Chapter Test

1. Find the values of:

(i) $\sin^2 60° - \cos^2 45° + 3\tan^2 30°$

(ii) $\frac{2\cos^2 45°+3\tan^2 30°}{\sqrt{3}\cos 30°+\sin 30°}$

(iii) sec 30° tan 60° + sin 45° cosec 45° + cos 30° cot 60°

Solution:

(i) $\sin^2 60° - \cos^2 45° + 3\tan^2 30°$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{3}{4} - \frac{1}{2} + 3 \times \frac{1}{3} = \frac{3}{4} - \frac{1}{2} + \frac{1}{1}$$

$$= \frac{3-2+4}{4} = \frac{7-2}{4} = \frac{5}{4} = 1\frac{1}{4}$$

Therefore, $\sin^2 60° - \cos^2 45° + 3\tan^2 30° = 1\frac{1}{4}$

(ii) $$\frac{2\cos^2 45° + 3\tan^2 30°}{\sqrt{3}\cos 45° + \sin 30°}$$

$$= \frac{2\left(\frac{1}{\sqrt{2}}\right)^2 + 3 \times \left(\frac{1}{\sqrt{3}}\right)^2}{\sqrt{3} \times \left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}}$$

$$= \frac{2 \times \frac{1}{2} + 3 \times \frac{1}{3}}{\frac{3}{2} + \frac{1}{2}} = \frac{1+1}{\frac{3+1}{2}} = \frac{2}{\frac{4}{2}} = \frac{2}{2} = 1$$

Hence, $\frac{2\cos^2 45°+3\tan^2 30°}{\sqrt{3}\cos 30°+\sin 30°} = 1$

(iii) sec 30° tan 60° + sin 45° cosec 45° + cos 30° cot 60°

$$= \frac{2}{\sqrt{3}} \times \sqrt{3} + \frac{1}{\sqrt{2}} \times \sqrt{2} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{2}{1} + \frac{1}{1} + \frac{1}{2}$$

= 2 + 1 + ½ = 3 + ½ = (6 + 1)/2

= 7/2 = 3½
Thus, sec 30° tan 60° + sin 45° cosec 45° + cos 30° cot 60° = 3½

2. Taking A = 30°, verify that
(i) $\cos^4 A - \sin^4 A = \cos 2A$
(ii) 4cos A cos (60° – A) cos (60° + A) = cos 3 A.
Solution:

(i) $\cos^4 A - \sin^4 A = \cos 2A$
Let's take A = 30°
so, we have
L.H.S.= $\cos^4 A - \sin^4 A = \cos^4 30° - \sin^4 30°$

$$= \left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{2}\right)^4$$

$$= \frac{\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}}{2\times2\times2\times2} - \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{9}{16} - \frac{1}{16}$$

$$= \frac{9-1}{16} = \frac{8}{16} = \frac{1}{2}$$

Now,
R.H.S. = **c**os 2A = cos 2(30°) = ½
Therefore, L.H.S. = R.H.S. hence verified.

(ii) 4 cos A cos (60°- A) cos (60° + A) = cos 3 A
Let's take A = 30°
L.H.S. = 4 cos A cos (60° – A) cos (60° + A)
= 4 cos 30° cos (60° – 30°) cos (60° + 30°)
= 4 cos 30° cos 30° cos 90°
= 4 × (√3/2) × (√3/2) × 0
= 0
Now,
R.H.S. = cos 3A
= cos (3 × 30°) = cos 90° = 0
Hence, L.H.S. = R.H.S. hence verified.

3. If A = 45° and B = 30°, verify that sin A/ (cos A + sin A + sin B) = 2/3
Solution:
Taking,

$$\text{L.H.S. } \frac{\sin A}{\cos A + \sin A \sin B}$$

$$= \frac{\sin 45^\circ}{\cos 45^\circ + \sin 45^\circ \sin 30^\circ}$$

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4}}$$

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{2\sqrt{2} + \sqrt{2}}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{3\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2}}{2} \times \frac{4}{3\sqrt{2}} = \frac{2}{3} = \text{R.H.S.}$$

Hence verified.

4. Taking A = 60° and B = 30°, verify that
(i) sin (A + B)/ cos A cos B = tan A + tan B
(ii) sin (A - B)/ sin A sin B = cot B – cot A
Solution:

(i) Here, A = 60° and B = 30°

$$\text{LHS} = \frac{\sin(A+B)}{\cos A \cos B} = \frac{\sin(60^\circ + 30^\circ)}{\cos 60^\circ \cos 30^\circ}$$

$$= \frac{\sin 90^\circ}{\cos 60^\circ \cos 30^\circ} = \frac{1}{\frac{1}{2} \times \frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\frac{\sqrt{3}}{4}} = \frac{4}{\sqrt{3}}$$

Now,

$$\text{R.H.S.} = \tan A + \tan B$$

$$= \tan 60^\circ + \tan 30^\circ$$

$$= \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$= \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$$\therefore \text{ L.H.S.} = \text{R.H.S.}$$

(ii) A = 60°, B = 30°

$$\text{L.H.S.} = \frac{\sin(A-B)}{\sin A \sin B} = \frac{\sin(60^\circ - 30^\circ)}{\sin 60^\circ \sin 30^\circ}$$

$$= \frac{\sin 30^\circ}{\sin 60^\circ \sin 30^\circ} = \frac{1}{\sin 60^\circ} = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2}{\sqrt{3}}$$

$$\text{R.H.S.} = \cot B - \cot A$$

$$= \cot 30^\circ - \cot 60^\circ$$

$$= \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

∴ L.H.S. = R.H.S.

5. If √2 tan 2θ = √6 and θ° < 2θ < 90°, find the value of sin θ + √3 cos θ – 2 tan² θ.
Solution:

Given,
$\sqrt{2} \tan 2\theta = \sqrt{6}$
$\tan 2\theta = \sqrt{6}/\sqrt{2}$
$= \sqrt{3}$
$= \tan 60^\circ$
$\Rightarrow 2\theta = 60^\circ$
$\theta = 30^\circ$
Now,
$\sin \theta + \sqrt{3} \cos \theta - 2 \tan^2 \theta$
$= \sin 30^\circ + \sqrt{3} \cos 30^\circ - 2 \tan^2 30^\circ$
$= ½ + \sqrt{3} \times \sqrt{3}/2 - 2 (1/\sqrt{3})^2$
$= ½ + 3/2 - 2/3$
$= 4/2 - 2/3$
$= (12 - 4)/6$
$= 8/6$
$= 4/3$

6. If 3θ is an acute angle, solve the following equations for θ:
(i) (cosec 3θ – 2) (cot 2θ – 1) = 0
(ii) (tan θ – 1) (cosec 3θ – 1) = 0
Solution:

(i) (cosec 3θ – 2) (cot 2θ – 1) = 0

Now, either
cosec 3θ – 2 or cot 2θ – 1 = 0
⇒ cosec 3θ = 2 or cot 2θ = 1
So,
cosec 3θ = cosec 3θ° or cot 2θ =cot 45°
⇒ 3θ = 30° or 2θ = 45°
Thus, θ = 30° or 45°.

(ii) (tan θ – 1) (cosec 3θ – 1) = 0
Now, either
tan θ – 1 = 0 or cosec 3θ – 1 = 0
⇒ tan θ = 1 or cosec 3θ = 1
So,
tan θ = tan45° or cosec 3 θ = cosec 90°
⇒ θ = 45° or 3θ = 90° i.e. θ = 30°
Thus, θ = 45° or 30°.

7. If tan (A + B) = √3 and tan (A – B) = 1 and A, B (B < A) are acute angles, find the values of A and B.
Solution:

Given, tan (A + B) = √3
So, tan (A + B) = tan 60° [Since, tan 60° = √3]
⇒ A + B = 60° ……….. (i)
Also, given
tan (A – B) = 1
So, tan (A – B) = tan 45° [tan 45° = 1]
⇒ A – B = 45° ………….. (ii)
From equation (1) and (2), we get
A + B = 60°
A – B = 45°

2A = 105°
A = 52½°
Now, on substituting the value of A in equation (i), we get
52½° + B = 60°
B = 60° - 52½° = 7½°
Therefore, the value of A = 52½° and B = 7½°

8. Without using trigonometrical tables, evaluate the following:
(i) $\sin^2 28° + \sin^2 62° – \tan^2 45°$
(ii) $2\frac{\cos 27°}{\sin 63°} + \frac{\tan 27°}{\cot 63°} + \cos 0°$
(iii) cos 18° sin 72° + sin 18° cos 72°
(iv) 5 sin 50° sec 40° – 3 cos 59° cosec 31°

Solution:

(i) $\sin^2 28° + \sin^2 62° – \tan^2 45°$
$= \sin^2 28° + \sin^2 (90° – 28°) – \tan^2 45°$
$= \sin^2 28° + \cos^2 28° – \tan^2 45°$
$= 1 – (1)^2$ $\quad$ ($\because \sin^2 \theta + \cos^2 \theta = 1$ and $\tan 45° = 1$)
$= 1 – 1$
$= 0$

$$(ii)\quad 2\frac{\cos 27°}{\sin 63°} + \frac{\tan 27°}{\cot 63°} + \cos 0°$$
$$= 2\frac{\cos 27°}{\sin(90° - 27°)} + \frac{\tan 27°}{\cot(90° - 27°)} + \cos 0°$$
$$= 2\frac{\cos 27°}{\cos 27°} + \frac{\tan 27°}{\tan 27°} + 1 \qquad (\because \cos 0° = 1)$$
$= 2 \times 1 + 1 + 1$
$= 2 + 1 + 1$
$= 4$

(iii) $\cos 18° \sin 12° + \sin 18° \cos 12°$
$= \cos (90° – 12°) \sin 72° + \sin (90° – 12°) \cos 12°$
$= \sin 72°.\sin 12° + \cos 12° \cos 12°$
$= \sin^2 12° + \cos^2 12°$
$= 1$ $\quad$ ($\because \sin^2 \theta + \cos^2 \theta = 1$)

(iv) $5 \sin 50° \sec 40° – 3 \cos 59° \operatorname{cosec} 31°$
$$= 5\frac{\sin 50°}{\cos 40°} - 3\frac{\cos 59°}{\sin 31°}$$
$$= 5\frac{\sin 50°}{\cos(90° - 50°)} - 3\frac{\cos 59°}{\sin(90° - 59°)}$$
$$= 5\frac{\sin 50°}{\sin 50°} - 3\frac{\cos 59°}{\cos 59°} = 5 \times 1 - 3 \times 1$$
$= 5 – 3$
$= 1$

9. Prove that:

$$\frac{\cos(90° - \theta)\sec(90° - \theta)\tan\theta}{\operatorname{cosec}(90° - \theta)\sin(90° - \theta)\cot(90° - \theta)} + \frac{\tan(90° - \theta)}{\cot\theta} = 2$$

Solution:

$$\text{L.H.S.} = \frac{\cos(90° - \theta)\sec(90° - \theta)\tan\theta}{\text{cosec}(90° - \theta)\sin(90° - \theta)\cot(90° - \theta)} + \frac{\tan(90° - \theta)}{\cot\theta}$$

$$= \frac{\sin\theta\,\text{cosec}\,\theta\tan\theta}{\sec\theta\cos\theta\tan\theta} + \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1 \times \tan\theta}{1 \times \tan\theta} + 1 = 1 + 1 = 2 = \text{R.H.S.}$$

Thus, L.H.S. = R.H.S.
Hence proved.

10. When 0° < A < 90°, solve the following equations :
(i) sin 3A = cos 2A
(ii) tan 5A = cotA
Solution:

(i) sin 3A = cos 2A
⇒ sin 3A = sin (90° – 2A)
So,
3A = 90° – 2A
3 A + 2A = 90°
5A = 90°
∴ A = 90°/5 = 18°

(ii) tan 5A = cot A
⇒ tan 5A = tan (90° – A)
So,
5A = 90°- A
5A + A = 90°
6A = 90°
∴ A = 90°/6 = 15°

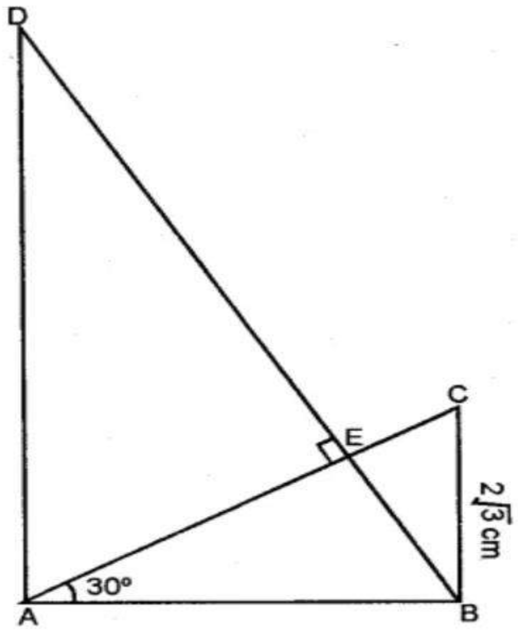
11. Find the value of θ if
(i) sin (θ + 36°) = cos θ, where θ and θ + 36° are acute angles.
(ii) sec 4θ = cosec (θ – 20°), where 4θ and θ – 20° are acute angles.
Solution:

(i) Given, θ and (θ + 36°) are acute angles
And,
sin (θ + 36°) = cos θ = sin (90° – θ)   [As, sin (90° – θ) = cos θ]
On comparing, we get
θ + 36° = 90° – θ
θ + θ = 90° – 36°
2θ = 54°

θ = 54°/2
∴ θ = 27°

(ii) Given, θ and (θ – 20°) are acute angles
And,
sec 4θ = cosec (θ – 20°)
cosec (90° – 4θ) = cosec (θ – 20°) [Since, cosec (90° – θ) = sec θ]
On comparing, we get
90° – 4θ = θ – 20°
90° + 20° = θ + 4θ
5θ = 110°
θ = 110°/5
∴ θ = 22°

12. In the adjoining figure, ABC is right-angled triangle at B and ABD is right angled triangle at A. If BD ⊥ AC and BC = 2cm, find the length of AD.



Solution:

Given, ΔABC and ΔABD are right angled triangles in which ∠A = 90° and ∠B = 90°
And,
BC = 2√3 cm. AC and BD intersect each other at E at right angle and ∠CAB = 30°.
Now in right ΔABC, we have
tan θ = BC/AB
⇒ tan 30° = 2√3/ AB
⇒ 1/√3 = 2√3/ AB
⇒ AB = 2√3 × √3 = 2 × 3 = 6 cm.
In ΔABE, ∠EAB = 30° and ∠EAB = 90°
Hence,

$\angle ABE$ or $\angle ABD = 180° - 90° - 30°$

$= 60°$

Now in right ΔABD, we have

$\tan 60° = AD/AB$

$\Rightarrow \sqrt{3} = AD/6$

Thus, $AD = 6\sqrt{3}$ cm.