/RN = PQ/MN
(3 + x)/3 = 6/1.5
(3 + x)/3 = 60/15
(3 + x)/3 = 4/1

$(3 + x) = 12$
$X = 12 – 3$
$X = 9m$
Therefore, women is 9m away from the pole.

EXERCISE 13.2

1. (a) In the figure (i) given below if DE || BG, AD = 3 cm, BD = 4 cm and BC = 5 cm. Find (i) AE : EC (ii) DE.



Solution:-

From the figure,

DE || BG, AD = 3 cm, BD = 4 cm and BC = 5 cm

(i) AE: EC

So, AD/BD = AE/EC

AE/EC = AD/BD

AE/EC = ¾

AE: EC = 3: 4

(ii) consider ΔADE and ΔABC

∠D = ∠B

∠E = ∠C

Therefore, ΔADE ~ ΔABC

Then, DE/BC = AD/AB

DE/5 = 3/(3 + 4)

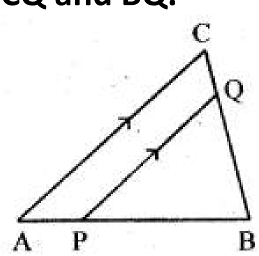
DE/5 = 3/7

DE = (3 × 5)/7

DE = 15/7

DE = $2\frac{1}{7}$

(b) In the figure (ii) given below, PQ || AC, AP = 4 cm, PB = 6 cm and BC = 8 cm. Find CQ and BQ.



Solution:-

From the figure,

PQ || AC, AP = 4 cm, PB = 6 cm and BC = 8 cm

∠BQP = ∠BCA … [because alternate angles are equal]

Also, ∠B = ∠B … [common for both the triangles]

Therefore, ΔABC ~ ΔBPQ

Then, BQ/BC = BP/AB = PQ/AC

BQ/BC = 6/(6 + 4) = PQ/AC

BQ/BC = 6/10 = PQ/AC

BQ/8 = 6/10 = PQ/AC … [because BC = 8 cm given]

Now, BQ/8 = 6/10

BQ = (6/10) ×8

BQ = 48/10

BQ = 4.8 cm

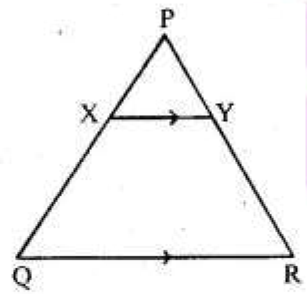
Also, CQ = BC – BQ

CQ = (8 – 4.8) cm

CQ = 3.2cm

Therefore, CQ = 3.2 cm and BQ = 4.8 cm

(c) In the figure (iii) given below, if XY || QR, PX = 1 cm, QX = 3 cm, YR = 4.5 cm and QR = 9 cm, find PY and XY.



Solution:-

From the figure,

XY || QR, PX = 1 cm, QX = 3 cm, YR = 4.5 cm and QR = 9 cm,

So, PX/QX = PY/YR

1/3 = PY/4.5

By cross multiplication we get,

(4.5 × 1)/3 = PY

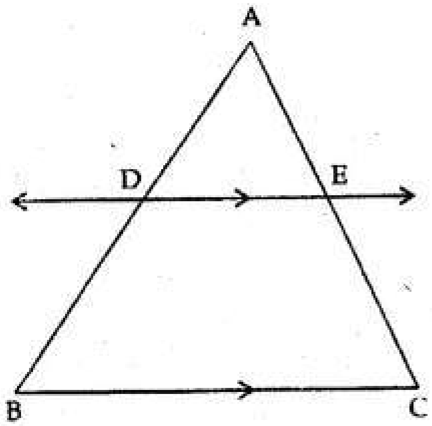
PY = 45/30

PY = 1.5

Then, ∠X = ∠Q

∠Y = ∠R
So, ΔPXY ~ ΔPQR
Therefore, XY/QR = PX/PQ
XY/9 = 1/(1 + 3)
XY/9 = ¼
XY = 9/4
XY = 2.25

2. In the given figure, DE || BC.
(i) If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.
(ii) If DB = x – 3, AB = 2x, EC = x – 2 and AC = 2x + 3, find the value of x.

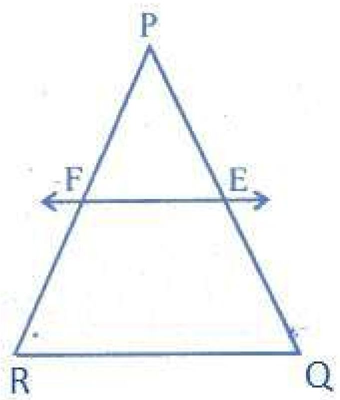


Solution:-
(i) From the figure, it is given that,
Consider the ΔABC,
AD/DB = AE/EC
x/(x - 2) = (x + 2)/(x - 1)
By cross multiplication we get,
X(x - 1) = (x - 2) (x + 2)
$x^2 – x = x^2 – 4$
-x = -4
x = 4

(ii) From the question it is given that,
DB = x – 3, AB = 2x, EC = x – 2 and AC = 2x + 3
Consider the ΔABC,
AD/DB = AE/EC
2x/(x - 2) = (2x + 3)/(x - 3)

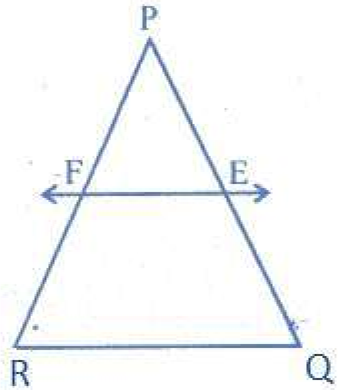
By cross multiplication we get,
$2x(x - 2) = (2x + 3)(x - 3)$
$2x^2 – 4x = 2x^2 – 6x + 3x – 9$
$2x^2 – 4x – 2x^2 + 6x – 3x = -9$
$-7x + 6x = -9$
$-x = -9$
$x = 9$

3. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR:
(i) PE = 3.9 cm, EQ = 3 cm, PF = 8 cm and RF = 9 cm.
Solution:-
From the given dimensions,
Consider the ΔPQR



So, PE/EQ = 3.9/3
= 39/30
= 13/10
Then, PF/FR = 8/9
By comparing both the results,
13/10 ≠ 8/9
Therefore, PE/EQ ≠ PF/FR
So, EF is not parallel to QR

(ii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.
Solution:-
From the dimensions given in the question,
Consider the ΔPQR

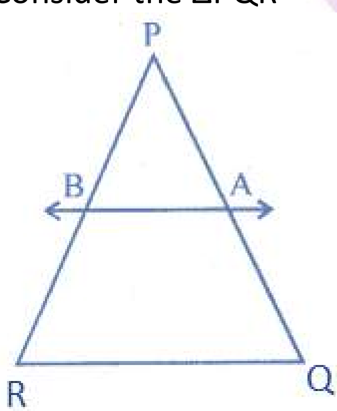


So, PQ/PE = 1.28/0.18
= 128/18
= 64/9
Then, PR/PF = 2.56/0.36
= 256/36
= 64/9
By comparing both the results,
64/9 = 64/9
Therefore, PQ/PE = PR/PF
So, EF is parallel to QR.

4. A and B are respectively the points on the sides PQ and PR of a triangle PQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB || QR? Give reasons for your answer.

Solution:-

From the dimensions given in the question,
Consider the ΔPQR



So, PQ/PA = 12.5/5
= 2.5/1

PR/PB = (PB + BR)/PB

= (4 + 6)/4

= 10/4

= 2.5

By comparing both the results,

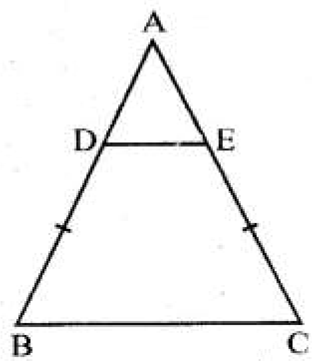
2.5 = 2.5

Therefore, PQ/PA = PR/PB

So, AB is parallel to QR.

5.

(a) In figure (i) given below, DE || BC and BD = CE. Prove that ABC is an isosceles triangle.



Solution:-

From the question it is given that,

DE || BC and BD = CE

So, we have to prove that ABC is an isosceles triangle.

Consider the triangle ABC,

AD/DB = AE/EC

Given, DB = EC … [equation (i)]

Then, AD = AE … [equation (ii)]

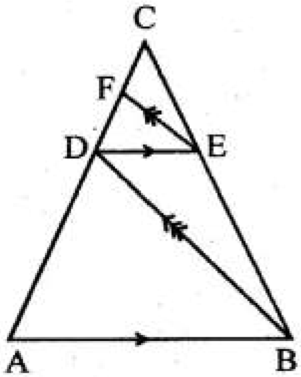
By adding equation (i) and equation (ii) we get,

AD + DB = AE + EC

So, AB = AC

Therefore, ΔABC is an isosceles triangle.

(b) In figure (ii) given below, AB || DE and BD || EF. Prove that DC^2 = CF x AC.



Solution:-

From the figure it is given that, AB || DE and BD || EF.

We have to prove that, $DC^2 = CF \times AC$

Consider the ΔABC,

DC/CA = CE/CB … [equation (i)]

Now, consider ΔCDE

CF/CD = CE/CB … [equation (ii)]

From equation (i) and equation (ii),

DC/CA = CF/CD

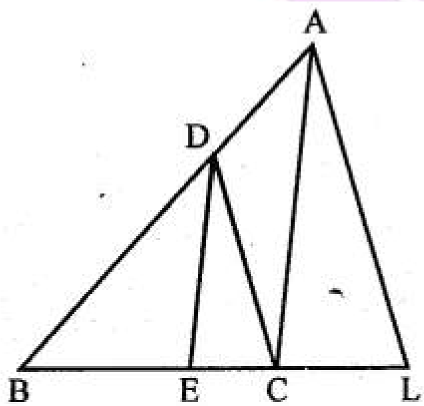
DC/AC = CF/DC

By cross multiplication we get,

$DC^2 = CF \times AC$

6.

(a) In the figure (i) given below, CD || LA and DE || AC. Find the length of CL if BE = 4 cm and EC = 2 cm.



Solution:-

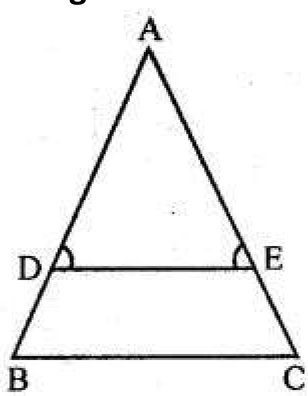
From the given figure, CD || LA and DE || AC,

Consider the ΔBCA,

BE/BC = BD/BA

By using the corollary of basic proportionality theorem,
BE/(BE + EC) = BD/AB
4/(4 + 2) = BD/AB … [equation (i)]
Then, consider the ΔBLA
BC/BL = BD/AB
By using the corollary of basic proportionality theorem,
6/(6 + CL) = BD/AB … [equation (ii)]
Now, combining the equation (i) and equation (ii), we get
6/(6 + CL) = 4/6
By cross multiplication we get,
6 x 6 = 4 x (6 + CL)
24 + 4CL = 36
4CL = 36 – 24
CL = 12/4
CL = 3 cm
Therefore, the length of CL is 3 cm.

(b) In the give figure, ∠D = ∠E and AD/BD = AE/EC. Prove that BAC is an isosceles triangle.



Solution:-
From the given figure, ∠D = ∠E and AD/BD = AE/EC,
We have to prove that, BAC is an isosceles triangle
So, consider the ΔADE
∠D = ∠E … [from the question]
AD = AE … [sides opposite to equal angles]
Consider the ΔABC,
Then, AD/DB = AE/EC … [equation (i)]
Therefore, DE parallel to BC

Because AD = AE

DB = EC … [equation (ii)]

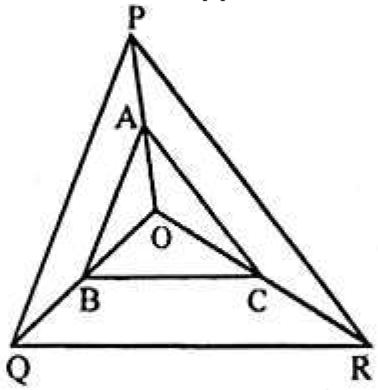
By adding equation (i) and equation (ii) we get,

AD + DB = AE + EC

AB = AC

Therefore, ΔABC is an isosceles triangle.

7. In the adjoining given below, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. show that BC || QR.



Solution:-

Consider the ΔPOQ

AB || PQ … [given]

So, OA/AP = OB/BQ … [equation (i)]

Then, consider the ΔOPR

AC || PR

OA/AP = OC/CR … [equation (ii)]

Now by comparing both equation (i) and equation (ii),

OB/BQ = OC/CR

Then, in ΔOQR

OB/BQ = OC/CR

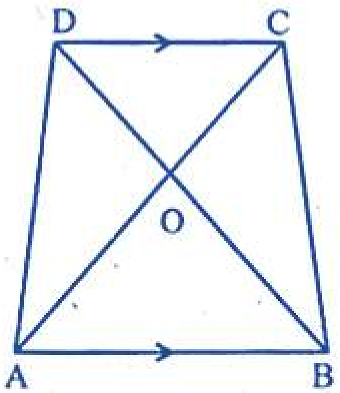
Therefore, BC || QR

8. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at O. Using Basic Proportionality theorem, prove that AO/BO = CO/DO

Solution:-

From the question it is given that,

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at O



Now consider the ΔOAB and ΔOCD,

∠AOB = ∠COD [because vertically opposite angles are equal]

∠OBA = ∠ODC [because alternate angles are equal]

∠OAB = ∠OCD [because alternate angles are equal]

Therefore, ΔOAB ~ ΔOCD

Then, OA/OC = OB/OD

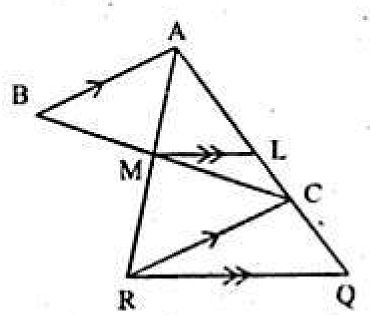
AO/OB = CO/DO … [by alternate angles]

9.

(a) In the figure (1) given below, AB || CR and LM || QR.

(i) Prove that BM/MC = AL/LQ

(ii) Calculate LM : QR, given that BM : MC = 1 : 2.



(1)

Solution:-

From the question it is given that, AB || CR and LM || QR

(i) We have to prove that, BM/MC = AL/LQ

Consider the ΔARQ

LM || QR … [from the question]

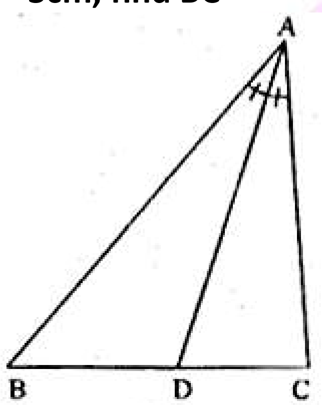
So, AM/MR = AL/LQ … [equation (i)]

Now, consider the ΔAMB and ΔMCR

∠AMB = ∠CMR … [because vertically opposite angles are equal]
∠MBA = ∠MCR … [because alternate angles are equal]
Therefore, AM/MR = BM/MC … [equation (ii)]
From equation (i) and equation (ii) we get,
BM/MR = AL/LQ

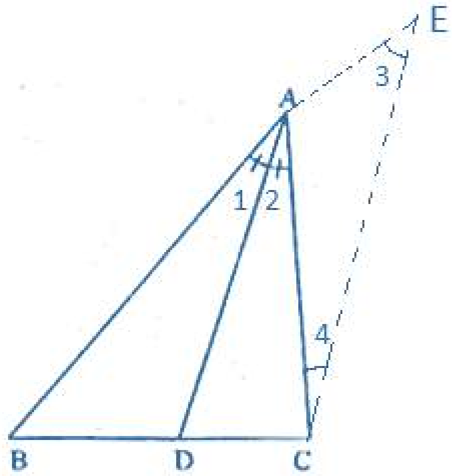
(ii) Given, BM : MC = 1 : 2
AM/MR = BM/MC
AM/MR = ½ … [equation (iii)]
LM || QR … [given from equation]
AM/MR = LM/QR … [equation (iv)]
AR/AM = QR/LM
(AM + MR)/AM = QR/LM
1 + MR/AM = QR/LM
1 + (2/1) = QR/LM
3/1 = QR/LM
LM/QR = 1/3
Therefore, the ratio of LM: QR is 1: 3.

(b) In the figure (2) given below AD is bisector of ∠BAC. If AB = 6 cm, AC = 4 cm and BD = 3cm, find BC



(2)

Solution:-
From the question it is given that,
AD is bisector of ∠BAC
AB = 6 cm, AC = 4 cm and BD = 3cm
Construction, from C draw a straight line CE parallel to DA and join AE



∠1 = ∠2 … [equation (i)]

By construction CE || DE

So, ∠2 = ∠4 … [because alternate angles are equal] [equation (ii)]

Again by construction CE || DE

∠1 = ∠3 … [because corresponding angles are equal] [equation (iii)]

By comparing equation (i), equation (ii) and equation(iii) we get,

∠3 = ∠4

So, AC = AE … [equation (iv)]

Now, consider the ΔBCE,

CE || DE

BD/DC = AB/AE

BD/DC = AB/AC

3/DC = 6/4

By cross multiplication we get,

3 × 4 = 6 × DC

DC = (3 × 4)/6

DC = 12/6

DC = 2

Therefore, BC = BD + DC

= 3 + 2

= 5 cm

EXERCISE 13.3

1. Given that Δs ABC and PQR are similar.
Find:
(i) The ratio of the area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1 : 3.
(ii) the ratio of their corresponding sides if area of ΔABC : area of ΔPQR = 25 : 36.
Solution:-
From the question it is given that,
(i) The area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1 : 3
Then, ΔABC ~ ΔPQR
area of ΔABC/area of ΔPQR = BC^2/QR^2
So, BC : QR = 1 : 3
Therefore, ΔABC/area of ΔPQR = $1^2/3^2$
= 1/9
Hence the ratio of the area of ΔABC to the area of ΔPQR is 1: 9
(ii) The area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 25 : 36
Then, ΔABC ~ ΔPQR
area of ΔABC/area of ΔPQR = BC^2/QR^2
area of ΔABC/area of ΔPQR = BC^2/QR^2 = 25/36
= $(BC/QR)^2 = (5/6)^2$
BC/QR = 5/6
Hence the ratio of their corresponding sides is 5 : 6

2. ΔABC ~ DEF. If area of ΔABC = 9 sq. cm., area of ΔDEF =16 sq. cm and BC = 2.1 cm., find the length of EF.
Solution:-
From the question it is given that,
ΔABC ~ DEF
Area of ΔABC = 9 sq. cm
Area of ΔDEF =16 sq. cm
We know that,
area of ΔABC/area of ΔDEF = BC^2/EF^2
area of ΔABC/area of ΔDEF = BC^2/EF^2
9/16 = BC^2/EF^2
9/16 = $(2.1)^2/x^2$
2.1/x = √9/√16

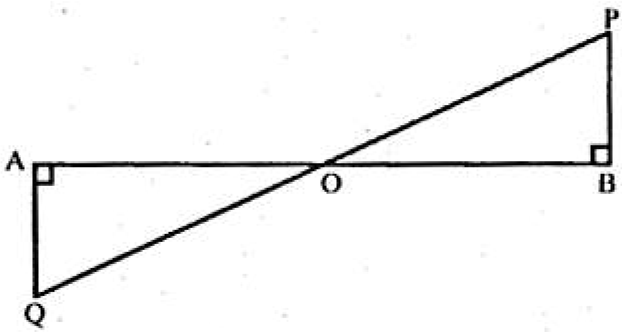
$2.1/x = 3/4$
By cross multiplication we get,
$2.1 \times 4 = 3 \times x$
$8.4 = 3x$
$x = 8.4/3$
$x = 2.8$
Therefore, EF = 2.8 cm

3. ΔABC ~ ΔDEF. If BC = 3 cm, EF = 4 cm and area of ΔABC = 54 sq. cm. Determine the area of ΔDEF.
Solution:-
From the question it is given that,
ΔABC ~ ΔDEF
BC = 3 cm, EF = 4 cm
Area of ΔABC = 54 sq. cm.
We know that,
Area of ΔABC/ area of ΔDEF = BC^2/EF^2
54/area of ΔDEF = $3^2/4^2$
54/area of ΔDEF = 9/16
By cross multiplication we get,
Area of ΔDEF = $(54 \times 16)/9$
$= 6 \times 16$
= 96 cm

4. The area of two similar triangles are 36 cm² and 25 cm². If an altitude of the first triangle is 2.4 cm, find the corresponding altitude of the other triangle.
Solution:-
From the question it is given that,
The area of two similar triangles are 36 cm^2 and 25 cm^2.
Let us assume ΔPQR ~ ΔXYZ, PM and XN are their altitudes.
So, area of ΔPQR = 36 cm^2
Area of ΔXYZ = 25 cm^2
PM = 2.4 cm
Assume XN = a
We know that,
area of ΔPQR/area of ΔXYZ = PM^2/XN^2
$36/25 = (2.4)^2/a^2$

By cross multiplication we get,
$36a^2 = 25 (2.4)^2$
$a^2 = 5.76 \times 25/36$
$a^2 = 144/36$
$a^2 = 4$
$a = \sqrt{4}$
a = 2 cm
Therefore, altitude of the other triangle XN = 2 cm.

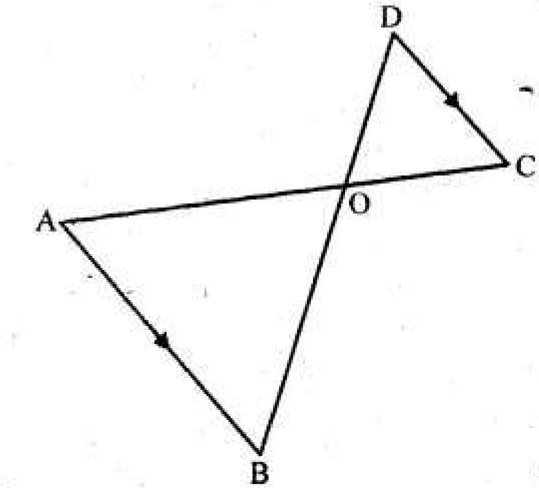
5.
(a) In the figure, (i) given below, PB and QA are perpendiculars to the line segment AB. If PO = 6 cm, QO = 9 cm and the area of ΔPOB = 120 cm², find the area of ΔQOA.



Solution:-
From the question it is given that, PO = 6 cm, QO = 9 cm and the area of ΔPOB = 120 cm²
From the figure,
Consider the ΔAOQ and ΔBOP,
∠OAQ = ∠OBP … [both angles are equal to 90°]
∠AOQ = ∠BOP … [because vertically opposite angles are equal]
Therefore, ΔAOQ ~ ΔBOP
Then, area of ΔAOQ/area of ΔBOP = OQ^2/PO^2
Area of ΔAOQ/120 = $9^2/6^2$
Area of ΔAOQ/120 = 81/36
Area of ΔAOQ = (81 × 120)/36
Area of ΔAOQ = 270 cm^2

b) In the figure (ii) given below, AB || DC. AO = 10 cm, OC = 5cm, AB = 6.5 cm and OD = 2.8 cm.
(i) Prove that ΔOAB ~ ΔOCD.
(ii) Find CD and OB.

(iii) Find the ratio of areas of ΔOAB and ΔOCD.



Solution:-

From the question it is given that,

AB || DC. AO = 10 cm, OC = 5cm, AB = 6.5 cm and OD = 2.8 cm

(i) We have to prove that, ΔOAB ~ ΔOCD

So, consider the ΔOAB and ΔOCD

∠AOB = ∠COD … [because vertically opposite angles are equal]

∠OBA = ∠OCD … [because alternate angles are equal]

Therefore, ΔOAB ~ ΔOCD … [from AAA axiom]

(ii) Consider the ΔOAB and ΔOCD

OA/OC = OB/OD = AB/CD

Now consider OA/OC = OB/OD

10/5 = OB/2.8

OB = (10 × 2.8)/5

OB = 2 × 2.8

OB = 5.6 cm

Then, consider OA/OC = AB/CD

10/5 = 6.5/CD

CD = (6.5 × 5)/10

CD = 32.5/10

CD = 3.25 cm

(iii) We have to find the ratio of areas of ΔOAB and ΔOCD.

From (i) we proved that, ΔOAB ~ ΔOCD

Then, area of (ΔOAB)/area of ΔOCD

$AB^2/CD^2 = (6.5)^2/(3.25)^2$

$= (6.5 \times 6.5)/(3.25 \times 3.25)$

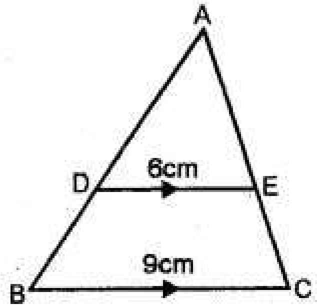
= 2 × 2/1

= 4/1

Therefore, the ratio of areas of ΔOAB and ΔOCD = 4: 1.

6.

(a) In the figure (i) given below, DE || BC. If DE = 6 cm, BC = 9 cm and area of ΔADE = 28 sq. cm, find the area of ΔABC.



Solution:-

From the question it is given that,

DE || BC, DE = 6 cm, BC = 9 cm and area of ΔADE = 28 sq. cm

From the fig, ∠D = ∠B and ∠E = ∠C … [corresponding angles are equal]

Now consider the ΔADE and ΔABC,

∠A = ∠A … [common angles for both triangles]

Therefore, ΔADE ~ ΔABC

Then, area of ΔADE/area of ΔABC = $(DE)^2/(BC)^2$

28/area of ΔABC = $(6)^2/(9)^2$

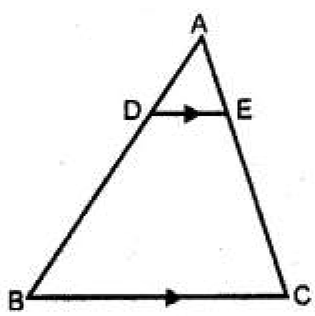
28/area of ΔABC = 36/81

area of ΔABC = (28 × 81)/36

area of ΔABC = 2268/36

area of ΔABC = 63 cm^2

(b) In the figure (ii) given below, DE || BC and AD : DB = 1 : 2, find the ratio of the areas of ΔADE and trapezium DBCE.



Solution:-

From the question it is given that, DE || BC and AD : DB = 1 : 2,

∠D = ∠B, ∠E = ∠C … [corresponding angles are equal]

Consider the ΔADE and ΔABC,

∠A = ∠A … [common angles for both triangles]

Therefore, ΔADE ~ ΔABC

But, AD/DB = ½

Then, DB/AD = 2/1

Now, adding 1 for both side LHS and RHS,

(DB/AD) + 1 = (2/1) + 1

(DB + AD)/AD = (2 + 1)

Therefore, ΔADE ~ ΔABC

Then, area of ΔADE/area of ΔABC = AD^2/AB^2

Area of ΔADE/area of ΔABC = $(1/3)^2$

Area of ΔADE/area of ΔABC = 1/9

Area of ΔABC = 9 area of ΔADE

Area of trapezium DBCE

Area of ΔABC – area of ΔADE

9 area of ΔADE – area of ΔADE

8 area of ΔADE

Therefore, area of ΔADE/area of trapezium = 1/8

Then area of ΔADE : area of trapezium DBCE = 1: 8

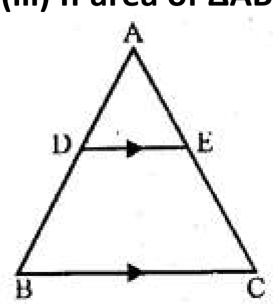
7.

In the given figure, DE || BC.

(i) Prove that ΔADE and ΔABC are similar.

(ii) Given that AD = ½ BD, calculate DE if BC = 4.5 cm.

(iii) If area of ΔABC = 18cm^2, find the area of trapezium DBCE



Solution:-

(i) From the question it is given that, DE || BC

We have to prove that, ΔADE and ΔABC are similar

∠A = ∠A … [common angle for both triangles]

∠ADE = ∠ABC … [because corresponding angles are equal]

Therefore, ΔADE ~ ΔABC … [AA axiom]

(ii) From (i) we proved that, ΔADE ~ ΔABC

Then, AD/AB = AB/AC = DE/BC

So, AD/(AD + BD) = DE/BC

(½ BD)/ ((½BD) + BD) = DE/4.5

(½ BD)/ ((3/2)BD) = DE/4.5

½ × (2/3) = DE/4.5

1/3 = DE/4.5

Therefore, DE = 4.5/3

DE = 1.5 cm

(iii) From the question it is given that, area of ΔABC = 18 cm^2

Then, area of ΔADE/area of ΔABC = DE^2/BC^2

area of ΔADE/18 = $(DE/BC)^2$

area of ΔADE/18 = $(AD/AB)^2$

area of ΔADE/18 = $(1/3)^2$ = 1/9

area of ΔADE = 18 × 1/9

area of ΔADE = 2

So, area of trapezium DBCE = area of ΔABC – area of ΔADE

= 18 – 2

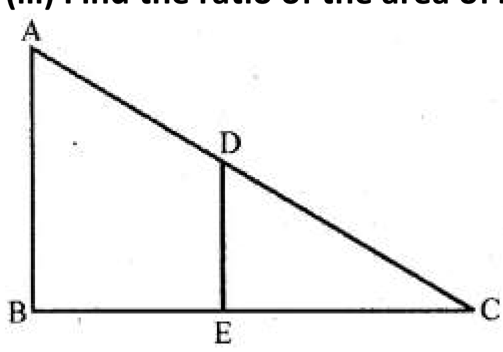
= 16 cm^2

8. In the given figure, AB and DE are perpendicular to BC.

(i) Prove that ΔABC ~ ΔDEC

(ii) If AB = 6 cm: DE = 4 cm and AC = 15 cm, calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC.



Solution:-

(i) Consider the ΔABC and ΔDEC,

∠ABC = ∠DEC … [both angles are equal to 90^o]

∠C = ∠C … [common angle for both triangles]

Therefore, ΔABC ~ ΔDEC … [by AA axiom]

(ii) AC/CD = AB/DE

Corresponding sides of similar triangles are proportional

15/CD = 6/4

CD = (15 × 4)/6

CD = 60/6

CD = 10 cm

(iii) we know that, area of ΔABC/area of ΔDEC = AB^2/DE^2

area of ΔABC/area of ΔDEC = $6^2/4^2$

area of ΔABC/area of ΔDEC = 36/16

area of ΔABC/area of ΔDEC = 9/4

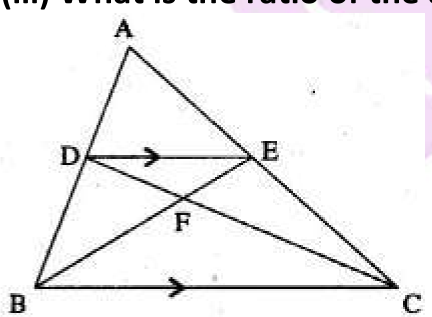
Therefore, the ratio of the area of ΔABC : area of ΔDEC is 9 : 4.

9. In the adjoining figure, ABC is a triangle. DE is parallel to BC and AD/DB = 3/2,

(i) Determine the ratios AD/AB, DE/BC

(ii) Prove that ΔDEF is similar to ΔCBF. Hence, find EF/FB.

(iii) What is the ratio of the areas of ΔDEF and ΔCBF?



Solution:-

(i) We have to find the ratios AD/AB, DE/BC,

From the question it is given that, AD/DB = 3/2

Then, DB/AD = 2/3

Now add 1 for both LHS and RHS we get,

(DB/AD) + 1 = (2/3) + 1

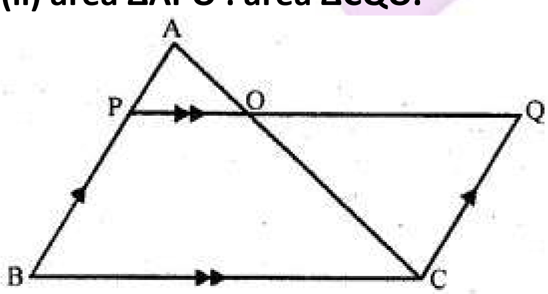
(DB + AD)/AD = (2 + 3)/3

From the figure (DB + AD) = AB
So, AB/AD = 5/3
Now, consider the ΔADE and ΔABC,
∠ADE = ∠B … [corresponding angles are equal]
∠AED = ∠C … [corresponding angles are equal]
Therefore, ΔADE ~ ΔABC … [by AA similarity]
Then, AD/AB = DE/BC = 3/5

(ii) Now consider the ΔDEF and ΔCBF
∠EDF = ∠BCF … [because alternate angles are equal]
∠DEF = ∠FBC … [because alternate angles are equal]
∠DFE = ∠ABFC … [because vertically opposite angles are equal]
Therefore, ΔDEF ~ ΔCBF
So, EF/FB = DE/BC = 3/5

(iii) we have to find the ratio of the areas of ΔDEF and ΔCBF,
We know that, Area of ΔDFE/Area of ΔBFC = DE^2/BC^2
Area of ΔDFE/Area of ΔBFC = $(DE/BC)^2$
Area of ΔDFE/Area of ΔBFC = $(3/5)^2$
Area of ΔDFE/Area of ΔBFC = 9/25
Therefore, the ratio of the areas of ΔDEF and ΔCBF is 9: 25.

10. In ΔABC, AP : PB = 2 : 3. PO is parallel to BC and is extended to Q so that CQ is parallel to BA. Find:
(i) area ΔAPO : area ΔABC.
(ii) area ΔAPO : area ΔCQO.

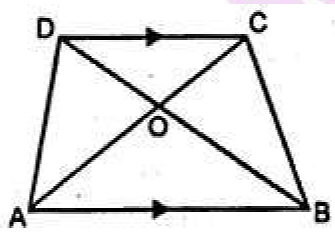


Solution:-
From the question it is given that,
PB = 2: 3
PO is parallel to BC and is extended to Q so that CQ is parallel to BA.
(i) we have to find the area ΔAPO: area ΔABC,

Then,
∠A = ∠A … [common angles for both triangles]
∠APO = ∠ABC … [because corresponding angles are equal]
Then, ΔAPO ~ ΔABC … [AA axiom]
We know that, area of ΔAPO/area of ΔABC = AP^2/AB^2
$= AP^2/(AP + PB)^2$
$= 2^2/(2 + 3)^2$
$= 4/5^2$
$= 4/25$
Therefore, area ΔAPO: area ΔABC is 4: 25

(ii) we have to find the area ΔAPO: area ΔCQO
Then, ∠AOP = ∠COQ … [because vertically opposite angles are equal]
∠APQ = ∠OQC … [because alternate angles are equal]
Therefore, area of ΔAPO/area of ΔCQO = AP^2/CQ^2
area of ΔAPO/area of ΔCQO = AP^2/PB^2
area of ΔAPO/area of ΔCQO = $2^2/3^2$
area of ΔAPO/area of ΔCQO = 4/9
Therefore, area ΔAPO: area ΔCQO is 4: 9

11.
(a) In the figure (i) given below, ABCD is a trapezium in which AB || DC and AB = 2 CD. Determine the ratio of the areas of ΔAOB and ΔCOD.



Solution:-
From the question it is given that,
ABCD is a trapezium in which AB || DC and AB = 2 CD,
Then, ∠OAB = ∠OCD … [because alternate angles are equal]
∠OBA = ∠ODC
Then, ΔAOB ~ ΔCOD
So, area of ΔAOB/area of ΔCOD = AB^2/CD^2
$= (2CD)^2/CD^2$ … [because AB = 2 CD]
$= 4CD^2/CD^2$

= 4/1

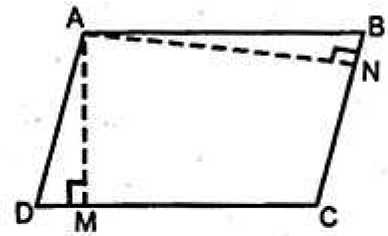
Therefore, the ratio of the areas of ΔAOB and ΔCOD is 4: 1.

(b) In the figure (ii) given below, ABCD is a parallelogram. AM ⊥ DC and AN ⊥ CB. If AM = 6 cm, AN = 10 cm and the area of parallelogram ABCD is 45 cm², find

(i) AB

(ii) BC

(iii) area of ΔADM : area of ΔANB.



Solution:-

From the question it is given that,

ABCD is a parallelogram, AM ⊥ DC and AN ⊥ CB

AM = 6 cm

AN = 10 cm

The area of parallelogram ABCD is 45 cm²

Then, area of parallelogram ABCD = DC × AM = BC × AN

45 = DC × 6 = BC × 10

(i) DC = 45/6

Divide both numerator and denominator by 3 we get,

= 15/2

= 7.5 cm

Therefore, AB = DC = 7.5 cm

(ii) BC × 10 = 45

BC = 45/10

BC = 4.5 cm

(iii) Now, consider ΔADM and ΔABN

∠D = ∠B … [because opposite angles of a parallelogram]

∠M = ∠N … [both angles are equal to 90^o]

Therefore, ΔADM ~ ΔABN

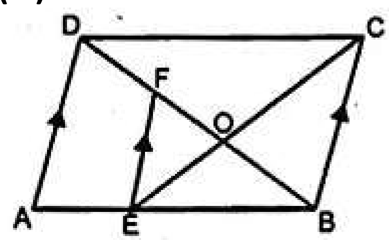
Therefore, area of ΔADM/area of ΔABN = AD^2/AB^2

= BC^2/AB^2

$= 4.5^2/7.5^2$
$= 20.25/56.25$
$= 2025/5625$
$= 81/225$
$= 9/25$

Therefore, area of ΔADM : area of ΔANB is 9: 25

(c) In the figure (iii) given below, ABCD is a parallelogram. E is a point on AB, CE intersects the diagonal BD at O and EF || BC. If AE : EB = 2 : 3, find
(i) EF : AD
(ii) area of ΔBEF : area of ΔABD
(iii) area of ΔABD : area of trapezium AFED
(iv) area of ΔFEO : area of ΔOBC.



Solution:-
From the question it is given that, ABCD is a parallelogram.
E is a point on AB, CE intersects the diagonal BD at O.
AE : EB = 2 : 3
(i) We have to find EF : AD
So, AB/BE = AD/EF
EF/AD = BE/AB
AE/EB = 2/3 … [given]
Now add 1 to both LHS and RHS we get,
(AE/EB) + 1 = (2/3) + 1
(AE + EB)/EB = (2 + 3)/3
AB/EB = 5/3
EB/AB = 3/5
Therefore, EF : AD is 3: 5

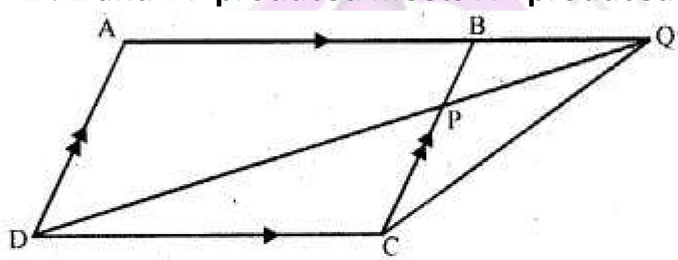
(ii) we have to find area of ΔBEF: area of ΔABD,
Then, area of ΔBEF/area of ΔABD = $(EF)^2/(AD)^2$
area of ΔBEF/area of ΔABD = $3^2/5^2$
$= 9/25$

Therefore, area of ΔBEF: area of ΔABD is 9: 25

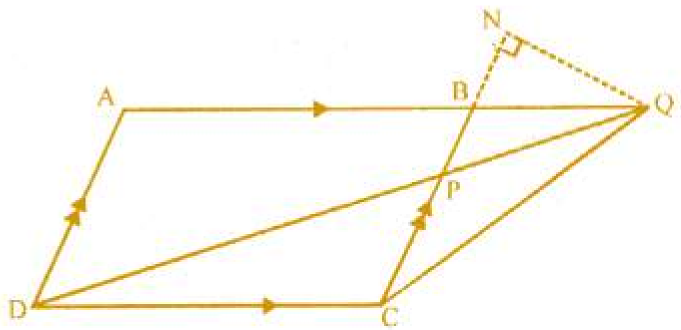
(iii) From (ii) area of ΔABD/area of ΔBEF = 25/9
25 area of ΔBEF = 9 area of ΔABD
25(area of ΔABD – area of trapezium AEFD) = 9 area of ΔABD
25 area of ΔABD – 25 area of trapezium AEFD = 9 area of ΔABD
25 area of trapezium AEFD = 25 area of ΔABD – 9 area of ΔABD
25 area of trapezium AEFD = 16 area of ΔABD
area of ΔABD/area of trapezium AEFD = 25/16
Therefore, area of ΔABD : area of trapezium AFED = 25: 16

(iv) Now we have to find area of ΔFEO : area of ΔOBC
So, consider ΔFEO and ΔOBC,
∠EOF = ∠BOC … [because vertically opposite angles are equal]
∠F = ∠OBC … [because alternate angles are equal]
ΔFEO ~ ΔOBC
Then, area of FEO/area of ΔOBC = EF^2/BC^2
$EF^2/AD^2 = 9/25$
Therefore, area of ΔFEO: area of ΔOBC = 9: 25.

12. In the adjoining figure, ABCD is a parallelogram. P is a point on BC such that BP : PC = 1 : 2 and DP produced meets AB produced at Q. If area of ΔCPQ = 20 cm², find



(i) area of ΔBPQ.
(ii) area ΔCDP.
(iii) area of parallelogram ABCD.
Solution:-
From the question it is given that, ABCD is a parallelogram.
BP: PC = 1: 2
area of ΔCPQ = 20 cm²
Construction: draw QN perpendicular CB and Join BN.



Then, area of ΔBPQ/area of ΔCPQ = ((½BP) × QN)/((½PC) × QN)

= BP/PC = ½

(i) So, area ΔBPQ = ½ area of ΔCPQ

= ½ × 20

Therefore, area of ΔBPQ = 10 cm^2

(ii) Now we have to find area of ΔCDP,

Consider the ΔCDP and ΔBQP,

Then, ∠CPD = ∠QPD … [because vertically opposite angles are equal]

∠PDC = ∠PQB … [because alternate angles are equal]

Therefore, ΔCDP ~ ΔBQP … [AA axiom]

area of ΔCDP/area of ΔBQP = PC^2/BP^2

area of ΔCDP/area of ΔBQP = $2^2/1^2$

area of ΔCDP/area of ΔBQP = 4/1

area of ΔCDP = 4 × area ΔBQP

Therefore, area of ΔCDP = 4 × 10

= 40 cm^2

(iii) We have to find the area of parallelogram ABCD,

Area of parallelogram ABCD = 2 area of ΔDCQ

= 2 area (ΔDCP + ΔCPQ)

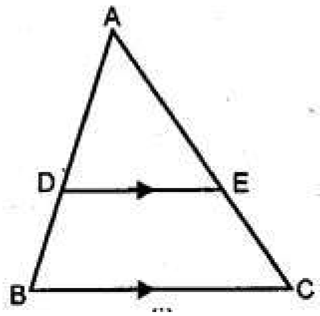
= 2 (40 + 20) cm^2

= 2 × 60 cm^2

= 120 cm^2

Therefore, the area of parallelogram ABCD is 120 cm^2.

13. (a) In the figure (i) given below, DE || BC and the ratio of the areas of ΔADE and trapezium DBCE is 4 : 5. Find the ratio of DE : BC.



Solution:-

From the question it is given that,

DE || BC

The ratio of the areas of ΔADE and trapezium DBCE is 4 : 5

Now, consider the ΔABC and ΔADE

∠A = ∠A … [common angle for both triangles]

∠D = ∠B and

∠E = ∠C … [because corresponding angles are equal]

Therefore, ΔADE ~ ΔABC

So, area of ΔADE/area of ΔABC = $(DE)^2/(BC)^2$ … [equation (i)]

Then, area of ΔADE/area of trapezium DBCE = 4/5

area of trapezium DBCE/area of ΔADE = 5/4

Add 1 for both LHS and RHS we get,

(area of trapezium DBCE/area of ΔADE) + 1 = (5/4) + 1

(area of trapezium DBCE + area of ΔADE)/area of ΔADE = (5 + 4)/4

area of ΔABC/area of ΔADE = 9/4

area of ΔADE/area of ΔABC = 4/9

From equation (i),

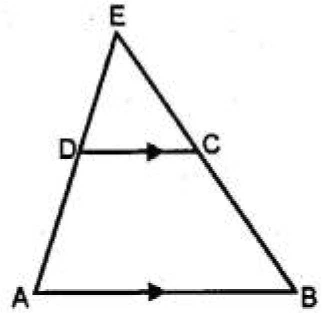
area of ΔADE/area of ΔABC = $(DE)^2/(BC)^2$

area of ΔADE/area of ΔABC = $(DE)^2/(BC)^2 = 4^2/9^2$

area of ΔADE/area of ΔABC = $(DE)^2/(BC)^2 = 2/3$

Therefore, DE: BC = 2: 3

(b) In the figure (ii) given below, AB || DC and AB = 2 DC. If AD = 3 cm, BC = 4 cm and AD, BC produced meet at E, find (i) ED (ii) BE (iii) area of ΔEDC : area of trapezium ABCD.



Solution:-

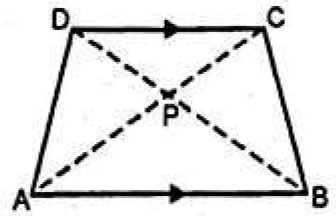
From the question it is given that,
AB || DC
AB = 2 DC, AD = 3 cm, BC = 4 cm
Now consider ΔEAB,
EA/DA = EB/CB = AB/DC = 2DC/DC = 2/1
(i) EA = 2, DA = 2 × 3 = 6 cm
Then, ED = EA – DA
= 6 – 3
= 3 cm

(ii) EB/CB = 2/1
EB = 2 CB
EB = 2 × 4
EB = 8 cm

(iii) Now, consider the ΔEAB, DC || AB
So, ΔEDC ~ ΔEAB
Therefore, area of ΔEDC/area of ΔABE = DC^2/AB^2
area of ΔEDC/area of ΔABE = $DC^2/(2DC)^2$
area of ΔEDC/area of ΔABE = $DC^2/4DC^2$
area of ΔEDC/area of ΔABE = ¼
Therefore, area of ABE = 4 area of ΔEDC
Then, area of ΔEDC + area of trapezium ABCD = 4 area of ΔEDC
Area of trapezium ABCD = 3 area of ΔEDC
So, area of ΔEDC/area of trapezium ABCD = 1/3
Therefore, area of ΔEDC: area of trapezium ABCD = 1: 3

14. (a) In the figure given below, ABCD is a trapezium in which DC is parallel to AB. If AB = 9 cm, DC = 6 cm and BB = 12 cm., find (i) BP (ii) the ratio of areas of ΔAPB and

ΔDPC.



Solution:-

From the question it is given that,

DC is parallel to AB

AB = 9 cm, DC = 6 cm and BB = 12 cm

(i) Consider the ΔAPB and ΔCPD

∠APB = ∠CPD … [because vertically opposite angles are equal]

∠PAB = ∠PCD … [because alternate angles are equal]

So, ΔAPB ~ ΔCPD

Then, BP/PD = AB/CD

BP/(12 - BP) = 9/6

6BP = 108 – 9BP

6BP + 9BP = 108

15BP = 108

BP = 108/15

Therefore, BP = 7.2 cm

(ii) We know that, area of ΔAPB/area of ΔCPD = AB^2/CD^2

area of ΔAPB/area of ΔCPD = $9^2/6^2$

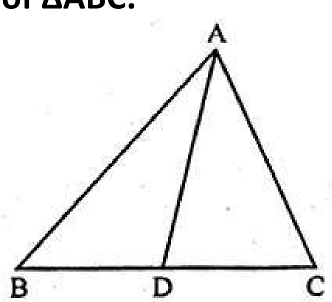
area of ΔAPB/area of ΔCPD = 81/36

By dividing both numerator and denominator by 9, we get,

area of ΔAPB/area of ΔCPD = 9/4

Therefore, the ratio of areas of ΔAPB and ΔDPC is 9: 4

(b) In the figure given below, ∠ABC = ∠DAC and AB = 8 cm, AC = 4 cm, AD = 5 cm. (i) Prove that ΔACD is similar to ΔBCA (ii) Find BC and CD (iii) Find the area of ΔACD : area of ΔABC.



Solution:-

From the question it is given that,

∠ABC = ∠DAC

AB = 8 cm, AC = 4 cm, AD = 5 cm

(i) Now, consider ΔACD and ΔBCA

∠C = ∠C … [common angle for both triangles]

∠ABC = ∠CAD … [from the question]

So, ΔACD ~ ΔBCA … [by AA axiom]

(ii) AC/BC = CD/CA = AD/AB

Consider AC/BC = AD/AB

4/BC = 5/8

BC = (4 × 8)/5

BC = 32/5

BC = 6.4 cm

Then, consider CD/CA = AD/AB

CD/4 = 5/8

CD = (4 × 5)/8

CD = 20/8

CD = 2.5 cm

(iii) from (i) we proved that, ΔACD ~ ΔBCA

area of ΔACB/area of ΔBCA = AC^2/AB^2

$= 4^2/8^2$

= 16/64

By dividing both numerator and denominator by 16, we get,

= ¼

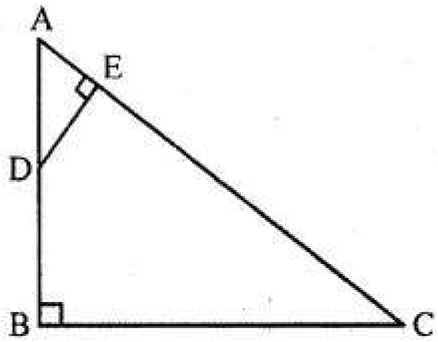
Therefore, the area of ΔACD : area of ΔABC is 1: 4.

15. ABC is a right angled triangle with ∠ABC = 90°. D is any point on AB and DE is perpendicular to AC. Prove that:

(i) ΔADE ~ ΔACB.

(ii) If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.

(iii) Find, area of ΔADE : area of quadrilateral BCED.



Solution:-

From the question it is given that,

∠ABC = 90°

AB and DE is perpendicular to AC

(i) Consider the ΔADE and ΔACB,

∠A = ∠A … [common angle for both triangle]

∠B = ∠E … [both angles are equal to 90^o]

Therefore, ΔADE ~ ΔACB

(ii) from (i) we proved that, ΔADE ~ ΔACB

So, AE/AB = AD/AC = DE/BC … [equation (i)]

Consider the ΔABC, is a right angle triangle

From Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 5^2$

$169 = AB^2 + 25$

$AB^2 = 169 – 25$

$AB^2 = 144$

AB = √144

AB = 12 cm

Consider the equation (i),

AE/AB = AD/AC = DE/BC

Take, AE/AB = AD/AC

4/12 = AD/13

1/3 = AD/13

(1 × 13)/3 = AD

AD = 4.33 cm

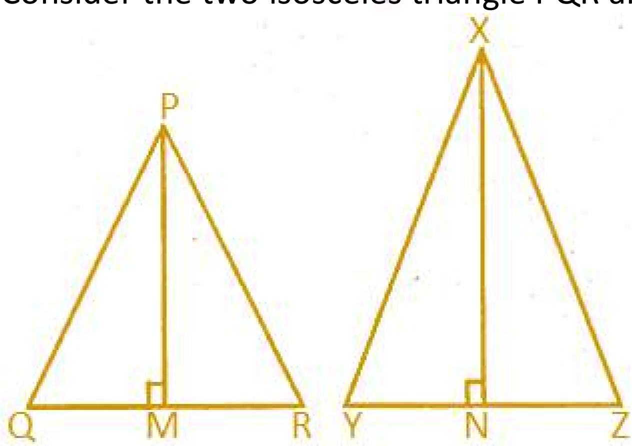
Now, take AE/AB = DE/BC

4/12 = DE/5

1/3 = DE/5
DE = (5 × 1)/3
DE = 5/3
DE = 1.67 cm

(iii) Now, we have to find area of ΔADE : area of quadrilateral BCED,
We know that, Area of ΔADE = ½ × AE × DE
= ½ × 4 × (5/3)
= 10/3 cm^2
Then, area of quadrilateral BCED = area of ΔABC – area of ΔADE
= ½ × BC × AB – 10/3
= ½ × 5 × 12 – 10/3
= 1 × 5 × 6 – 10/3
= 30 – 10/3
= (90 - 10)/3
= 80/3 cm^2
So, the ratio of area of ΔADE : area of quadrilateral BCED = (10/3)/(80/3)
= (10/3) × (3/80)
= (10 × 3)/(3 × 80)
= (1 × 1)/(1 × 8)
= 1/8
Therefore, area of ΔADE : area of quadrilateral BCED is 1: 8.

16. Two isosceles triangles have equal vertical angles and their areas are in the ratio 7: 16. Find the ratio of their corresponding height.
Solution:-
Consider the two isosceles triangle PQR and XYZ,



$\angle P = \angle X$ … [from the question]
So, $\angle Q + \angle R = \angle Y + \angle Z$
$\angle Q = \angle R$ and $\angle Y = \angle Z$ [because opposite angles of equal sides]
Therefore, $\angle Q = \angle Y$ and $\angle R = \angle Z$
$\Delta PQR \sim \Delta XYZ$
Then, area of ΔPQR/area of $\Delta XYZ = PM^2/XN^2$ … [from corollary of theorem]
$PM^2/XN^2 = 7/16$
$PM/XN = \sqrt{7}/\sqrt{16}$
$PM/XN = \sqrt{7}/4$
Therefore, ratio of PM: DM = √7: 4

17. On a map drawn to a scale of 1 : 250000, a triangular plot of land has the following measurements : AB = 3 cm, BC = 4 cm and ∠ABC = 90°. Calculate
(i) the actual length of AB in km.
(ii) the area of the plot in sq. km:
Solution:-
From the question it is given that,
Map drawn to a scale of 1: 250000
AB = 3 cm, BC = 4 cm and $\angle ABC = 90^o$
(i) We have to find the actual length of AB in km.
Let us assume scale factor K = 1: 250000
K = 1/250000
Then, length of AB of actual plot = 1/k × length of AB on the map
= (1/(1/250000)) × 3
= 250000 × 3
To covert cm into km divide by 100000
= (250000 × 3)/(100 × 1000)
= 15/2
length of AB of actual plot = 7.5 km
(ii) We have to find the area of the plot in sq. km
Area of plot on the map = ½ × AB × BC
= ½ × 3 × 4
= ½ × 12
= 1 × 6
= 6 cm^2
Then, area of actual plot = $1/k^2 \times 6\ cm^2$
= $250000^2 \times 6$

To covert cm into km divide by $(100000)^2$

$= (250000 \times 250000 \times 6)/(100000 \times 100000)$

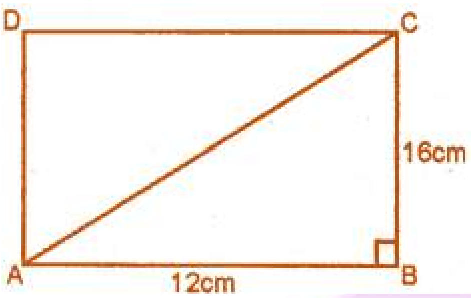
$= (25/4) \times 6$

$= 75/2$

$= 37.5 \text{ km}^2$

18. On a map drawn to a scale of 1 : 25000, a rectangular plot of land, ABCD has the following measurements AB = 12 cm and BG = 16 cm. Calculate:
(i) the distance of a diagonal of the plot in km.
(ii) the area of the plot in sq. km.
Solution:-

From the question it is given that,
Map drawn to a scale of 1: 25000
AB = 12 cm, BG = 16 cm



Consider the ΔABC,
From the Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC = \sqrt{(AB^2 + BC^2)}$

$= \sqrt{((12)^2 + (16)^2)}$

$= \sqrt{144 + 256}$

$= \sqrt{400}$

$= 20 \text{ cm}$

Then, area of rectangular plot ABCD = AB × BC

$= 12 \times 16$

$= 192 \text{ cm}^2$

(i) We have to find the distance of a diagonal of the plot in km. .

Let us assume scale factor K = 1: 25000

K = 1/25000

Then, length of AB of actual plot = 1/k × length of diagonal of rectangular plot
= (1/(1/25000)) × 3
= 25000 × 20
To covert cm into km divide by 100000
= (25000 × 20)/(100 × 1000)
= 5 km

(ii) We have to find the area of the plot in sq. km.
Then, area of actual plot = $1/k^2 \times 192\ cm^2$
= $25000^2 \times 192$
To covert cm into km divide by $(100000)^2$
= (25000 × 25000 × 192)/(100000 × 100000)
= $12\ km^2$

19. The model of a building is constructed with the scale factor 1 : 30.
(i) If the height of the model is 80 cm, find the actual height of the building in metres.
(ii) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model.
Solution:-
From the question it is given that,
The model of a building is constructed with the scale factor 1 : 30
So, Height of the model/Height of actual building = 1/30
(i) Given, the height of the model is 80 cm
Then, 80/H = 1/30
H = (80 × 30)
H = 2400 cm
H = 2400/100
H = 24 m

(ii) Given, the actual volume of a tank at the top of the building is 27 m^3
Volume of model/Volume of tank = $(1/30)^3$
V/27 = 1/27000
V = 27/27000
V = $1/1000\ m^3$
Therefore, Volume of model = $1000\ cm^3$

20. A model of a ship is made to a scale of 1 : 200.

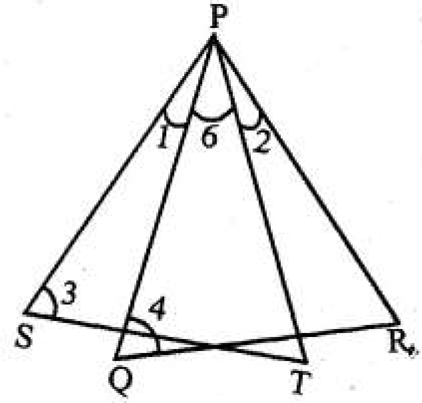
(i) If the length of the model is 4 m, find the length of the ship.
(ii) If the area of the deck of the ship is 160000 m^2, find the area of the deck of the model.
(iii) If the volume of the model is 200 liters, find the volume of the ship in m^3. (100 liters = 1 m^3)
Solution:-
From the question it is given that, a model of a ship is made to a scale of 1 : 200
(i) Given, the length of the model is 4 m
Then, length of the ship = (4 × 200)/1
= 800 m

(ii) Given, the area of the deck of the ship is 160000 m^2
Then, area of deck of the model = 160000 × $(1/200)^2$
= 160000 × (1/40000)
= 4 m^2

(iii) Given, the volume of the model is 200 liters
Then, Volume of ship = 200 × $(200/1)^3$
= 200 × 8000000
= (200 × 8000000)/100
= 1600000 m^3

CHAPTER TEST

1. In the adjoining figure, ∠1 = ∠2 and ∠3 = ∠4. Show that PT x QR = PR x ST.



Solution:-

From the question it is given that,

∠1 = ∠2 and ∠3 = ∠4

We have to prove that, PT x QR = PR x ST

Given, ∠1 = ∠2

Adding ∠6 to both LHS and RHS we get,

∠1 + ∠6 = ∠2 + ∠6

∠SPT = ∠QPR

Consider the ΔPQR and ΔPST,

From above ∠SPT = ∠QPR

∠3 = ∠4

Therefore, ΔPQR ~ ΔPST

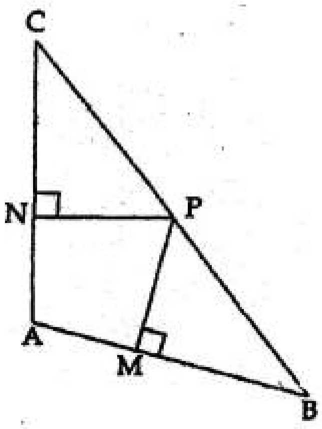
So, PT/PR = ST/QR

By cross multiplication we get,

PT x QR = PR x ST

Hence, it is proved that PT x QR = PR x ST

2. In the adjoining figure, AB = AC. If PM ⊥ AB and PN ⊥ AC, show that PM x PC = PN x PB.



Solution:-

From the given figure,

AB = AC. If PM ⊥ AB and PN ⊥ AC

We have to show that, PM x PC = PN x PB

Consider the ΔABC,

AB = AC … [given]

∠B = ∠C

Then, consider ΔCPN and ΔBPM

∠N = ∠M … [both angles are equal to 90^o]

∠C = ∠B … [from above]

Therefore, ΔCPN ~ ΔBPM … [from AA axiom]

So, PC/PB = PN/PM

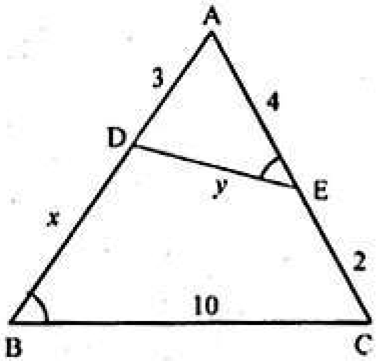
By cross multiplication we get,

PC x PM = PN x PB

Therefore, it is proved that, PM x PC = PN x PB

3.

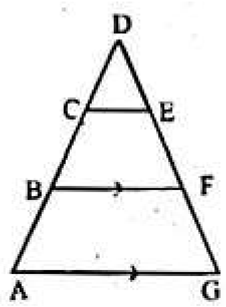
(a) In the figure given below. ∠AED = ∠ABC. Find the values of x and y.



Solution:-

From the figure it is given that,
∠AED = ∠ABC
Consider the ΔABC and ΔADE
∠AED = ∠ABC … [from the figure]
∠A = ∠A … [common angle for both triangles]
Therefore, ΔABC ~ ΔADE … [by AA axiom]
Then, AD/AC = DE/BC
3/(4 + 2) = y/10
3/6 = y/10
By cross multiplication we get,
y = (3 x 10)/6
y = 30/6
y = 5
Now, consider AB/AE = BC/DE
(3 + x)/4 = 10/y
Substitute the value of y,
(3 + x)/4 = 10/5
By cross multiplication,
5(3 + x) = 10 x 4
15 + 5x = 40
5x = 40 – 15
5x = 25
X = 25/5
x = 5
Therefore, the value of x = 5 cm and y = 5 cm

(b) In the figure given below, CD = ½ AC, B is mid-point of AC and E is mid-point of DF. If BF || AG, prove that :
(i) CE || AG
(ii) 3 ED = GD



Solution:-

From the question it is given that,

CD = ½ AC

BF || AG

(i) We have to prove that, CE || AG

Consider, CD = ½ AC

AC = 2BC … [because from the figure B is mid-point of AC]

So, CD = ½ (2BC)

CD = BC

Hence, CE || BF … [equation (i)]

Given, BF || AG … [equation (ii)]

By comparing the results of equation (i) and equation (ii) we get,

CE || AG

(ii) We have to prove that, 3 ED = GD

Consider the ΔAGD,

CE || AG … [above it is proved]

So, ED/GD = DC/AD

AD = AB + BC + DC

= DC + DC + DC

= 3DC

So, ED/GD = DC/(3DC)

ED/GD = 1/(3(1))

ED/GD = 1/3

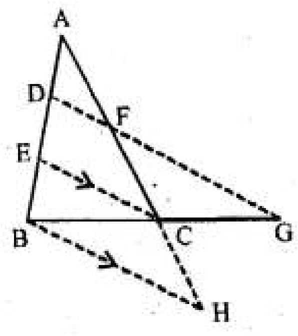
3ED = GD

Hence it is proved that, 3ED = GD

4. In the adjoining figure, 2 AD = BD, E is mid-point of BD and F is mid-point of AC and EC || BH. Prove that:

(i) DF || BH

(ii) AH = 3 AF.



Solution:-

From the question it is given that, 2 AD = BD, EC || BH

(i) Given, E is mid-point of BD

2DE = BD … [equation (i)]

2AD = BD … [equation (ii)]

From equation (i) and equation (ii) we get,

2DE = 2AD

DE = AD

Also given that, F is mid-point of AC

DF || EC … [equation (iii)]

Given, EC || BH … [equation (iv)]

By comparing equation (iii) and equation (iv) we get,

DF || BH

(ii) We have to prove that, AH = 3 AF,

Given, E is mid-point of BD and EC || BH

And c is midpoint of AH,

Then, FC = CH … [equation (v)]

Also given F is mid-point of AC

AF = FC … [equation (vi)]

By comparing both equation (v) and equation (vi) we get,

FC = AF = CH

AF = (1/3)AH

By cross multiplication we get,

3AF = AH

Therefore, it is proved that 3AF = AH

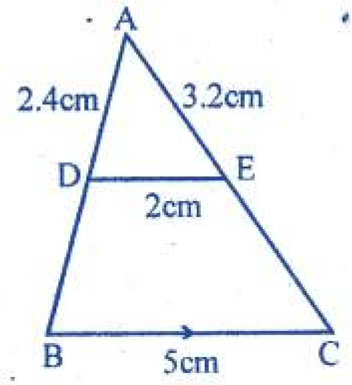
5. In a ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm, find BD and CE.

Solution:-

From the question it is given that, In a ΔABC, D and E are points on the sides AB and AC respectively.

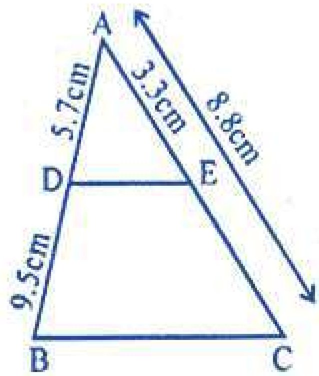
DE || BC

AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm



Consider the ΔABC,
Given, DE || BC
So, AD/AB = AE/AC = DE/BC
Now, consider AD/AB = DE/BC
2.4/AB = 2/5
AB = (2.4 x 5)/2
AB = 12/2
AB = 6 cm
Then, consider AE/AC = DE/BC
3.2/AC = 2/5
AC = (3.2 x 5)/2
AC = 16/2
AC = 8 cm
Hence, BD = AB – AD
= 6 – 2.4
= 3.6 cm
CE = AC – AE
= 8 – 3.2
= 4.8 cm

6. In a ΔABC, D and E are points on the sides AB and AC respectively such that AD = 5.7cm, BD = 9.5cm, AE = 3.3cm and AC = 8.8cm. Is DE || BC? Justify your answer.
Solution:-
From the question it is given that,
In a ΔABC, D and E are points on the sides AB and AC respectively.
AD = 5.7cm, BD = 9.5cm, AE = 3.3cm and AC = 8.8cm



Consider the ΔABC,

EC = AC – AE

= 8.8 – 3.3

= 5.5 cm

Then, AD/DB = 5.7/9.5

= 57/95

By dividing both numerator and denominator by 19 we get,

= 3/5

AE/EC = 3.3/5.5

= 33/55

By dividing both numerator and denominator by 11 we get,

= 3/5

So, AD/DB = AE/EC

Therefore, DE || BC

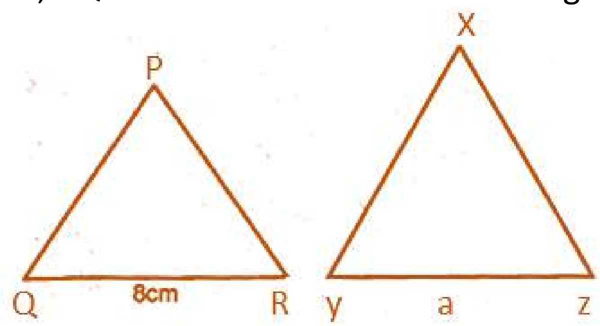
7. If the areas of two similar triangles are 360 cm² and 250 cm² and if one side of the first triangle is 8 cm, find the length of the corresponding side of the second triangle.

Solution:-

From the question it is given that, the areas of two similar triangles are 360 cm² and 250 cm².

one side of the first triangle is 8 cm

So, PQR and XYZ are two similar triangles,



So, let us assume area of ΔPQR = 360 cm^2, QR = 8 cm

And area of ΔXYZ = 250 cm^2

Assume YZ = a

We know that, area of ΔPQR/area of ΔXYZ = QR^2/yz^2

$360/250 = (8)^2/a^2$

$360/250 = 64/a^2$

By cross multiplication we get,

$a^2 = (250 \times 64)/360$

$a^2 = 400/9$

$a = \sqrt{(400/9)}$

$a = 20/3$

$a = 6\frac{2}{3}$

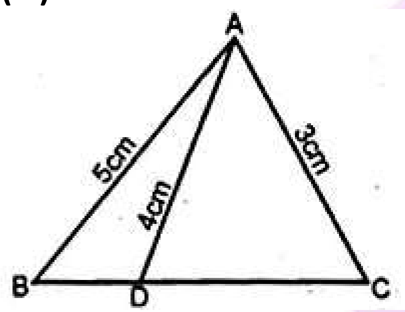
Therefore, the length of the corresponding side of the second triangle YZ = $6\frac{2}{3}$

8. In the adjoining figure, D is a point on BC such that ∠ABD = ∠CAD. If AB = 5 cm, AC = 3 cm and AD = 4 cm, find

(i) BC

(ii) DC

(iii) area of ΔACD : area of ΔBCA.



Solution:-

From the question it is given that,

∠ABD = ∠CAD

AB = 5 cm, AC = 3 cm and AD = 4 cm

Now, consider the ΔABC and ΔACD

∠C = ∠C … [common angle for both triangles]

∠ABC = ∠CAD … [from the question]

So, ΔABC ~ ΔACD

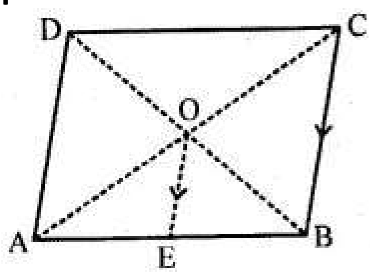
Then, AB/AD = BC/AC = AC/DC

(i) Consider AB/AD = BC/AC

5/4 = BC/3

BC = (5 x 3)/4
BC = 15/4
BC = 3.75 cm
(ii) Consider AB/AD = AC/DC
5/4 = 3/DC
DC = (3 x 4)/5
DC = 12/5
Dc = 2.4 cm
(iii) Consider the ΔABC and ΔACD
∠CAD = ∠ABC … [from the question]
∠ACD = ∠ACB … [common angle for both triangle]
Therefore, ΔACD ~ ΔABC
Then, area of ΔACD/area of ΔABC = AD^2/AB^2
= $4^2/5^2$
= 16/25
Therefore, area of ΔACD : area of ΔBCA is 16: 25.

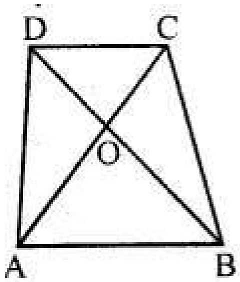
9. In the adjoining figure, the diagonals of a parallelogram intersect at O. OE is drawn parallel to CB to meet AB at E, find area of ΔAOE : area of parallelogram ABCD.



Solution:-
From the given figure,
The diagonals of a parallelogram intersect at O.
OE is drawn parallel to CB to meet AB at E.
In the figure four triangles have equal area.
So, area of ΔOAB = ¼ area of parallelogram ABCD
Then, O is midpoint of AC of ΔABC and DE || CB
E is also midpoint of AB
Therefore, OE is the median of ΔAOB
Area of ΔAOE = ½ area of ΔAOB
= ½ × ¼ area of parallelogram ABCD
= 1/8 area of parallelogram ABCD

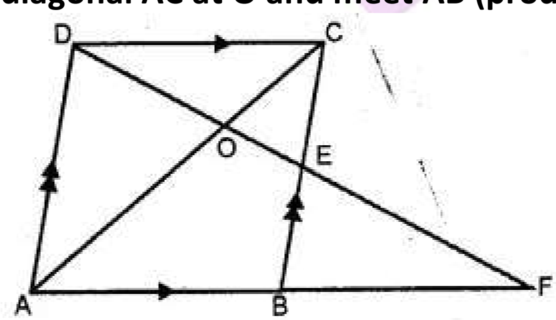
So, area of ΔAOE/area of parallelogram ABCD = 1/8
Therefore, area of ΔAOE: area of parallelogram ABCD is 1: 8.

10. In the given figure, ABCD is a trapezium in which AB || DC. If 2AB = 3DC, find the ratio of the areas of ΔAOB and ΔCOD.



Solution:-
From the question it is given that, ABCD is a trapezium in which AB || DC. If 2AB = 3DC.
So, 2AB = 3DC
AB/DC = 3/2
Now, consider ΔAOB and ΔCOD
∠AOB = ∠COD … [because vertically opposite angles are equal]
∠OAB = ∠OCD … [because alternate angles are equal]
Therefore, ΔAOB ~ ΔCOD … [from AA axiom]
Then, area of ΔAOB/area of ΔCOD = AB^2/DC^2
= $3^2/2^2$
= 9/4
Therefore, the ratio of the areas of ΔAOB and ΔCOD is 9: 4

11. In the adjoining figure, ABCD is a parallelogram. E is mid-point of BC. DE meets the diagonal AC at O and meet AB (produced) at F. Prove that



Solution:-
From the question it is given that,
ABCD is a parallelogram. E is mid-point of BC.

DE meets the diagonal AC at O.

(i) Now consider the ΔAOD and ΔEDC,

∠AOD = ∠EOC … [because Vertically opposite angles are equal]

∠OAD = ∠OCB … [because alternate angles are equal]

Therefore, ΔAOD ~ ΔEOC

Then, OA/OC = DO/OE = AD/EC = 2EC/EC

OA/OC = DO/OE = 2/1

Therefore, OA: OC = 2: 1

(ii) From (i) we proved that, ΔAOD ~ ΔEOC

So, area of ΔOEC/area of ΔAOD = OE^2/DO^2

area of ΔOEC/area of ΔAOD = $1^2/2^2$

area of ΔOEC/area of ΔAOD = ¼

Therefore, area of ΔOEC: area of ΔAOD is 1: 4.

13. A model of a ship is made to a scale of 1: 250 calculate:

(i) The length of the ship, if the length of model is 1.6 m.

(ii) The area of the deck of the ship, if the area of the deck of model is 2.4 m^2.

(iii) The volume of the model, if the volume of the ship is 1 km^3.

Solution:-

From the question it is given that, a model of a ship is made to a scale of 1 : 250

(i) Given, the length of the model is 1.6 m

Then, length of the ship = (1.6 × 250)/1

= 400 m

(ii) Given, the area of the deck of the ship is 2.4 m^2

Then, area of deck of the model = 2.4 × $(1/250)^2$

= 1,50,000 m^2 = 4 m^2

(iii) Given, the volume of the model is 1 km^3

Then, Volume of ship = $(1/250^3)$ × 1 km^3

= $1/(250)^3 \times 1000^3$

= 4^3

= 64 m^3

Therefore, volume of ship is 64 m^3.