EXERCISE 13.1

1. State which pairs of triangles in the figure given below are similar. Write the similarity rule used and also write the pairs of similar triangles in symbolic form (all lengths of sides are in cm):

Solution:-
(i) From the ΔABC and ΔPQR
AB/PQ = 3.2/4
   = 32/40
   Divide both numerator and denominator by 8 we get,  
   = 4/5
AC/PR = 3.6/4.5
   = 36/45
   Divide both numerator and denominator by 9 we get,  
   = 4/5
BC/QR = 3/5.4
Divide both numerator and denominator by 6 we get,

\[ \frac{5}{9} \]

By comparing all the results, the sides are not equal. Therefore, the triangles are not equal.

(ii) From the $\triangle DEF$ and $\triangle LMN$

$\angle E = \angle N = 40^\circ$

Then, $DE/LN = 4/2$

\[ \frac{DE}{LN} = \frac{4}{2} \]

Divide both numerator and denominator by 2 we get,

\[ \frac{2}{1} \]

$EF/MN = 4.8/2.4$

\[ \frac{EF}{MN} = \frac{4.8}{2.4} \]

\[ \frac{48}{24} \]

Divide both numerator and denominator by 24 we get,

\[ \frac{2}{1} \]

Therefore, $\triangle DEF \sim \triangle LMN$

2. It is given that $\triangle DEF \sim \triangle RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$? Why?

Solution:-

From the question is given that, $\triangle DEF \sim \triangle RPQ$

$\angle D = \angle R$ and $\angle F = \angle Q$ not $\angle P$

No, $\angle F \neq \angle P$

3. If in two right triangles, one of the acute angle of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles are similar? Why?

Solution:-

From the figure, two line segments are intersecting each other at P.

In $\triangle BCP$ and $\triangle DPE$
5/10 = 6/12
Dividing LHS and RHS by 2 we get, 
½ = ½
Therefore, \( \triangle BCD \sim \triangle DEP \)

5. It is given that \( \triangle ABC \sim \triangle EDF \) such that \( AB = 5 \text{ cm}, AC = 7 \text{ cm}, DF = 15 \text{ cm} \) and \( DE = 12 \text{ cm} \).

Find the lengths of the remaining sides of the triangles.

Solution:
As per the dimensions given in the questions,

From the question it is given that,
\( \triangle DEF \sim \triangle LMN \)
So, \( \frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF} \)
Consider \( \frac{AB}{ED} = \frac{AC}{EF} \)
\( \frac{5}{12} = \frac{7}{EF} \)
By cross multiplication,
\( EF = \frac{(7 \times 12)}{5} \)
\( EF = 16.8 \text{ cm} \)

Now, consider \( \frac{AB}{ED} = \frac{BC}{DF} \)
\( \frac{5}{12} = \frac{BC}{15} \)
\( BC = \frac{(5 \times 15)}{12} \)
\( BC = 75/12 \)
\( BC = 6.25 \text{ cm} \)

6.
(a) If \( \triangle ABC \sim \triangle DEF, AB = 4 \text{ cm}, DE = 6 \text{ cm}, EF = 9 \text{ cm} \) and \( FD = 12 \text{ cm} \), then find the perimeter of \( \triangle ABC \).

Solution:-
As per the dimensions given in the questions,

Now, we have to find out the perimeter of $\triangle ABC$

Let $\triangle ABC \sim \triangle DEF$

So, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Consider, $\frac{AB}{DE} = \frac{AC}{DF}$

$\frac{4}{6} = \frac{AC}{12}$

By cross multiplication we get,

$AC = \frac{(4 \times 12)}{6}$

$AC = \frac{48}{6}$

$AC = 8$ cm

Then, consider $\frac{AB}{DE} = \frac{BC}{EF}$

$\frac{4}{6} = \frac{BC}{9}$

$BC = \frac{(4 \times 9)}{6}$

$BC = \frac{36}{6}$

$BC = 6$ cm

Therefore, the perimeter of $\triangle ABC = AB + BC + AC$

$= 4 + 6 + 8$

$= 18$ cm

(b) If $\triangle ABC \sim \triangle PQR$, Perimeter of $\triangle ABC = 32$ cm, perimeter of $\triangle PQR = 48$ cm and PR = 6 cm, then find the length of AC.

Solution:-

From the question it is given that,

$\triangle ABC \sim \triangle PQR$

Perimeter of $\triangle ABC = 32$ cm

Perimeter of $\triangle PQR = 48$ cm

So, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

Then, perimeter of $\triangle ABC$/perimeter of $\triangle PQR = \frac{AC}{PR}$

$\frac{32}{48} = \frac{AC}{6}$
AC = (32 × 6)/48
AC = 4
Therefore, the length of AC = 4 cm.

7. Calculate the other sides of a triangle whose shortest side is 6 cm and which is similar to a triangle whose sides are 4 cm, 7 cm and 8 cm.
Solution:-
Let us assume that, \( \triangle ABC \sim \triangle DEF \)

\( \triangle ABC \) is BC = 6 cm
\( \triangle ABC \sim \triangle DEF \)
So, \( AB/DE = BC/EF = AC/DF \)
Consider \( AB/DE = BC/EF \)
\[ \frac{AB}{8} = \frac{6}{4} \]
\[ AB = \frac{(6 \times 8)}{4} \]
\[ AB = \frac{48}{4} \]
\[ AB = 12 \]

Now, consider \( BC/EF = AC/DF \)
\[ \frac{6}{4} = \frac{AC}{7} \]
\[ AC = \frac{(6 \times 7)}{4} \]
\[ AC = \frac{42}{4} \]
\[ AC = 21/2 \]
\[ AC = 10.5 \text{ cm} \]

8.
(a) In the figure given below, AB \parallel DE, AC = 3 cm, CE = 7.5 cm and BD = 14 cm. Calculate CB and DC.
Solution:-
From the question it is given that,
AB || DE
AC = 3 cm
CE = 7.5 cm
BD = 14 cm

From the figure,
∠ACB = ∠DCE [because vertically opposite angles]
∠BAC = ∠CED [alternate angles]

Then, \( \triangle ABC \sim \triangle CDE \)
So, \( \frac{AC}{CE} = \frac{BC}{CD} \)
\( \frac{3}{7.5} = \frac{BC}{CD} \)
By cross multiplication we get,
\( 7.5BC = 3CD \)
Let us assume \( BC = x \) and \( CD = 14 - x \)
\( 7.5x = 3 \times (14 - x) \)
\( 7.5x = 42 - 3x \)
\( 7.5x + 3x = 42 \)
\( 10.5x = 42 \)
\( x = \frac{42}{10.5} \)
\( x = 4 \)
Therefore, \( BC = x = 4 \) cm
\( CD = 14 - x \)
\( = 14 - 4 \)
\( = 10 \) cm

(b) In the figure (2) given below, CA || BD, the lines AB and CD meet at G.
(i) Prove that \( \triangle ACO \sim \triangle BDO \).
(ii) If BD = 2.4 cm, OD = 4 cm, OB = 3.2 cm and AC = 3.6 cm, calculate OA and OC.
Solution:
(i) We have to prove that, \( \triangle ACO \sim \triangle BDO \).
So, from the figure
Consider \( \triangle ACO \) and \( \triangle BDO \)
Then,
\[ \angle AOC = \angle BOD \]  [from vertically opposite angles]
\[ \angle A = \angle B \]
Therefore, \( \triangle ACO \sim \triangle BDO \)
Given, \( BD = 2.4 \text{ cm, } OD = 4 \text{ cm, } OB = 3.2 \text{ cm, } AC = 3.6 \text{ cm,} \)
\( \triangle ACO \sim \triangle BOD \)
So, \( AO/OB = CO/OD = AC/BD \)
Consider \( AC/BD = AO/OB \)
\[ 3.6/2.4 = AO/3.2 \]
\[ AO = (3.6 \times 3.2)/2.4 \]
\[ AO = 4.8 \text{ cm} \]
Now, consider \( AC/BD = CO/OD \)
\[ 3.6/2.4 = CO/4 \]
\[ CO = (3.6 \times 4)/2.4 \]
\[ CO = 6 \text{ cm} \]

9. (a) In the figure
(i) given below, \( \angle P = \angle RTS \).
Prove that \( \triangle RPQ \sim \triangle RTS \).
Solution:-
From the given figure, ∠P = ∠RTS
So we have to prove that ΔRPQ ~ ΔRTS
In ΔRPQ and ΔRTS
∠R = ∠R (common angle for both triangle)
∠P = ∠RTS (from the question)
ΔRPQ ~ ΔRTS

(b) In the figure (ii) given below, ∠ADC = ∠BAC. Prove that CA² = DC x BC

Solution:-
From the figure, ∠ADC = ∠BAC
So, we have to prove that, CA² = DC x BC
In ΔABC and ΔADC
∠C = ∠C (common angle for both triangle)
∠BAC = ∠ADC (from the question)
ΔABC ~ ΔADC
Therefore, CA/DC = BC/CA
We know that, corresponding sides are proportional,
Therefore, CA² = DC x BC

10. (a) In the figure (1) given below, AP = 2PB and CP = 2PD.
(i) Prove that ΔACP is similar to ΔBDP and AC || BD.
(ii) If AC = 4.5 cm, calculate the length of BD.
Solution:
From the question it is given that,
AP = 2PB, CP = 2PD
(i) We have to prove that, \( \Delta ACP \) is similar to \( \Delta BDP \) and \( AC \parallel BD \)
AP = 2PB
\[
\frac{AP}{PB} = \frac{2}{1}
\]
Then, CP = 2PD
\[
\frac{CP}{PD} = \frac{2}{1}
\]
\( \angle APC = \angle BPD \) \[from \text{vertically opposite angles}\]
So, \( \Delta ACP \sim \Delta BDP \) \[\text{from vertically opposite angles}\]
Therefore, \( \angle CAP = \angle PBD \) \[\text{from vertically opposite angles}\]
Hence, \( AC \parallel BD \)
(ii) \( \frac{AP}{PB} = \frac{AC}{BD} = \frac{2}{1} \)
AC = 2BD
2BD = 4.5 cm
BD = 4.5/2
BD = 2.25 cm

(b) In the figure (2) given below,
\( \angle ADE = \angle ACB \).
(i) Prove that \( \Delta s \) ABC and AED are similar.
(ii) If \( AE = 3 \text{ cm}, BD = 1 \text{ cm} \) and \( AB = 6 \text{ cm} \), calculate AC.

Solution:
From the given figure,
(i) \( \angle A = \angle A \) \[\text{common angle for both triangles}\]
\( \angle ACB = \angle ADE \) \[\text{given}\]
Therefore, \( \Delta ABC \sim \Delta AED \)
(ii) from (i) proved that, \( \Delta ABC \sim \Delta AED \)
So, \( \frac{BC}{DE} = \frac{AB}{AE} = \frac{AC}{AD} \)
AD = AB – BD
= 6 – 1 = 5
Consider, AB/AE = AC/AD
6/3 = AC/5
AC = (6 × 5)/3
AC = 30/3
AC = 10 cm

(c) In the figure (3) given below, ∠PQR = ∠PRS. Prove that triangles PQR and PRS are similar. If PR = 8 cm, PS = 4 cm, calculate PQ.

Solution:
From the figure,
∠P = ∠P \text{ (common angle for both triangles)}
∠PQR = ∠PRS \text{ [from the question]}
So, ∆PQR ~ ∆PRS
Then, PQ/PR = PR/PS = QR/SR
Consider PQ/PR = PR/PS
PQ/8 = 8/4
PQ = (8 × 8)/4
PQ = 64/4
PQ = 16 cm

11. In the given figure, ABC is a triangle in which AB = AC. P is a point on the side BC such that PM \perp AB and PN \perp AC. Prove that BM \times NP = CN \times MP.
From the question it is given that, \( \triangle ABC \) is a triangle in which \( AB = AC \). P is a point on the side BC such that PM \( \perp AB \) and PN \( \perp AC \). We have to prove that, \( BM \times NP = CN \times MP \)

Consider the \( \triangle ABC \)

\( AB = AC \) \[\text{[from the question]}\]

\( \angle B = \angle C \) \[\text{[angles opposite to equal sides]}\]

Then, consider \( \triangle BMP \) and \( \triangle CNP \)

\( \angle M = \angle N \)

Therefore, \( \triangle BMP \sim \triangle CNP \)

So, \( BM/CN = MP/NP \)

By cross multiplication we get,

\( BM \times NP = CN \times MP \)

Hence it is proved.

12. Prove that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

Solution:-

Consider the two triangles, \( \triangle MNO \) and \( \triangle XYZ \)

From the question it is given that, two triangles are similar triangles

So, \( \triangle MNO \sim \triangle XYZ \)

If two triangles are similar, the corresponding angles are equal and their corresponding
sides are proportional.
\[ \frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ} \]
Perimeter of \( \triangle MNO = MN + NO + MO \)
Perimeter of \( \triangle XYZ = XY + YZ + XZ \)
Therefore, \( (\frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ}) = (\frac{MN}{XY} + \frac{NO}{YZ} + \frac{MO}{XZ}) = \frac{\text{Perimeter of } \triangle MNO}{\text{perimeter of } \triangle XYZ} \)

13. In the adjoining figure, ABCD is a trapezium in which \( AB \parallel DC \). The diagonals AC and BD intersect at O. Prove that \( \frac{AO}{OC} = \frac{BO}{OD} \)

Using the above result, find the values of \( x \) if \( OA = 3x - 19, OB = x - 4, OC = x - 3 \) and \( OD = 4 \).

Solution:-
From the given figure, ABCD is a trapezium in which \( AB \parallel DC \),
The diagonals AC and BD intersect at O.
So we have to prove that, \( \frac{AO}{OC} = \frac{BO}{OD} \)
Consider the \( \triangle AOB \) and \( \triangle COD \),
\[ \angle AOB = \angle COD \] ... [vertically opposite angles]
\[ \angle OAB = \angle OCD \]
Therefore, \( \triangle AOB \sim \triangle COD \)
So, \( \frac{OA}{OC} = \frac{OB}{OD} \)
Now by using above result we have to find the value of \( x \) if \( OA = 3x - 19, OB = x - 4, OC = x - 3 \) and \( OD = 4 \).
\[ \frac{OA}{OC} = \frac{OB}{OD} \]
\[ \frac{(3x - 19)}{(x - 3)} = \frac{(x - 4)}{4} \]
By cross multiplication we get,
\[ (x - 3)(x - 4) = 4(3x - 19) \]
\[ x^2 - 4x - 3x + 12 = 12x - 76 \]
\[ x^2 - 7x + 12 - 12x + 76 = 0 \]
\[ x^2 - 19x + 88 = 0 \]
\[ x^2 - 8x - 11x + 88 = 0 \]
\[ x(x - 8) - 11(x - 8) = 0 \]
\[ (x - 8)(x - 11) = 0 \]
Take \( x - 8 = 0 \)
\[ x = 8 \]
Then, \( x - 11 = 0 \)
\[ x = 11 \]
Therefore, the value of \( x \) is 8 and 11.

14. In \( \triangle ABC \), \( \angle A \) is acute. BD and CE are perpendicular on AC and AB respectively. Prove that \( AB \times AE = AC \times AD \).

Solution:
Consider the \( \triangle ABC \),

So, we have to prove that, \( AB \times AE = AC \times AD \)

Now, consider the \( \triangle ADB \) and \( \triangle AEC \),

\( \angle A = \angle A \) \quad [\text{common angle for both triangles}]

\( \angle ADB = \angle AEC \) \quad [\text{both angles are equal to 90°}]

\( \triangle ADB \sim \triangle AEC \)

So, \( AB/AC = AD/AE \)

By cross multiplication we get,

\( AB \times AE = AC \times AD \)

15. In the given figure, DB \( \perp \) BC, DE \( \perp \) AB and AC \( \perp \) BC. Prove that \( BE/DE = AC/BC \).
Solution:
From the figure, DB ⊥ BC, DE ⊥ AB and AC ⊥ BC
We have to prove that, BE/DE = AC/BC
Consider the ∆ABC and ∆DEB,
∠C = 90°
∠A + ∠ABC = 90° [from the figure equation (i)]
Now in ∆DEB
∠DBE + ∠ABC = 90° [from the figure equation (ii)]
From equation (i), we get
∠A = ∠DBE
Then, in ∆ABC and ∆DBE
∠C = ∠E [both angles are equal to 90°]
So, ∆ABC ~ ∆DBE
Therefore, AC/BE = BC/DE
By cross multiplication, we get
AC/BC = BE/DE

16.
(a) In the figure (1) given below, E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. show that ∆ABE ~ ∆CFB.

Solution:-
From the figure, ABCD is a parallelogram,
Then, E is a point on AD and produced and BE intersects CD at F.
We have to prove that ∆ABE ~ ∆CFB
Consider $\triangle ABE$ and $\triangle CFB$

$\angle A = \angle C$  
[opp. angles of a parallelogram]

$\angle ABE = \angle BFC$  
[alt. angles are equal]

$\triangle ABE \sim \triangle CFB$

(b) In the figure (2) given below, PQRS is a parallelogram; PQ = 16 cm, QR = 10 cm. L is a point on PR such that $RL : LP = 2 : 3$. QL produced meets RS at M and PS produced at N.

(i) Prove that triangle RLQ is similar to triangle PLN. Hence, find PN.

Solution:
From the question it is given that,
Consider the $\triangle RLQ$ and $\triangle PLN$,

$\angle RLQ = \angle NLP$  
[vertically opposite angles are equal]

$\angle RQL = \angle LNP$  
[alt. angles are equal]

Therefore, $\triangle RLQ \sim \triangle PLN$

So, $QR/PN = RL/LP = 2/3$

$QR/PN = 2/3$

$10/PN = 2/3$

$PN = (10 \times 3)/2$

$PN = 30/2$

$PN = 15$ cm

Therefore, $PN = 15$ cm

(ii) Name a triangle similar to triangle RLM. Evaluate RM.

Solution:
From the figure,
Consider $\triangle RLM$ and $\triangle QLP$

Then, $\angle RLM = \angle QLP$  
[vertically opposite angles are equal]

$\angle LRM = \angle LPQ$  
[alt. angles are equal]

Therefore, $\triangle RLM \sim \triangle QLP$

Then, $RM/PQ = RL/LP = 2/3$

So, $RM/16 = 2/3$
RM = \( \frac{(16 \times 2)}{3} \)
RM = \( \frac{32}{3} \)
RM = \( 10\frac{2}{3} \)

17. The altitude BN and CM of \( \triangle ABC \) meet at H. Prove that
(i) \( CN \times HM = BM \times HN \)
(ii) \( HC/ HB = \sqrt{\left(\frac{CN \times HN}{BM \times HM}\right)} \)
(iii) \( \triangle MHN \sim \triangle BHC \)
Solution:-
Consider the \( \triangle ABC \),
Where, the altitude BN and CM of \( \triangle ABC \) meet at H.
and construction: join MN

(i) We have to prove that, \( CN \times HM = BM \times HN \)
In \( \triangle BHM \) and \( \triangle CHN \)
\( \angle BHM = \angle CHN \) \([\text{because vertically opposite angles are equal}]\)
\( \angle M = \angle N \) \([\text{both angles are equal to } 90^\circ]\)
Therefore, \( \triangle BH M \sim \triangle CHN \)
So, \( HM/HN = BM/CN = HB/HC \)
Then, by cross multiplication we get
\( CN \times HM = BM \times HN \)

(ii) Now, \( HC/ HB = \sqrt{\left(\frac{CN \times HN}{BM \times HM}\right)} \)
Because, \( M \) and \( N \) divide \( AB \) and \( AC \) in the same ratio.
(iii) Now consider \( \triangle MHN \) and \( \triangle BHC \)
\( \angle MHN = \angle BHC \) \([\text{because vertically opposite angles are equal}]\)
\( \angle MNH = \angle HBC \) \([\text{because alternate angles are equal}]\)
Therefore, \( \triangle MHN \sim \triangle BHC \)
18. In the given figure, CM and RN are respectively the medians of \( \triangle ABC \) and \( \triangle PQR \). If \( \triangle ABC \sim \triangle PQR \), prove that:

(i) \( \triangle AMC \sim \triangle PQR \)

(ii) \( \frac{CM}{RN} = \frac{AB}{PQ} \)

(iii) \( \triangle CMB \sim \triangle RNQ \)

**Solution:**

From the given figure it is given that, CM and RN are respectively the medians of \( \triangle ABC \) and \( \triangle PQR \).

(i) We have to prove that, \( \triangle AMC \sim \triangle PQR \)

Consider the \( \triangle ABC \) and \( \triangle PQR \)

As \( \triangle ABC \sim \triangle PQR \)

\( \angle A = \angle P, \angle B = \angle Q \) and \( \angle C = \angle R \)

And also corresponding sides are proportional

\( \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \)

Then, consider the \( \triangle AMC \) and \( \triangle PNR \),

\( \angle A = \angle P \)

\( \frac{AC}{PR} = \frac{AM}{PN} \)

Because, \( \frac{AB}{PQ} = \frac{1}{2} \frac{AB}{PQ} \)

\( \frac{AB}{PQ} = \frac{AM}{PN} \)

Therefore, \( \triangle AMC \sim \triangle PNR \)

(ii) From solution(i) \( \frac{CM}{RN} = \frac{AM}{PN} \)

\( \frac{CM}{RN} = \frac{2AM}{2PN} \)

\( \frac{CM}{RN} = \frac{AB}{PQ} \)

(iii) Now consider the \( \triangle CMB \) and \( \triangle RNQ \)

\( \angle B = \angle Q \)

\( \frac{BC}{QP} = \frac{BM}{QN} \)

Therefore, \( \triangle CMB \sim \triangle RNQ \)
19. In the adjoining figure, medians AD and BE of \( \triangle ABC \) meet at the point G, and DF is drawn parallel to BE. Prove that

(i) \( EF = FC \)

(ii) \( AG : GD = 2 : 1 \)

Solution:

From the figure it is given that, medians AD and BE of \( \triangle ABC \) meet at the point G, and DF is drawn parallel to BE.

(i) We have to prove that, \( EF = FC \)

From the figure, D is the midpoint of BC and also DF parallel to BE.

So, F is the midpoint of EC

Therefore, \( EF = FC = \frac{1}{2} EC \)

\( EF = \frac{1}{2} AE \)

(ii) Now consider the \( \triangle AGE \) and \( \triangle ADF \)

Then, (BG or GE) \( \parallel \) DF

Therefore, \( \triangle AGE \sim \triangle ADF \)

So, \( \frac{AG}{GD} = \frac{AE}{EF} \)

\( \frac{AG}{GD} = \frac{1}{\frac{1}{2}} \)

\( \frac{AG}{GD} = 1 \times (2/1) \)

Therefore, \( AG : GD = 2 : 1 \)

20. (a) In the figure given below, AB, EF and CD are parallel lines. Given that AB = 15 cm, EG = 5 cm, GC = 10 cm and DC = 18 cm. Calculate

(i) EF

(ii) AC.
Solution:-
From the figure it is given that, AB, EF and CD are parallel lines.
(i) Consider the $\triangle EFG$ and $\triangle CGD$
$\angle EGF = \angle CGD$ [Because vertically opposite angles are equal]
$\angle FEG = \angle GCD$ [alternate angles are equal]
Therefore, $\triangle EFG \sim \triangle CGD$
Then, $\frac{EG}{GC} = \frac{EF}{CD}$
$\frac{5}{10} = \frac{EF}{18}$
$EF = \frac{(5 \times 18)}{10}$
Therefore, $EF = 9 \text{ cm}$
(ii) Now, consider the $\triangle ABC$ and $\triangle EFC$
$EF \parallel AB$
So, $\triangle ABC \sim \triangle EFC$
Then, $\frac{AC}{EC} = \frac{AB}{EF}$
$AC/(5 + 10) = \frac{15}{9}$
$AC/15 = \frac{15}{9}$
$AC = \frac{(15 \times 15)}{9}$
Therefore, $AC = 25 \text{ cm}$

(b) In the figure given below, AF, BE and CD are parallel lines. Given that AF = 7.5 cm, CD = 4.5 cm, ED = 3 cm, BE = $x$ and AE = $y$. Find the values of $x$ and $y$. 
Solution:
From the figure, AF, BE and CD are parallel lines.
Consider the \( \triangle AEF \) and \( \triangle CED \)
\( \angle AEF \) and \( \angle CED \) [because vertically opposite angles are equal]
\( \angle F = \angle C \) [alternate angles are equal]
Therefore, \( \triangle AEF \sim \triangle CED \)
So, \( \frac{AF}{CD} = \frac{AE}{ED} \)
\[ \frac{7.5}{4.5} = \frac{y}{3} \]
By cross multiplication,
\[ y = \frac{(7.5 \times 3)}{4.5} \]
\[ y = 5 \text{ cm} \]
So, similarly in \( \triangle ACD \), BE \parallel CD
Therefore, \( \triangle ABE \sim \triangle ACD \)
\[ \frac{EB}{CD} = \frac{AE}{AD} \]
\[ \frac{x}{4.5} = \frac{5}{5 + 3} \]
\[ \frac{x}{4.5} = \frac{5}{8} \]
\[ x = \frac{(4.5 \times 5)}{8} \]
\[ x = 22.5/8 \]
\[ x = 225/80 \]
\[ x = \frac{45}{16} \]
\[ x = 2 \frac{13}{16} \]

21. In the given figure, \( \angle A = 90^\circ \) and AD \( \perp BC \) If BD = 2 cm and CD = 8 cm, find AD.

Solution:
From the figure, consider \( \triangle ABC \),
So, \( \angle A = 90^\circ \)
And AD \( \perp BC \)
\( \angle BAC = 90^\circ \)
Then, \( \angle BAD + \angle DAC = 90^\circ \) ... [equation (i)]
Now, consider \( \triangle ADC \)
\(\angle ADC = 90^\circ\)

So, \(\angle DCA + \angle DAC = 90^\circ\) \hspace{1cm} \ldots \text{[equation (ii)]}

From equation (i) and equation (ii)
We have,
\(\angle BAD + \angle DAC = \angle DCA + \angle DAC\)
\(\angle BAD = \angle DCA\) \hspace{1cm} \ldots \text{[equation (iii)]}

So, from \(\triangle BDA\) and \(\triangle ADC\)
\(\angle BDA = \angle ADC\) \hspace{1cm} \ldots \text{[both the angles are equal to } 90^\circ\]
\(\angle BAD = \angle DCA\) \hspace{1cm} \ldots \text{[from equation (iii)]}

Therefore, \(\triangle BDA \sim \triangle ADC\)
\(BD/AD = AD/DC = AB/AC\)
Because, corresponding sides of similar triangles are proportional
\(BD/AD = AD/DC\)
By cross multiplication we get,
\(AD^2 = BD \times CD\)
\(AD^2 = 2 \times 8 = 16\)
\(AD = \sqrt{16}\)
\(AD = 4\)

22. A 15 metres high tower casts a shadow of 24 metres long at a certain time and at the same time, a telephone pole casts a shadow 16 metres long. Find the height of the telephone pole.

Solution:-
From the question it is given that,
Height of a tower PQ = 15m
It’s shadow QR = 24 m
Let us assume the height of a telephone pole MN = x
It’s shadow NO = 16 m

![Diagram of a 15m high tower casting a 24m shadow and a telephone pole with a 16m shadow]
Given, at the same time, 
\( \triangle PQR \sim \triangle MNO \)

Therefore, 
\[
\frac{PQ}{MN} = \frac{ON}{RQ}
\]

\[
15/x = 24/16
\]

By cross multiplication we get,
\[
x = \frac{(15 \times 16)}{24}
\]

\[
x = \frac{240}{24}
\]

\[
x = 10
\]

Therefore, height of pole = 10 m.

23. A street light bulb is fixed on a pole 6 m above the level of street. If a woman of height casts a shadow of 3 m, find how far she is away from the base of the pole?

Solution:

From the question it is given that,
Height of pole (PQ) = 6m
Height of a woman (MN) = 1.5m

So, shadow NR = 3m

Therefore, pole and woman are standing in the same line

\( PM \parallel MR \)

\( \triangle PRQ \sim \triangle MNR \)

So, 
\[
\frac{RQ}{RN} = \frac{PQ}{MN}
\]

\[
\frac{3 + x}{3} = \frac{6}{1.5}
\]

\[
(3 + x)/3 = 6/1.5
\]

\[
(3 + x)/3 = 60/15
\]

\[
(3 + x)/3 = 4/1
\]
(3 + x) = 12
X = 12 – 3
X = 9m
Therefore, women is 9m away from the pole.
EXERCISE 13.2

1. (a) In the figure (i) given below if DE || BG, AD = 3 cm, BD = 4 cm and BC = 5 cm. Find (i) AE : EC (ii) DE.

Solution:-

From the figure,
DE || BG, AD = 3 cm, BD = 4 cm and BC = 5 cm
(i) AE: EC
So, AD/BD = AE/EC
AE/EC = AD/BD
AE/EC = 3/4
AE: EC = 3: 4
(ii) Consider \( \triangle ADE \) and \( \triangle ABC \)
\( \angle D = \angle B \)
\( \angle E = \angle C \)
Therefore, \( \triangle ADE \sim \triangle ABC \)
Then, \( \frac{DE}{BC} = \frac{AD}{AB} \)
\( \frac{DE}{5} = \frac{3}{3+4} \)
\( \frac{DE}{5} = \frac{3}{7} \)
\( DE = \frac{3 \times 5}{7} \)
\( DE = \frac{15}{7} \)
\( DE = 2\frac{1}{7} \)

(b) In the figure (ii) given below, PQ || AC, AP = 4 cm, PB = 6 cm and BC = 8 cm. Find CQ and BQ.
Solution:

From the figure,
PQ || AC, AP = 4 cm, PB = 6 cm and BC = 8 cm
∠BQP = ∠BCA  ... [because alternate angles are equal]
Also, ∠B = ∠B  ... [common for both the triangles]

Therefore, ΔABC ~ ΔBPQ

Then, BQ/BC = BP/AB = PQ/AC

BQ/BC = 6/(6 + 4) = PQ/AC

BQ/BC = 6/10 = PQ/AC

BQ/8 = 6/10 = PQ/AC  ... [because BC = 8 cm given]

Now, BQ/8 = 6/10

BQ = (6/10) \times 8

BQ = 48/10

BQ = 4.8 cm

Also, CQ = BC − BQ

CQ = (8 − 4.8) cm

CQ = 3.2 cm

Therefore, CQ = 3.2 cm and BQ = 4.8 cm

(c) In the figure (iii) given below, if XY || QR, PX = 1 cm, QX = 3 cm, YR = 4.5 cm and QR = 9 cm, find PY and XY.

Solution:

From the figure,
XY || QR, PX = 1 cm, QX = 3 cm, YR = 4.5 cm and QR = 9 cm,
So, PX/QX = PY/YR

1/3 = PY/4.5

By cross multiplication we get,

(4.5 \times 1)/3 = PY

PY = 45/30

PY = 1.5

Then, ∠X = ∠Q
\[ \angle Y = \angle R \]
So, \( \triangle PXY \sim \triangle PQR \)
Therefore, \( \frac{XY}{QR} = \frac{PX}{PQ} \)
\[ \frac{XY}{9} = \frac{1}{1 + 3} \]
\[ \frac{XY}{9} = \frac{1}{4} \]
\[ XY = \frac{9}{4} \]
\[ XY = 2.25 \]

2. In the given figure, \( DE \parallel BC \).

(i) If \( AD = x, DB = x - 2, AE = x + 2 \) and \( EC = x - 1 \), find the value of \( x \).
(ii) If \( DB = x - 3, AB = 2x, EC = x - 2 \) and \( AC = 2x + 3 \), find the value of \( x \).

Solution:-

(i) From the figure, it is given that,
Consider the \( \triangle ABC \),
\[ \frac{AD}{DB} = \frac{AE}{EC} \]
\[ \frac{x}{x - 2} = \frac{x + 2}{x - 1} \]
By cross multiplication we get,
\[ x(x - 1) = (x - 2)(x + 2) \]
\[ x^2 - x = x^2 - 4 \]
\[ -x = -4 \]
\[ x = 4 \]

(ii) From the question it is given that,
\( DB = x - 3, AB = 2x, EC = x - 2 \) and \( AC = 2x + 3 \)
Consider the \( \triangle ABC \),
\[ \frac{AD}{DB} = \frac{AE}{EC} \]
\[ \frac{2x}{x - 2} = \frac{2x + 3}{x - 3} \]
By cross multiplication we get,
\[2x(x - 2) = (2x + 3)(x - 3)\]
\[2x^2 - 4x = 2x^2 - 6x + 3x - 9\]
\[2x^2 - 4x - 2x^2 + 6x - 3x = -9\]
\[-7x + 6x = -9\]
\[-x = -9\]
\[x = 9\]

3. E and F are points on the sides PQ and PR respectively of a \( \triangle PQR \). For each of the following cases, state whether EF \( \parallel \) QR:

(i) PE = 3.9 cm, EQ = 3 cm, PF = 8 cm and RF = 9 cm.

Solution:
From the given dimensions, consider the \( \triangle PQR \)

So, \( \frac{PE}{EQ} = \frac{3.9}{3} \)
\[= \frac{39}{30}\]
\[= \frac{13}{10}\]
Then, \( \frac{PF}{FR} = \frac{8}{9}\)
By comparing both the results,
\[\frac{13}{10} \neq \frac{8}{9}\]
Therefore, \( \frac{PE}{EQ} \neq \frac{PF}{FR} \)
So, EF is not parallel to QR

(ii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.

Solution:
From the dimensions given in the question, consider the \( \triangle PQR \)
So, \( \frac{PQ}{PE} = \frac{1.28}{0.18} \)
\[ = \frac{128}{18} \]
\[ = \frac{64}{9} \]

Then, \( \frac{PR}{PF} = \frac{2.56}{0.36} \)
\[ = \frac{256}{36} \]
\[ = \frac{64}{9} \]

By comparing both the results,
\( \frac{64}{9} = \frac{64}{9} \)
Therefore, \( \frac{PQ}{PE} = \frac{PR}{PF} \)
So, \( EF \) is parallel to \( QR \).

4. A and B are respectively the points on the sides \( PQ \) and \( PR \) of a triangle \( PQR \) such that \( PQ = 12.5 \) cm, \( PA = 5 \) cm, \( BR = 6 \) cm and \( PB = 4 \) cm. Is \( AB \parallel QR \)? Give reasons for your answer.

Solution:-
From the dimensions given in the question,
Consider the \( \triangle PQR \)

So, \( \frac{PQ}{PA} = \frac{12.5}{5} \)
\[ = \frac{2.5}{1} \]
PR/PB = (PB + BR)/PB
= (4 + 6)/4
= 10/4
= 2.5

By comparing both the results,
2.5 = 2.5
Therefore, PQ/PA = PR/PB
So, AB is parallel to QR.

5.
(a) In figure (i) given below, DE || BC and BD = CE. Prove that ABC is an isosceles triangle.

Solution:-
From the question it is given that,
DE || BC and BD = CE
So, we have to prove that ABC is an isosceles triangle.
Consider the triangle ABC,
AD/DB = AE/EC
Given, DB = EC ... [equation (i)]
Then, AD = AE ... [equation (ii)]
By adding equation (i) and equation (ii) we get,
AD + DB = AE + EC
So, AB = AC
Therefore, ∆ABC is an isosceles triangle.

(b) In figure (ii) given below, AB || DE and BD || EF. Prove that DC² = CF x AC.
Solution:
From the figure it is given that, AB || DE and BD || EF.
We have to prove that, $DC^2 = CF \times AC$

Consider the $\triangle ABC$,

$\frac{DC}{CA} = \frac{CE}{CB}$ \hspace{1cm} \text{[equation (i)]}$

Now, consider $\triangle CDE$

$\frac{CF}{CD} = \frac{CE}{CB}$ \hspace{1cm} \text{[equation (ii)]}$

From equation (i) and equation (ii),

$\frac{DC}{CA} = \frac{CF}{CD}$

$\frac{DC}{AC} = \frac{CF}{DC}$

By cross multiplication we get,

$DC^2 = CF \times AC$

6.
(a) In the figure (i) given below, CD || LA and DE || AC. Find the length of CL if $BE = 4$ cm and $EC = 2$ cm.

Solution:-
From the given figure, CD || LA and DE || AC,

Consider the $\triangle BCA$,

$\frac{BE}{BC} = \frac{BD}{BA}$
By using the corollary of basic proportionality theorem,

$$\frac{BE}{BE + EC} = \frac{BD}{AB}$$

$$\frac{4}{4 + 2} = \frac{BD}{AB}$$  \[\text{... (equation (i))}\]

Then, consider the \(\Delta BLA\)

$$\frac{BC}{BL} = \frac{BD}{AB}$$

By using the corollary of basic proportionality theorem,

$$\frac{6}{6 + CL} = \frac{BD}{AB}$$  \[\text{... (equation (ii))}\]

Now, combining the equation (i) and equation (ii), we get

$$\frac{6}{6 + CL} = \frac{4}{6}$$

By cross multiplication we get,

$$6 \times 6 = 4 \times (6 + CL)$$

$$24 + 4CL = 36$$

$$4CL = 36 - 24$$

$$CL = \frac{12}{4}$$

$$CL = 3 \text{ cm}$$

Therefore, the length of CL is 3 cm.

(b) In the given figure, \(\angle D = \angle E\) and \(AD/BD = AE/EC\). Prove that \(BAC\) is an isosceles triangle.

![Diagram of triangle](https://byjus.com)

**Solution:-**

From the given figure, \(\angle D = \angle E\) and \(AD/BD = AE/EC\),

We have to prove that, \(BAC\) is an isosceles triangle

So, consider the \(\Delta ADE\)

\(\angle D = \angle E\)  \[\text{... [from the question]}\]

\(AD = AE\)  \[\text{... [sides opposite to equal angles]}\]

Consider the \(\Delta ABC\),

Then, \(AD/DB = AE/EC\)  \[\text{... (equation (i))}\]

Therefore, \(DE\) parallel to \(BC\)
Because \( AD = AE \)
\( DB = EC \) ... [equation (ii)]
By adding equation (i) and equation (ii) we get,
\( AD + DB = AE + EC \)
\( AB = AC \)
Therefore, \( \Delta ABC \) is an isosceles triangle.

7. In the adjoining given below, A, B and C are points on OP, OQ and OR respectively such that \( AB \parallel PQ \) and \( AC \parallel PR \). show that \( BC \parallel QR \).

Solution:
Consider the \( \Delta POQ \)
\( AB \parallel PQ \) ... [given]
So, \( OA/\ AP = OB/BQ \) ... [equation (i)]
Then, consider the \( \Delta OPR \)
\( AC \parallel PR \)
\( OA/\ AP = OC/CR \) ... [equation (ii)]
Now by comparing both equation (i) and equation (ii),
\( OB/BQ = OC/CR \)
Then, in \( \Delta OQR \)
\( OB/BQ = OC/CR \)
Therefore, \( BC \parallel QR \)

8. ABCD is a trapezium in which \( AB \parallel DC \) and its diagonals intersect each other at O. Using Basic Proportionality theorem, prove that \( AO/BO = CO/DO \)
Solution:-
From the question it is given that,
ABCD is a trapezium in which \( AB \parallel DC \) and its diagonals intersect each other at O
Now consider the \( \triangle OAB \) and \( \triangle OCD \),

\[ \angle AOB = \angle COD \quad [\text{because vertically opposite angles are equal}] \]

\[ \angle OBA = \angle ODC \quad [\text{because alternate angles are equal}] \]

\[ \angle OAB = \angle OCD \quad [\text{because alternate angles are equal}] \]

Therefore, \( \triangle OAB \sim \triangle OCD \)

Then, \( \frac{OA}{OC} = \frac{OB}{OD} \)

\[ \frac{AO}{OB} = \frac{CO}{DO} \quad \ldots \quad [\text{by alternate angles}] \]

9. (a) In the figure (1) given below, \( AB \parallel CR \) and \( LM \parallel QR \).

(i) Prove that \( \frac{BM}{MC} = \frac{AL}{LQ} \)

(ii) Calculate \( LM : QR \), given that \( BM : MC = 1 : 2 \).

Solution:-

From the question it is given that, \( AB \parallel CR \) and \( LM \parallel QR \)

(i) We have to prove that, \( \frac{BM}{MC} = \frac{AL}{LQ} \)

Consider the \( \triangle ARQ \)

\( LM \parallel QR \quad \ldots \quad [\text{from the question}] \)

So, \( \frac{AM}{MR} = \frac{AL}{LQ} \quad \ldots \quad [\text{equation (i)}] \)

Now, consider the \( \triangle AMB \) and \( \triangle MCR \)
\[ \angle AMB = \angle CMR \quad \ldots \text{[because vertically opposite angles are equal]} \]
\[ \angle MBA = \angle MCR \quad \ldots \text{[because alternate angles are equal]} \]
Therefore, \( \frac{AM}{MR} = \frac{BM}{MC} \quad \ldots \text{[equation (ii)]} \]

From equation (i) and equation (ii) we get,
\[ \frac{BM}{MR} = \frac{AL}{LQ} \quad \text{(ii)} \]

Given, \( \frac{BM}{MC} = \frac{1}{2} \)
\[ \frac{AM}{MR} = \frac{BM}{MC} \]
\[ \frac{AM}{MR} = \frac{1}{2} \quad \ldots \text{[equation (iii)]} \]

LM || QR \quad \ldots \text{[given from equation]}
\[ \frac{AM}{MR} = \frac{LM}{QR} \quad \ldots \text{[equation (iv)]} \]

\[ \frac{AR}{AM} = \frac{QR}{LM} \]
\[ \frac{(AM + MR)}{AM} = \frac{QR}{LM} \]
\[ 1 + \frac{MR}{AM} = \frac{QR}{LM} \]
\[ 1 + \left(\frac{2}{1}\right) = \frac{QR}{LM} \]
\[ \frac{3}{1} = \frac{QR}{LM} \]
\[ \frac{LM}{QR} = \frac{1}{3} \]

Therefore, the ratio of \( LM : QR \) is 1: 3.

(b) In the figure (2) given below \( AD \) is bisector of \( \angle BAC \). If \( AB = 6 \text{ cm}, AC = 4 \text{ cm} \) and \( BD = 3 \text{ cm} \), find \( BC \)

\[ \text{Solution:-} \]
From the question it is given that,
\( AD \) is bisector of \( \angle BAC \)
\( AB = 6 \text{ cm}, AC = 4 \text{ cm} \) and \( BD = 3 \text{ cm} \)
Construction, from C draw a straight line CE parallel to DA and join AE
\[ \angle 1 = \angle 2 \quad \text{[equation (i)]} \]

By construction \( CE \parallel DE \)
So, \( \angle 2 = \angle 4 \quad \text{[because alternate angles are equal] [equation (ii)]} \]

Again by construction \( CE \parallel DE \)
\[ \angle 1 = \angle 3 \quad \text{[because corresponding angles are equal] [equation (iii)]} \]

By comparing equation (i), equation (ii) and equation (iii) we get,
\[ \angle 3 = \angle 4 \]
So, \( AC = AE \quad \text{[equation (iv)]} \]

Now, consider the \( \triangle BCE \),
\( CE \parallel DE \)
\( \frac{BD}{DC} = \frac{AB}{AE} \)
\( \frac{BD}{DC} = \frac{AB}{AC} \)
\( \frac{3}{DC} = \frac{6}{4} \)
By cross multiplication we get,
\[ 3 \times 4 = 6 \times DC \]
\[ DC = \frac{(3 \times 4)}{6} \]
\[ DC = 12/6 \]
\[ DC = 2 \]
Therefore, \( BC = BD + DC \]
\[ = 3 + 2 \]
\[ = 5 \text{ cm} \]
EXERCISE 13.3

1. Given that Δs ABC and PQR are similar.
   Find:
   (i) The ratio of the area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1 : 3.
   (ii) the ratio of their corresponding sides if area of ΔABC : area of ΔPQR = 25 : 36.
   Solution:-
   From the question it is given that,
   (i) The area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 1 : 3
   Then, ΔABC ~ ΔPQR
   area of ΔABC/area of ΔPQR = BC²/QR²
   So, BC : QR = 1 : 3
   Therefore, ΔABC/area of ΔPQR = 1²/3²
   = 1/9
   Hence the ratio of the area of ΔABC to the area of ΔPQR is 1: 9
   (ii) The area of ΔABC to the area of ΔPQR if their corresponding sides are in the ratio 25 : 36
   Then, ΔABC ~ ΔPQR
   area of ΔABC/area of ΔPQR = BC²/QR²
   area of ΔABC/area of ΔPQR = BC²/QR² = 25/36
   = (BC/QR)² = (5/6)²
   BC/QR = 5/6
   Hence the ratio of their corresponding sides is 5 : 6

2. ΔABC ~ DEF. If area of ΔABC = 9 sq. cm., area of ΔDEF =16 sq. cm and BC = 2.1 cm., find the length of EF.
   Solution:-
   From the question it is given that,
   ΔABC ~ DEF
   Area of ΔABC = 9 sq. cm
   Area of ΔDEF =16 sq. cm
   We know that,
   area of ΔABC/area of ΔDEF = BC²/EF²
   area of ΔABC/area of ΔDEF = BC²/EF²
   9/16 = BC²/EF²
   9/16 = (2.1)²/x²
   2.1/x = √9/√16
2.1/x = ¾
By cross multiplication we get,
2.1 × 4 = 3 × x
8.4 = 3x
x = 8.4/3
x = 2.8
Therefore, EF = 2.8 cm

3. ΔABC ~ ΔDEF. If BC = 3 cm, EF = 4 cm and area of ΔABC = 54 sq. cm. Determine the area of ΔDEF.
Solution:-
From the question it is given that, ΔABC ~ ΔDEF
BC = 3 cm, EF = 4 cm
Area of ΔABC = 54 sq. cm.
We know that,
Area of ΔABC/ area of ΔDEF = BC^2/EF^2
54/area of ΔDEF = 3^2/4^2
54/area of ΔDEF = 9/16
By cross multiplication we get,
Area of ΔDEF = (54 × 16)/9
= 6 × 16
= 96 cm

4. The area of two similar triangles are 36 cm² and 25 cm². If an altitude of the first triangle is 2.4 cm, find the corresponding altitude of the other triangle.
Solution:-
From the question it is given that,
The area of two similar triangles are 36 cm² and 25 cm².
Let us assume ΔPQR ~ ΔXYZ, PM and XN are their altitudes.
So, area of ΔPQR = 36 cm²
Area of ΔXYZ = 25 cm²
PM = 2.4 cm
Assume XN = a
We know that,
area of ΔPQR/area of ΔXYZ = PM^2/XN^2
36/25 = (2.4)^2/a^2
By cross multiplication we get,
\[36a^2 = 25 \times (2.4)^2\]
\[a^2 = \frac{5.76 \times 25}{36}\]
\[a^2 = \frac{144}{36}\]
\[a^2 = 4\]
\[a = \sqrt{4}\]
\[a = 2 \text{ cm}\]

Therefore, altitude of the other triangle \(XN = 2 \text{ cm}\).

5.
(a) In the figure, (i) given below, PB and QA are perpendiculars to the line segment AB. If \(PO = 6 \text{ cm}, QO = 9 \text{ cm}\) and the area of \(\Delta POB = 120 \text{ cm}^2\), find the area of \(\Delta QOA\).

Solution:
From the question it is given that, \(PO = 6 \text{ cm}, QO = 9 \text{ cm}\) and the area of \(\Delta POB = 120 \text{ cm}^2\) From the figure,
Consider the \(\Delta AOQ\) and \(\Delta BOP\),
\[\angle OAQ = \angle OBP\] [both angles are equal to \(90^\circ\)]
\[\angle AOQ = \angle BOP\] [because vertically opposite angles are equal]
Therefore, \(\Delta AOQ \sim \Delta BOP\)
Then, area of \(\Delta AOQ/\text{area of } \Delta BOP = \frac{OQ^2}{PO^2}\)
Area of \(\Delta AOQ/120 = \frac{9^2}{6^2}\)
Area of \(\Delta AOQ/120 = \frac{81}{36}\)
Area of \(\Delta AOQ = \frac{81\times120}{36}\)
Area of \(\Delta AOQ = 270 \text{ cm}^2\)

b) In the figure (ii) given below, \(AB \parallel DC\). \(AO = 10 \text{ cm}, OC = 5 \text{ cm}, AB = 6.5 \text{ cm}\) and \(OD = 2.8 \text{ cm}\).

(i) Prove that \(\Delta OAB \sim \Delta OCD\).
(ii) Find \(CD\) and \(OB\).
(iii) Find the ratio of areas of $\triangle OAB$ and $\triangle OCD$.

Solution:
From the question it is given that,
$AB \parallel DC$. $AO = 10$ cm, $OC = 5$ cm, $AB = 6.5$ cm and $OD = 2.8$ cm
(i) We have to prove that, $\triangle OAB \sim \triangle OCD$
So, consider the $\triangle OAB$ and $\triangle OCD$
$\angle AOB = \angle COD$ ... [because vertically opposite angles are equal]
$\angle OBA = \angle OCD$ ... [because alternate angles are equal]
Therefore, $\triangle OAB \sim \triangle OCD$ ... [from AAA axiom]
(ii) Consider the $\triangle OAB$ and $\triangle OCD$
$\frac{OA}{OC} = \frac{OB}{OD} = \frac{AB}{CD}$
Now consider $\frac{OA}{OC} = \frac{OB}{OD}$
$\frac{10}{5} = \frac{OB}{2.8}$
$OB = \frac{(10 \times 2.8)}{5}$
$OB = 2 \times 2.8$
$OB = 5.6$ cm
Then, consider $\frac{OA}{OC} = \frac{AB}{CD}$
$\frac{10}{5} = \frac{6.5}{CD}$
$CD = \frac{(6.5 \times 5)}{10}$
$CD = 32.5/10$
$CD = 3.25$ cm
(iii) We have to find the ratio of areas of $\triangle OAB$ and $\triangle OCD$.
From (i) we proved that, $\triangle OAB \sim \triangle OCD$
Then, area of $\triangle OAB$ / area of $\triangle OCD$
$\frac{AB^2}{CD^2} = \frac{(6.5)^2}{(3.25)^2}$
$= \frac{6.5 \times 6.5}{3.25 \times 3.25}$

Therefore, the ratio of areas of $\triangle OAB$ and $\triangle OCD = 4: 1$.

6.
(a) In the figure (i) given below, $DE \parallel BC$. If $DE = 6 \text{ cm}$, $BC = 9 \text{ cm}$ and area of $\triangle ADE = 28 \text{ sq. cm}$, find the area of $\triangle ABC$.

Solution:-
From the question it is given that,
$DE \parallel BC$, $DE = 6 \text{ cm}$, $BC = 9 \text{ cm}$ and area of $\triangle ADE = 28 \text{ sq. cm}$
From the fig, $\angle D = \angle B$ and $\angle E = \angle C$ ... [corresponding angles are equal]
Now consider the $\triangle ADE$ and $\triangle ABC$,
$\angle A = \angle A$ ... [common angles for both triangles]
Therefore, $\triangle ADE \sim \triangle ABC$
Then, area of $\triangle ADE$/area of $\triangle ABC = (DE)^2/(BC)^2$
\[
\frac{28}{\text{area of } \triangle ABC} = \left(\frac{6}{9}\right)^2
\]
\[
\text{area of } \triangle ABC = \frac{28 \times 81}{36}
\]
\[
\text{area of } \triangle ABC = 63 \text{ cm}^2
\]

(b) In the figure (ii) given below, $DE \parallel BC$ and $AD : DB = 1 : 2$, find the ratio of the areas of $\triangle ADE$ and trapezium $DBCE$.

Solution:-
From the question it is given that,
$DE \parallel BC$, $AD : DB = 1 : 2$
Area of $\triangle ADE = \text{area of trapezium DBCE}$
Therefore, $\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \left(\frac{1}{2}\right)^2$
\[
\frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{1}{4}
\]
\[
\text{area of trapezium DBCE} = \frac{3}{4} \times 63 = 47.25 \text{ cm}^2
\]
Solution:-
From the question it is given that, \( DE \parallel BC \) and \( AD : DB = 1 : 2 \),
\[ \angle D = \angle B, \angle E = \angle C \] ... [corresponding angles are equal]
Consider the \( \triangle ADE \) and \( \triangle ABC \),
\[ \angle A = \angle A \] ... [common angles for both triangles]
Therefore, \( \triangle ADE \sim \triangle ABC \)
But, \( AD/DB = \frac{1}{2} \)
Then, \( DB/AD = \frac{2}{1} \)
Now, adding 1 for both side LHS and RHS,
\[ (DB/AD) + 1 = (2/1) + 1 \]
\[ (DB + AD)/AD = (2 + 1) \]
Therefore, \( \triangle ADE \sim \triangle ABC \)
Then, area of \( \triangle ADE \)/area of \( \triangle ABC \) = \( AD^2/AB^2 \)
Area of \( \triangle ADE \)/area of \( \triangle ABC \) = \( (1/3)^2 \)
Area of \( \triangle ADE \)/area of \( \triangle ABC \) = \( 1/9 \)
Area of \( \triangle ABC \) = 9 area of \( \triangle ADE \)
Area of \( \triangle ADE \) area of \( \triangle ABC \) - area of \( \triangle ADE \)
9 area of \( \triangle ADE \) - area of \( \triangle ADE \)
8 area of \( \triangle ADE \)
Therefore, area of \( \triangle ADE \)/area of trapezium = \( 1/8 \)
Then area of \( \triangle ADE \) : area of trapezium DBCE = 1 : 8

7.
In the given figure, \( DE \parallel BC \).
(i) Prove that \( \triangle ADE \) and \( \triangle ABC \) are similar.
(ii) Given that \( AD = \frac{1}{2} BD \), calculate \( DE \) if \( BC = 4.5 \) cm.
(iii) If area of \( \triangle ABC = 18\text{cm}^2 \), find the area of trapezium DBCE

Solution:-
(i) From the question it is given that, \( DE \parallel BC \)
We have to prove that, \( \triangle ADE \) and \( \triangle ABC \) are similar

\[ \angle A = \angle A \quad \text{... [common angle for both triangles]} \]

\[ \angle ADE = \angle ABC \quad \text{... [because corresponding angles are equal]} \]

Therefore, \( \triangle ADE \sim \triangle ABC \quad \text{... [AA axiom]} \)

(ii) From (i) we proved that, \( \triangle ADE \sim \triangle ABC \)

Then, \( \frac{AD}{AB} = \frac{AB}{AC} = \frac{DE}{BC} \)

So, \( \frac{AD}{AD + BD} = \frac{DE}{BC} \)

\[ \frac{(\frac{1}{2} BD)}{((\frac{1}{2} BD) + BD)} = \frac{DE}{4.5} \]

\[ \frac{(\frac{1}{2} BD)}{((\frac{3}{2} BD)} = \frac{DE}{4.5} \]

\[ \frac{1}{3} = \frac{DE}{4.5} \]

Therefore, \( DE = 4.5/3 \)

\[ DE = 1.5 \text{ cm} \]

(iii) From the question it is given that, area of \( \triangle ABC = 18 \text{ cm}^2 \)

Then, \( \frac{\text{area of } \triangle ADE}{\text{area of } \triangle ABC} = \frac{DE}{BC} \)

\[ \frac{\text{area of } \triangle ADE}{18} = \left(\frac{DE}{BC}\right)^2 \]

\[ \frac{\text{area of } \triangle ADE}{18} = \left(\frac{AD}{AB}\right)^2 \]

\[ \frac{\text{area of } \triangle ADE}{18} = \left(\frac{1}{3}\right)^2 = 1/9 \]

\[ \text{area of } \triangle ADE = 18 \times 1/9 \]

\[ \text{area of } \triangle ADE = 2 \]

So, area of trapezium DBCE = area of \( \triangle ABC \) – area of \( \triangle ADE \)

\[ = 18 - 2 \]

\[ = 16 \text{ cm}^2 \]

8. In the given figure, \( AB \) and \( DE \) are perpendicular to \( BC \).

(i) Prove that \( \triangle ABC \sim \triangle DEC \)

(ii) If \( AB = 6 \text{ cm} \): \( DE = 4 \text{ cm} \) and \( AC = 15 \text{ cm} \), calculate \( CD \).

(iii) Find the ratio of the area of \( \triangle ABC : \) area of \( \triangle DEC \).
Solution:-
(i) Consider the $\triangle ABC$ and $\triangle DEC,$
$\angle ABC = \angle DEC \quad \ldots \quad \text{[both angles are equal to 90°]}$
$\angle C = \angle C \quad \ldots \quad \text{[common angle for both triangles]}$
Therefore, $\triangle ABC \sim \triangle DEC \quad \ldots \quad \text{[by AA axiom]}$

(ii) $AC/CD = AB/DE$
Corresponding sides of similar triangles are proportional
$15/CD = 6/4$
$CD = (15 \times 4)/6$
$CD = 60/6$
$CD = 10 \text{ cm}$

(iii) we know that, area of $\triangle ABC$/area of $\triangle DEC = AB^2/DE^2$
area of $\triangle ABC$/area of $\triangle DEC = 6^2/4^2$
area of $\triangle ABC$/area of $\triangle DEC = 36/16$
area of $\triangle ABC$/area of $\triangle DEC = 9/4$
Therefore, the ratio of the area of $\triangle ABC : \text{area of } \triangle DEC$ is $9 : 4.$

9. In the adjoining figure, $ABC$ is a triangle. $DE$ is parallel to $BC$ and $AD/DB = 3/2,$
(i) Determine the ratios $AD/AB,$ $DE/BC$
(ii) Prove that $\triangle DEF$ is similar to $\triangle CBF.$ Hence, find $EF/FB.$
(iii) What is the ratio of the areas of $\triangle DEF$ and $\triangle CBF$?

Solution:-
(i) We have to find the ratios $AD/AB,$ $DE/BC,$
From the question it is given that, $AD/DB = 3/2$
Then, $DB/AD = 2/3$
Now add 1 for both LHS and RHS we get,
$(DB/AD) + 1 = (2/3) + 1$
$(DB + AD)/AD = (2 + 3)/3$
From the figure \((DB + AD) = AB\)
So, \(AB/AD = 5/3\)
Now, consider the \(\triangle ADE\) and \(\triangle ABC\),
\[\angle ADE = \angle B \quad \ldots\] [corresponding angles are equal]
\[\angle AED = \angle C \quad \ldots\] [corresponding angles are equal]
Therefore, \(\triangle ADE \sim \triangle ABC\) \quad \ldots\ [by AA similarity]
Then, \(AD/AB = DE/BC = 3/5\)

(ii) Now consider the \(\triangle DEF\) and \(\triangle CBF\)
\[\angle EDF = \angle BCF \quad \ldots\] [because alternate angles are equal]
\[\angle DEF = \angle FBC \quad \ldots\] [because alternate angles are equal]
\[\angle DFE = \angle ABFC \quad \ldots\] [because vertically opposite angles are equal]
Therefore, \(\triangle DEF \sim \triangle CBF\)
So, \(EF/FB = DE/BC = 3/5\)

(iii) we have to find the ratio of the areas of \(\triangle DEF\) and \(\triangle CBF\),
We know that, \(\text{Area of } \triangle DFE / \text{Area of } \triangle BFC = DE^2 / BC^2\)
\(\text{Area of } \triangle DFE / \text{Area of } \triangle BFC = (DE/BC)^2\)
\(\text{Area of } \triangle DFE / \text{Area of } \triangle BFC = (3/5)^2\)
\(\text{Area of } \triangle DFE / \text{Area of } \triangle BFC = 9/25\)
Therefore, the ratio of the areas of \(\triangle DEF\) and \(\triangle CBF\) is \(9:25\).

10. In \(\triangle ABC\), \(AP : PB = 2 : 3\). \(PO\) is parallel to \(BC\) and is extended to \(Q\) so that \(CQ\) is parallel to \(BA\). Find:
(i) \(\text{area } \triangle APO : \text{area } \triangle ABC\).
(ii) \(\text{area } \triangle APO : \text{area } \triangle CQO\).

Solution:-
From the question it is given that,
\(PB = 2:3\)
\(PO\) is parallel to \(BC\) and is extended to \(Q\) so that \(CQ\) is parallel to \(BA\).
(i) we have to find the area \(\triangle APO\) : \(\triangle ABC\),
Then,
\[ \angle A = \angle A \]  
[common angles for both triangles]
\[ \angle APO = \angle ABC \]  
[because corresponding angles are equal]
Then, \( \triangle APO \sim \triangle ABC \)  
[AA axiom]
We know that, area of \( \triangle APO \)/area of \( \triangle ABC \) = \( \frac{AP^2}{AB^2} \)
\[ = \frac{AP^2}{(AP + PB)^2} \]
\[ = \frac{2^2}{(2 + 3)^2} \]
\[ = \frac{4}{5^2} \]
\[ = \frac{4}{25} \]
Therefore, area \( \triangle APO \): area \( \triangle ABC \) is 4: 25

(ii) we have to find the area \( \triangle APO \): area \( \triangle CQO \)
Then, \( \angle AOP = \angle COQ \)  
[because vertically opposite angles are equal]
\[ \angle APQ = \angle OQC \]  
[because alternate angles are equal]
Therefore, area of \( \triangle APO \)/area of \( \triangle CQO \) = \( \frac{AP^2}{CQ^2} \)
area of \( \triangle APO \)/area of \( \triangle CQO \) = \( \frac{AP^2}{PB^2} \)
area of \( \triangle APO \)/area of \( \triangle CQO \) = \( \frac{2^2}{3^2} \)
area of \( \triangle APO \)/area of \( \triangle CQO \) = \( \frac{4}{9} \)
Therefore, area \( \triangle APO \): area \( \triangle CQO \) is 4: 9

11.
(a) In the figure (i) given below, \( ABCD \) is a trapezium in which \( AB \parallel DC \) and \( AB = 2 \, CD \).
Determine the ratio of the areas of \( \triangle AOB \) and \( \triangle COD \).

Solution:-
From the question it is given that,
\( ABCD \) is a trapezium in which \( AB \parallel DC \) and \( AB = 2 \, CD \),
Then, \( \angle OAB = \angle OCD \)  
[because alternate angles are equal]
\[ \angle OBA = \angle ODC \]
Then, \( \triangle AOB \sim \triangle COD \)
So, area of \( \triangle AOB \)/area of \( \triangle COD \) = \( \frac{AB^2}{CD^2} \)
\[ = \frac{(2CD)^2}{CD^2} \]  
[because \( AB = 2 \, CD \)]
\[ = \frac{4CD^2}{CD^2} \]
Therefore, the ratio of the areas of \( \triangle AOB \) and \( \triangle COD \) is 4:1.

(b) In the figure (ii) given below, \( ABCD \) is a parallelogram. \( AM \perp DC \) and \( AN \perp CB \). If \( AM = 6 \text{ cm} \), \( AN = 10 \text{ cm} \) and the area of parallelogram \( ABCD \) is 45 cm\(^2\), find

(i) \( AB \)
(ii) \( BC \)
(iii) area of \( \triangle ADM \): area of \( \triangle ANB \).

Solution:

From the question it is given that,

\( ABCD \) is a parallelogram, \( AM \perp DC \) and \( AN \perp CB \)

\( AM = 6 \text{ cm} \)
\( AN = 10 \text{ cm} \)

The area of parallelogram \( ABCD \) is 45 cm\(^2\)

Then, area of parallelogram \( ABCD = DC \times AM = BC \times AN \)

\[
45 = DC \times 6 = BC \times 10
\]

(i) \( DC = \frac{45}{6} \)
Divide both numerator and denominator by 3 we get,

\[
= \frac{15}{2} = 7.5 \text{ cm}
\]

Therefore, \( AB = DC = 7.5 \text{ cm} \)

(ii) \( BC \times 10 = 45 \)
\( BC = \frac{45}{10} \)
\( BC = 4.5 \text{ cm} \)

(iii) Now, consider \( \triangle ADM \) and \( \triangle ANB \)

\( \angle D = \angle B \) \hspace{1cm} [because opposite angles of a parallelogram]

\( \angle M = \angle N \) \hspace{1cm} [both angles are equal to 90\(^\circ\)]

Therefore, \( \triangle ADM \sim \triangle ANB \)

Therefore, area of \( \triangle ADM \)/area of \( \triangle ANB = AD^2/AB^2 \)

\[
= BC^2/AB^2
\]
= \frac{4.5^2}{7.5^2}
= \frac{20.25}{56.25}
= \frac{2025}{5625}
= \frac{81}{225}
= \frac{9}{25}

Therefore, area of $\triangle ADM : area$ of $\triangle ANB$ is 9: 25

(c) In the figure (iii) given below, $ABCD$ is a parallelogram. $E$ is a point on $AB$, $CE$ intersects the diagonal $BD$ at $O$ and $EF \parallel BC$. If $AE : EB = 2 : 3$, find

(i) $EF : AD$
(ii) $area$ of $\triangle BEF : area$ of $\triangle ABD$
(iii) $area$ of $\triangle ABD : area$ of trapezium $AFED$
(iv) $area$ of $\triangle FEO : area$ of $\triangle OBC$.

Solution:-
From the question it is given that, $ABCD$ is a parallelogram.
$E$ is a point on $AB$, $CE$ intersects the diagonal $BD$ at $O$.
$AE : EB = 2 : 3$

(i) We have to find $EF : AD$

So, $\frac{AB}{BE} = \frac{AD}{EF}$

$\frac{EF}{AD} = \frac{BE}{AB}$

$AE/EB = 2/3 \ldots [given]$

Now add 1 to both LHS and RHS we get,

\[
(AE/EB) + 1 = (2/3) + 1
\]

\[
(AE + EB)/EB = (2 + 3)/3
\]

\[
AB/EB = 5/3
\]

\[
EB/AB = 3/5
\]

Therefore, $EF : AD$ is 3: 5

(ii) we have to find area of $\triangle BEF : area$ of $\triangle ABD$,

Then, area of $\triangle BEF/area$ of $\triangle ABD = (EF)^2/(AD)^2$

area of $\triangle BEF/area$ of $\triangle ABD = \frac{3^2}{5^2}$

$= 9/25$
Therefore, area of $\triangle BEF$: area of $\triangle ABD$ is 9: 25

(iii) From (ii) area of $\triangle ABD$ / area of $\triangle BEF = 25/9$
25 area of $\triangle BEF$ = 9 area of $\triangle ABD$
25(area of $\triangle ABD$ – area of trapezium $AEFD$) = 9 area of $\triangle ABD$
25 area of $\triangle ABD$ – 25 area of trapezium $AEFD$ = 9 area of $\triangle ABD$
25 area of trapezium $AEFD$ = 16 area of $\triangle ABD$
area of $\triangle ABD$ / area of trapezium $AEFD = 25/16$
Therefore, area of $\triangle ABD$ : area of trapezium $AFED$ = 25: 16

(iv) Now we have to find area of $\triangle FEO$ : area of $\triangle OBC$
So, consider $\triangle FEO$ and $\triangle OBC$,
$\angle EOF = \angle BOC \ldots [\text{because vertically opposite angles are equal}]
\angle F = \angle OBC \ldots [\text{because alternate angles are equal}]
\triangle FEO \sim \triangle OBC$
Then, area of $\triangle FEO$ / area of $\triangle OBC = EF^2/BC^2$
$EF^2/AD^2 = 9/25$
Therefore, area of $\triangle FEO$: area of $\triangle OBC$ = 9: 25.

12. In the adjoining figure, $ABCD$ is a parallelogram. $P$ is a point on $BC$ such that $BP : PC = 1 : 2$ and DP produced meets $AB$ produced at $Q$. If area of $\triangle CPQ = 20 \text{ cm}^2$, find

(i) area of $\triangle BPQ$.
(ii) area $\triangle CDP$.
(iii) area of parallelogram $ABCD$.

Solution:-
From the question it is given that, $ABCD$ is a parallelogram.
$BP:PC = 1:2$
area of $\triangle CPQ = 20 \text{ cm}^2$
Construction: draw $QN$ perpendicular $CB$ and Join $BN$. 
Then, area of ∆BPQ/area of ∆CPQ = ((½BP) \times QN)/((½PC) \times QN)

= BP/PC = ½

(i) So, area ∆BPQ = ½ area of ∆CPQ

= ½ \times 20

Therefore, area of ∆BPQ = 10 cm²

(ii) Now we have to find area of ∆CDP,
Consider the ∆CDP and ∆BQP,
Then, ∠CPD = ∠QPD

∠PDC = ∠PQB

Therefore, ∆CDP ~ ∆BQP

area of ∆CDP/area of ∆BQP = PC²/BP²

area of ∆CDP/area of ∆BQP = 2²/1²

area of ∆CDP/area of ∆BQP = 4/1

area of ∆CDP = 4 \times area ∆BQP

Therefore, area of ∆CDP = 4 \times 10

= 40 cm²

(iii) We have to find the area of parallelogram ABCD,
Area of parallelogram ABCD = 2 area of ∆DCQ

= 2 area (ΔDCP + ∆CPQ)

= 2 (40 + 20) cm²

= 2 \times 60 cm²

= 120 cm²

Therefore, the area of parallelogram ABCD is 120 cm².

13. (a) In the figure (i) given below, DE || BC and the ratio of the areas of ∆ADE and trapezium DBCE is 4 : 5. Find the ratio of DE : BC.
Solution:
From the question it is given that,
DE || BC
The ratio of the areas of ∆ADE and trapezium DBCE is 4 : 5
Now, consider the ∆ABC and ∆ADE
∠A = ∠A … [common angle for both triangles]
∠D = ∠B and ∠E = ∠C … [because corresponding angles are equal]
Therefore, ∆ADE ~ ∆ABC
So, area of ∆ADE/area of ∆ABC = (DE)^2/(BC)^2 … [equation (i)]
Then, area of ∆ADE/area of trapezium DBCE = 4/5
area of trapezium DBCE/area of ∆ADE = 5/4
Add 1 for both LHS and RHS we get,
(area of trapezium DBCE/area of ∆ADE) + 1 = (5/4) + 1
(area of trapezium DBCE + area of ∆ADE)/area of ∆ADE = (5 + 4)/4
area of ∆ABC/area of ∆ADE = 9/4
area of ∆ADE/area of ∆ABC = 4/9
From equation (i),
area of ∆ADE/area of ∆ABC = (DE)^2/(BC)^2
area of ∆ADE/area of ∆ABC = (DE)^2/(BC)^2 = 4^2/9^2
area of ∆ADE/area of ∆ABC = (DE)^2/(BC)^2 = 2/3
Therefore, DE: BC = 2: 3

(b) In the figure (ii) given below, AB || DC and AB = 2 DC. If AD = 3 cm, BC = 4 cm and AD, BC produced meet at E, find (i) ED (ii) BE (iii) area of ∆EDC : area of trapezium ABCD.
Solution:
From the question it is given that,

AB || DC
AB = 2 DC, AD = 3 cm, BC = 4 cm

Now consider ∆EAB,

\[
\frac{EA}{DA} = \frac{EB}{CB} = \frac{AB}{DC} = \frac{2DC}{DC} = \frac{2}{1}
\]

(i) EA = 2, DA = 2 × 3 = 6 cm
Then, ED = EA − DA
     = 6 − 3
     = 3 cm

(ii) \( \frac{EB}{CB} = \frac{2}{1} \)
EB = 2 CB
EB = 2 × 4
EB = 8 cm

(iii) Now, consider the ∆EAB, DC || AB
So, ∆EDC ~ ∆EAB
Therefore, area of ∆EDC/area of ∆ABE = DC^2/AB^2
\[
\text{area of } \triangle EDC/\text{area of } \triangle ABE = \frac{DC^2}{(2DC)^2} = \frac{DC^2}{4DC^2} = \frac{1}{4}
\]
Therefore, area of ABE = 4 area of ∆EDC
Then, area of ∆EDC + area of trapezium ABCD = 4 area of ∆EDC
Area of trapezium ABCD = 3 area of ∆EDC
So, area of ∆EDC/area of trapezium ABCD = 1/3
Therefore, area of ∆EDC: area of trapezium ABCD = 1: 3

14. (a) In the figure given below, ABCD is a trapezium in which DC is parallel to AB. If AB = 9 cm, DC = 6 cm and BB = 12 cm., find (i) BP (ii) the ratio of areas of ∆APB and
Solution:

From the question it is given that,
DC is parallel to AB
AB = 9 cm, DC = 6 cm and BB = 12 cm

(i) Consider the \( \triangle APB \) and \( \triangle CPD \)

\[ \angle APB = \angle CPD \]  [because vertically opposite angles are equal]
\[ \angle PAB = \angle PCD \]  [because alternate angles are equal]

So, \( \triangle APB \sim \triangle CPD \)

Then, \( \frac{BP}{PD} = \frac{AB}{CD} \)

\[ \frac{BP}{12 - BP} = \frac{9}{6} \]
\[ 6BP = 108 - 9BP \]
\[ 6BP + 9BP = 108 \]
\[ 15BP = 108 \]
\[ BP = \frac{108}{15} \]

Therefore, \( BP = 7.2 \) cm

(ii) We know that, area of \( \triangle APB \)/area of \( \triangle CPD \) = \( \frac{AB^2}{CD^2} \)

area of \( \triangle APB \)/area of \( \triangle CPD \) = \( \frac{9^2}{6^2} \)

area of \( \triangle APB \)/area of \( \triangle CPD \) = \( \frac{81}{36} \)

By dividing both numerator and denominator by 9, we get,
area of \( \triangle APB \)/area of \( \triangle CPD \) = \( \frac{9}{4} \)

Therefore, the ratio of areas of \( \triangle APB \) and \( \triangle DPC \) is 9:4

(b) In the figure given below, \( \angle ABC = \angle DAC \) and \( AB = 8 \) cm, \( AC = 4 \) cm, \( AD = 5 \) cm. (i) Prove that \( \triangle DAC \) is similar to \( \triangle BCA \) (ii) Find \( BC \) and \( CD \) (iii) Find the area of \( \triangle DAC \) : area of \( \triangle ABC \).
**Solution:-**

From the question it is given that,

∠ABC = ∠DAC

AB = 8 cm, AC = 4 cm, AD = 5 cm

(i) Now, consider ΔACD and ΔBCA

∠C = ∠C \[\text{[common angle for both triangles]}\]

∠ABC = ∠CAD \[\text{[from the question]}\]

So, ΔACD \sim ΔBCA \[\text{[by AA axiom]}\]

(ii) \[\frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{AB}\]

Consider \[\frac{AC}{BC} = \frac{AD}{AB}\]

4/BC = 5/8

BC = (4 \times 8)/5

BC = 32/5

BC = 6.4 cm

Then, consider \[\frac{CD}{CA} = \frac{AD}{AB}\]

CD/4 = 5/8

CD = (4 \times 5)/8

CD = 20/8

CD = 2.5 cm

(iii) from (i) we proved that, ΔACD \sim ΔBCA

area of ΔACB/area of ΔBCA = \frac{AC^2}{AB^2}

= \frac{4^2}{8^2}

= 16/64

By dividing both numerator and denominator by 16, we get,

= \frac{1}{4}

Therefore, the area of ΔACD : area of ΔABC is 1: 4.

15. ABC is a right angled triangle with ∠ABC = 90°. D is any point on AB and DE is perpendicular to AC. Prove that:

(i) ΔADE \sim ΔACB.

(ii) If AC = 13 cm, BC = 5 cm and AE = 4 cm. Find DE and AD.

(iii) Find, area of ΔADE : area of quadrilateral BCED.
Solution:

From the question it is given that,
\[ \angle ABC = 90^\circ \]

AB and DE is perpendicular to AC

(i) Consider the \( \triangle ADE \) and \( \triangle ACB \),

\[ \angle A = \angle A \quad \text{[common angle for both triangle]} \]
\[ \angle B = \angle E \quad \text{[both angles are equal to 90°]} \]

Therefore, \( \triangle ADE \sim \triangle ACB \)

(ii) from (i) we proved that, \( \triangle ADE \sim \triangle ACB \)

So, \( AE/AB = AD/AC = DE/BC \) \quad \text{[equation (i)]}

Consider the \( \triangle ABC \), is a right angle triangle

From Pythagoras theorem, we have

\[ AC^2 = AB^2 + BC^2 \]
\[ 13^2 = AB^2 + 5^2 \]
\[ 169 = AB^2 + 25 \]
\[ AB^2 = 169 - 25 \]
\[ AB^2 = 144 \]
\[ AB = \sqrt{144} \]
\[ AB = 12 \text{ cm} \]

Consider the equation (i),

\[ AE/AB = AD/AC = DE/BC \]

Take, \( AE/AB = AD/AC \)

\[ 4/12 = AD/13 \]
\[ 1/3 = AD/13 \]
\[ (1 \times 13)/3 = AD \]
\[ AD = 4.33 \text{ cm} \]

Now, take \( AE/AB = DE/BC \)

\[ 4/12 = DE/5 \]
\[ \frac{1}{3} = \frac{DE}{5} \]
\[ DE = \frac{(5 \times 1)}{3} \]
\[ DE = \frac{5}{3} \]
\[ DE = 1.67 \text{ cm} \]

(iii) Now, we have to find area of \( \triangle ADE : \) area of quadrilateral \( \text{BCED} \).

We know that, Area of \( \triangle ADE = \frac{1}{2} \times \text{AE} \times \text{DE} \)
\[ = \frac{1}{2} \times 4 \times \left( \frac{5}{3} \right) \]
\[ = \frac{10}{3} \text{ cm}^2 \]

Then, area of quadrilateral \( \text{BCED} = \text{area of } \triangle \text{ABC} - \text{area of } \triangle \text{ADE} \)
\[ = \frac{1}{2} \times \text{BC} \times \text{AB} - \frac{10}{3} \]
\[ = \frac{1}{2} \times 5 \times 12 - \frac{10}{3} \]
\[ = 1 \times 5 \times 6 - \frac{10}{3} \]
\[ = 30 - \frac{10}{3} \]
\[ = \frac{(90 - 10)}{3} \]
\[ = \frac{80}{3} \text{ cm}^2 \]

So, the ratio of area of \( \triangle ADE : \) area of quadrilateral \( \text{BCED} = \frac{\frac{10}{3}}{\frac{80}{3}} \)
\[ = \frac{10}{3} \times \frac{3}{80} \]
\[ = \frac{1 \times 1}{1 \times 8} \]
\[ = \frac{1}{8} \]

Therefore, area of \( \triangle ADE : \) area of quadrilateral \( \text{BCED} \) is 1: 8.

16. Two isosceles triangles have equal vertical angles and their areas are in the ratio 7: 16. Find the ratio of their corresponding height.

Solution:-
Consider the two isosceles triangle \( \text{PQR} \) and \( \text{XYZ} \),

![Diagram of two isosceles triangles PQR and XYZ](https://byjus.com)
\[ \angle P = \angle X \]  \[ \text{... [from the question]} \]

So, \( \angle Q + \angle R = \angle Y + \angle Z \)

\[ \angle Q = \angle R \text{ and } \angle Y = \angle Z \]  \[ \text{[because opposite angles of equal sides]} \]

Therefore, \( \angle Q = \angle Y \text{ and } \angle R = \angle Z \)

\( \triangle PQR \sim \triangle XYZ \)

Then, area of \( \triangle PQR \)/area of \( \triangle XYZ = \frac{PM^2}{XN^2} \)  \[ \text{... [from corollary of theorem]} \]

\[ \frac{PM^2}{XN^2} = \frac{7}{16} \]

\[ \frac{PM}{XN} = \frac{\sqrt{7}}{\sqrt{16}} \]

\[ \frac{PM}{XN} = \frac{\sqrt{7}}{4} \]

Therefore, ratio of \( PM: DM = \sqrt{7}: 4 \)

17. On a map drawn to a scale of 1 : 250000, a triangular plot of land has the following measurements: \( AB = 3 \text{ cm}, \ BC = 4 \text{ cm} \) and \( \angle ABC = 90^\circ \). Calculate (i) the actual length of \( AB \) in km. (ii) the area of the plot in sq. km:

Solution:-

From the question it is given that,

Map drawn to a scale of 1: 250000

\( AB = 3 \text{ cm}, \ BC = 4 \text{ cm} \) and \( \angle ABC = 90^\circ \)

(i) We have to find the actual length of \( AB \) in km.

Let us assume scale factor \( K = 1: 250000 \)

\[ K = \frac{1}{250000} \]

Then, length of \( AB \) of actual plot = \( \frac{1}{k} \times \text{length of } AB \text{ on the map} \)

\[ = \left(\frac{1}{\frac{1}{250000}}\right) \times 3 \]

\[ = 250000 \times 3 \]

To convert cm into km divide by 100000

\[ = \frac{250000 \times 3}{100 \times 1000} \]

\[ = \frac{15}{2} \]

\[ = 7.5 \text{ km} \]

(ii) We have to find the area of the plot in sq. km

Area of plot on the map = \( \frac{1}{2} \times AB \times BC \)

\[ = \frac{1}{2} \times 3 \times 4 \]

\[ = \frac{1}{2} \times 12 \]

\[ = 1 \times 6 \]

\[ = 6 \text{ cm}^2 \]

Then, area of actual plot = \( \frac{1}{k^2} \times \text{area of plot on the map} \)

\[ = \frac{1}{250000^2} \times 6 \]

\[ = \frac{6}{250000^2} \]

\[ = \frac{6}{250000^2} \]
To convert cm into km divide by \((100000)^2\)

\[
= \frac{250000 \times 250000 \times 6}{100000 \times 100000}
\]

\[
= \frac{25}{4} \times 6
\]

\[
= \frac{75}{2}
\]

\[
= 37.5 \text{ km}^2
\]

18. On a map drawn to a scale of 1 : 25000, a rectangular plot of land, ABCD has the following measurements AB = 12 cm and BG = 16 cm. Calculate:
(i) the distance of a diagonal of the plot in km.
(ii) the area of the plot in sq. km.

Solution:-
From the question it is given that,
Map drawn to a scale of 1: 25000
AB = 12 cm, BG = 16 cm

Consider the \(\triangle ABC\),
From the Pythagoras theorem,
\[AC^2 = AB^2 + BC^2\]
\[AC = \sqrt{AB^2 + BC^2}\]
\[= \sqrt{(12)^2 + (16)^2}\]
\[= \sqrt{144 + 256}\]
\[= \sqrt{400}\]
\[= 20 \text{ cm}\]

Then, area of rectangular plot ABCD = \(AB \times BC\)
\[= 12 \times 16\]
\[= 192 \text{ cm}^2\]

(i) We have to find the distance of a diagonal of the plot in km.
Let us assume scale factor \(K = 1: 25000\)
\[K = \frac{1}{25000}\]
Then, length of AB of actual plot = \(\frac{1}{k} \times \text{length of diagonal of rectangular plot}\)

\[\frac{1}{k} \times 3 = 25000 \times 20\]

To convert cm into km divide by 100000

\[\frac{25000 \times 20}{100 \times 1000} = 5 \text{ km}\]

(ii) We have to find the area of the plot in sq. km.

Then, area of actual plot = \(\frac{1}{k^2} \times 192 \text{ cm}^2\)

\[\frac{25000^2 \times 192}{100000^2} = 12 \text{ km}^2\]

19. The model of a building is constructed with the scale factor 1 : 30.

(i) If the height of the model is 80 cm, find the actual height of the building in metres.

(ii) If the actual volume of a tank at the top of the building is 27 m³, find the volume of the tank on the top of the model.

Solution:

From the question it is given that,

The model of a building is constructed with the scale factor 1 : 30

So, \(\frac{\text{Height of the model}}{\text{Height of actual building}} = \frac{1}{30}\)

(i) Given, the height of the model is 80 cm

Then, \(\frac{80}{H} = \frac{1}{30}\)

\[H = 80 \times 30\]

\[H = 2400 \text{ cm}\]

\[H = 2400/100\]

\[H = 24 \text{ m}\]

(ii) Given, the actual volume of a tank at the top of the building is 27 m³

\(\frac{\text{Volume of model}}{\text{Volume of tank}} = \left(\frac{1}{30}\right)^3\)

\[\frac{V}{27} = \frac{1}{27000}\]

\[V = 27/27000\]

\[V = 1/1000 \text{ m}^3\]

Therefore, Volume of model = 1000 cm³

20. A model of a ship is made to a scale of 1 : 200.
(i) If the length of the model is 4 m, find the length of the ship.
(ii) If the area of the deck of the ship is 160000 m², find the area of the deck of the model.
(iii) If the volume of the model is 200 liters, find the volume of the ship in m³. (100 liters = 1 m³)

Solution:

From the question it is given that, a model of a ship is made to a scale of 1 : 200
(i) Given, the length of the model is 4 m
Then, length of the ship = \( \frac{4 \times 200}{1} \)

= 800 m

(ii) Given, the area of the deck of the ship is 160000 m²
Then, area of deck of the model = \( 160000 \times \left(\frac{1}{200}\right)^2 \)

= \( 160000 \times \left(\frac{1}{40000}\right) \)

= 4 m²

(iii) Given, the volume of the model is 200 liters
Then, Volume of ship = \( 200 \times \left(\frac{200}{1}\right)^3 \)

= \( 200 \times 8000000 \)

= \( \frac{200 \times 8000000}{100} \)

= 1600000 m³
1. In the adjoining figure, \( \angle 1 = \angle 2 \) and \( \angle 3 = \angle 4 \). Show that \( PT \times QR = PR \times ST \).

Solution:
From the question it is given that,
\( \angle 1 = \angle 2 \) and \( \angle 3 = \angle 4 \)
We have to prove that, \( PT \times QR = PR \times ST \)
Given, \( \angle 1 = \angle 2 \)
Adding \( \angle 6 \) to both LHS and RHS we get,
\( \angle 1 + \angle 6 = \angle 2 + \angle 6 \)
\( \angle SPT = \angle QPR \)
Consider the \( \triangle PQR \) and \( \triangle PST \),
From above \( \angle SPT = \angle QPR \)
\( \angle 3 = \angle 4 \)
Therefore, \( \triangle PQR \sim \triangle PST \)
So, \( PT/PR = ST/QR \)
By cross multiplication we get,
\( PT \times QR = PR \times ST \)
Hence, it is proved that \( PT \times QR = PR \times ST \)

2. In the adjoining figure, \( AB = AC \). If \( PM \perp AB \) and \( PN \perp AC \), show that \( PM \times PC = PN \times PB \).
Solution:

From the given figure,
AB = AC. If PM \perp AB and PN \perp AC
We have to show that, PM \times PC = PN \times PB
Consider the \( \triangle ABC \),
\( AB = AC \) ... [given]
\( \angle B = \angle C \)
Then, consider \( \triangle CPN \) and \( \triangle BPM \)
\( \angle N = \angle M \) ... [both angles are equal to 90°]
\( \angle C = \angle B \) ... [from above]
Therefore, \( \triangle CPN \sim \triangle BPM \) ... [from AA axiom]
So, \( \frac{PC}{PB} = \frac{PN}{PM} \)
By cross multiplication we get,
\( PC \times PM = PN \times PB \)
Therefore, it is proved that, \( PM \times PC = PN \times PB \)

3.
(a) In the figure given below. \( \angle AED = \angle ABC \). Find the values of \( x \) and \( y \).

Solution:-

From the figure it is given that,
\[ \angle AED = \angle ABC \]
Consider the \( \triangle ABC \) and \( \triangle ADE \)
\[ \angle AED = \angle ABC \]
\[ \angle A = \angle A \]  
[common angle for both triangles]
Therefore, \( \triangle ABC \sim \triangle ADE \)  
[by AA axiom]

Then, \( \frac{AD}{AC} = \frac{DE}{BC} \)
\[ \frac{3}{4 + 2} = \frac{y}{10} \]
\[ \frac{3}{6} = \frac{y}{10} \]
By cross multiplication we get,
\[ y = \frac{(3 \times 10)}{6} \]
\[ y = 30/6 \]
\[ y = 5 \]

Now, consider \( \frac{AB}{AE} = \frac{BC}{DE} \)
\[ \frac{3 + x}{4} = \frac{10}{y} \]
Substitute the value of \( y \),
\[ \frac{3 + x}{4} = \frac{10}{5} \]
By cross multiplication,
\[ 5(3 + x) = 10 \times 4 \]
\[ 15 + 5x = 40 \]
\[ 5x = 40 - 15 \]
\[ 5x = 25 \]
\[ x = 25/5 \]
\[ x = 5 \]

Therefore, the value of \( x = 5 \) cm and \( y = 5 \) cm

(b) In the figure given below, \( CD = \frac{1}{2} AC \), \( B \) is mid-point of \( AC \) and \( E \) is mid-point of \( DF \).
If \( BF \parallel AG \), prove that:
(i) \( CE \parallel AG \)
(ii) \( 3 \times ED = GD \)

![Diagram](https://byjus.com)
Solution:-
From the question it is given that,
CD = ½ AC
BF || AG
(i) We have to prove that, CE || AG
Consider, CD = ½ AC
AC = 2BC … [because from the figure B is mid-point of AC]
So, CD = ½ (2BC)
CD = BC
Hence, CE || BF … [equation (i)]
Given, BF || AG … [equation (ii)]
By comparing the results of equation (i) and equation (ii) we get,
CE || AG
(ii) We have to prove that, 3 ED = GD
Consider the ∆AGD,
CE || AG … [above it is proved]
So, ED/GD = DC/AD
AD = AB + BC + DC
= DC + DC + DC
= 3DC
So, ED/GD = DC/(3DC)
ED/GD = 1/(3(1))
ED/GD = 1/3
3ED = GD
Hence it is proved that, 3ED = GD

4. In the adjoining figure, 2 AD = BD, E is mid-point of BD and F is mid-point of AC and EC || BH. Prove that:
(i) DF || BH
(ii) AH = 3 AF.

[Diagram]
Solution:-
From the question it is given that, \(2AD = BD\), \(EC \parallel BH\)

(i) Given, \(E\) is mid-point of \(BD\)
\[
2DE = BD \quad \text{... [equation (i)]}
\]
\[
2AD = BD \quad \text{... [equation (ii)]}
\]
From equation (i) and equation (ii) we get,
\[
2DE = 2AD
\]
\[
DE = AD
\]

Also given that, \(F\) is mid-point of \(AC\)
\[
DF \parallel EC \quad \text{... [equation (iii)]}
\]
Given, \(EC \parallel BH \quad \text{... [equation (iv)]}
By comparing equation (iii) and equation (iv) we get,
\[
DF \parallel BH
\]

(ii) We have to prove that, \(AH = 3 AF\),
Given, \(E\) is mid-point of \(BD\) and \(EC \parallel BH\)
And \(c\) is midpoint of \(AH\),
Then, \(FC = CH \quad \text{... [equation (v)]}
Also given \(F\) is mid-point of \(AC\)
\[
AF = FC \quad \text{... [equation (vi)]}
\]
By comparing both equation (v) and equation (vi) we get,
\[
FC = AF = CH
\]
\[
AF = (1/3)AH
\]
By cross multiplication we get,
\[
3AF = AH
\]
Therefore, it is proved that \(3AF = AH\)

5. In a \(\triangle ABC\), \(D\) and \(E\) are points on the sides \(AB\) and \(AC\) respectively such that \(DE \parallel BC\). If \(AD = 2.4\) cm, \(AE = 3.2\) cm, \(DE = 2\) cm and \(BC = 5\) cm, find \(BD\) and \(CE\).

Solution:-
From the question it is given that, In a \(\triangle ABC\), \(D\) and \(E\) are points on the sides \(AB\) and \(AC\) respectively.
\(DE \parallel BC\)
\(AD = 2.4\) cm, \(AE = 3.2\) cm, \(DE = 2\) cm and \(BC = 5\) cm
Consider the \( \triangle ABC \),

**Given**, \( DE \parallel BC \)

So, \( \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \)

Now, consider \( \frac{AD}{AB} = \frac{DE}{BC} \)

\[
2.4/AB = 2/5
\]

\[
AB = \frac{(2.4 \times 5)}{2} = 12/2 = 6 \text{ cm}
\]

Then, consider \( \frac{AE}{AC} = \frac{DE}{BC} \)

\[
3.2/AC = 2/5
\]

\[
AC = \frac{(3.2 \times 5)}{2} = 16/2 = 8 \text{ cm}
\]

Hence, \( BD = AB - AD \)

\[
= 6 - 2.4 = 3.6 \text{ cm}
\]

\( CE = AC - AE \)

\[
= 8 - 3.2 = 4.8 \text{ cm}
\]

6. In a \( \triangle ABC \), D and E are points on the sides AB and AC respectively such that \( AD = 5.7 \text{ cm}, BD = 9.5 \text{ cm}, AE = 3.3 \text{ cm} \) and \( AC = 8.8 \text{ cm} \). Is \( DE \parallel BC \)? Justify your answer.

**Solution:**

From the question it is given that,

In a \( \triangle ABC \), D and E are points on the sides AB and AC respectively.

\( AD = 5.7 \text{ cm}, BD = 9.5 \text{ cm}, AE = 3.3 \text{ cm} \) and \( AC = 8.8 \text{ cm} \)
Consider the $\triangle ABC$.

EC = AC – AE

\[= 8.8 - 3.3\]

\[= 5.5 \text{ cm}\]

Then, $\frac{AD}{DB} = \frac{5.7}{9.5}$

\[= \frac{57}{95}\]

By dividing both numerator and denominator by 19 we get,

\[= \frac{3}{5}\]

$\frac{AE}{EC} = \frac{3.3}{5.5}$

\[= \frac{33}{55}\]

By dividing both numerator and denominator by 11 we get,

\[= \frac{3}{5}\]

So, $\frac{AD}{DB} = \frac{AE}{EC}$

Therefore, $DE \parallel BC$

7. If the areas of two similar triangles are 360 cm$^2$ and 250 cm$^2$ and if one side of the first triangle is 8 cm, find the length of the corresponding side of the second triangle.

Solution:-

From the question it is given that, the areas of two similar triangles are 360 cm$^2$ and 250 cm$^2$.

one side of the first triangle is 8 cm

So, PQR and XYZ are two similar triangles,
So, let us assume area of $\Delta PQR = 360 \text{ cm}^2$, $QR = 8 \text{ cm}$
And area of $\Delta XYZ = 250 \text{ cm}^2$
Assume $YZ = a$
We know that, area of $\Delta PQR$/area of $\Delta XYZ = QR^2/yz^2$
\[
360/250 = (8)^2/a^2
\]
360/250 = 64/a^2
By cross multiplication we get,
\[
a^2 = (250 \times 64)/360
\]
a^2 = 400/9
a = $\sqrt{400/9}$
a = 20/3
Therefore, the length of the corresponding side of the second triangle $YZ = \frac{20}{3}$

8. In the adjoining figure, D is a point on BC such that $\angle ABD = \angle CAD$. If $AB = 5 \text{ cm}$, $AC = 3 \text{ cm}$ and $AD = 4 \text{ cm}$, find
(i) $BC$
(ii) $DC$
(iii) area of $\Delta ACD$ : area of $\Delta BCA$.

**Solution:**
From the question it is given that,
$\angle ABD = \angle CAD$
$AB = 5 \text{ cm}$, $AC = 3 \text{ cm}$ and $AD = 4 \text{ cm}$
Now, consider the $\Delta ABC$ and $\Delta ACD$
$\angle C = \angle C$ ... [common angle for both triangles]
$\angle ABC = \angle CAD$ ... [from the question]
So, $\Delta ABC \sim \Delta ACD$
Then, $AB/AD = BC/AC = AC/DC$
(i) Consider $AB/AD = BC/AC$
\[
5/4 = BC/3
\]
(i) Consider \( \frac{AB}{AD} = \frac{AC}{DC} \)
\[
\frac{5}{4} = \frac{3}{DC}
\]
\[
DC = \frac{3 \times 4}{5} = \frac{12}{5} = 2.4 \text{ cm}
\]

(ii) Consider \( \triangle ABC \) and \( \triangle ACD \)
\[
\angle CAD = \angle ABC \quad \ldots \quad \text{[from the question]}
\]
\[
\angle ACD = \angle ACB \quad \ldots \quad \text{[common angle for both triangles]}
\]
Therefore, \( \triangle ACD \sim \triangle ABC \)
Then, \( \frac{\text{area of } \triangle ACD}{\text{area of } \triangle ABC} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25} \)
Therefore, the area of \( \triangle ACD : \text{area of } \triangle BCA \) is 16 : 25.

9. In the adjoining figure, the diagonals of a parallelogram intersect at \( O \). \( OE \) is drawn parallel to \( CB \) to meet \( AB \) at \( E \), find area of \( \triangle AOE : \text{area of parallelogram } ABDC \).

![Parallelogram Diagram](https://byjus.com)

**Solution:-**
From the given figure, the diagonals of a parallelogram intersect at \( O \).
OE is drawn parallel to \( CB \) to meet \( AB \) at \( E \).
In the figure four triangles have equal area.
So, area of \( \triangle OAB = \frac{1}{4} \text{ area of parallelogram } ABDC \)
Then, \( O \) is midpoint of \( AC \) of \( \triangle ABC \) and \( DE \parallel CB \)
E is also midpoint of \( AB \)
Therefore, \( OE \) is the median of \( \triangle AOB \)
Area of \( \triangle AOE = \frac{1}{2} \text{ area of } \triangle AOB \)
\[
= \frac{1}{2} \times \frac{1}{4} \text{ area of parallelogram } ABDC
= \frac{1}{8} \text{ area of parallelogram } ABDC
So, area of $\triangle AOE/\text{area of parallelogram } ABCD = 1/8$
Therefore, area of $\triangle AOE$: area of parallelogram $ABCD$ is $1:8$.

10. In the given figure, $ABCD$ is a trapezium in which $AB \parallel DC$. If $2AB = 3DC$, find the ratio of the areas of $\triangle AOB$ and $\triangle COD$.

Solution:-
From the question it is given that, $ABCD$ is a trapezium in which $AB \parallel DC$. If $2AB = 3DC$.
So, $2AB = 3DC$
$AB/DC = 3/2$
Now, consider $\triangle AOB$ and $\triangle COD$
$\angle AOB = \angle COD$ ... [because vertically opposite angles are equal]
$\angle OAB = \angle OCD$ ... [because alternate angles are equal]
Therefore, $\triangle AOB \sim \triangle COD$ ... [from AA axiom]
Then, area of $\triangle AOB/\text{area of } \triangle COD = AB^2/DC^2$
$= (3/2)^2$
$= 9/4$
Therefore, the ratio of the areas of $\triangle AOB$ and $\triangle COD$ is $9:4$

11. In the adjoining figure, $ABCD$ is a parallelogram. $E$ is mid-point of $BC$. $DE$ meets the diagonal $AC$ at $O$ and meet $AB$ (produced) at $F$. Prove that

Solution:-
From the question it is given that,
$ABCD$ is a parallelogram. $E$ is mid-point of $BC$. 
DE meets the diagonal AC at O.

(i) Now consider the \( \triangle AOD \) and \( \triangle EDC \),

\[ \angle AOD = \angle EOC \quad \text{... [because Vertically opposite angles are equal]} \]

\[ \angle OAD = \angle OCB \quad \text{... [because alternate angles are equal]} \]

Therefore, \( \triangle AOD \sim \triangle EOC \)

Then, \( \frac{OA}{OC} = \frac{DO}{OE} = \frac{AD}{EC} = \frac{2EC}{EC} \)

\[ \frac{OA}{OC} = \frac{DO}{OE} = \frac{2}{1} \]

Therefore, \( OA: OC = 2: 1 \)

(ii) From (i) we proved that, \( \triangle AOD \sim \triangle EOC \)

So, area of \( \triangle OEC \)/area of \( \triangle AOD = \frac{OE^2}{DO^2} \)

area of \( \triangle OEC \)/area of \( \triangle AOD = \frac{1^2}{2^2} \)

area of \( \triangle OEC \)/area of \( \triangle AOD = \frac{1}{4} \)

Therefore, area of \( \triangle OEC \): area of \( \triangle AOD \) is 1: 4.

13. A model of a ship is made to a scale of 1: 250 calculate:

(i) The length of the ship, if the length of model is 1.6 m.

(ii) The area of the deck of the ship, if the area of the deck of model is 2.4 m².

(iii) The volume of the model, if the volume of the ship is 1 km³.

Solution:-

From the question it is given that, a model of a ship is made to a scale of 1 : 250

(i) Given, the length of the model is 1.6 m

Then, length of the ship = \( (1.6 \times 250)/1 \)

\[ = 400 \text{ m} \]

(ii) Given, the area of the deck of the ship is 2.4 m²

Then, area of deck of the model = \( 2.4 \times (1/250)^2 \)

\[ = 1,50,000 \text{ m}^2 \]

\[ = 4 \text{ m}^2 \]

(iii) Given, the volume of the model is 1 km³

Then, Volume of ship = \( (1/250)^3 \times 1 \text{ km}^3 \)

\[ = 1/(250)^3 \times 1000^3 \]

\[ = 4^3 \]

\[ = 64 \text{ m}^3 \]

Therefore, volume of ship is 64 m³.