EXERCISE 8.1

1. Classify the following matrices:

(i) $\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}$

Solution:

It is square matrix of order 2

(ii) $[2 \ \ 3 \ \ - \ \ 7]$

Solution:

It is row matrix of order 1 × 3

(iii) $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

Solution:

It is column matrix of order 3 × 1

(iv) $\begin{bmatrix} 2 & -4 \\ 0 & 0 \\ 1 & 7 \end{bmatrix}$

Solution:

It is a matrix of order 3 × 2

(v) $\begin{bmatrix} 2 & 7 & 8 \\ -1 & \sqrt{2} & 0 \end{bmatrix}$

Solution:

It is a matrix of order 2 × 3

(vi) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Solution:
It is zero matrix of order 2 × 3

2. (i) If a matrix has 4 elements, what are the possible order it can have?

Solution:
It can have 1 × 4, 4 × 1 or 2 × 2 order.

(ii) If a matrix has 4 elements, what are the possible orders it can have?

Solution:
It can have 1 × 8, 8 × 1, 2 × 4 or 4 × 2 order.

3. Construct a 2 × 2 matrix whose elements a_{ij} are given by
(i) $a_{ij} = 2i – j$
(ii) $a_{ij} = i.j$

Solution:
(i) Given $a_{ij} = 2i – j$
Therefore matrix of order 2 × 2 is
$$\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

(ii) Given $a_{ij} = i.j$
Therefore matrix of order 2 × 2 is
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

4. Find the values of x and y if:

$$\begin{bmatrix} 2x+y \\ 3x-2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 2x+y \\ 3x-2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now by comparing the corresponding elements,

2x + y = 5 ….. i

3x – 2y = 4 ….ii

Multiply (i) by 2 and (ii) by 1 we get

4x + 2y = 10 and 3x – 2y = 4

By adding we get

7x = 14

x = 14/7

x = 2

Substituting the value of x in (i)

4 + y = 5

y = 5 – 4

y = 1

Hence x = 2 and y = 1

5. Find the value of x if

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$$

Comparing the corresponding terms of given matrix we get

-y = 2

Therefore y = -2

Again we have

3x + y = 1
3x = 1 – y
Substituting the value of y we get
3x = 1 – (-2)
3x = 1 + 2
3x = 3
x = 3/3
x = 1
Hence x = 1 and y = -2

6. If

$$\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$$

Find the values of x and y.

Solution:
Given

$$\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$$

Comparing the corresponding terms, we get
x + 3 = 5
x = 5 – 3
x = 2
Again we have
y – 4 = 3
y = 3 + 4
y = 7
Hence x = 2 and y = 7

7. Find the values of x, y and z if

$$\begin{bmatrix} x+2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2+y \\ 3 & -20 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} x+2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2+y \\ 3 & -20 \end{bmatrix}$$

Comparing the corresponding elements of given matrix, then we get

x + 2 = -5

x = -5 – 2

x = -7

Also we have 5z = -20

z = -20/5

z = - 4

Again from given matrix we have

y^2 + y – 6 = 0

The above equation can be written as

y^2 + 3y – 2y – 6 = 0

y (y + 3) – 2 (y + 3) = 0

y + 3 = 0 or y – 2 = 0

y = -3 or y = 2

Hence x = -7, y = -3, 2 and z = -4

8. Find the values of x, y, a and b if

$$\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$$

Comparing the corresponding elements

x – 2 = 3 and y = 1

x = 2 + 3

x = 5

again we have

a + 2b = 5….. i

3a – b = 1 ……ii

Multiply (i) by 1 and (ii) by 2

a + 2b = 5

6a – 2b = 2

Now by adding above equations we get

7a = 7

a = 7/7

a = 1

Substituting the value of a in (i) we get

1 + 2b = 5

2b = 5 -1

2b = 4

b = 4/2

b = 2

9. Find the values of a, b, c and d if

$$\begin{bmatrix} a+b & 3 \\ 5+c & ab \end{bmatrix} = \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} a+b & 3 \\ 5+c & ab \end{bmatrix} = \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$$

Comparing the corresponding terms, we get

3 = d

d = 3

Also we have

5 + c = -1

c = -1 – 5

c = -6

Also we have,

a + b = 6 and a b = 8

we know that,

$(a – b)^2 = (a + b)^2 – 4$ ab

$(6)^2 – 32 = 36 – 32 = 4 = (\pm 2)^2$

a – b = ± 2

If a – b = 2
a + b = 6
Adding the above two equations we get
2a = 4
a = 4/2
a = 2
b = 6 – 4
b = 2
Again we have a – b = -2
And a + b = 6
Adding above equations we get
2a = 4
a = 4/2
a = 2
Also, b = 6 – 2 = 4
a = 2 and b = 4

10. Find the values of x, y, a and b, if

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Solution:
Given

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Comparing the corresponding terms, we get
3x + 4y = 2….i
x – 2y = 4 …..ii
Multiply (i) by 1 and (ii) by 2
3x + 4y = 2, 2x – 4y = 8
Adding the above equations we get
5x = 10
x = 10/5
x = 2
Substituting the value of x in (i)

we get
$6 + 4y = 2$
$4y = 2 – 6$
$4y = -4$
$y = -1$

EXERCISE 8.2

1. Given that M = $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ and N = $\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$, find M + 2N

Solution:

Given

$$M = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$$
$$N = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

Now we have to find M + 2N

$$M + 2N = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

On simplifying we get,

$$= \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -2 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 2+4 & 0+0 \\ 1-2 & 2+4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -1 & 6 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

find 2A – 3B

Solution:

Given

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

Now we have to find,

$$\therefore 2A - 3B = 2\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

On simplifying we get

$$= \begin{bmatrix} 4 & 0 \\ -6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix} = \begin{bmatrix} 4-0 & 0-3 \\ -6+6 & 2-9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}$$

3. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

Compute $3A + 4B$

Solution:

Given

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

Now we have to find $3A + 4B$

$$3A + 4B = 3\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

On simplifying we get

$$= \begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 12 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+4 & 12+8 \\ 6+12 & 9+4 \end{bmatrix} = \begin{bmatrix} 7 & 20 \\ 18 & 13 \end{bmatrix}$$

4. Given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$

(i) find the matrix 2A + B

(ii) find a matrix C such that $C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution:

Given

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$$

Now we have to find 2A + B,

$$(i)\quad 2A + B = 2\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4 & 8-1 \\ 4-3 & 6-2 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 1 & 4 \end{bmatrix}$$

Again we have to find C + B

$$(ii)\quad C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$$

On simplifying we get

$$= \begin{bmatrix} 0-(-4) & 0-(-1) \\ 0-(-3) & 0-(-2) \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

5. $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

Find $A + 2B - 3C$

Solution:

Given

$A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

Now we have to find A + 2B – 3C

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + 2\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix}$$

On simplifying we get

$$= \begin{bmatrix} 1-4-0 & 2-2-9 \\ -2+2-6 & 3+4+3 \end{bmatrix} = \begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

Find the matrix X if:

(i) 3A + X = B

(ii) X – 3B = 2A

Solution:

Given

$A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

Now we have to find

(i) 3A + X = B

X = B – 3A

Substituting the values we get

$$X=\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}-3\begin{bmatrix}0 & -1\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}-\begin{bmatrix}0 & -3\\ 3 & 6\end{bmatrix}$$

$$=\begin{bmatrix}1-0 & 2+3\\ -1-3 & 1-6\end{bmatrix}=\begin{bmatrix}1 & 5\\ -4 & -5\end{bmatrix}$$

(ii) X – 3B = 2A

X = 2A + 3B

Now substituting the values A and B we get

$$X=2\begin{bmatrix}0 & -1\\ 1 & 2\end{bmatrix}+3\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0 & -2\\ 2 & 4\end{bmatrix}+\begin{bmatrix}3 & 6\\ -3 & 3\end{bmatrix}$$

$$=\begin{bmatrix}0+3 & -2+6\\ 2-3 & 4+3\end{bmatrix}=\begin{bmatrix}3 & 4\\ -1 & 7\end{bmatrix}$$

7. Solve the matrix equation

$$\begin{bmatrix}2 & 1\\ 5 & 0\end{bmatrix}-3X=\begin{bmatrix}-7 & 4\\ 2 & 6\end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix}2 & 1\\ 5 & 0\end{bmatrix}-3X=\begin{bmatrix}-7 & 4\\ 2 & 6\end{bmatrix}$$

On rearranging we get

$$\begin{bmatrix} 2 & 1 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} -7 & 4 \\ 2 & 6 \end{bmatrix} = 3X$$

On simplification we get

$$X = \frac{1}{3}\begin{bmatrix} 9 & -3 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$$

EXERCISE 8.3

1. If A = $\begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$, is the product AB possible ? Give a reason. If yes, find AB.

Solution:

Yes, the product is possible because of number of column in A = number of row in B

That is order of matrix is 2 × 1

$$AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3\times 2+5\times 4 \\ 4\times 2+(-2)\times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+20 \\ 8-8 \end{bmatrix} = \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$

2. If A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$,B = $\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$, find AB and BA, Is AB = BA ?

Solution:

Given

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

Now we have to find A × B

$$\therefore A \times B = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\times\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-15 & -2+10 \\ 1-9 & -1+6 \end{bmatrix} = \begin{bmatrix} -13 & 8 \\ -8 & 5 \end{bmatrix}$$

Again have to find B × A

$$B \times A = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5-3 \\ -6+2 & -15+6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -4 & -9 \end{bmatrix}$$

Hence AB is not equal to BA

3. If $P = \begin{bmatrix} 4 & 6 \\ 2 & -8 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$

Find 2PQ

Solution:

Given

$P = \begin{bmatrix} 4 & 6 \\ 2 & -8 \end{bmatrix}$,

$Q = \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$

$$2PQ = 2\begin{bmatrix} 4 & 6 \\ 2 & -8 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}$$

$$= 2\begin{bmatrix} 8-6 & -12+6 \\ 4+8 & -6-8 \end{bmatrix} = 2\begin{bmatrix} 8 & -6 \\ 12 & -14 \end{bmatrix} = \begin{bmatrix} 4 & -12 \\ 24 & -28 \end{bmatrix}$$

4. Given $A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$, evaluate $A^2 - 4A$

Solution:

Given

$$A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$

Now consider,

$$A^2 - 4A = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$$

On simplifying we get

$$= \begin{bmatrix} 1+8 & 1+3 \\ 8+24 & 8+9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4 & 4-4 \\ 32-32 & 17-12 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

5. If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

Find AB – 5C

Solution:

Given

$A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$$

On simplification we get

$$= \begin{bmatrix} 3\times0+7\times5 & 3\times2+7\times3 \\ 2\times0+4\times5 & 2\times2+4\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$5C = 5\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find A(BA)

Solution:

Given

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$A(BA) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix} = \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$$

7. Given matrices:

$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$

Find the products of

(i) ABC

(ii) ACB and state whether they are equal.

Solution:

Given

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix},$$

$$C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

Now consider,

$$ABC = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-1 & 8-2 \\ 12-2 & 16-4 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix} \times \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -15+0 & 5-12 \\ -30+0 & 10-24 \end{bmatrix} = \begin{bmatrix} -15 & -7 \\ -30 & -14 \end{bmatrix}$$

$$ACB = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+0 & 2-2 \\ -12+0 & 4-4 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -18+0 & -24+0 \\ -36+0 & -48+0 \end{bmatrix} = \begin{bmatrix} -18 & -24 \\ -36 & -48 \end{bmatrix}$$

$\therefore ABC \neq ACB.$

8. Evaluate : $\begin{bmatrix} 4\sin 30^o & 2cos60^o \\ sin90^o & 2cos0^o \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

Solution:

Given

$$\begin{bmatrix} 4\sin 30^o & 2cos60^o \\ sin90^o & 2cos0^o \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$sin30^o = \frac{1}{2}, cos60^o = \frac{1}{2}$

$\sin 90^o = 1$ and $\cos 0^o = 1$

$$\therefore \begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4+1\times5 & 2\times5+1\times4 \\ 1\times4+2\times5 & 1\times5+2\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix} = \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

9. If $A = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$ find the matrix AB + BA

Solution:

Given

$A = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$,

$B = \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -2-12 & 3-18 \\ 4-16 & -6-24 \end{bmatrix} = \begin{bmatrix} -14 & -15 \\ -12 & -30 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & -3 \\ -4 & -6 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-6 & 6-12 \\ 4-12 & -12-24 \end{bmatrix} = \begin{bmatrix} -8 & -6 \\ -8 & -36 \end{bmatrix}$$

$\therefore AB + BA$

$$=\begin{bmatrix}-14 & -15\\ -12 & -30\end{bmatrix}+\begin{bmatrix}-8 & -6\\ -8 & -36\end{bmatrix}$$

$$=\begin{bmatrix}-14-8 & -15-6\\ -12-8 & -30-36\end{bmatrix}=\begin{bmatrix}-22 & -21\\ -20 & -66\end{bmatrix}$$

10. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}6 & 1\\ 1 & 1\end{bmatrix}$, $C=\begin{bmatrix}-2 & -3\\ 0 & 1\end{bmatrix}$

Find each of the following and state if they are equal.
(i) CA + B
(ii) A + CB

Solution:

(i) CA + B

$$CA=\begin{bmatrix}-2 & -3\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-2-9 & -4-12\\ 0+3 & 0+4\end{bmatrix}=\begin{bmatrix}-11 & -16\\ 3 & 4\end{bmatrix}$$

$$CA+B=\begin{bmatrix}-11 & -16\\ 3 & 4\end{bmatrix}+\begin{bmatrix}6 & 1\\ 1 & 1\end{bmatrix}$$

(ii) A + CB

$$=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}-2 & -3\\ 0 & 1\end{bmatrix}\times\begin{bmatrix}6 & 1\\ 1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}-12-3 & -2-3\\ 0+1 & 0+1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}-15 & -5\\ 1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1-15 & 2-5\\ 3+1 & 4+1\end{bmatrix}=\begin{bmatrix}-14 & -3\\ 4 & 5\end{bmatrix}$$

We can say that $CA + B \neq A + CB$

CHAPTER TEST

1. Find the values of a and b if

$$\begin{bmatrix} a+3 & b^2+2 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} 2a+1 & 3b \\ 0 & b^2-5b \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} a+3 & b^2+2 \\ 0 & -6 \end{bmatrix} = \begin{bmatrix} 2a+1 & 3b \\ 0 & b^2-5b \end{bmatrix}$$

comparing the corresponding elements

a + 3 = 2a + 1

⇒ 2a – a =3 – 1

⇒ a = 2 b² + 2 = 3b

⇒ b² – 3b + 2 = 0

⇒ b² – b – 2b + 2 = 0

⇒ b (b – 1) – 2 (b – 1) = 0

⇒ (b – 1) (b – 2) = 0.

Either b – 1 = 0,

then b = 1 or b – 2 = 0,

then b = 2

Hence a = 2, b = 2 or 1

2. Find a, b, c and d if $3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix} + \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix}$

Solution:

Given

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix} + \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix}$$

Now comparing the corresponding elements

3a = 4 + a

a – a = 4
2a = 4
Therefore, a = 2
3b = a + b + 6
3b – b = 2 + 6
2b = 8
Therefore, b = 4
3d = 3 + 2d
3d – 2d = 3
Therefore, d = 3
3c = c + d – 1
3c – c = 3 – 1
2c = 2
Therefore, c = 1
Hence a = 2,b = 4, c = 1 and d= 3

3. Find X if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Solution:

Given

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = 2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - Y$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$