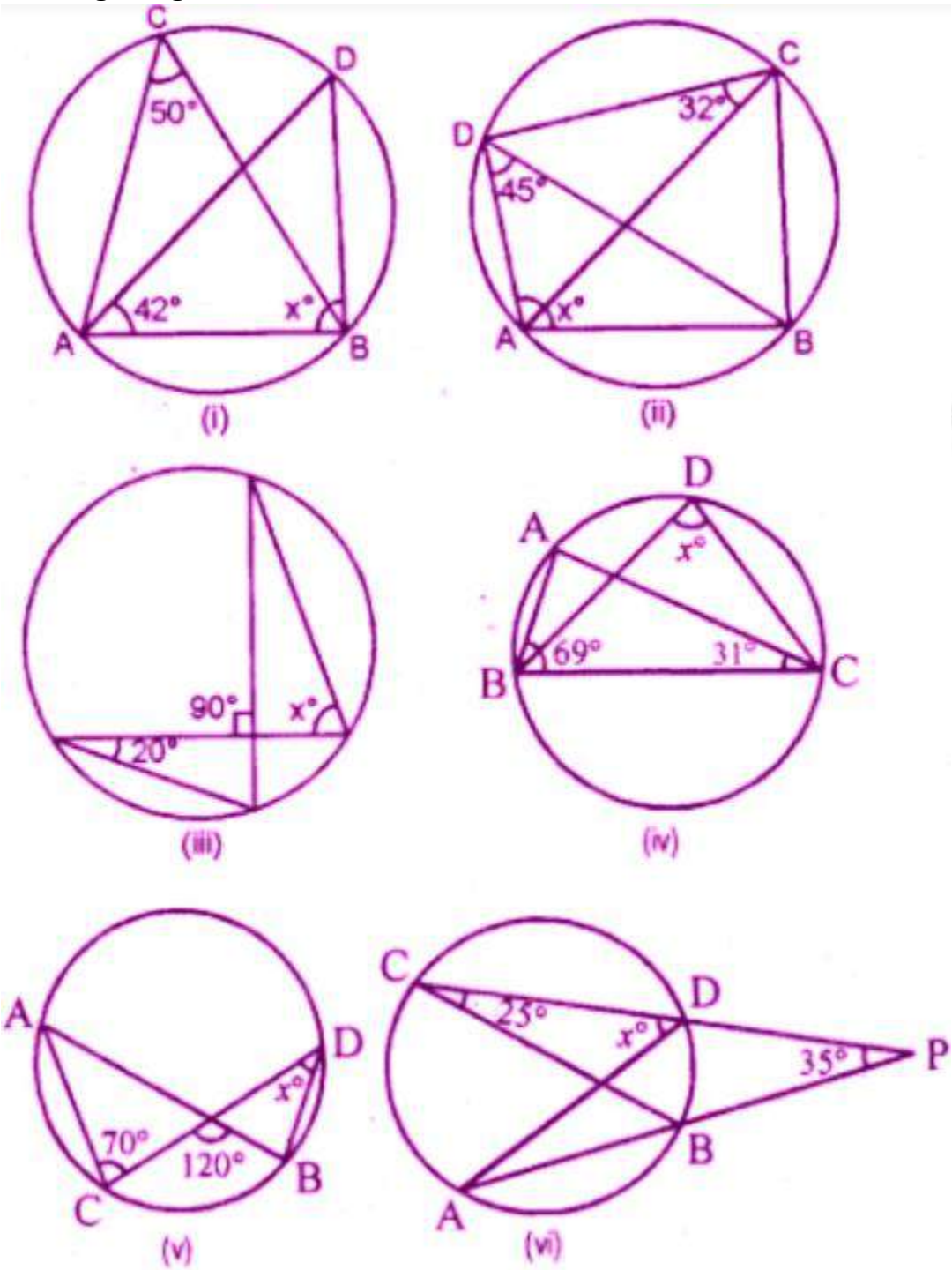
EXERCISE 15.1

1. Using the given information, find the value of x in each of the following figures:

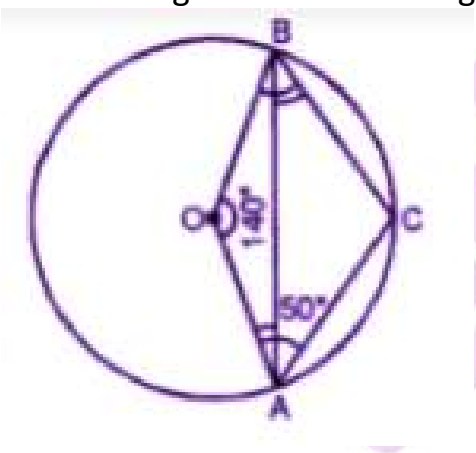


Solution:

(i) ∠ADB and ∠ACB are in the same segment.
∠ADB = ∠ACB = 50°
Now in ΔADB,
∠DAB + X + ∠ADB = 180°
= 42° + x + 50° = 180°
= 92° + x = 180°
x = 180° - 92°
x = 88°

(ii) In the given figure we have
= 32° + 45° + x = 180°
= 77° + x = 180°
x = 103°

(iii) From the given number we have
∠BAD = ∠BCD
Because angles in the same segment



But ∠BAD = 20°
∠BAD = 20°
∠BCD = 20°
∠CEA = 90°
∠CED = 90°
Now in triangle CED,
∠CED + ∠BCD + ∠CDE = 180°
90° + 20° + x = 180°
= 110° + x = 180°
x = 180° – 110°

$x = 70^o$

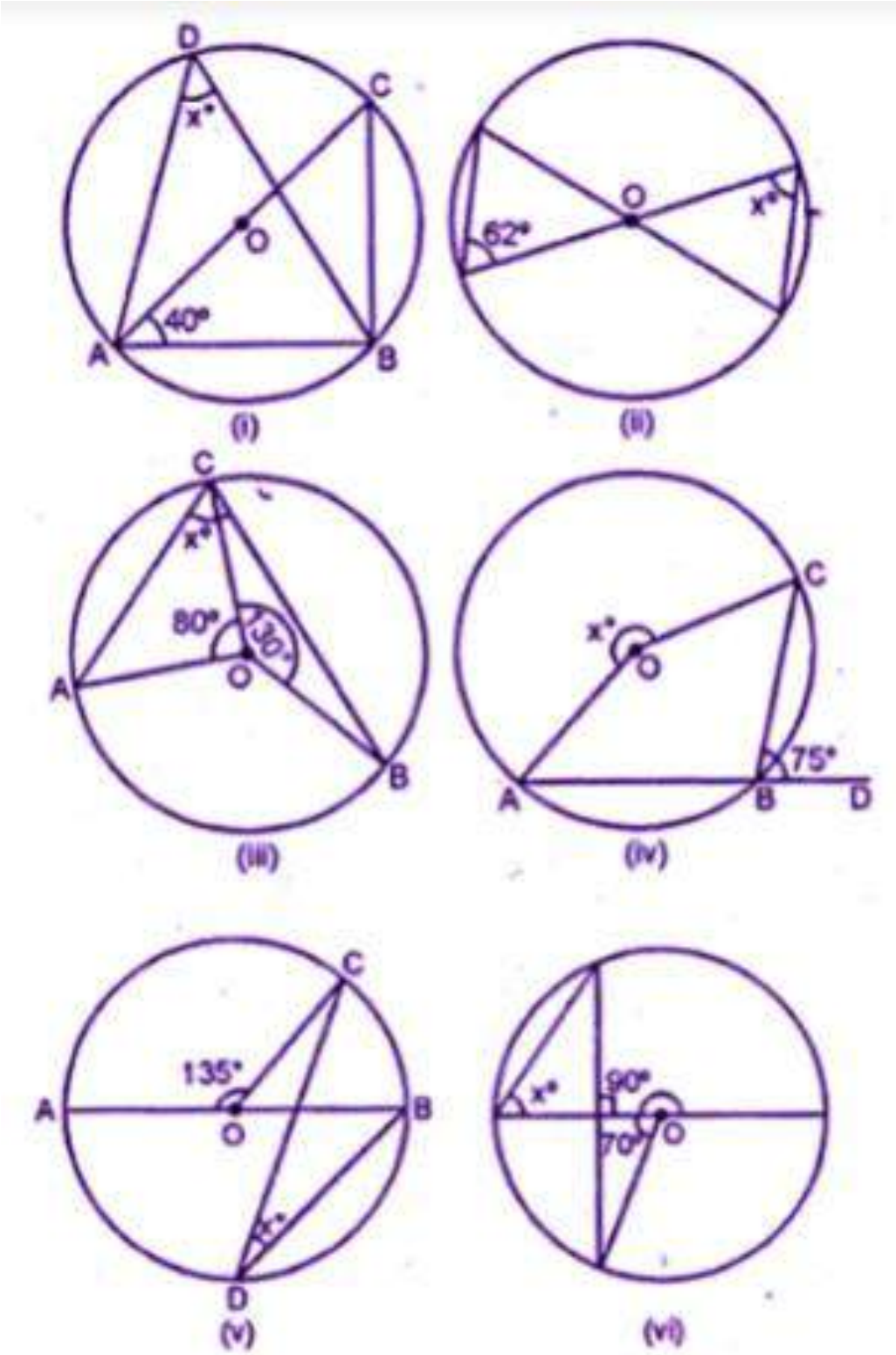
(iv) In ΔABC
$\angle ABC + \angle ABC + \angle BAC = 180^o$
(Because sum of a triangle)
$69^o + 31^o + \angle BAC = 180^o$
$\angle BAC = 180^o – 100^o$
$\angle BAC = 80^o$
Since ∠BAC and ∠BAD are in the same Segment.
$\angle BAD = x^o = 80^o$

(v) Given $\angle CPB = 120^o$, $\angle ACP = 70^o$
To find, x^o i,e., ∠PBD
Reflex ∠CPB = ∠BPO + ∠CPA
$120^0 = \angle BPD + \angle BPD$
(BPD = CPA are vertically opposite ∠s)
$2\angle BPD = 120^o$ $\angle PBD = 120^0/2 = 60^o$
Also ∠ACP and PBD are in the same segment
$\angle PBD + \angle ACP = 70^0$
Now, In ΔPBD
$\angle PBD + \angle PDB + \angle BPD = 180^o$
(sum of all ∠s in a triangle)
$70^0 + x^o + 60^0 = 180^o$
$x = 180^o - 130^o$
$x = 50^o$

(vi) ∠DAB = ∠BCD
(Angles in the same segment of the circle)
$\angle DAB = 25^0$ ($\angle BCD = 25^0$ given)
In ΔDAP,
Ex, ∠CDA = ∠DAP + ∠DPA
$x^o = \angle DAB + \angle DPA$
$x^o = 25^o + 35^o$
$x^o = 60^o$

2. If O is the center of the circle, find the value of x in each of the following figures

(using the given information):



Solution:

(i) $\angle ACB = \angle ADB$

(Angles in the same segment of a circle)

But $\angle ADB = x°$

$\angle ABC = x^o$

Now in ΔABC

$\angle CAB + \angle ABC + \angle ACB = 180^o$

$40^o + 90^0 + x^o = 180^o$

(AC is the diameter)

$130^o + x^o = 180^o$

$x^o = 180^0 – 130^o = 50^o$

(ii) $\angle ACD = \angle ABD$

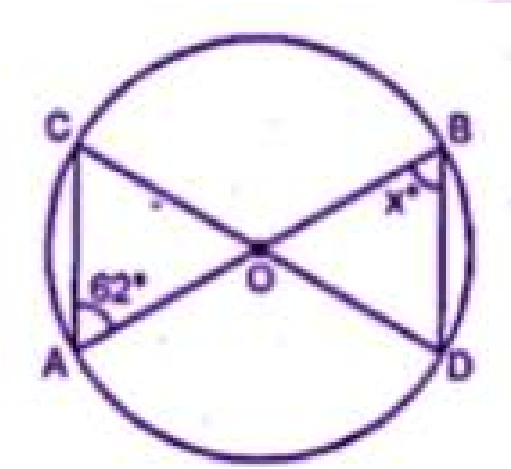
(angles in the same segment)

$\angle ACD = x^o$

Now in triangle OAC,

OA = OC

(radii of the same circle)



$\angle ACO = \angle AOC$

(opposite angles of equal sides)

Therefore, $x^o = 62^o$

(iii) $\angle AOB + \angle AOC + \angle BOC = 360^o$

(sum of angles at a point)

$\angle AOB + 80^o + 130^o = 360^o$

$\angle AOB + 210^o = 360^o$

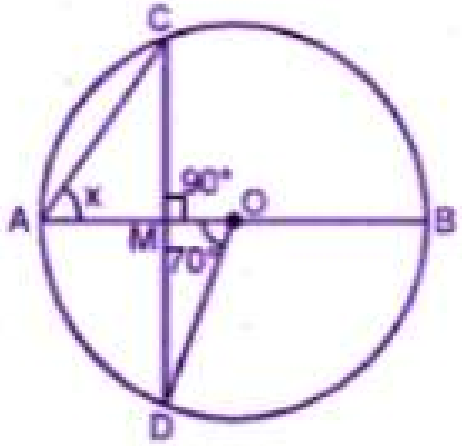
$\angle AOB = 360^o – 210^o = 150^0$

Now arc AB subtends ∠AOB at the centre ∠ACB at the remaining part of the circle
∠AOB = 2 ∠ACB
∠ACB = ½ ∠AOB = ½ × 150° = 75°

(iv) ∠ACB + ∠CBD = 180°
∠ABC + 75° = 180°
∠ABC = 180° – 75°- = 105°
Now arc AC Subtends reflex ∠AOC at the centre and ∠ ABC at the remaining part of the circle.
Reflex ∠AOC = 2 ∠ABC
= 2× 105° =210°

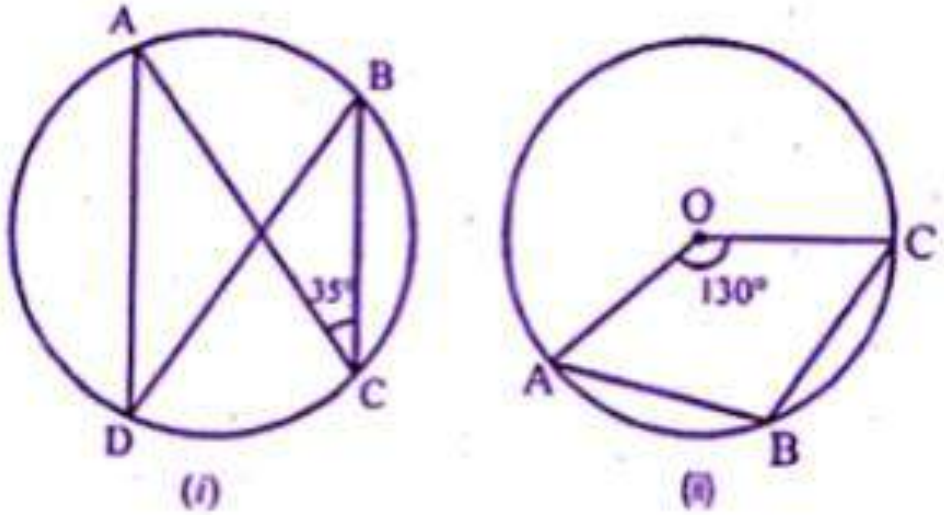
(v) ∠AOC + ∠COB = 180°
135° + ∠COB = 180°
∠COB = 180^0 – 135° = 45°
Now arc BC Subtends reflex ∠COB at the centre and ∠ CDB at the remaining part of the circle.
∠COB = 2 ∠CDB
∠CDB = ½ ∠COB
= ½ × 45° = 45°/2 = 22 1/2°

(vi) Arc AB subtends ∠AOD at the centre and ∠ACD at the remaining part of the Circle
∠AOD = 2 ∠ACB
∠ACB = ½ ∠AOD = ½ × 70° = 35°
∠CMO = 90°
∠AMC = 90°
(∠AMC + ∠CMO = 180°)



Now in ΔACM
∠ACM + ∠AMC + ∠CAM = 180°
35° + 90° + x° = 180°
125° + x° = 180°
X° = 180 – 125° = 55°

3. (a) In the figure (i) given below, AD || BC. If ∠ACB = 35°. Find the measurement of ∠DBC.
(b) In the figure (ii) given below, it is given that O is the centre of the circle and ∠AOC = 130°. Find ∠ ABC

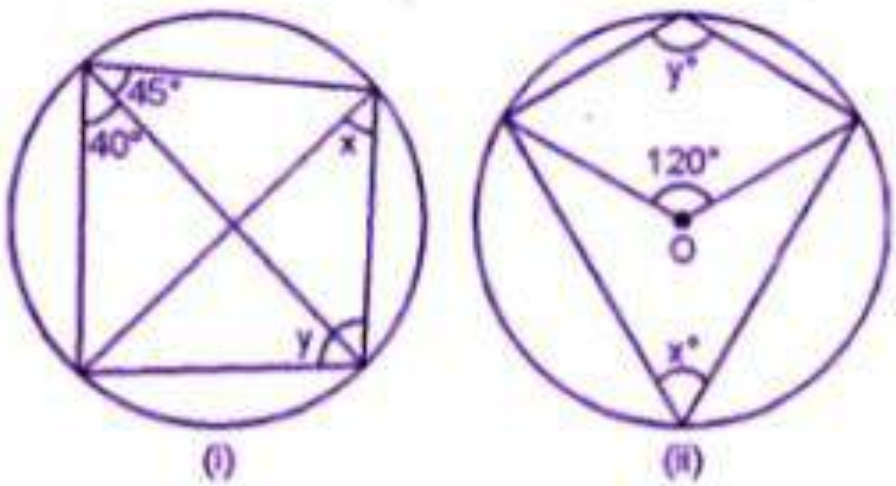


Solution:
(a) Construction: Join AB
∠A = ∠C = 35^0 (Alt Angles)

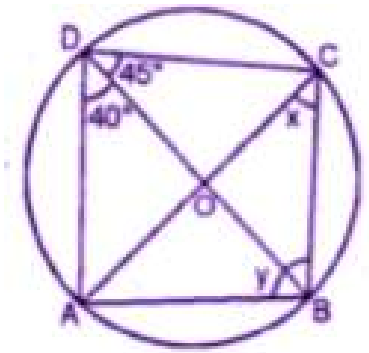
∠ABC = 35°

(b) ∠AOC + reflex ∠AOC = 360°
130° + Reflex ∠AOC = 360°
Reflex ∠AOC = 360° – 130° = 230°
Now arc BC Subtends reflex ∠AOC at the centre and ∠ ABC at the remaining part of the circle.
Reflex ∠AOC = 2 ∠ABC
∠ABC =1/2 reflex ∠AOC
= ½ × 230° = 115°

4. a) In the figure (i) given below, calculate the values of x and y.
(b) In the figure (ii) given below, O is the centre of the circle. Calculate the values of x and y.



Solution:
(a) ABCD is cyclic Quadrilateral

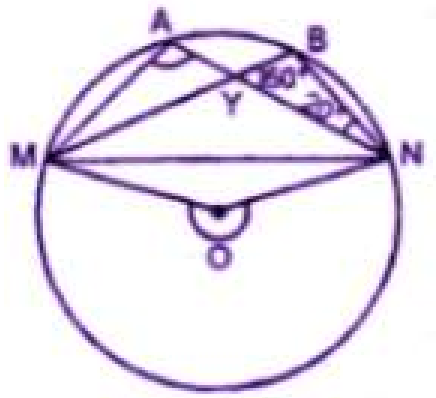


$\angle B + \angle D = 180^0$
$Y + 40^0 + 45^o = 180^o$
$(y + 85^o = 180^o)$
$Y = 180^o – 85^o = 95^o$
$\angle ACB = \angle ADB$
$x^o = 40$

(a) Arc ADC Subtends $\angle AOC$ at the centre and $\angle$ ABC at the remaining part of the circle
$\angle AOC = 2 \angle ABC$
$x^o = 60^o$
Again ABCD is a Cyclic quadrilateral
$\angle B + \angle D = 180^o$
$(60^o + y^o = 180^o)$
$y = 180^o – 60^o = 120^o$

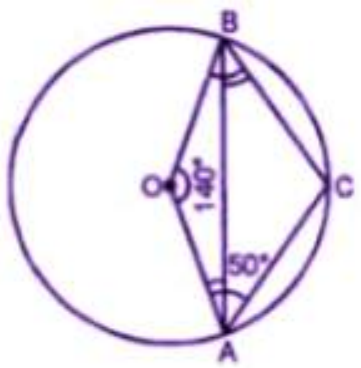
5. (a) In the figure (i) given below, M, A, B, N are points on a circle having centre O. AN and MB cut at Y. If ∠NYB = 50° and ∠YNB = 20°, find ∠MAN and the reflex angle MON.
(b) In the figure (ii) given below, O is the centre of the circle. If ∠AOB = 140° and ∠OAC = 50°, find
(i) ∠ACB
(ii) ∠OBC
(iii) ∠OAB
(iv) ∠CBA

Solution
(a) ∠NYB = 50°, ∠YNB = 20°.



In ΔYNB,
∠NYB + ∠YNB + ∠YBN = 180o
$50^o + 20^o +$ ∠YBN = 180°
∠YBN + $70^o = 180^o$
∠YBN = $180^o – 70^o = 110^o$
But ∠MAN = ∠YBN
(Angles in the same segment)
∠MAN = 110°
Major arc MN subtend reflex ∠MON at the Centre and ∠MAN at the remaining part of the choice.
Reflex ∠MAN at the remaining part of the circle
Reflex ∠MON = 2 ∠MAN = $2 \times 110^o = 220^o$

(b) (i) ∠AOB + reflex ∠AOB = 360°
(Angles at the point)
140° + reflex ∠AOB = 360°
Reflex ∠AOB = $360^o – 140^o = 220^o$



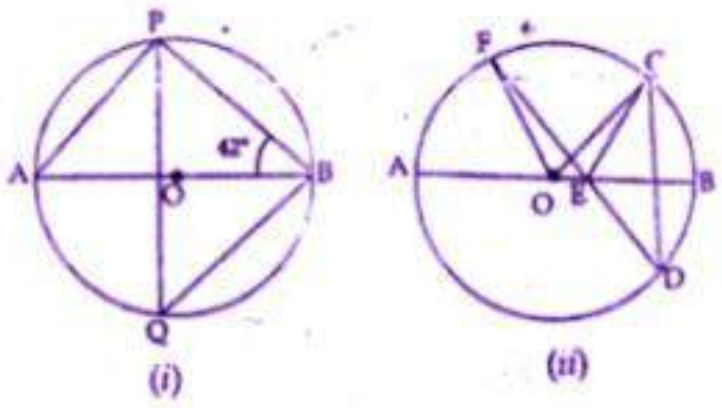
Now major arc AB subtends ∠AOB + ∠OBC = 360°
50° + 110° + 140° + ∠OBC = 360°
300° + ∠OBC = 360°
∠300° + ∠OBC = 360°
∠OBC = 360° – 300°
∠OBC = 60°

(ii) In Quadrilateral .OACB
∠OAC + ∠ACB + ∠AOB + ∠OBC = 360°
50° + 110° + 140° + ∠OBC = 360°
300° + ∠OBC = 360°
∠OBC = 360° – 300°
∠OBC =60°

(iii) in ΔOAB,
OA = OB
(Radii of the same circle)
∠OAB + ∠OBA = 180°
2 ∠OAB = 180° – 140° = 40°
∠OAB = 40°/2 = 20°
But ∠OBC = 60°
∠CBA = ∠OBC - ∠OBA
= 60° – 20° = 40°

6. (a)In the figure (i) given below, O is the centre of the circle and ∠PBA = 42°. Calculate the value of ∠PQB
(b) In the figure (ii) given below, AB is a diameter of the circle whose centre is O. Given that ∠ECD = ∠EDC = 32°, calculate
(i) ∠CEF

(ii) ∠COF.



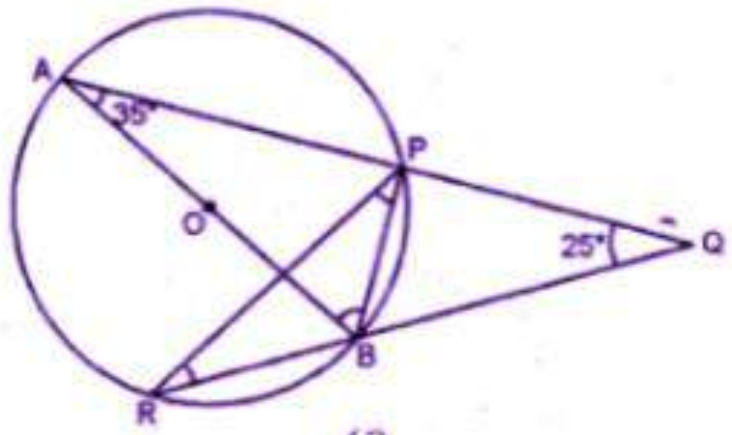
Solution:
In ΔAPB,
∠APB = 90° (Angle in a semi-circle)
But ∠A + ∠APB + ∠ABP = 180° (Angles of a triangle)
∠A + 90° + 42°= 180°
∠A + 132° = 180°
⇒ ∠A = 180° – 132° = 48°
But ∠A = ∠PQB
(Angles in the same segment of a circle)
∠PQB = 48°

(b) (i) in ΔEDC,
(Ext, angle of a triangle is equal to the sum
of its interior opposite angels)

(ii) arc CF subtends ∠COF at the centre and
∠CDF at the remaining part of the circle
∠COF = 2 ∠CDF = 2 ∠CDE
=2 × 32° = 2 ∠CDE
= 2 × 32° = 64°

7. (a) In the figure (i) given below, AB is a diameter of the circle APBR. APQ and RBQ are straight lines, ∠A = 35°, ∠Q = 25°. Find (i) ∠PRB (ii) ∠PBR (iii) ∠BPR.

(b) In the figure (ii) given below, it is given that ∠ABC = 40° and AD is a diameter of the circle. Calculate ∠DAC.



(i)

(ii)

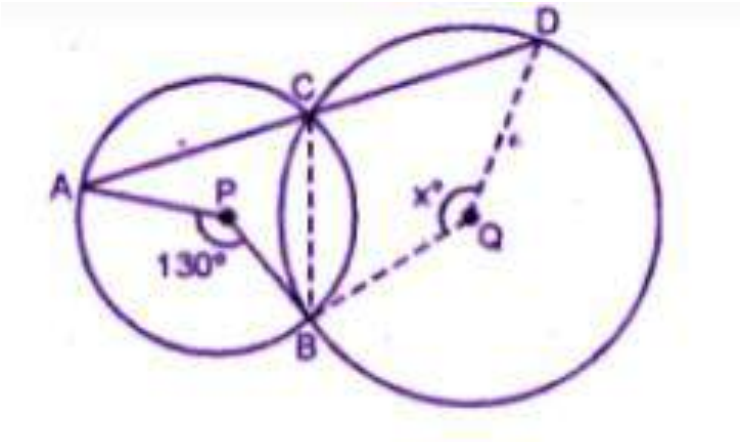
Solution

(a) (i) ∠PRB = ∠BAP

(Angles in the same segment of the circle)

∴ ∠PRB = 35° (∵ ∠BAP = 35° given)

8. (a) In the figure given below, P and Q are centers of two circles intersecting at B and C. ACD is a straight line. Calculate the numerical value of x.

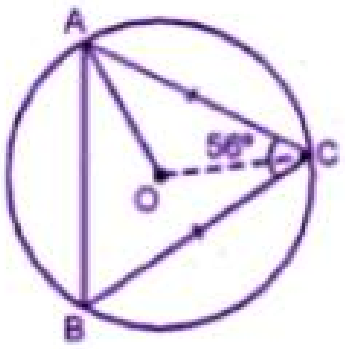


(b) In the figure given below, O is the circumcenter of triangle ABC in which AC = BC. Given that ∠ACB = 56°, calculate
(i)∠CAB
(ii)∠OAC

Solution:
Given that
(a) Arc AB subtends ∠APB at the center
and ∠ACB at the remaining part of the circle
∠ACB = ½ ∠APB = ½ × 130° = 65°
But ∠ACB + ∠BCD = 180°
(Linear Pair)
65° + ∠BCD = 180°
∠BCD = 180° – 65° = 115°
Major arc BD subtends reflex ∠BQD at the
Centre and ∠BCD at the remaining part of
the circle
reflex ∠BQD = 2 ∠BCD
=2 × 115° = 230°
But reflex ∠BQD + x = 360°
(Angles at a point)
230° + x = 360°
x = 360° – 230° = 130°

(b) Join OC
In ΔABC,AC = BC

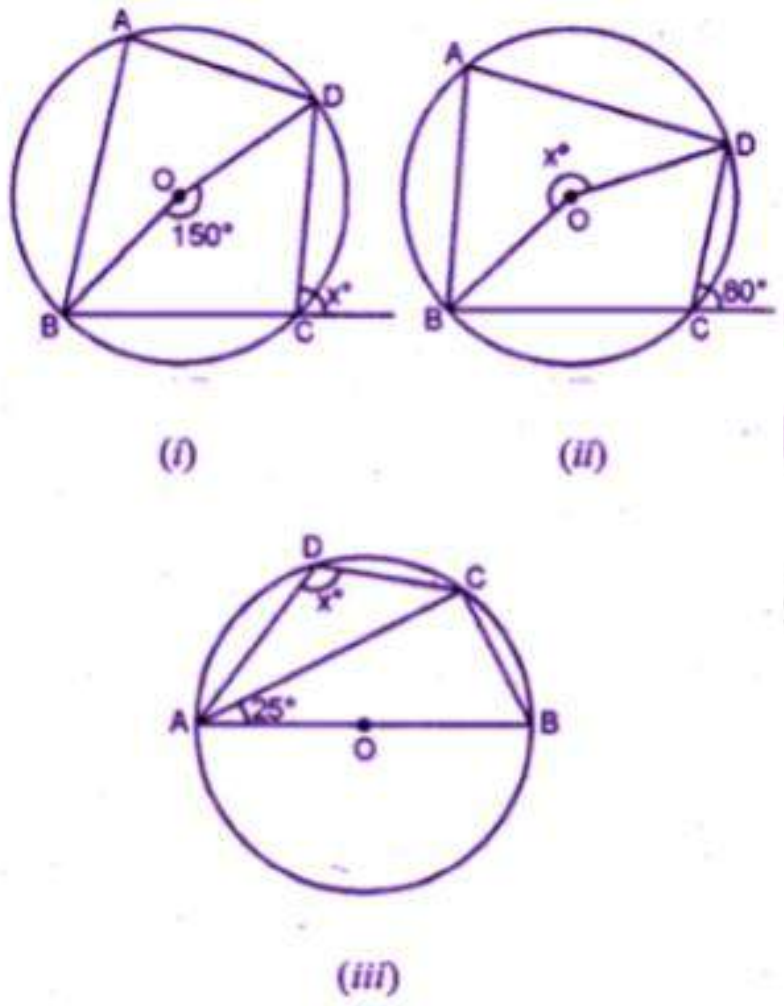
$\angle A = \angle B$
But $\angle A + \angle B + \angle C = 180^o$
$\angle A + \angle A + 56^0 = 180^0$
$2\ \angle A\ 180^o - 56^0 = 124^o$



$\angle A = 124/2 = 62^o$ or $\angle CAB = 62^0$
OC is the radius of the circle
OC bisects $\angle ACB$
$\angle OCA = ½\ \angle ACB = ½ \times 56^o = 28^o$
Now in ΔOAC
OA = OC
(radii of the same Circle)
$\angle OAC = \angle OCA = 28^o$

EXERCISE 15.2

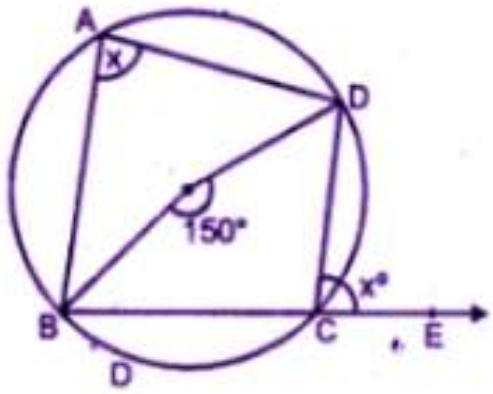
1. If O is the center of the circle, find the value of x in each of the following figures (using the given information)



Solution:

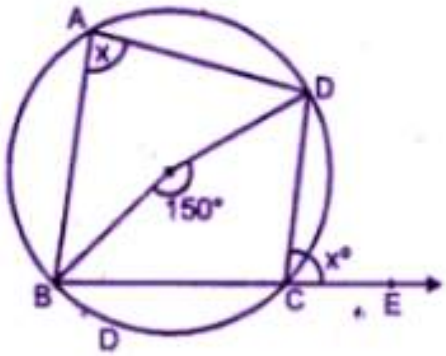
From the figure

(i) ABCD is a cyclic quadrilateral



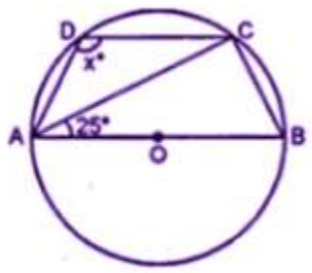
Ext. ∠DCE = ∠BAD
∠BAD = x^o
Now arc BD subtends ∠BOD at the center
And ∠BAD at the remaining part of the circle.
∠BOD = 2 ∠BAD = 2 x
2 x = 150° (x = 75^0)

(ii) ∠BCD + ∠DCE = 180°
(Linear pair)



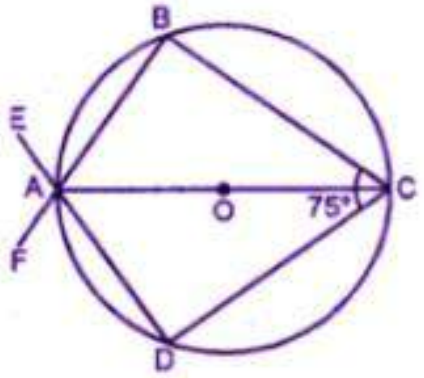
∠BCD + 80^0 = 180°
∠BCD = $180^0 – 80^0 = 100^o$
Arc BAD subtends reflex ∠BOD at the
Centre and ∠BCD at the remaining part of the circle
Reflex ∠BOD = 2 ∠BCD
$X^o = 2 \times 100^o = 200^o$

(iii) In ΔACB,



∠CAB + ∠ABC + ∠ACB = 180°
(Angles of a triangle)
But ∠ACB = 90°
((Angles of a semicircle)
25° + 90° + ∠ABC = 180°
=115° + ∠ABC = 180°
∠ABC = 180° – 115°=65°
ABCD is a cyclic quadrilateral
∠ABC + ∠ADC = 180°
(Opposite angles of a cyclic quadrilateral)
65° + x° =180°
x° = 180° -65° = 115°

2. (a) In the figure (i) given below, O is the center of the circle. If ∠AOC = 150°, find (i) ∠ABC (ii) ∠ADC
(b) In the figure (i) given below, AC is a diameter of the given circle and ∠BCD = 75°. Calculate the size of (i) ∠ABC (ii) ∠EAF.



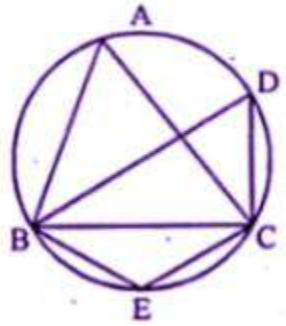
Solution:
(a) Given, ∠AOC = 150° and AD = CD

We know that an angle subtends by an arc of a circle at the center is twice the angle subtended by the same arc at any point on the remaining part of the circle.

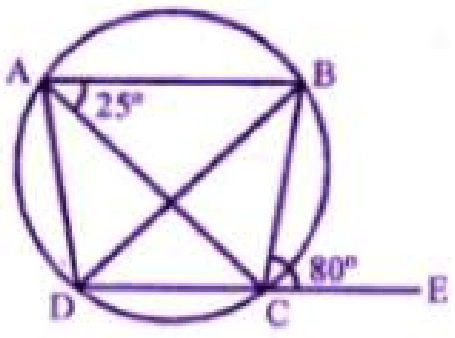
(i) $\angle AOC = 2 \times \angle ABC$
$\angle ABC = \angle AOC/2 = 150^o/2 = 75^o$
(ii) From the figure, ABCD is a cyclic quadrilateral
$\angle ABC + \angle ADC = 180^o$
(Sum of opposite angels in a cyclic quadrilateral Is 180°)
$75^o + \angle ADC = 180^o$
$\angle ADC + 180^o – 75^o$
$\angle ADC = 105^o$
(b) (i) AC is the diameter of the circle
$\angle ABC = 90^o$ (Angle in a semi-circle)

(ii) ABCD is a cyclic quadrilateral
$\angle BAD + \angle BCD = 180^o$
$\angle BAD + 75^o = 180^o$
($\angle BCD = 75^o$)
$\angle BAD = 180^o - 75^o = 105^o$
But $\angle EAF = \angle BAD$
(Vertically opposite angles)
$\angle EAF = 105^o$

3. (a) In the figure, (i) given below, if $\angle DBC = 58°$ and BD is a diameter of the circle, calculate:
(i) $\angle BDC$ (ii) $\angle BEC$ (iii) $\angle BAC$

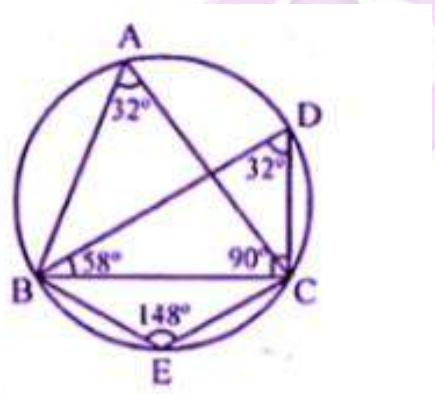


(b) In the figure (if) given below, AB is parallel to DC, ∠BCE = 80° and ∠BAC = 25°. Find:
(i) ∠CAD (ii) ∠CBD (iii) ∠ADC (2008)



Solution:
(a) ∠DBC = 58°
BD is diameter
∠DCB = 90° (Angle in semi-circle)

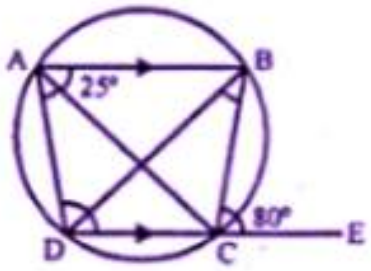
(i) In ΔBDC
∠BDC + ∠DCB + ∠CBD = 180°
∠BDC = 180°- 90° – 58° = 32°



(ii) BEC = 180° – 32° = 148°
(opposite angles of cyclic quadrilateral)

(iii) ∠BAC = ∠BDC = 32°
(Angles in same segment)

(b) in the figure, AB ‖DC
∠BCE = 80° and ∠BAC = 25°
ABCD is a cyclic Quadrilateral and DC is Production to E

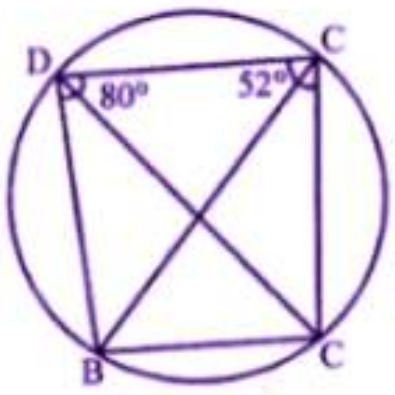


(i) Ext, ∠BCE = interior ∠A
80° = ∠BAC + ∠CAD
80° = 25° + ∠CAD
∠CAD = 80° – 25° = 55°

(ii) But ∠CAD = ∠CBD
(Alternate angels)
∠CBD = 55°

(iii) ∠BAC = ∠BDC
(Angles in the same segments)
∠BDC = 25°
(∠BAC = 25°)
Now AB ‖ DC and BD is the transversal
∠BDC = ∠ABD
∠ABD = 25°
∠ABC = ∠ABD + ∠CBD = 25° + 55° = 80°
But ∠ABC + ∠ADC = 180°
(opposite angles of a cyclic quadrilateral)
80° + ∠ADC = 180°
∠ADC = 180° – 80° = 100°

4. (a) In the figure given below, ABCD is a cyclic quadrilateral. If ∠ADC = 80° and ∠ACD = 52°, find the values of ∠ABC and ∠CBD.



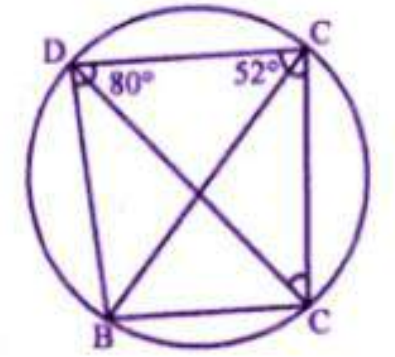
(b) In the figure given below, O is the center of the circle. ∠AOE =150°, ∠DAO = 51°. Calculate the sizes of ∠BEC and ∠EBC.

Solution:

(a) In the given figure, ABCD is a cyclic quadrilateral

∠ADC = 80° and ∠ACD = 52°

To find the measure of ∠ABC and ∠CBD



ABCD is a Cyclic Quadrilateral

∠ABC + ∠ADC = 180°

(Sum of opposite angles = 180°)

∠ABC + 80° = 180°

∠AOE = 150°, ∠DAO = 51°

To find ∠BEC and ∠EBC

ABED is a cyclic quadrilateral
Ext. ∠BEC = ∠DAB = 51°
∠ AOE = 150°
Ref ∠AOE = 360° – 150° = 51°
∠AOE = 150°
Ref ∠AOE = 360° – 150° = 210°
Now arc ABE subtends ∠AOE at the Centre
And ∠ADE at the remaining part of the circle.
∠ADE = ½ ref ∠AOE = ½ × 210° = 105°
But Ext ∠EBC = ∠ADE = 105°
Hence ∠BEC = 51° and ∠EBC = 105°

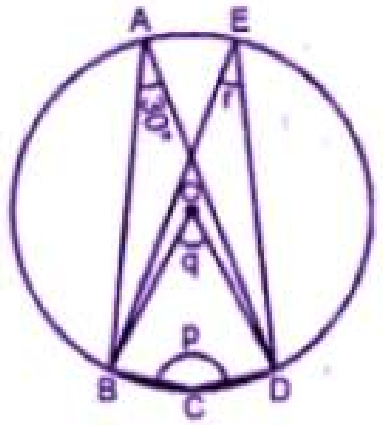
5. (a) In the figure (i) given below, ABCD is a parallelogram. A circle passes through A and D and cuts AB at E and DC at F. Given that ∠BEF = 80°, find ∠ABC.
(b) In the figure (ii) given below, ABCD is a cyclic trapezium in which AD is parallel to BC and ∠B = 70°, find:
(i)∠BAD (ii) DBCD.

Solution:
(a) ADFE is a cyclic quadrilateral
Ext. ∠FEB = ∠ADF
⇒ ∠ADF = 80°
ABCD is a parallelogram
∠B = ∠D = ∠ADF = 80°
or ∠ABC = 80°

(b)In trapezium ABCD, AD || BC
(i) ∠B + ∠A = 180°
⇒ 70° + ∠A = 180°
⇒ ∠A = 180° – 70° = 110°
∠BAD = 110°

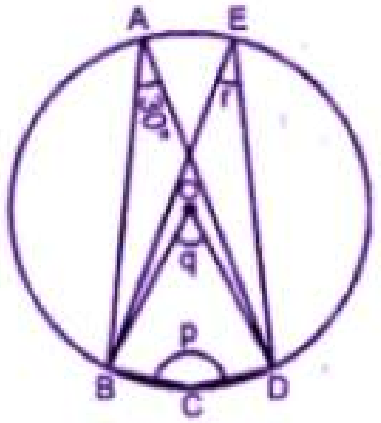
(ii) ABCD is a cyclic quadrilateral
∠A + ∠C = 180°
⇒ 110° + ∠C = 180°
⇒ ∠C = 180° – 110° = 70°
∠BCD = 70°

6. (a) In the figure given below, O is the center of the circle. If ∠BAD = 30°, find the values of p, q and r.



(a) In the figure given below, two circles intersect at points P and Q. If ∠A = 80° and ∠D = 84°, calculate
(i) ∠QBC
(ii) ∠BCP

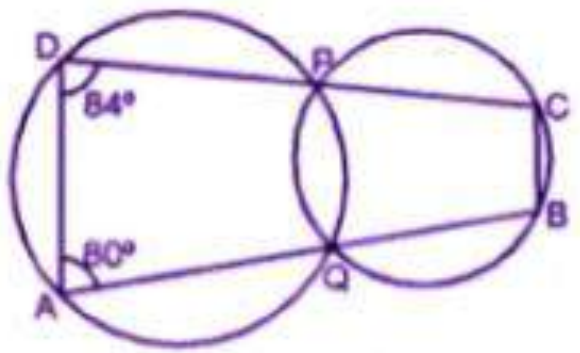
Solution:
(a) (i) ABCD is a cyclic quadrilateral



∠A + ∠C = 180°
30° + p = 180°
P=180° – 30° = 150°

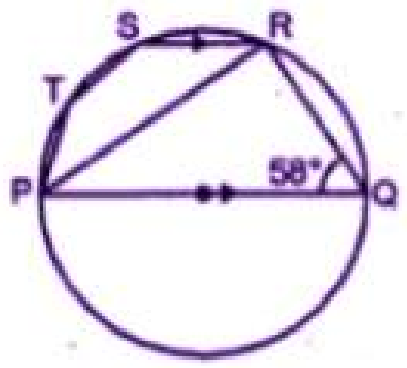
(ii) Arc BD subtends ∠BOD at the center
And ∠BAD at the remaining part of the circle
∠BOD = 2 ∠BAD
q = 2 × 30° = 60°
∠BAD = ∠BED are in the same segment of the circle
∠BAD = ∠BED
30° = r
r = 30°

(b) Join PQ
AQPD is a cyclic quadrilateral



∠A + ∠QPD = 180°

7. (a) In the figure given below, PQ is a diameter. Chord SR is parallel to PQ.Given ∠PQR = 58°, calculate (i) ∠RPQ (ii) ∠STP
(T is a point on the minor arc SP)



(b) In the figure given below, if ∠ACE = 43° and ∠CAF = 62°, find the values of a, b and c (2007)

Solution:

(a) In ΔPQR,

∠PRQ = 90° (Angle in a semi-circle) and ∠PQR = 58°

∠RPQ = 90° – ∠PQR = 90° – 58° = 32°

SR || PQ (given)

∠SRP = ∠RPQ = 32° (Alternate angles)

Now PRST is a cyclic quadrilateral,

∠STP + ∠SRP = 180°

∠STP = $180^o – 32^o = 148^o$

(b) In the given figure,

∠ACE 43° and ∠CAF = 62^0

Now, in ΔAEC

∠ACE + ∠CAE + ∠AEC = 180°

$43^o + 62^o$ + ∠AEC = 180°

105° + ∠AEC = 180°

∠AEC = $180^o – 105^0 = 75^o$

But ∠ABD + ∠AED = 180^0

(sum of opposite angles of acyclic quadrilateral)

and ∠AED = ∠AEC

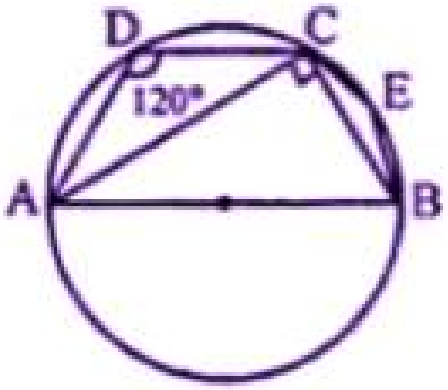
a + $75^o = 180^o$

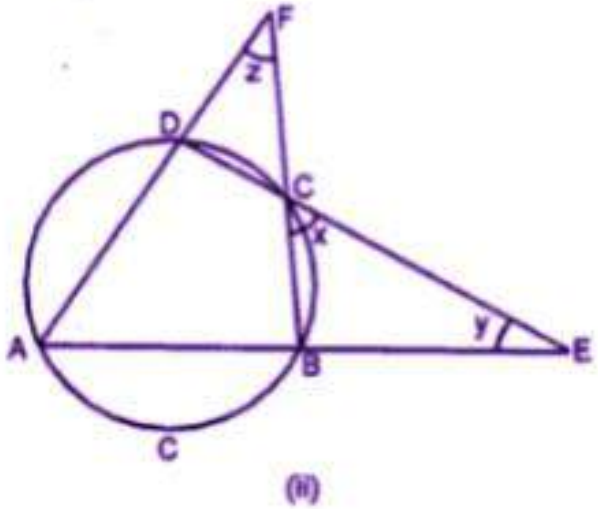
a = $180^o – 75^o – 105^o$

but ∠EDF = ∠BAE

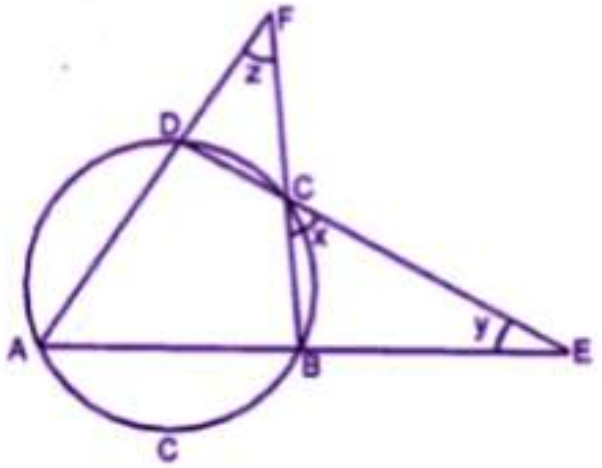
(Angles in the alternate segment)

8. (a) In the figure (i) given below, AB is a diameter of the circle. If ∠ADC = 120°, find ∠CAB.
(b) In the figure (ii) given below, sides AB and DC of a cyclic quadrilateral ABCD are produced to meet at E, the sides AD and BC are produced to meet at F. If x : y : z = 3 : 4 : 5, find the values of x, y and z.





Solution:
(a) Construction: Join BC, and AC then ABCD is a cyclic quadrilateral.

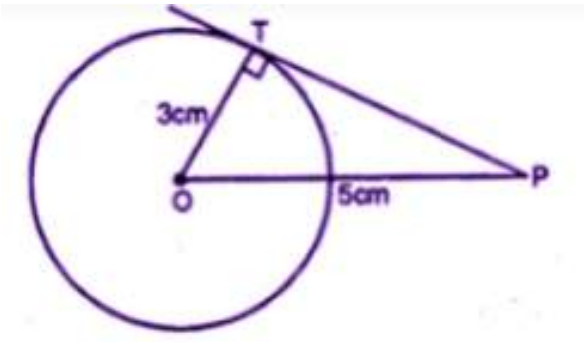


Now in ΔDCF
Ext. ∠2 = x + z
and in ΔCBE
Ext. ∠1 = x + y
Adding (i) and (ii)
x + y + x + z = ∠1 + ∠2
2 x + y + z = 180°
(ABCD is a cyclic quadrilateral)
But x : y : z = 3 : 4 : 5
x/y = ¾ (y = 4/3 x)
x/z = 3/5 (z = 5/3 x.

Exercise 15.3

1. Find the length of the tangent drawn to a circle of radius 3cm, from a point distnt 5cm from the center.

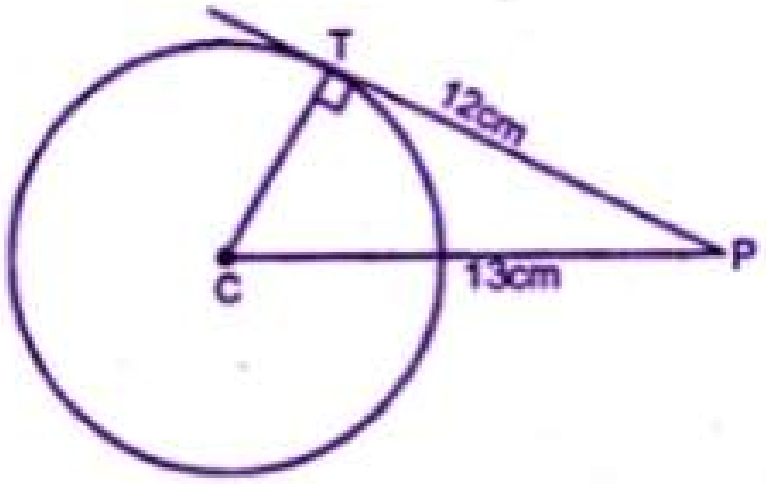
Solution:



In a circle with center O and radius 3cm and p is at a distance of 5cm.
That is OT = 3 cm, OP = 5 cm
OT is the radius of the circle
OT ⊥ PT
Now in right Δ OTP, by Pythagoras axiom,
$OP^2 = OT^2 + PT^2$
$(5)^2 = (3)^2 + PT^2$
$PT^2 = (5)^2 – (3)^2 = 25 – 9 = 16 = (4)^2$
PT = 4 cm.

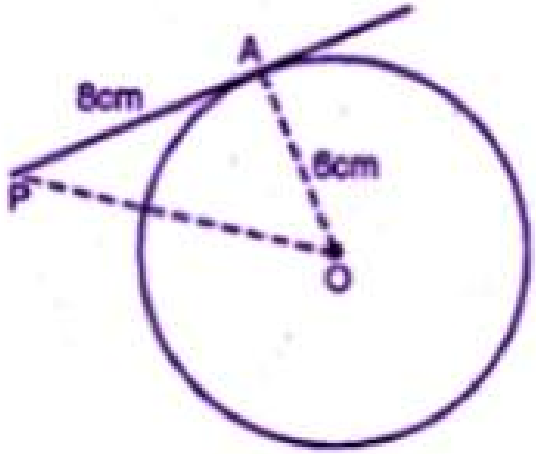
2. A point P is at a distance 13 cm from the center C of a circle and PT is a tangent to the given circle. If PT = 12 cm, find the radius of the circle.

Solution:
CT is the radius
CP = 13 cm and tangent PT = 12 cm



CT is the radius and TP is the tangent
CT is perpendicular TP
Now in right angled triangle CPT,
$CP^2 = CT^2 + PT^2$ [using Pythagoras axiom]
$(13)^2 = (CT)^2 + (12)^2$
$169 = (CT)^2 + 144$
$(CT)^2 = 169 - 144 = 25 = (5)^2$
CT = 5 cm.
Hence the radius of the circle is 5cm

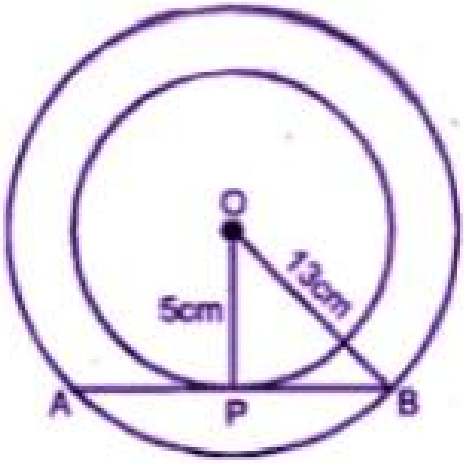
3. The tangent to a circle of radius 6 cm from an external point P, is of length 8 cm. Calculate the distance of P from the nearest point of the circle.



Solution:
Radius of the circle = 6 cm
and length of tangent = 8 cm
Let OP be the distance
i.e. OA = 6 cm, AP = 8 cm .
OA is the radius
OA ⊥ AP
Now In right ΔOAP,
$OP^2 = OA^2 + AP^2$
(By Pythagoras axiom)
$= (6)^2 + (8)^2$
=36 + 64
= 100
$= (10)^2$
OP = 10 cm.

4. Two concentric circles are of the radii 13 cm and 5 cm. Find the length of the chord of the outer circle which touches the inner circle.

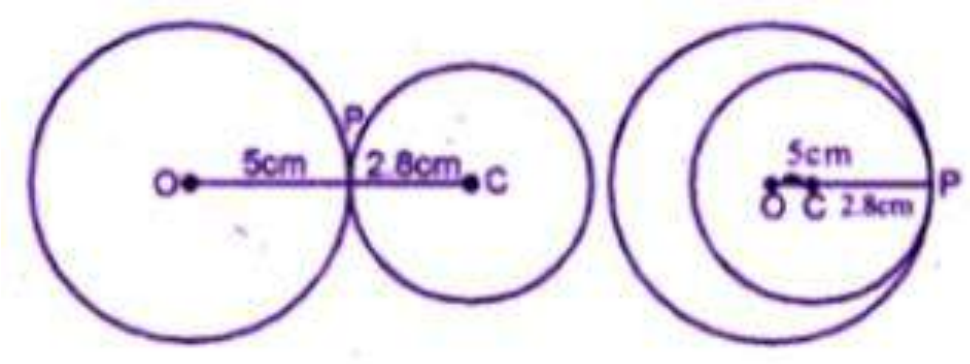
Solution:
Two concentric circles with center O
OP and OB are the radii of the circles respectively, then
OP = 5 cm, OB = 13 cm.



Ab is the chord of outer circle which touches the inner circle at P.
OP is the radius and APB is the tangent to the inner circle.
In the right angled triangle OPB, by Pythagoras axiom,
$OB^2 = OP^2 + PB^2$
$13^2 = 5^2 + PB^2$
$169 = 25 + PB^2$
$PB^2 = 169 – 25$
= 144
PB = 12 cm
But P is the mid-point of AB.
AB = 2PB
= 24 cm

5. Two circles of radii 5 cm and 2-8 cm touch each other. Find the distance between their centers if they touch :
(i) externally
(ii) internally.

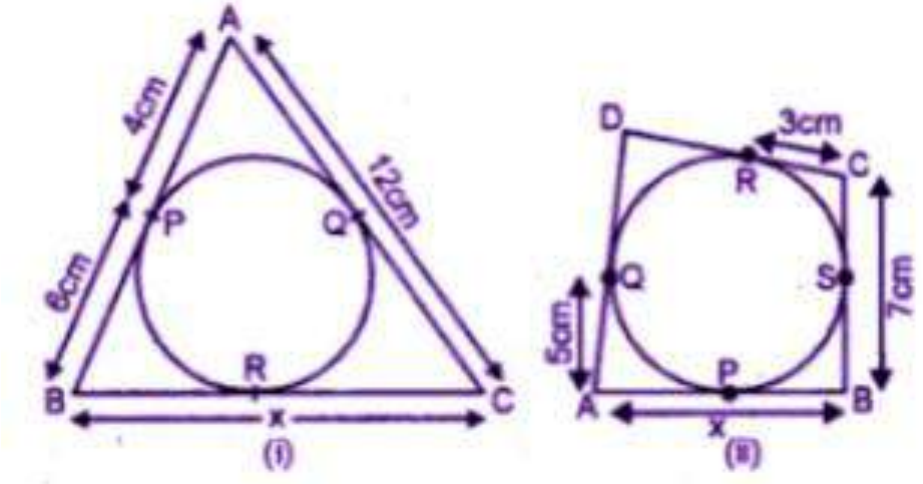
Solution:
Radii of the circles are 5 cm and 2.8 cm.
i.e. OP = 5 cm and CP = 2.8 cm.



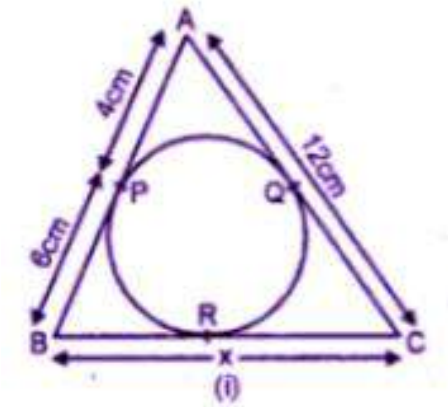
(i) When the circles touch externally,
then the distance between their centers = OC = 5 + 2.8
= 7.8 cm.

(ii) When the circles touch internally,
then the distance between their centers = OC = 5.0 – 2.8
= 2.2 cm

6. (a) In figure (i) given below, triangle ABC is circumscribed, find x.
(b) In figure (ii) given below, quadrilateral ABCD is circumscribed, find x.

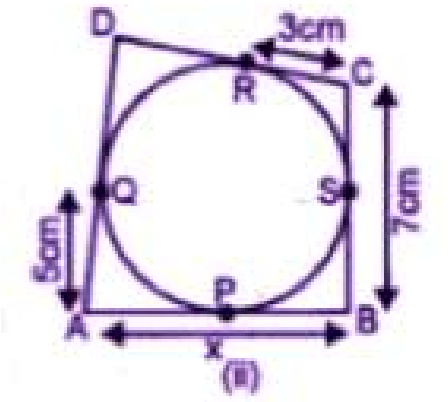


Solution:
(a) From A, AP and AQ are the tangents to the circle
∴ AQ = AP = 4cm



But AC =12 cm
CQ = 12 – 4 = 8 cm.
From B, BP and BR are the tangents to the circle
BR = BP = 6 cm.
Similarly, from C,
CQ and CR the tangents
CR = CQ = 8 cm
x = BC = BR + CR = 6 cm + 8 cm = 14 cm

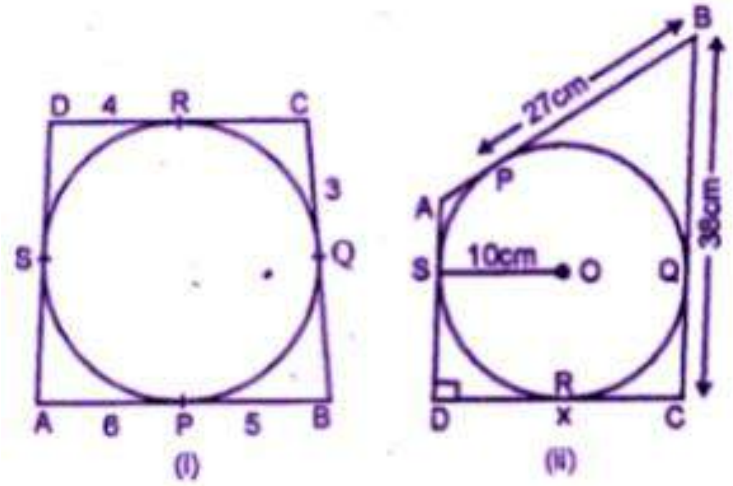
(b) From C, CR and CS are the tangents to the circle.



CS = CR = 3 cm.
But BC = 7 cm.
BS = BC – CS = 7 – 3 =4 cm.
Now from B,BP and BS are the tangents to the circle.
BP = BS = 4 cm
From A, AP and AQ are the tangents to the circle.
AP = AQ = 5cm

x = AB = AP + BP = 5 + 4
= 9 cm

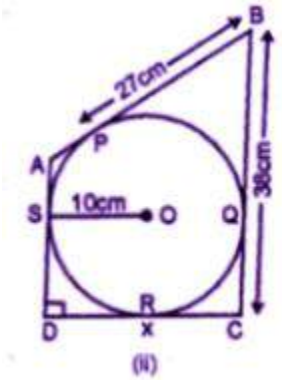
7. (a) In figure (i) given below, quadrilateral ABCD is circumscribed; find the perimeter of quadrilateral ABCD.
(b) In figure (ii) given below, quadrilateral ABCD is circumscribed and AD ⊥ DC ; find x if radius of incircle is 10 cm.



Solution:
(a) From A, AP and AS are the tangents to the circle
∴AS = AP = 6
From B, BP and BQ are the tangents
∴BQ = BP = 5
From C, CQ and CR are the tangents
CR = CQ
From D, DS and DR are the tangents
DS = DR = 4
Therefore, perimeter of the quadrilateral ABCD
= 6 + 5 + 5 + 3 + 3 + 4 + 4 + 6
= 36 cm

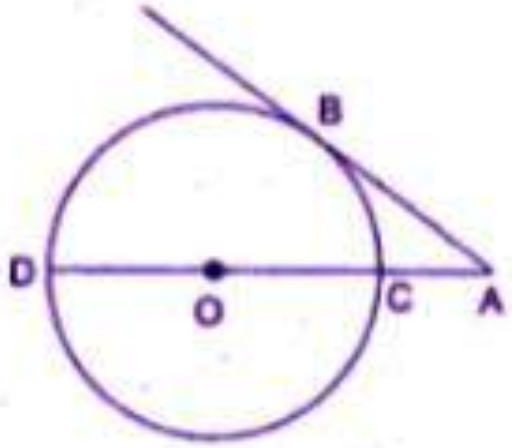
(b) in the circle with center O, radius OS = 10 cm
PB = 27 cm, BC = 38 cm
OS id the radius and AD is the tangent.
Therefore, OS perpendicular to AD.

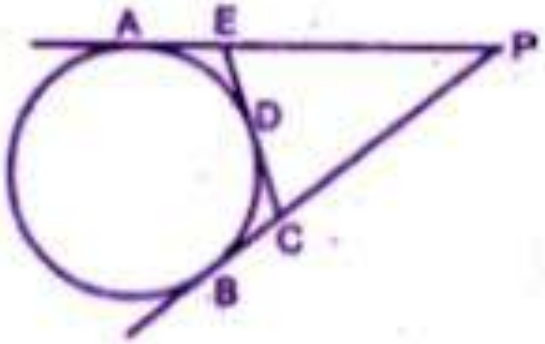
SD = OS = 10 cm.



Now from D,DR and DS are the tangents
To the circle
DR = DS = 10 cm
From B, BP and BQ are tangents to the circle.
BQ = BP = 27 cm.
CQ = CB – BQ = 38 – 27 = 11 cm.
Now from C, CQ and CR are the tangents to the circle
CR = CQ = 11 cm.
DC = x =DR + CR
= 10 + 11 = 21 cm

8. (a) In the figure (i) given below, O is the center of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7.5 cm, find the radius of the circle.
(b) In the figure (ii) given below, from an external point P, tangents PA and PB are drawn to a circle. CE is a tangent to the circle at D. If AP = 15 cm, find the perimeter of the triangle PEC.



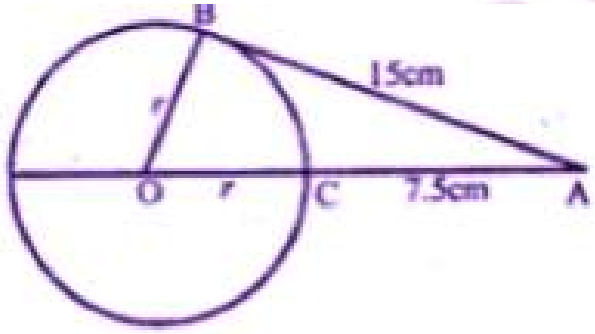


Solution:

(i) Join OB

∠OBA = 90°

(Radius through the point of contact is perpendicular to the tangent)



$OB^2 = OA^2 – AB^2$

$r^2 = (r + 7.5)^2 – 15^2$

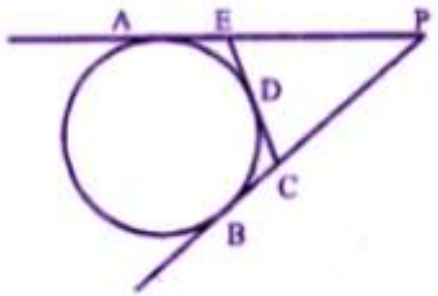
$r^2 = r^2 + 56.25 + 15r – 225$

15r = 168.75

r = 11.25

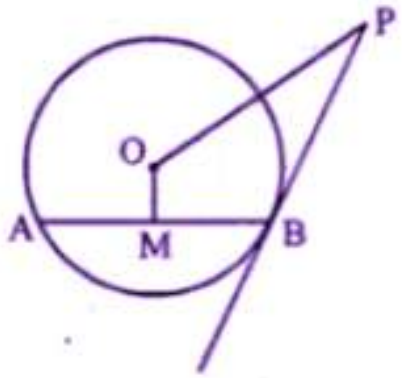
Hence, radius of the circles = 11.25 cm

(ii) In the figure, PA and PB are the tangents Drawn from P to the circle.



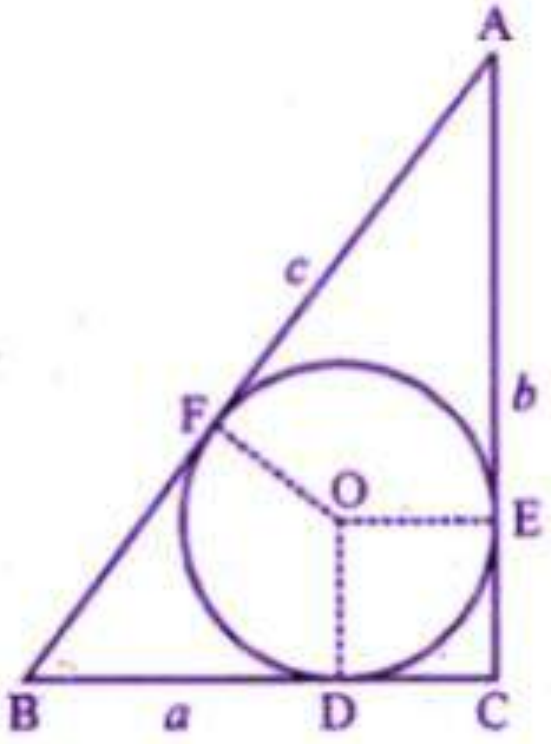
CE is tangent at D
AP = 15 cm
PA and PB are tangents to the circle
AP = BP = 15 cm
Similarly EA and ED are tangents
EA = ED
Similarly BC = CD
Now perimeter of triangle PEC,
= PE + EC + PC
= PE + ED + CD + PC
PE + EA + CB + PC
(ED = EA and CB = CD)
=AP + PB = 15 + 15
= 30 cm.

9. (a) If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by
r = /frac (a + b – c) – (2)
(b) In the given figure, PB is a tangent to a circle with center O at B. AB is a chord of length 24 cm at a distance of 5 cm from the center. If the length of the tangent is 20 cm, find the length of OP.



Solution:

(a) Let the circle touch the sides BC, CA and AB
of the right triangle ABC at points D, E and F respectively,
where BC = a, CA = b
and AB = c (as showing in the given figure).

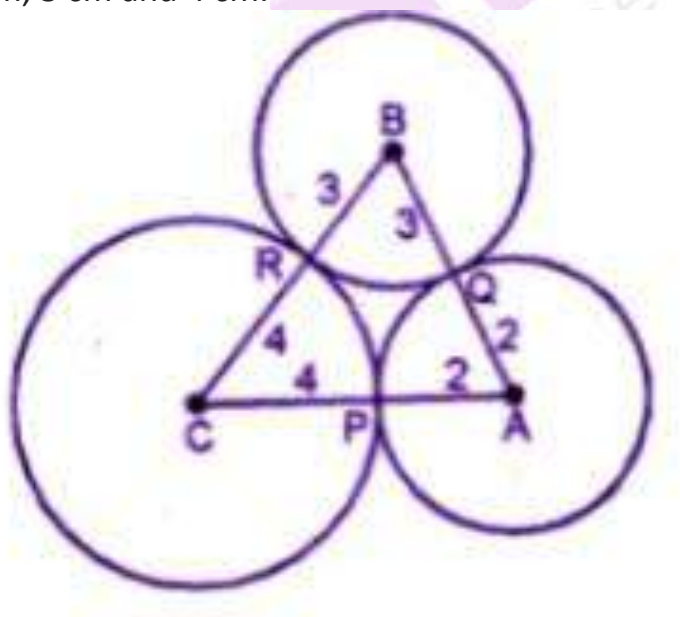


As the lengths of tangents drawn from an
External point to a circle are equal
AE = AF, BD = BF and CD = DE
OD ⊥ BC and OE ⊥ CA

(tangents is ⊥ to radius)
OD ⊥ BC and OE ⊥ CA
(tangents is ⊥ to radius)
ODCE is a square of side r
DC = CE = r
AF = AE = AC – EC = b – r and
BF = BD = BC – DC = a – r
Now, AB = AF + BF
C = (b – r) + (a – r)
2r = a + b – c
r = a + b – c/2
OP^2 = 400 + 169
OP = a−√569cm

10. Three circles of radii 2 cm, 3 cm and 4 cm touch each other externally. Find the perimeter of the triangle obtained on joining the centers of these circles.

Solution:
Three circles with centers A, B and C touch each other externally at P, Q and R respectively and the radii of these circles are 2 cm, 3 cm and 4 cm.

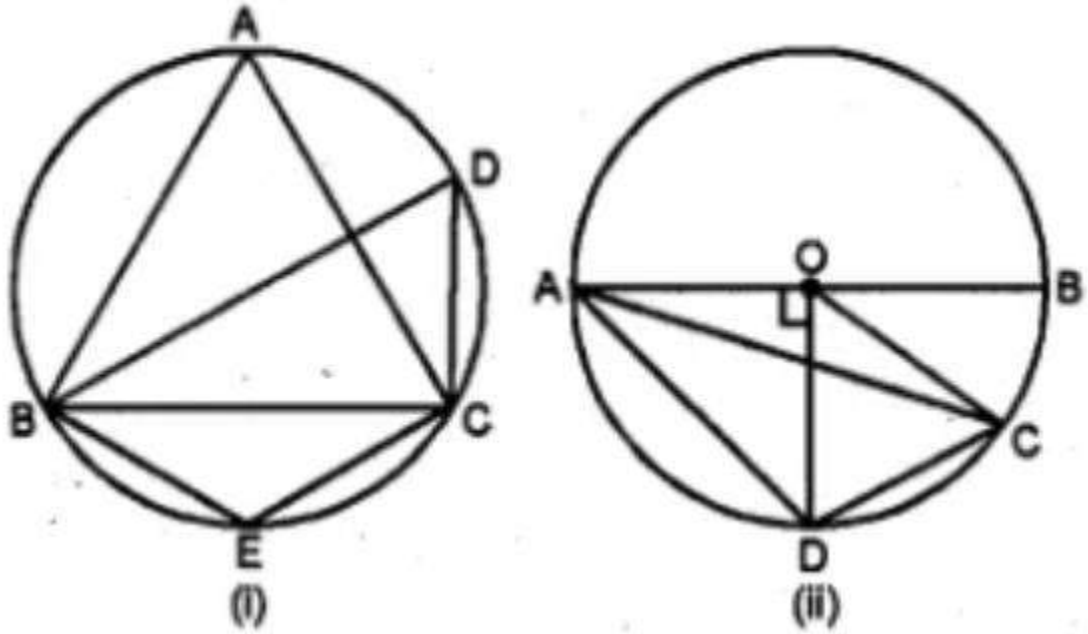


By joining the centers of triangle ABC formed in which,
AB = 2 + 3 = 5 cm

BC = 3 + 4 = 7 cm
CA = 4 + 2 = 6 cm
Therefore, perimeter of the triangle ABC = AB + BC + CA
= 5 + 7 + 6
= 18 cm

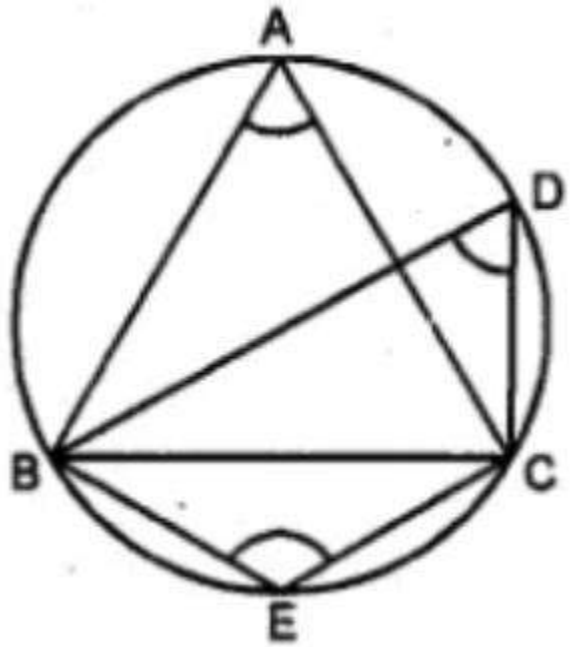
CHAPTER TEST

1. (a) In the figure (i) given below, triangle ABC is equilateral. Find ∠BDC and ∠BEC. (b) In the figure (ii) given below, AB is a diameter of a circle with center O. OD is perpendicular to AB and C is a point on the arc DB. Find ∠BAD and ∠ACD



Solution:

(a) triangle ABC is an equilateral triangle



Each angle = 60°

$\angle A = 60^o$

But $\angle A = \angle D$

(Angles in the same segment)

$\angle D = 60^0$

Now ABEC is a cyclic quadrilateral,

$\angle A = \angle E = 180^o$

$60^o + \angle E = 180^o$

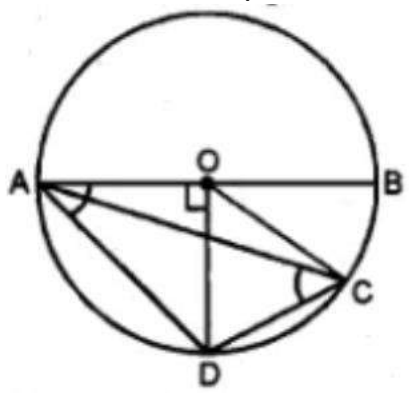
$60^0 + \angle E = 180^o$ ($\angle E = 180^o - 60^o$

$\angle E = 120^o$

Hence $\angle BDC = 60^o$ and $\angle BEC = 120^o$

(c) AB is diameter of circle with centre O.

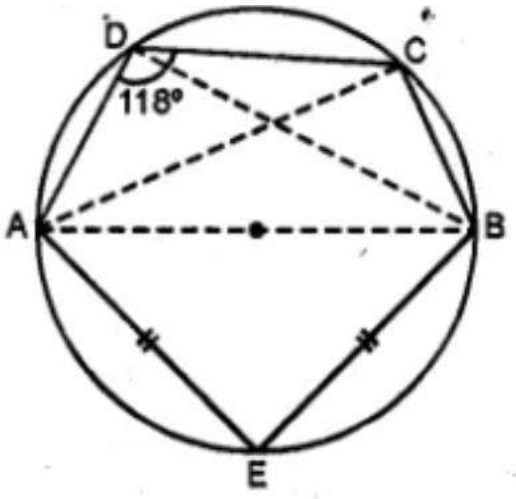
OD ⊥ AB and C is a point on arc DB.



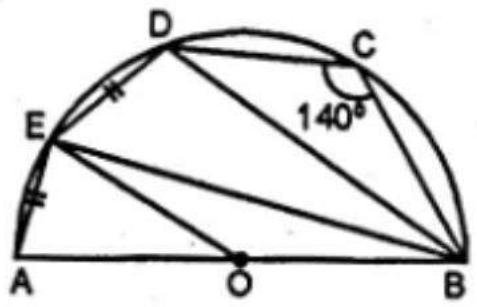
In ΔAOD, $\angle AOD = 90^0$

OA = OD (raddi of the semi – circle)
∠OAD = ∠ODA
But ∠OAD + ∠ODA = 90°
∠OAD + ∠ODA = 90°
2∠OAD = 90°
∠OAD = 90°/2 = 45°
Or ∠BAD = 45°
(ii) Arc AD subtends ∠AOD at the centre and
∠ACD at the remaining part of the circle
∠AOD = 2 ∠ACD
90° = 2 ∠ACD (OD ⊥ AB)
∠ACD = 90°/2 = 45°

2. **(a) In the figure given below, AB is a diameter of the circle. If AE = BE and ∠ADC = 118°, find (i) ∠BDC (ii) ∠CAE**



(B) inthe figure given below, AB is the diameter of the semi-circle ABCDE with centre O. If AE = ED and ∠BCD = 140°, find ∠AED and ∠EBD. Also Prove that OE is parallel to BD.



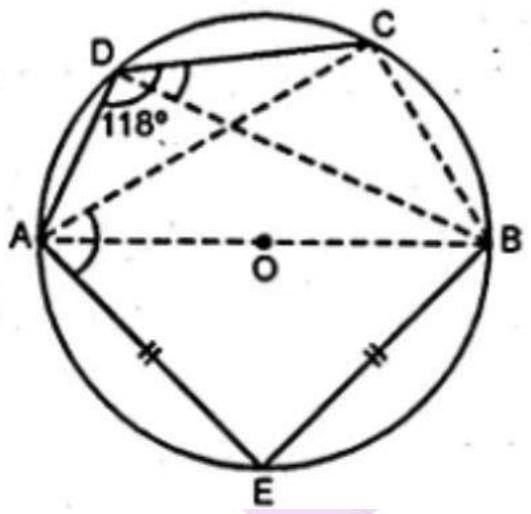
Solution:

(a) Join DB, CA and CB. ∠ADC = 118° (given) and ∠ADB = 90° (Angles in a semi-circle)

∠BDC = ∠ADC - ∠ADB

= $118^0 – 90^o = 28^o$

∠ABCD is a cyclic quadrilateral)



∠ADC + ∠ABC = 180°

118° + ∠ABC = 180°

∠ABC = $180^o – 118^o = 62^o$

But in ΔAEB

∠AEB = 90°

(Angles in a semi-circle)

∠EAB = ∠ABE (AE = BE)

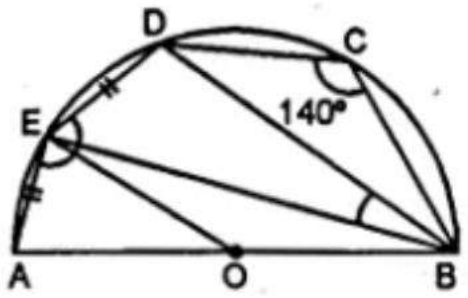
∠EAB + ∠ABE = 90°

∠EAB = 90° × ½ = 45°

∠CBE = ∠ABC + ∠ABE

$= 62^o + 45^o = 107^o$
But AEBD is a cyclic quadrilateral
$\angle CAE + \angle CBE = 180^o$
$\angle CAE + 107^o = 180^o$
$\angle CAE = 180^o - 107^o = 73^o$

(b) AB is the diameter of semi-circle ABCDE
With center O.AE = ED and $\angle BCD = 140^o$
In cyclic quadrilateral EBCD.

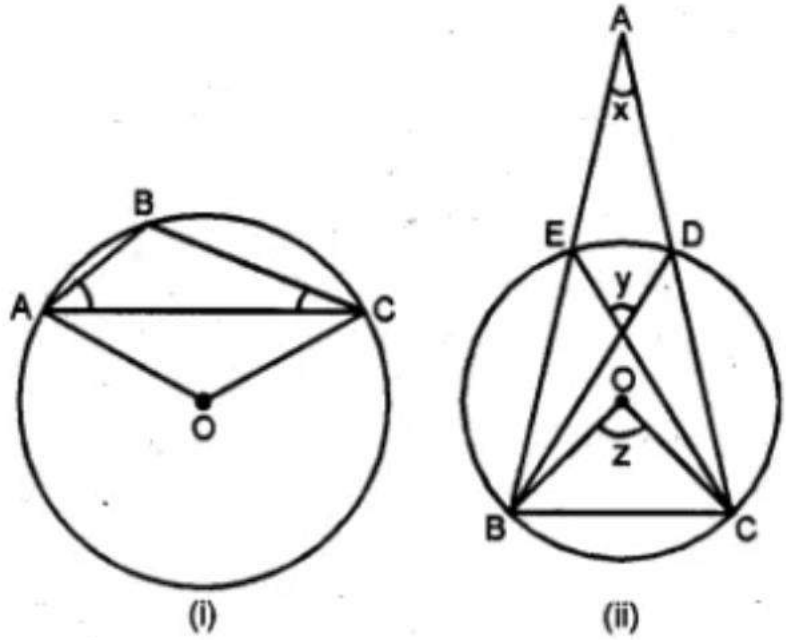


(i) $\angle BCD + \angle BED = 180o$
$140o + \angle BED = 180o$
$\angle BED = 180o - 140^o = 40^0$
But $\angle AED = 90^o$
(Angles in a semi circle)
$\angle AED = \angle AEB + \angle BED$
$= 90^o + 40^o = 130^o$

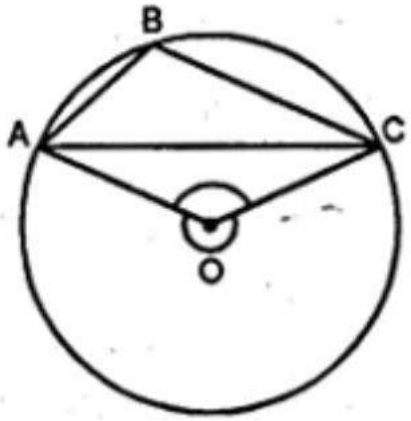
(ii) Now in cyclic quadrilateral AEDB
$\angle AED + \angle DBA = 180^o$
$130^o + \angle DBA = 180^o$
$\angle BDA = 180^o - 130^o = 50^o$
Chord AE = ED (given)
$\angle DBE = \angle EBA$
But $\angle DBE + \angle EBA = 50^o$
$DBE + \angle DBE = 50^o$
$2\angle DBE = 50^o$
$\angle DBE = 25^o$ or $\angle EBD = 25^o$
In $\Delta OEB, OE = OB$
(raddi of the same circle)
$\angle OEB = \angle EBO = \angle DBE$

But these are ultimate angles
OE ∥ BD

3. a) In the figure (i) given below, O is the centre of the circle. Prove that ∠AOC = 2 (∠ACB + ∠BAC). (b) In the figure (ii) given below, O is the centre of the circle. Prove that x + y = z



Solution :
(a) Given: O is the center of the circle. To Prove : ∠AOC = 2 (∠ACB + ∠BAC). Proof: In ΔABC, ∠ACB + ∠BAC + ∠ABC = 180° (Angles of a triangle)
∠ABC = 180° – (∠ACB + ∠BAC)….(i)
In the circle, arc AC subtends ∠AOC at
The center and ∠ABC at the remaining part of the circle.
Reflex ∠AOC = 2 ∠ABC …(ii)
Reflex AOC = 2 { (180° – (ACB + BAC)}
But ∠AOC = 360° – 2(∠ACB + ∠BAC)
But ∠AOC = 360° – reflex ∠AOC



=360 – (360° – 2(∠ACB + ∠BAC)

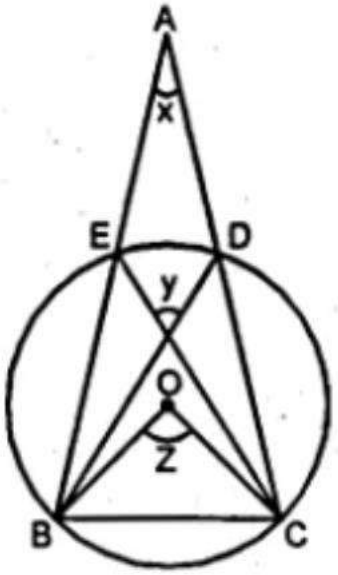
=360° – 360° + 2 (∠ACB + ∠BAC)

=2 (∠ACB + ∠BAC)

Hence ∠AOC = 2 (∠ACB + ∠BAC)

(b) Given : in the figure, O is the center of the circle

To Prove : × + y = z.



Proof : Arc BC subtends ∠AOB at the center and ∠BEC at the remaining part of the circle.

∠BOC = 2 ∠BEC

But ∠BEC = ∠BDC

(Angles in the same segment)

∠BOC = ∠BEC + ∠BDC ……. (ii)

Similarly in ΔABD

Ext. ∠BDC = x + ∠ABD

= x + ∠EBD .....(iii)

Substituting the value of (ii) and (iii) in (i)

∠BOC = y - ∠EBD + x + ∠EBD = x + y

Z = x + y