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VOLUME - 2

MATHEMATICS

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FOREWORD

The Colourful world of children is full of excitement and spectacular thoughts! Their imaginative power can even attract the wild creatures to accompany them in a friendly manner. Their enthusiasum and innovative prescription can even trigger the non-living entities and enchant the poetic Tamil. It is nothing but a bundle of joy blended with emotions when you travel into their creative world.

We have tried our level best to achieve the following objectives through the new text books by gently holding the tender hands of those little lads.

- To tune their mind away from rote-learning and guide them into the world of creativity.
- To make the children be proud of their ancient history, culture, art and rich Tamil literature.
- To march triumphantly with confidence into the modern world with the help of Science and Technology.
- To facilitate them to extend their journey of learning beyond the text book into the world of wisdom.

These new text books are studded with innovative design, richer content blended with appropriate psychological approach meant for children. We firmly believe that these newly designed text books will certainly create a sparkle in the mind of the children and make them explore the world afresh.

Mathematics is a unique symbolic language in which the whole world works and acts accordingly. This text book is an attempt to make learning of Mathematics easy for the students community.

Mathematics is not about numbers, equations, computations or algorithms; it is about understanding

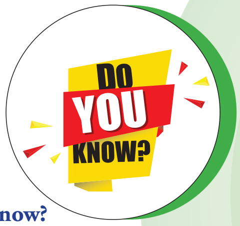
— William Paul Thurston

Activity

To enjoy learning Mathematics by doing

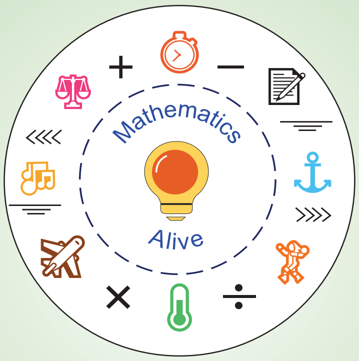
Try this/these

A few questions which provide scope for reinforcement of the content learnt.



Do you know?

To know additional information related to the topic to create interest.



Think

To think deep and explore yourself!

To understand that Mathematics can be experienced everywhere in nature and real life.

Miscellaneous and Challenging problems

To give space for learning more and to face higher challenges in Mathematics and to face Competitive Examinations.

Note

To know important facts and concepts

ICT Corner

Go, Search the content and Learn more!

The main goal of Mathematics in School Education is to mathematise the child's thought process. It will be useful to know how to mathematise than to know a lot of Mathematics.

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QR GUIDE

Let's use the QR code in the text books!

- Download DIKSHA app from the Google Play Store.
- Tap the QR code icon to scan QR codes in the textbook.
- Point the device and focus on the QR code.
- On successful scan, content linked to the QR code gets listed.
- Note: For ICT corner, Digi Links QR codes use any other QR scanner.

1 LIFE MATHEMATICS

B1Q6R5

Learning Objectives

- To recall the concepts of profit and loss and simple interest.
- To solve problems involving applications of percentage, profit and loss, overhead expenses, discount and GST.
- To know what compound interest is and to be able to find compound interest through patterns and formula and use them in simple problems.
- To find the difference between simple and compound interest for 2 years and 3 years.

Recap

1. If the selling price of an article is less than the cost price of the article then, there is a ________.
2. An article costing ₹5000 is sold for ₹4850. Is there a profit or loss? What is the percentage?
3. If the ratio of cost price and the selling price is 5:7, then the profit/ gain is__________%.
4. The formula to find the simple interest for a given principal is __________.
5. Find the simple interest on ₹900 for 73 days at 8% p.a.
6. In how many years will ₹2000 become ₹3600 at 10% p.a simple interest?

Try these

Find the indicated percentage value of the given numbers.

% \ Number	60	240	660	852	1200
10%					
20%					
25%					
$33\frac{1}{3}\%$					

1.1 Introduction

The following conversation happens in a Math class of VIII std.

Teacher : Dear students, money collection is being made for the Flag Day, and so far 32 out of 40 students in VII std and 42 out of 50 students from our class have contributed. Can anyone of you say, whose contribution is better?

Sankar : Teacher, 32 out of 40 is $\frac{32}{40}$ and 42 out of 50 is $\frac{42}{50}$ .The corresponding like fractions of them are $\frac{160}{200}$ and $\frac{168}{200}$ respectively. So, our class students contribution is better.

Teacher : Good, Sankar. Is there any other way to compare?

Bumrah : Yes Teacher, percentages will help. $\frac{32}{40}=\frac{32}{40}\times 100=80\ \%$ and $\frac{42}{50}=\frac{42}{50}\times 100=84\%$

Our class contribution is 4% more than VII std.

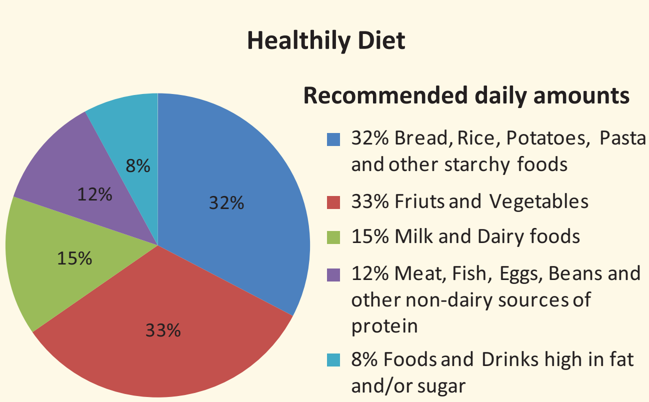
Teacher : Well done, Bumrah. You are exactly correct. Can anyone of you say where the use of percentages is seen more?

Bhuvi : Yes Teacher, I have learnt from my father that percentages are used in the calculation of profit and loss, discount, calculation of tax(eg: GST), interest, growth and depreciation and almost everywhere where comparison is made. He also told me that it is an easy tool which is helpful to compare values.

Teacher : Well said, Bhuvi. These are the topics what we are going to see in this chapter using percentages.

The above conversation leads the way to know the application of percentages in various situations and the commonly seen problems in our day-to-day life.

Mathematics Alive—Life Mathematics in Real Life



Percentages are used in diet recommendations and in many daily life situations.

Money grows faster by compound interest.

1.2 Applications of Percentage in Problems

We know that Per Cent means per hundred or out of a hundred. It is denoted by the symbol %. $x\%$ denotes the fraction $\frac{x}{100}$. It is very useful in comparing quantities easily. Let us see its uses in the word problems as given below.

Example: 1

900 boys and 600 girls appeared in an examination of which 70% of the boys and 85% of the girls passed out in the examination. Find the total percentage of students who did not pass.

Solution:

Number of students who did not pass $= 30\%$ of boys + 15% of girls

$$= \frac{30}{100}\times 900 + \frac{15}{100}\times 600$$

$$= 270 + 90 = 360$$

$\therefore$ Percentage of students who did not pass $= \frac{360}{1500}\times 100 = 24\%$

Example: 2

If $x\%$ of 600 is 450 then, find the value of x.

Solution:

$$x\% \text{ of } 600 = 450$$

$$\frac{x}{100}\times 600 = 450$$

$$x = \frac{450}{6}$$

$$x = 75$$

Example: 3

When a number is decreased by 25% it becomes 120. Find the number.

Solution:

Let the number be x.

Given, $x - \frac{25}{100}x = 120$

$$\frac{100x - 25x}{100} = 120$$

$$\frac{75x}{100} = 120$$

$$\Rightarrow x = \frac{120\times 100}{75}$$

$$x = 160$$

Example: 4

If the price of Orid dhall after 20% increase is ₹96 per kg, find the original price of Orid dhall per kg.

Solution:

Let the original price of Orid dhall be ₹ x .

$$\text{New price after price of 20\% increase} = x + \frac{20}{100}x = \frac{120x}{100}$$

$$\text{Given that, } 96 = \frac{120x}{100}$$

$$\therefore x = \frac{96\times100}{200}$$

$$\therefore \text{ Original price of Orid dhall} = ₹80$$

DO YOU KNOW?

(i) If we start with a quantity A and increase that quantity by $x\%$, we will get the increased quantity as,

$$\mathrm{I} = \left(1+\frac{x}{100}\right)\mathrm{A}$$

(ii) If we start with a quantity A and decrease that quantity by $x\%$, we will get the decreased quantity as,

$$\mathrm{D} = \left(1-\frac{x}{100}\right)\mathrm{A}$$

We shall apply the above formulae for examples 3 and 4 and check the answers.

Aliter for Example: 3

We have,

$$\mathrm{D} = \left(1-\frac{x}{100}\right)A$$

$$\mathrm{D} = \left(1-\frac{25}{100}\right)A$$

$$\Rightarrow 120 = \frac{75}{100}\times A$$

$$A = 120\times\frac{100}{75}$$

$$A = 160$$

Aliter for Example: 4

We have,

$$\mathrm{I} = \left(1+\frac{x}{100}\right)A$$

$$\mathrm{I} = \left(1+\frac{20}{100}\right)A$$

$$\Rightarrow 96 = \frac{120}{100}\times A$$

$$A = 96\times\frac{100}{120}$$

$$A = ₹80$$

Example: 5

In a leadership election between two persons A and B, A wins by a margin of 192 votes. If A gets 58% of the total votes, find the total votes polled.

Solution:

Let the total votes polled be x.

Votes polled in favour in A = 58% of $x = \frac{58x}{100}$

Votes polled in favour of B = $(100 - 58)\%$ of $x = 42\%$ of $x = \frac{42x}{100}$

Given, Winning margin $\quad A - B = 192$

$$\text{That is,} \quad \frac{58x}{100} - \frac{42x}{100} = 192$$

$$\Rightarrow \frac{16x}{100} = 192$$

$$\Rightarrow x = 192 \times \frac{100}{16}$$

$$x = 1200 \text{ votes}$$

Example: 6

The income of a person is increased by 10% and then decreased by 10%. Find the change in his income.

Solution:

Let his income be ₹ x.

After 10% increase, income is ₹ $x + \frac{10}{100} \times x =$ ₹ $\frac{110}{100}$ (or) ₹ $\frac{11}{10}x$

Now, after 10% decrease, income is ₹ $\frac{11x}{10} - \frac{10}{100}\left(\frac{11x}{10}\right)$

$$\text{i.e.} \quad \frac{11x}{10} - \frac{11x}{100} = \frac{110x - 11x}{100} = ₹\,\frac{99x}{100}$$

$\therefore$ Net change in his income $= x - \frac{99x}{100} = \frac{x}{100}$

$$\therefore \text{ Percentage change } = \frac{\frac{x}{100}}{x} \times 100 = 1\%$$

That is, income is reduced by 1%.

(or)

Aliter

Let his income be ₹100

After 10% increase, income is $100 + 100 \times \frac{10}{100}$ = ₹110.

Now, after 10% decrease, income is $110 - 110 \times \frac{10}{100} = 110 - 11 =$ ₹99

$\therefore$ Net change in his income $= 100 - 99 = 1$

$\therefore$ Percentage change $= \frac{1}{100} \times 100 = 1\%$.

That is, income is reduced by 1%.

Note

Given any two numbers, if one of them is increased or decreased by $x\%$ and the other one is increased or decreased by $y\%$, then the product of the number will be increased or decreased by $\left(x + y + \frac{xy}{100}\right)\%$. Use 'negative' sign for decrease and assume 'decrease' if the sign is negative.

Use this note to check the answer for Example 6.

Example: 7

If the population in a town has increased from 20000 to 25000 in a year, find the percentage increase in population.

Solution:

Increase in population $= 25000 - 20000$

$= 5000$

$\therefore$ Percentage increase in population $= \frac{5000}{20000} \times 100$

$= 25\%$

Think

An increase from 200 to 600 is clearly a 200% increase. Isn' it? (check!). With a lot of pride, the traffic police commissioner of a city reported that the accidents had decreased by 200% in one year. He came up with this number stating that the accidents had gone down from 600 last year to 200 this year. Is the decrease from 600 to 200, the same 200% as above? Justify.

Example: 8

Akila scored 80% in an examination. If her score was 576 marks,find the maximum marks of the examination.

Solution:

Let the maximum marks be x.

Now, 80% of $x = 576$

$$\frac{80}{100} \times x = 576$$

$$\Rightarrow x = 576 \times \frac{100}{80}$$

$$x = 720 \text{ marks}$$

Therefore, the total marks in the examination =720.

Try these

1. What percentage of a day is 10 hours?
2. Divide ₹350 among P, Q and R such that P gets 50% of what Q gets and Q gets 50% of what R gets.

Exercise 1.1

1. Fill in the blanks:

(i) If 30% of x is 150, then x is __________.

(ii) 2 minutes is __________% to an hour.

(iii) If x % of x = 25, then x = __________.

(iv) In a school of 1400 students, there are 420 girls. The percentage of boys in the school is __________.

(v) 0.5252 is __________%.

2. Rewrite each underlined part using percentage language.

(i) <u>One half</u> of the cake is distributed to the children.

(ii) Aparna scored <u>7.5 points out of 10</u> in a competition.

(iii) The statue was made of <u>pure silver.</u>

(iv) <u>48 out of 50</u> students participated in sports.

(v) Only <u>2 persons out of 3</u> will be selected in the interview.

3. 48 is 32% of what number?.

4. A bank pays ₹240 as interest for 2 years for a sum of ₹3000 deposited as savings. Find the rate of interest given by the bank.

5. A Welfare Association has a sports club where 30% of the members play cricket, 28% play volleyball, 22% play badminton and the rest play indoor games. If 30 member play indoor games.

(i) How many members are there in the sports club?

(ii) How many play cricket, volleyball and badminton?

6. What is 25% of 30% of 400?

7. If the difference between 75% of a number and 60% of the same number is 82.5, then find 20% of the number.

8. If a car is sold for ₹200000 from its original price of ₹300000, find the percentage for decrease in the value of the car.

9. A number when increased by 18% gives 236. Find the number.

10. A number when decreased by 20% gives 80. Find the number.

11. A number is increased by 25% and then decreased by 20%. Find the change in that number.

12. If the numerator of a fraction is increased by 25% and the denominator is increased by 10%, it becomes $\frac{2}{5}$. Find the original fraction.

13. A fruit vendor bought some mangoes of which 10% were rotten. He sold $33\frac{1}{3}\%$ of the rest. Find the total number of mangoes bought by him initially, if he still has 240 mangoes with him.

14. A student gets 31% marks in an examination but fails by 12 marks. If the pass percentage is 35%, find the maximum marks of the examination.

15. The ratio of boys and girls in a class is 5:3. If 16% of boys and 8% of girls failed in an examination, then find the percentage of passed students.

Objective Type Questions

16. 12% of 250 litres is the same as ________of 150 litres.

    (a) 10% (b) 15% (c) 20% (d) 30%

17. If three candidates A, B and C is a 3 school election got 153,245 and 102 votes respectively, the percentage of votes for the winner is___________.

    (a) 48% (b) 49% (c) 50% (d) 45%

18. 15% of 25% of 10000 =___________.

    (a) 375 b) 400 c) 425 d) 475

19. When 60 is subtracted from 60% of a number to give 60, the number is

    (a) 60 (b) 100 (c) 150 (d) 200

20. If 48% of $48 = 64\%$ of x, then $x =$

    (a) 64 (b) 56 (c) 42 (d) 36

1.3 Profit and Loss, Discount, Overhead Expenses and GST

1.3.1 Profit and Loss:

Cost Price (C.P)

The amount for which an article is bought is called its ***Cost Price*** (C.P)

Selling Price (S.P)

The amount for which an article is sold is called its ***Selling Price*** *(S.P)*

Profit or Gain

When the *S.P* is more than the *C.P*, then there is a ***profit or gain***.

Therefore, *Gain/Profit =S.P – C.P*

Loss

When the *S.P* is less than the *C.P*, then there is a ***loss***.

Therefore, *Loss = C.P – S.P*

It is to be noted that the profit or loss is always calculated on the cost price.

Formulae:

(i) Gain or Profit % $=\left(\frac{Profit}{C.P}\times 100\right)\%$

(ii) Loss % $=\left(\frac{Loss}{C.P}\times 100\right)\%$

(iii) $\text{S.P} = \frac{(100+Profit\%)}{100}\times \text{C.P}$ (or) $\text{C.P} = \left(\frac{100}{100+Profit\%}\right)\text{S.P}$

(iv) $\text{S.P} = \frac{(100-Loss\%)}{100}\times \text{C.P}$ (or) $\text{C.P} = \frac{100}{(100-Loss\%)}\times \text{S.P}$

1.3.2 Discount:

In order to increase the sale and also to clear the old stock during the month of Aadi and festival seasons, shopkeepers offer a certain percentage of rebates on the marked price of the articles. This rebate is known as ***discount***.

Marked price

In big shops and departmental stores, we see that every product is tagged with a card with a price written on it. The price marked on it is called the ***marked price***.

Based on this marked price only, the shopkeeper offers a discount of certain percentage. The price payable by the customer after deduction of discount is called the ***selling price***.

That is, Selling Price = Marked Price – Discount

1.3.3 Overhead Expenses:

Shopkeepers, traders and retailers are involved in the buying and selling of goods. Sometimes, when articles like machinery, furniture, electronic items etc., are bought, some expenses on repairs, transportation and labour charges are incurred. These expenses are included in the cost price and are called as overhead expenses. Thus,

$\therefore$ Total Cost Price = Cost Price + Overhead Expenses

1.3.4 Goods and Services Tax (GST):

The goods and services tax (GST) is the only common tax in India levied on almost all the goods and the services meant for domestic consumption. The GST is remitted by the consumers and the traders and is one of the sources of income to the government. There are 3 types of GST namely Central GST, (CGST), State GST (SGST) and Integrated GST (IGST). For union territories, there is UTGST.

The GST is shared by the Central and State Governments equally. There are also many products like Eggs, Honey, Milk, Salt etc., which are exempted from GST. Products like petrol, diesel etc., do not come under GST and they are taxed separately. The GST council has fitted over 1300 goods and 500 services under four tax slabs. They are: 5%, 12%, 18% and 28%.

Example: 9

If the selling price of a LED TV is equal to $\frac{5}{4}$ of its cost price, then find the gain / profit percentage.

Solution:

Let the C.P of the LED TV be ₹x.

$$\therefore \text{S.P} = \frac{5}{4}x$$

$$\text{Profit} = \text{S.P} - \text{C.P} = \frac{5}{4}x - x = \frac{x}{4}$$

$$\therefore \text{Profit } \% = \left(\frac{Profit}{C.P} \times 100\right)\%$$

$$= \left(\frac{x/4}{x} \times 100\right)\%$$

$$= \left(\frac{1}{4} \times 100\right)\% = 25\%$$

Example: 10

By selling a bicycle for ₹4275, a shopkeeper loses 5%. For how much should he sell it to have a profit of 5%?

Solution:

S.P of the bicycle = ₹4275

Loss = 5%

$$\therefore \text{C.P} = \frac{100}{100 - loss\%} \times \text{S.P}$$

$$= \frac{100}{95} \times 4275 = ₹4500$$

Now,

$$C.P = ₹4500 \text{ and the desired profit } = 5\%$$

$$\therefore \text{ Desired S.P } = \frac{100 + gain\%}{100} \times \text{ C.P}$$

$$= \frac{100+5}{100} \times 4500$$

$$= 105 \times 45$$

$$= ₹4725$$

Hence, the desired selling price is ₹4725.

Example: 11

Ranjith bought a washing machine for ₹16150 and paid ₹1350 for its transportation. Then, he sold it for ₹19250. Find his gain or loss percentage.

Solution:

Total C.P of the washing machine

= C.P + Overhead Expenses

= 16150 + 1350 = ₹17500

S.P = ₹19250

Therefore, we find $S.P > C.P.$

$$\text{Gain } \% = \left(\frac{Gain}{C.P} \times 100\right)\% = \left(\frac{19250 - 17500}{17500} \times 100\right)\%$$

$$= \frac{1750}{17500} \times 100 = 10\%$$

Example: 12

A pre-owned car was bought for ₹240000. On repairs ₹15000 was spent, ₹8500 was paid for its insurance. Then, it was sold for ₹258230. What is the gain or loss percentage?

Solution:

Total C.P of the car

= C.P + Overhead Expenses

= 24000 + 15000 +8500

= ₹263500

S.P = ₹258230

As S.P < C.P , there is a loss.

$$\therefore \text{ Loss } \% = \left(\frac{Loss}{C.P} \times 100\right)\%$$

$$= \left(\frac{263500-258230}{263500} \times 100 \right) \%$$

$$= \left(\frac{5270}{263500} \times 100 \right) \% = 2\%$$

$\therefore$ Loss percentage = 2%

Example: 13

The cost price of 16 boxes of strawberries is equal to the selling price of 20 boxes of strawberries. Find the gain or loss percentage.

Solution:

Let the C.P of one strawberry box be ₹ x.

Then C.P of 20 strawberry boxes = $20x$ and

S.P of 20 strawberry boxes = C.P of 16 strawberry boxes = $16x$

Thus, S.P < C.P, hence there is a loss.

$$\text{Loss} = \text{C.P} - \text{S.P} = 20x - 16x = 4x$$

$$\therefore \text{Loss } \% = \left(\frac{Loss}{C.P} \times 100 \right) \%$$

$$= \left(\frac{4x}{20x} \times 100 \right) \%$$

$$= 20 \%$$

(or)

Aliter

S.P of 20 strawberry boxes = C.P of 20 strawberry boxes + Profit

$\Rightarrow$ C.P of 16 strawberry boxes = C.P of 20 strawberry boxes + Profit

$\Rightarrow$ Profit = $-$ C.P of 4 boxes

That is, Loss = C.P of 4 boxes

$$\therefore \text{Loss } \% = \left(\frac{Loss}{C.P} \times 100 \right) \%$$

$$= \left(\frac{4}{20} \times 100 \right) \%$$

$$= 20\%$$

Example: 14

The marked price of an LED tube light is ₹550 and the shopkeeper offers a discount of 8% on it. Find the selling price of the LED tube light.

Solution:

$$\text{Marked price} = ₹550 \text{ and Discount} = 8\%$$

$$\therefore \text{ Discount} = \frac{8}{100} \times 550 = ₹44$$

$$\therefore \text{ Selling Price} = \text{Marked price} - \text{Discount}$$

$$= ₹550 - 44$$

$$= ₹506$$

$\therefore$ The selling price of the tube light is ₹506

Example: 15

The price of a rain coat was slashed from ₹1060 to ₹901 by a shopkeeper in the winter season to boost the sales. Find the rate of discount given by him.

Solution:

Given,

$$\text{Discount} = \text{Marked Price} - \text{Selling Price}$$

$$= 1060 - 901$$

$$= ₹159$$

$$\therefore \text{ Discount} = \frac{159}{1060} \times 100\%$$

$$= 15\%$$

Think

A shopkeeper marks the price of a marker board 15% above the cost price and then allows a discount of 15% on the marked price. Does he gain or lose in the transaction?

Example: 16

Find the single discount which is equivalent to two successive discounts of 25% and 20% given on an article.

Solution:

Let the marked price of an article be ₹100.

First discount of 25% $= 100 \times \frac{25}{100} = ₹25.$

$\therefore$ Price after first discount $= 100 - 25 = ₹75.$

Second discount of 20% $= 75 \times \frac{20}{100} = ₹15.$

$\therefore$ Price after second discount $= 75 - 15 = ₹60.$

Net selling price = ₹60.

$\therefore$ Single discount equivalent to two given successive discounts = (100-60)% = 40%.

Note

- By selling x articles, if one gains the cost price of y articles, then the profit % $= \left(\frac{y}{x} \times 100\right)\%$.
- By selling x articles, if one gains the selling price of y articles, then the profit % $= \left(\frac{y}{x-y} \times 100\right)\%$.
- If the cost price of x articles = selling price of y articles, then profit% $= \left(\frac{x-y}{y} \times 100\right)\%$.
  If the answer is negative, then it is treated as loss.
  (Use this formula for Example 1.11 and check the answer)
- If there are 2 successive discounts of a % and b % respectively, then
  $\text{S.P} = \left(1-\frac{a}{100}\right)\left(1-\frac{b}{100}\right) \times \text{M.P.}$
- Single discount equivalent to 3 successive discounts of a %, b % and c % respectively
  $= \left\{1-\left(1-\frac{a}{100}\right)\left(1-\frac{b}{100}\right)\left(1-\frac{c}{100}\right)\right\} \times 100\%$.
  (Use this formula for Example: 6 and check the answer).

Try these

1. By selling 5 articles, a man gains the cost price of 1 article. Find his gain percentage.
2. By selling 8 articles, a shopkeeper gains the selling price of 3 articles. Find his gain percentage.
3. If the C.P of 20 articles is equal to the S.P of 15 articles, find the profit or loss percentage.

Example: 17

A woman bought some eggs at the rate of 4 eggs for ₹18 and sold them at the rate of 5 eggs for ₹24. She gained ₹90 in selling all the eggs. How many eggs did she buy?

Solution:

Let the number of eggs bought by her be x.

$$\text{Then, Cost Price} = ₹\frac{18}{4} \times x = ₹\frac{9x}{2}$$

$$\text{Selling Price} = ₹\frac{24}{5} \times x = ₹\frac{24x}{5}$$

$$\therefore \text{Gain} = \text{S.P} - \text{C.P} = ₹\frac{24x}{5} - \frac{9x}{2}$$

$$= \frac{48x - 45x}{10} = \frac{3x}{10}$$

Given, gain = ₹90

That is, $\frac{3x}{10}$ = ₹90

$$\therefore\ x = \frac{90 \times 10}{3} = 300$$

Example: 18

A water heater is sold by a trader for ₹10502 including GST at 18% and 11% profit. Find the M.P of the water heater, profit and GST.

Solution:

Let the marked price be ₹ x.

Now , $x + \frac{18x}{100} = 10502$

$$\frac{118x}{100} = 10502$$

$\therefore$ Marked price, x = ₹8900.

GST at 18% $= 8900 \times \frac{18}{100}$

= ₹10502 – ₹8900

= ₹1602

(or)

Let the C.P be ₹y.

$\therefore$ Profit of the water heater $= \frac{11y}{100}$

$$y + \frac{11y}{100} = 8900$$

$$\Rightarrow \frac{111y}{100} = 8900$$

$$y = 8900 \times \frac{100}{111}$$

$\therefore$ Cost price of the water heater = ₹8018.

Example: 19

A family went to a hotel and spent ₹350 for the food and paid 5% GST extra. Calculate the CGST and SGST.

Solution:

Cost of the food = ₹350

5 % GST is equally shared by Central and State Governments at 2.5% each

$\therefore$ CGST = SGST = $350 \times \frac{2.5}{100}$ = ₹8.75

Exercise 1.2

1. **Fill in the blanks:**

   (i) Loss or gain percentage is always calculated on the__________.

   (ii) A mobile phone is sold for ₹8400 at a gain of 20%. The cost price of the mobile phone is________.

   (iii) An article is sold for ₹555 at a loss of $7\frac{1}{2}\%$. The cost price of the article is ________.

   (iv) The marked price of a mixer grinder is ₹4500 is sold for ₹4140 after discount. The rate of discount is __________.

   (v) The total bill amount of a shirt costing ₹575 and a T-shirt costing ₹325 with GST of 5 % is_______.

2. If selling an article for ₹820 causes 10% loss on the selling price, find its cost price.

3. If the profit earned on selling an article for ₹810 is the same as loss on selling it for ₹530, then find the cost price of the article.

4. Some articles are bought at 2 for ₹15 and sold at 3 for ₹25. Find the gain percentage.

5. If the selling price of 10 rulers is the same as the cost price of 15 rulers, then find the gain percentage.

6. By selling a speaker for ₹768, a man loses 20%. In order to gain 20% how much should he sell the speaker?

7. A man sold two gas stoves for ₹8400 each. He sold one at a gain of 20% and the other at a loss of 20%. Find his gain or loss $\%$ in the whole transaction.

8. Find the unknowns x, y and z.

S.No	Name of the items	Marked Price	Selling Price	Discount
(i)	Book	₹225	x	8%
(ii)	LED TV	y	₹11970	5%
(iii)	Digital clock	₹750	₹615	z

9. Find the total bill amount for the data given below.

S.No	Name of the items	Marked Price	Discount	GST
(i)	School bag	₹500	5%	12%
(ii)	Stationery	₹250	4%	5%
(iii)	Cosmetics	₹1250	8%	18%
(iv)	Hair dryer	₹2000	10%	28%

10. A shopkeeper buys goods at $\frac{4}{5}$ of its marked price and sells them at $\frac{7}{5}$ of the marked price find his profit percentage.

11. A branded AC has a marked price of ₹37250. There are 2 options given for the customer.

(i) Selling Price is ₹37250 along with attractive gifts worth ₹3000 (or)

(ii) Discount of 8% but no free gifts.

Which offer is better?

12. If a mattress is marked for ₹7500 and is available at two successive discounts of 10% and 20%, find the amount to be paid by the customer.

Objective Type Questions

13. A fruit vendor sells fruits for ₹200 gaining ₹40. His gain percentage is

(a) 20% (b) 22% (c) 25% (d) $16\frac{2}{3}\%$

14. By selling a flower pot for ₹528, a woman gains 20%. At what price should she sell it to gain 25%?

(a) ₹500 (b) ₹550 (c) ₹553 (d) ₹573

15. A man buys an article for ₹150 and makes overhead expenses which are 12% of the cost price. At what price must he sell it to gain 5%?

(a) ₹180 (b) ₹168 (c) ₹176.40 (d) ₹85

16. The price of a hat is ₹210. What is the marked price of the hat if it is bought at 16% discount?

(a) ₹243 (b) ₹176 (c) ₹230 (d) ₹250

17. The single discount which is equivalent to two successive discount of 20% and 25% is

(a) 40% (b) 45% (c) 5% (d) 22.5%

1.4 Compound Interest

The most powerful force in this universe is_______. How would you finish this quote? The world renowned physicist Albert Einstein completed this quote with the word **Compound Interest.**

When money is borrowed or deposited on simple interest, then the interest is calculated evenly on the principal throughout the loan or deposit period.

In post offices, banks, insurance companies and other financial institutions, they also offer another type of interest calculation. Here, the interest accrued during the first time period (say, 6 months) is added to the original principal and the amount so obtained is taken as the principal for the second time period (say, next 6 months) and this keeps going on, up to the fixed time agreement between the lender and the borrower.

After a certain period, the difference between the amount and the money borrowed or deposited is called the compound interest which is abbreviated as C.I. Clearly, compound interest will be more than the simple interest just because the principal keeps on changing for every time period.

We call the time period after which the interest is added to the principal, as the 'conversion period'. For example, if the interest is compounded say, quarterly, there will be four conversion periods in a year each after 3 months. In such cases, the interest rate will be one fourth of the annual rate and the number of times that interest will be compounded is four times the number of years.

In case of simple interest, the principal remains the same for the whole duration where as in case of compound interest, the principal keeps on changing as per the conversion period. Simple interest and Compound interest remains the same for the first conversion period.

Illustration 1

To find the compound interest on ₹20000 for 4 years at 10% p.a compounded annually and compare it with the simple interest obtained for the same.

Calculating Compound Interest

Principal for the I year = ₹20000

Interest for the I year $\left(\dfrac{20000\times10\times1}{100}\right)$ = ₹2000

Amount at the end of I Year (P+I) = ₹22000

That is, Principal for the II year = ₹22000

Interest for the II year $\left(\dfrac{22000\times10\times1}{100}\right)$ = ₹2200

Amount at the end of II year = ₹24200

That is, Principal for the III year = ₹24200

Interest for the III year $\left(\dfrac{24200\times10\times1}{100}\right)$ = ₹2420

Amount at the end of III year = ₹26620

That is, Principal for the IV year = ₹26620

Interest for the IV year $\left(\dfrac{26620\times10\times1}{100}\right)$ = ₹2662

$\therefore$ Amount at the end of IV year = ₹29282

Calculating Simple Interest

Recall that,

Simple Interest, $I=\dfrac{PNR}{100}$

$=\dfrac{20000\times4\times10}{100}$

= ₹ 8000

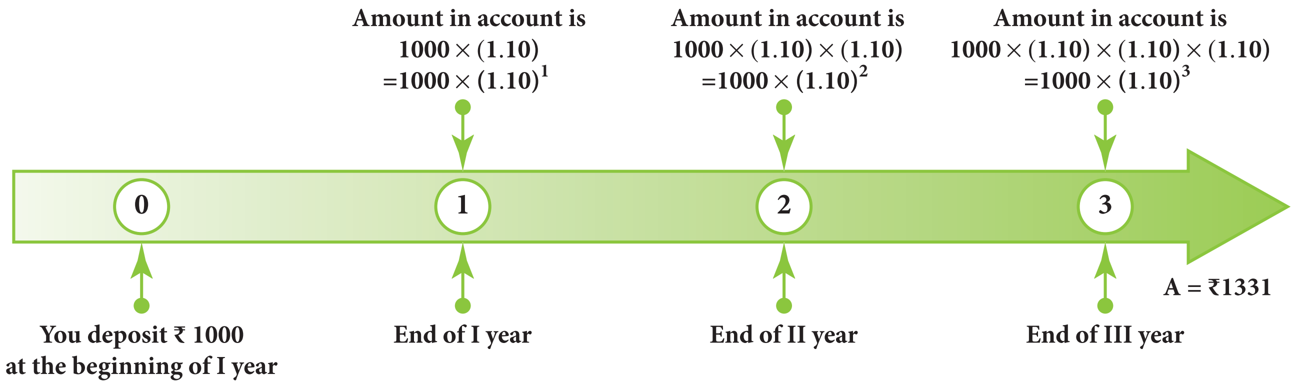
$\therefore$ Compound Interest for 4 years = A – P = 29282 – 20000

= ₹9282

What we observe from the calculation of C.I is a repeated multiplication by 1.1 as 20000, (×1.1) 22000, (×1.1) 24200, (×1.1) 26620, (×1.1)29282 for 4 years. We also note that Compound Interest (₹9282) grows faster and is clearly more than the simple interest (₹8000) obtained. When the time period is longer the above method is time consuming. So, we have a formula to find the amount and the compound interest easily.

Illustration 2

To calculate the amount and compound interest for ₹1000 for 3 years at 10% p.a compound annually.



Flow chart showing 10% interest being compounded annually

This leads to the pattern for Amount as $P\left(1+\frac{r}{100}\right)$ for the I year, $P\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)$ for the II year, $P\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)$ for the III year and so on. In general, $A = P\left(1+\frac{r}{100}\right)^n$ for the n^{th} year. Here $A = 1000\left(1+\frac{10}{100}\right)^3$ = ₹1331. So, C.I = ₹331.

The following formulae will be helpful in calculating the compound interest easily for the following periods.

(i) When the interest is compound annually, we have

$$A = P\left(1+\frac{r}{100}\right)^n$$

where A is the amount, P is the principal, r is the rate of interest per annum and n is the number of years and we shall get Compound Interest = Amount – Principal.

(ii) When the interest is compounded half yearly, we have

$$A = P\left(1+\frac{r}{200}\right)^{2n}$$

(iii) When the interest is compounded quarterly, we have

$$A = P\left(1+\frac{r}{400}\right)^{4n}$$

(iv) When the interest is compounded annually but rate of interest differs year by year, we have

$$A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)\ldots$$

where a, b and c are interest rates for I, II and III years respectively.

(v) When interest is compounded annually but time is in fraction say $a\frac{b}{c}$ years, we have

$$A = P\left(1+\frac{r}{100}\right)^{a}\left(1+\frac{\frac{b}{c}\times r}{100}\right)$$

(Use of calculators are permitted for lengthy calculations and also to verify answers).

DO YOU KNOW?

In 1626, Peter Minuit convinced the Wappinger Indians to sell him the Manhattan Island for 24 dollars. If the native Americans had put the 24 dollars into a bank account at 5% interest rate compounded monthly, by the year 2020 there would be well over 5.5 billion (550 crores) dollars in the account! This is the might of compound interest!

Example: 20

Find the C.I for the data given below:

(i) Principal = ₹4000, r = 5% p.a, n = 2 years, interest compounded annually.

(ii) Principal = ₹5000, r = 4% p.a, n = $1\frac{1}{2}$ years, interest compounded half-yearly.

(iii) Principal = ₹10000, r = 8% p.a, n = $2\frac{3}{4}$ years, interest compounded yearly.

(iv) Principal = ₹30000, r = 7% for I year, r = 8% for II year, compounded annually.

Solution:

(i) Amount, $A = P\left(1+\frac{r}{100}\right)^{n}$

$$= 4000\left(1+\frac{5}{100}\right)^{2}$$

$$= 4000\times\frac{21}{20}\times\frac{21}{20}$$

$$A = ₹4410$$

$\therefore$ C.I = A − P = 4410 − 4000 = ₹410

(ii) Amount, $A = P\left(1+\frac{r}{100}\right)^{2n} = 5000\left(1+\frac{4}{200}\right)^{2\times\frac{3}{2}} = 5000\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}$

$$= 51\times 10.2\times 10.2$$

$$= ₹5306.04$$

$\therefore$ C.I = A − P = ₹5306.04 − ₹5000

$$= ₹306.04$$

(iii) $\quad A = P\left(1+\frac{r}{100}\right)^{a}\left(1+\frac{\frac{b}{c}\times r}{100}\right) = 10000\left(1+\frac{8}{100}\right)^{2}\left(1+\frac{\frac{3}{4}\times 8}{100}\right)$

$$= 10000\left(\frac{27}{25}\right)^{2}\left(\frac{53}{50}\right)$$

$$= 10000 \times\frac{27}{25}\times\frac{27}{25}\times\frac{53}{50}$$

$$A = 12363.84$$

$$\therefore \text{C.I} = 12363.84 - 10{,}000$$

$$= ₹2363.84$$

(iv) $\quad A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)$

$$= 3000\left(1+\frac{7}{100}\right)\left(1+\frac{8}{100}\right)$$

$$= 30000 \times\frac{107}{100}\times\frac{108}{100}$$

$$= ₹34668$$

$\therefore$ C.I = A – I =34668-30000 = ₹4668.

1.4.1 Applications of Compound Interest formula:

The compound interest formula is used in the following situations.

(i) To find the increase or decrease in population.

(ii) To find the growth of cells when the rate of growth is given.

(iii) To find the depreciation in the values of machines, vehicles, utility appliances etc.,

Example: 21

The population of a town is increasing at the rate of 6% p.a. It was 238765 in the year 2018. Find the population in the year 2016 and 2020.

Solution:

Let the population in 2016 be 'P'.

Then, $\quad A = P\left(1+\frac{r}{100}\right)^{n}$

$$\Rightarrow 238765 = P\left(1+\frac{6}{100}\right)^{2} = P\left(\frac{53}{50}\right)^{2}$$

$$\Rightarrow P = 238765 \times\frac{50}{53}\times\frac{50}{53}$$

$$\therefore P = 212500$$

Let the population in 2020 be 'A'

Then, $A = P\left(1+\frac{r}{100}\right)^n$

$$\therefore\ A = 238765\left(1+\frac{6}{100}\right)^2$$

$$= 238765 \times \frac{53}{50} \times \frac{53}{50}$$

$$= 95.506 \times 53 \times 53$$

$$A = 268276$$

$\therefore$ The population in the year 2016 is 212500 and that in the year 2020 is 268276.

Example: 22

The value of a motor cycle 2 years ago was ₹70000. It depreciates at the rate of 4% p.a. Find its present value.

Solution:

$$\text{Depreciated value} = P\left(1-\frac{r}{100}\right)^n$$

$$= 70000\left(1-\frac{4}{100}\right)^2$$

$$= 70000 \times \frac{96}{100} \times \frac{96}{100}$$

$$= ₹64512$$

Example: 23

The bacteria in a culture grows by 5% in the first hour, decreases by 8% in the second hour and again increases by 10% in the third hour. Find the count of the bacteria at the end of 3 hours, if its initial count was 10000.

Solution:

Bacteria at the end of 3 hours

$$A = P\left(1+\frac{a}{100}\right)\left(1-\frac{b}{100}\right)\left(1+\frac{c}{100}\right) \text{ ('–' because 'decrease')}$$

$$= 10000\left(1+\frac{5}{100}\right)\left(1-\frac{8}{100}\right)\left(1+\frac{10}{100}\right)$$

$$= 10000 \times \frac{105}{100} \times \frac{92}{100} \times \frac{110}{100}$$

$$A = ₹10626$$

Try these

1. Find the principal which gives ₹420 as C.I at 20% p.a, compounded half yearly for one year.
2. The price of a laptop depreciates at 4% p.a. If its present price is ₹24000, find its price after 3 years.

1.4.2 Difference between S.I and C.I:

- There is no difference in S.I and C.I for the first conversion period.
- For 2 years, the difference in S.I and C.I is

$$C.I - S.I = P\left(\frac{r}{100}\right)^2$$

- For 3 years, the difference in S.I and C.I is

$$C.I - S.I = P\left(\frac{r}{100}\right)^2\left(3+\frac{r}{100}\right)$$

Example: 24

Find the difference in C.I and S.I for

(i) P = ₹5000, r = 4% p.a, n = 2 years.

(ii) P = ₹8000, r = 5% p.a, n = 3 years.

Solution:

(i) $C.I - S.I = P\left(\frac{r}{100}\right)^2 = 5000 \times \frac{4}{100} \times \frac{4}{100} = ₹8$

(ii) $C.I - S.I = P\left(\frac{r}{100}\right)^2\left(2+\frac{r}{100}\right)$

$$= 8000 \times \frac{5}{100} \times \frac{5}{100} \times \left(3+\frac{5}{100}\right)$$

$$= 20 \times \frac{61}{20} = ₹61$$

Activity

Mukunthan invests ₹30000 for 3 months in a bank which gives C.I at the rate of 12% p.a, compounded monthly. A private company offers his S.I at the rate of 12% p.a. What is the difference in the interests received by Mukunthan? Do by traditional method and verify your answer by calculator.

Exercise 1.3

1. **Fill in the blanks:**

   (i) The compound interest on ₹5000 at 12% p.a for 2 years compounded annually is ____________.

   (ii) The compound interest on ₹8000 at 10% p.a for 1 year, compounded half yearly is ____________.

   (iii) The annual rate of growth in population of a town is 10%. If its present population is 26620, the population 3 years ago was__________.

   (iv) The amount if the compound interest is calculated quarterly, is found using the formula ___________.

   (v) The difference between the S.I and C.I for 2 years for a principal of ₹5000 at the rate of interest 8% p.a is ____________.

2. **Say True or False.**

   (i) Depreciation value is calculated by the formula $P\left(1-\frac{r}{100}\right)^{n}$.

   (ii) If the present population of a city is P and it increases at the rate of $r\%$ p.a, then the population n years ago would be $P\left(1+\frac{r}{100}\right)^{n}$.

   (iii) The present value of a machine is ₹16800. It depreciates at 25% p.a. Its worth after 2 years is ₹9450.

   (iv) The time taken for ₹1000 to become ₹1331 at 20% p.a compounded annually is 3 years.

   (v) The compound interest on ₹16000 for 9 months at 20% p.a, compounded quarterly is ₹2522.

3. Find the compound interest on ₹3200 at 2.5% p.a for 2 years, compounded annually.
4. Find the compound interest for $2\frac{1}{2}$ years on ₹4000 at 10% p.a if the interest is compounded yearly.
5. Magesh invested ₹5000 at 12% p.a for one year. If the interest is compounded half yearly, find the amount he gets at the end of the year.
6. At what time will a sum of ₹3000 will amount to ₹3993 at 10% p.a compounded annually?
7. A principal becomes ₹2028 in 2 years at 4% p.a compound interest. Find the Principal.
8. At what rate percentage p.a will ₹5625 amount to ₹6084 in 2 years at compound interest?

9. In how many years will ₹3375 become ₹4096 at $13\frac{1}{3}$ % p.a where interest is compounded half-yearly?

10. Find the C.I on ₹15000 for 3 years if the rates of interest are 15%, 20% and 25% for I, II and III years respectively.

11. The present height of a tree is 847 cm. Find its height two years ago, if it increases at 10 % p.a.

12. Find the difference between C.I and S.I on ₹5000 for 1 year at 2% p.a, if the interest is compounded half yearly.

13. What is the difference in simple interest and compound interest on ₹15000 for 2 years at 6% p.a compounded annually.

14. Find the rate of interest if the difference between C.I and S.I on ₹8000 compounded annually for 2 years is ₹20.

15. Find the principal if the difference between C.I and S.I on it at 15% p.a for 3 years is ₹1134.

DO YOU KNOW?

- If a sum of money doubles in ***n*** years, then it will become ***m*** times in **(*m*-1)** × ***n*** years, if simple interest is applied.
- If a sum of money becomes ***m*** times in ***n*** years, then it will become m^a times in ***an*** years if compound interest is applied.
- Difference between the compound interest and the simple interest on a sum for 4 years at $r\%$ p.a is $Pi^2\left(6+4i+i^2\right)$ where i = $\frac{r}{100}$.

Objective Type Questions

16. The number of conversion periods, if the interest on a principal is compounded every two months is____________.

(a) 2 (b) 4 (c) 6 (d) 12

17. The time taken for ₹4400 to become ₹4851 at 10%, compounded half yearly is ________.

(a) 6 months (b) 1 year (c) $1\frac{1}{2}$ years (d) 2 years

18. The cost of a machine is ₹18000 and it depreciates at $16\frac{2}{3}$% annually. Its value after 2 years will be____________.

(a) ₹12000 (b) ₹12500 (c) ₹15000 (d) ₹16500

19. The sum which amounts to ₹2662 at 10% p.a in 3 years compounded yearly is________.

(a) ₹2000 (b) ₹1800 (c) ₹1500 (d) ₹2500

20. The difference between simple and compound interest on a certain sum of money for 2 years at 2% p.a is ₹1. The sum of money is ___________ .

(a) ₹2000 (b) ₹1500 (c) ₹3000 (d) ₹2500

Note

If P is $x\%$ or $\frac{x}{100}$ more efficient than Q, then Q is $\left(\frac{100x}{100+x}\right)\%$ less efficient than P?

If P is $x\%$ or $\frac{x}{100}$ less efficient than Q, then Q is $\left(\frac{100x}{100-x}\right)\%$ more efficient than P?

S6A5L1

Exercise 1.4

Miscellaneous Practice Problems

1. Nanda's marks in 3 Math tests T1, T2 and T3 were 38 out of 40, 27 out of 30 and 48 out of 50. In which test did he do well? Find his overall percentage in all the 3 tests.

2. Sultana bought the following things from a general store. Calculate the total bill amount to be paid by her.

(i) Medicines costing ₹800 with GST at 5%

(ii) Cosmetics costing ₹650 with GST at 12%

(iii) Cereals costing ₹900 with GST at 0%

(iv) Sunglass costing ₹1750 with GST at18%

(v) Air Conditioner costing ₹28500 with GST at 28%

3. P's income is 25% more than that of Q. By what percentage is Q's income less than P's?

4. Gopi sold a laptop at 12% gain. If it had been sold for ₹1200 more, the gain would have been 20%. Find the cost price of the laptop.

5. Vaidegi sold two sarees for ₹2200 each. On one she gains 10% and on the other she loses 12%. Calculate her gain or loss percentage in the sales.

6. A sum of money becomes ₹18000 in 2 years and ₹40500 in 4 years on compound interest. Find the sum.

7. Find the difference in the compound interest on ₹62500 for $1\frac{1}{2}$ years at 8% p.a compounded annually and when compounded half-yearly.

Challenging Problems

8. If the first number is 20% less than the second number and the second number is 25% more than 100, then find the first number.

9. A shopkeeper gives two successive discounts on an article whose marked price is ₹180 and selling price is ₹108. Find the first discount percentage if the second discount is 25%.

10. A man bought an article on 30% discount and sold it at 40% more than the marked price. Find the profit made by him.

11. Find the rate of compound interest at which a principal becomes 1.69 times itself in 2 years.

12. The simple interest on a certain principal for 3 years at 10% p.a is ₹300. Find the compound interest accrued in 3 years.

Summary

- Per Cent means per hundred or out of a hundred. It is denoted by the symbol %. $x\%$ denotes the fraction $\frac{x}{100}$.
- Percentages are useful in comparing quantities easily.
- The amount for which an article is bought is called its *Cost Price (C.P).*
- The amount for which an article is sold is called its *Selling Price (S.P).*
- When the S.P is more than the C.P, then there is a gain or profit. *Gain/Profit =S.P – C.P.*
- When the S.P is less than the C.P, then there is a loss. *Loss = C.P – S.P.*
- The profit or loss is always calculated on the cost price.
- **Marked price:** In big shops and departmental stores, we see that every product is tagged with a card with a price written on it. The price marked on it is called the *marked price.*
- The rebate which is given for an article on buying is known as *discount*.

❖ Selling price = Marked price – discount.

❖ **Formulae**

(i) Gain or Profit $\% = \left(\frac{Profit}{C.P} \times 100\right)\%$

(ii) Loss $\% = \left(\frac{Loss}{C.P} \times 100\right)\%$

(iii) $\text{S.P} = \frac{(100+Profit\%)}{100} \times C.P$ (or) $\text{C.P} = \left(\frac{100}{100+Profit\%}\right) \times S.P$

(iv) $\text{S.P} = \frac{(100-Loss\%)}{100} \times \text{C.P}$ (or) $\text{C.P} = \frac{100}{(100-Loss\%)} \times S.P$

❖ **Overhead Expenses:** Some expenses like repairs, transportation and labour charges incurred are included in the cost price and are called as overhead expenses.

❖ **Goods and Services Tax (GST):** The goods and services tax (GST) is the only common tax in India levied on almost all the goods and the services meant for domestic consumption.

❖ When the interest is compound annually, $A = P\left(1+\frac{r}{100}\right)^n$.

❖ When the interest is compounded half yearly, $A = P\left(1+\frac{r}{200}\right)^{2n}$.

❖ When the interest is compounded quarterly, $A = P\left(1+\frac{r}{400}\right)^{4n}$.

❖ When the interest is compounded annually but rate of interest differs year by year, $A=P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)$... where a, b and c are interest rates for I, II and III years respectively.

❖ When interest is compounded annually but time is in fraction say $a\frac{b}{c}$ years, $A=P\left(1+\frac{r}{100}\right)^a\left(1+\frac{\frac{b}{c}\times r}{100}\right)$.

❖ C.I = Amount – Principal (A–P).

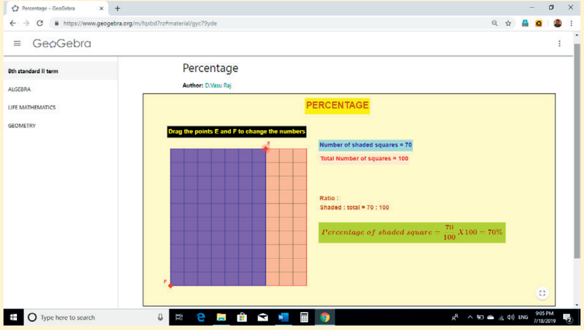
❖ Simple Interest and Compound Interest remains the same for the first conversion period.

❖ For 2 years, the difference in S.I and C.I is $\text{C.I} - \text{S.I} = P\left(\frac{r}{100}\right)^2$.

❖ For 3 years, the difference in S.I and C.I is $\text{C.I} - \text{S.I} = P\left(\frac{r}{100}\right)^2\left(3+\frac{r}{100}\right)$.

ICT CORNER

Expected Outcome

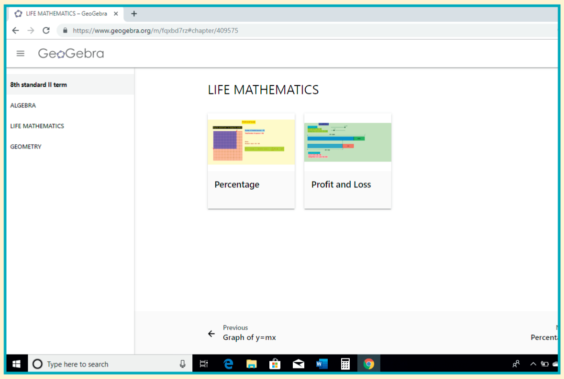


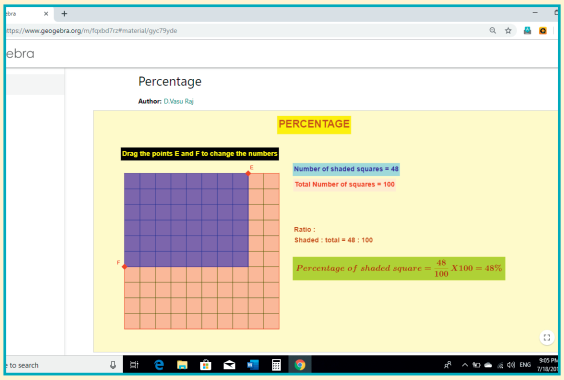
Step – 1

Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named "LIFE MATHEMATICS" will open. Click on the worksheet named "Percentage".

Step - 2

In the given worksheet you can drag the red points E and F to change the Blue rectangle. Find the ratio of Blue with the whole by counting the squares and check the ratio and percentage.





Step – 1 **Step – 2**

Go through the remaining worksheets given for this chapter

Browse in the link

Life Mathematics:

https://www.geogebra.org/m/fqxbd7rz#chapter/409575 or Scan the QR Code.

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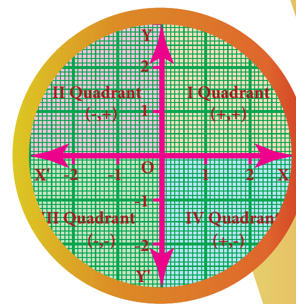
2 ALGEBRA



Learning Objectives

S8D7Z7

- To solve word problems that involve linear equations.
- To know how to plot the points in the graph.
- To draw graphs of simple linear functions

2.1 Introduction

We shall recall some earlier ideas in algebra.

What is the formula to find the perimeter of a rectangle? If we denote the length by l and breadth by b, the perimeter P is given as $P = 2(l + b)$. In this formula, 2 is a fixed number whereas the literal numbers P, l and b are not fixed because they depend upon the size of the rectangle and hence P, l and b are **variables.** For rectangles of different sizes, their values go on changing. 2 is a **constant** (which does not change whatever may be the size of the rectangle).

An algebraic **expression** is a mathematical phrase having one or more algebraic terms including variables, constants and operating symbols (such as plus and minus signs).

Example: $4x^2 + 5x + 7xy + 100$ is an algebraic expression; note that the first term $4x^2$ consists of constant 4 and variable x^2. What is the constant in the term $7xy$? Is there a variable in the last term of the expression?

The 'number parts' of the terms with variables are **coefficients**. In $4x^2 + 5x + 7xy + 100$, the coefficient of the first term is 4. What is the coefficient of the second term? It is 5. The coefficient of the xy term is 7.

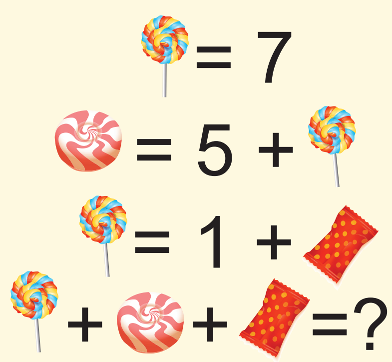
2.2 Forming algebraic expressions

We now to translate a few statements into an algebraic language and recall how to frame expressions. Here are some examples:

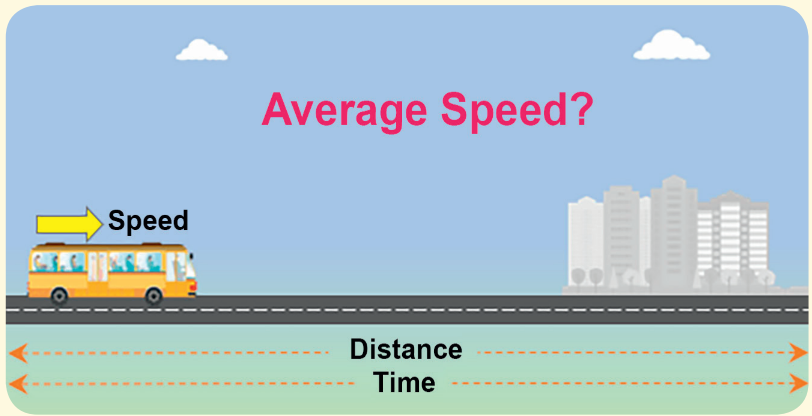
Statement	Expression	Comment
The sum of 8 and 7	$8 + 7$	When simplified, we get a single number. This is a numerical expression
The sum of x and 7	$x + 7$	We get an algebraic expression $x + 7$, since x is a variable.
16 divided by y	$\frac{16}{y}$	Here, y is a variable.

One more than thrice a number p	$3p+1$	Here p is a variable; 3 is the coefficient of p.
The product of a number and the same number less 5	$x(x-3)$	Note that here x stands for the same number throughout in the expression. (We use brackets to indicate multiplication).

Mathematics Alive—Algebra in Real Life



Linear equation are used to find unit price and total



Linear equation are used for speed, distance, time and average speed

2.3 Equations

An **equation** is a statement that asserts the equality of two expressions; the expressions are written one on each side of an "equal to" sign.

For example: $2x + 7 = 17$ is an equation (where x is a variable). $2x + 7$ forms the Left Hand Side (LHS) of the equation and 17 is its Right Hand Side (RHS).

2.3.1 Linear equations in one or more variables:

An equation is formed when a statement is put in the form of mathematical terms. Here are some Examples:

(i) A number is added to 5 to get 25

This statement can be written as $x+5=25$.

This equation $x+5=25$ is formed by one variable (x) whose highest power is 1. So it is called a linear equation in one variable.

Therefore, an equation containing only one variable with its highest power as one is called a linear equation in one variable.

Examples: $5x-2=8,\quad 3y+24=0$

This linear equation in one variable is also known as **simple equation**.

(ii) **Sum of two numbers is 45**

This statement can be written as $x + y = 45$.

This equation $x + y = 45$ is formed by two variables x and y whose highest power is 1. Hence, we call it as a linear equation in two variables.

Now, in this class we shall learn to solve linear equations in one variable only. You will learn to solve other type of equations in higher classes.

Note

The equations so formed with more than power 1 of its variables, (2,3....etc.) are called as quadratic, cubic equations and so on.

Examples: (i) $x^2 + 4x + 7 = 0$ is a quadratic equation.

(ii) $5x^3 - x^2 + 3x = 10$ is a cubic equation.

Try these

Identify which among the following are linear equations.

(i) $2 + x = 19$ (ii) $7x^2 - 5 = 3$ (iii) $4p^3 = 12$ (iv) 6m+2 (v) n=10

(vi) 7k – 12= 0 (vii) $\frac{6x}{8} + y = 1$ (viii) $5 + y = 3x$ (ix) 10p+2q=3 (x) $x^2 - 2x - 4$

Convert the following statements into linear equations:

Example:1

7 is added to a given number to give 19.

Solution:

Let the number be n.

When 7 is added to this number we get $n + 7$.

This result is to give 19.

Therefore, the equation is $n + 7 = 19$.

Think

(i) Is $t(t-5)=10$ a linear equation? Why?

(ii) Is $x^2 = 2x$, a linear equation? Why?

Example :2

The sum of 4 times a number and 18 is 28.

Solution:

Let the number be x.

4 times the number is $4x$.

Adding 18 now, we get $18 + 4x$.

The result now should be 28.

Thus, the equation has to be $18 + 4x = 28$.

Try these

Convert the following statements into linear equations:

1. On subtracting 8 from the product of 5 and a number, I get 32.
2. The sum of three consecutive integers is 78.
3. Peter had a Two hundred rupee note. After buying 7 copies of a book he was left with ₹60.
4. The base angles of an isosceles triangle are equal and the vertex angle measures 80°.
5. In a triangle ABC, ∠A is 10° more than ∠B. Also ∠C is three times ∠A. Express the equation in terms of angle B.

2.3.2 Solution of a linear equation:

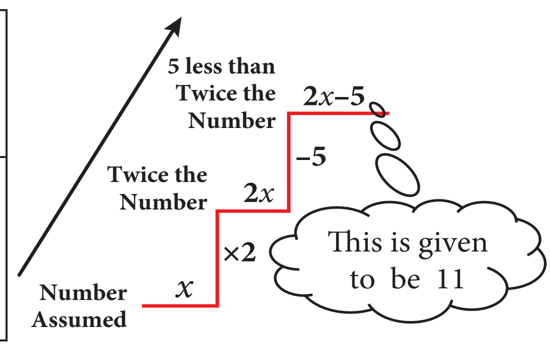
The value which replaces a variable in an equation so as to make the two sides of the equation equal is called a solution or root of the equation.

Example : $2x = 10$

We find that the equation is "satisfied" with the value $x = 5$. That is, if we put $x = 5$, in the equation, the value of the LHS will be equal to the RHS. Thus $x = 5$ is **a solution** of the equation. Note that no other value for x satisfies the equation. Thus one can say $x = 5$ is "**the**" solution of the equation.

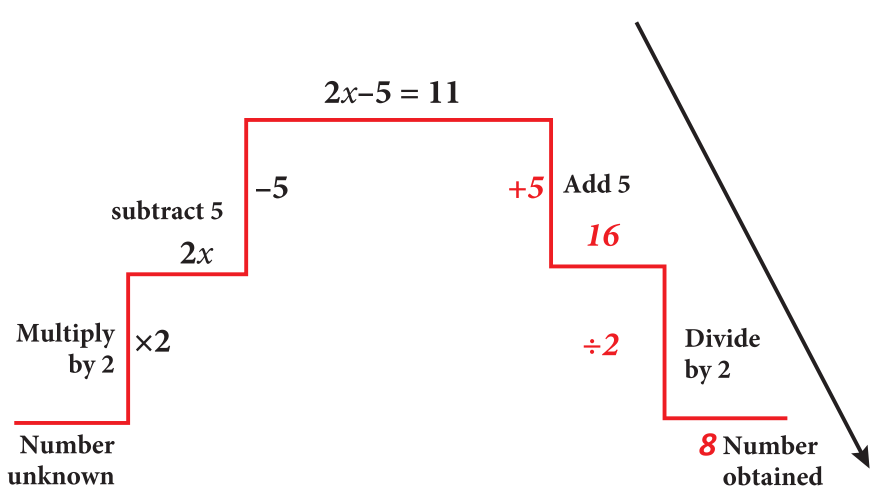
(i) The DO-UNDO Method:

Statement (given)	You  Think
5 less than twice a number is 11.	The number needed is unknown. Let it be x. Twice the number gives $2x$. 5 less is $2x - 5$. This result is given to be 11.



This formation of equation can be visualized as follows:

From the number x, we reached $2x - 5$ by performing operations like subtraction, multiplication etc. So when $2x - 5 = 11$ is given, to get back to the value of x, we have to *'undo'* all that we did! Thus, we *'do'* to form the equation and *'undo'* to get the solution.



Example: 3

Solve the equation: $x - 7 = 6$

Solution:

Given,

$$x - 7 = 6$$
$$x - 7 + 7 = 6 + 7$$
$$x = 13$$

Think

Can you get more than one solution for a linear equation?

(ii) Transposition method

The shifting of a number from one side of an equation to other is called transposition.

For the above example, doing addition of 7 on both sides is the same as changing the number –7 on the left hand side to its additive inverse +7 and add it on the right hand side.

$$x - 7 = 6$$
$$x = 6 + 7$$
$$x = 13$$

Example: 4

Solve the equation: $3x = 51$

Solution:

$$3x = 51 \quad \text{(Given)}$$
$$3 \times x = 51$$
$$\frac{3 \times x}{3} = \frac{51}{3} \quad (\div\, 3 \text{ on both sides})$$
$$x = 17$$

likewise, doing division by 3 on both sides is the same as changing the number 3 on the LHS to its reciprocal $\frac{1}{3}$ and multiply it on the RHS and vice-versa.

(i) $6x = 12$

$$x = \frac{12}{6}$$
$$x = 2$$

(ii) $\frac{y}{7} = 10$

$$y = 10 \times 7$$
$$y = 70$$

Example: 5

Solve $2x + 5 = 9$

Solution:

$$2x + 5 = 9$$
$$2x = 9 - 5$$
$$2x = 4$$
$$x = \frac{4}{2}$$
$$x = 2$$

Note

While rearranging the given linear equation, group the like terms on one side of the equality sign, and then do the basic arithmetic operations according to the signs that occur in the expression.

Example: 6

Solve $\frac{4y}{3} - 7 = \frac{2}{5}y$

Solution:

(Rearranging the like terms)

$$\frac{4y}{3} - \frac{2y}{5} = 7$$

$$\frac{20y - 6y}{15} = 7$$

$$14y = 7 \times 15$$

$$y = \frac{7 \times 15}{14}$$

$$y = \frac{15}{2}$$

Think

1. "An equation is multiplied or divided by a non zero number on either side." Will there be any change in the solution?
2. "An equation is multiplied or divided by two different numbers on either side". What will happen to the equation?

Exercise 2.1

1. Fill in the blanks:

Z6P5C3

(i) The value of x in the equation $x + 5 = 12$ is ————.

(ii) The value of y in the equation $y - 9 = (-5) + 7$ is ————.

(iii) The value of m in the equation $8m = 56$ is————.

(iv) The value of p in the equation $\frac{2p}{3} = 10$ is ————.

(v) The linear equation in one variable has ————solution.

2. Say True or False.

(i) The shifting of a number from one side of an equation to other is called transposition.

(ii) Linear equation in one variable has only one variable with power 2.

3. Match the following :

(a) $\frac{x}{2} = 10$	(i) $x = 4$
(b) $20 = 6x - 4$	(ii) $x = 1$
(c) $2x - 5 = 3 - x$	(iii) $x = 20$
(d) $7x - 4 - 8x = 20$	(iv) $x = \frac{8}{3}$
(e) $\frac{4}{11} - x = \frac{-7}{11}$	(v) $x = -24$

(A) (i),(ii), (iv) ,(iii),(v) (B) (iii), (iv) , (i) ,(ii), (v)

(C) (iii),(i) ,(iv), (v), (ii) (D) (iii) , (i) , (v) ,(iv) ,(ii)

4. Find x: (i) $\frac{2x}{3}-4=\frac{10}{3}$ (ii) $y+\frac{1}{6}-3y=\frac{2}{3}$ (iii) $\frac{1}{3}-\frac{1x}{3}=\frac{7x}{12}+\frac{5}{4}$

5. Find x: (i) $-3(4x+9)=21$ (ii) $20-2(5-p)=8$ (iii) $(7x-5)-4(2+5x)=10(2-x)$

6. Find x and m: (i) $\frac{3x-2}{4}-\frac{(x-3)}{5}=-1$ (ii) $\frac{m+9}{3m+15}=\frac{5}{3}$

2.4 Word problems that involve linear equations

The challenging part of solving word problems is translating the statements into equations. Collect as many such problems and attempt to solve them.

Example: 7

The sum of two numbers is 36 and one number exceeds another by 8. Find the numbers.

Solution:

Let the smaller number be x and the greater number be $x+8$

Given: the sum of two numbers = 36

$$x+(x+8) = 36$$
$$2x+8 = 36$$
$$2x = 36-8$$
$$2x = 28$$
$$x = \frac{28}{2}$$
$$x = 14$$

The smaller number, $x=14$

The greater number, $x+8=14+8=22$

Example: 8

A bus is carrying 56 passengers with some people having ₹8 tickets and the remaining having ₹10 tickets. If the total money received from these passengers is ₹500, find the number of passengers with each type of tickets.

Solution:

Let the number of passengers having ₹8 tickets be y. Then, the number of passengers with ₹10 tickets is (56–y).

Total money received from the passengers = ₹500

That is, $y\times ₹8+(56-y)\times ₹10 = 500$

$$8y+560-10y = 500$$
$$8y-10y = 500-560$$
$$-2y = -60$$
$$y = \frac{60}{2}$$
$$y = 30$$

Hence, the number of passengers having,

(i) ₹8 tickets =30

(ii) ₹10 tickets =56−30 =26

Example:9

The length of a rectangular field exceeds its breadth by 9 metres. If the perimeter of the field is 154*m*, find the length and breadth of the field.

Solution:

Let the breadth of the field be 'x' metres; then its length (x+9) metres.

Perimeter of the P = 2(length + breadth) = $2(x + 9 + x) = 2(2x + 9)$

Given that, $2(2x + 9) = 154$.

$$\begin{aligned} 4x + 18 &= 154 \\ 4x &= 154 - 18 \\ 4x &= 136 \\ x &= 34 \end{aligned}$$

Thus, Breadth of the rectangular field = 34*m*

Length of the rectangular field $=x+9 = 34+9 = 43m$

Example: 10

There is a wooden piece of length 2*m*. A carpenter wants to cut it into two pieces such that the first piece is 40 *cm* smaller than twice the other piece. What is the length of the smaller piece ?

Solution:

Let us assume that the length of the first piece is x *cm*.

Then the length of the second piece is (200*cm* – x *cm*) i.e., $(200 - x)$ *cm*.

According to the given statement (change *m* to *cm*),

First piece = 40 less than twice the second piece.

$$\begin{aligned} x &= 2\times(200 - x) - 40 \\ x &= 400 - 2x - 40 \\ x + 2x &= 360 \\ 3x &= 360 \\ x &= \frac{360}{3} \\ x &= 120 \end{aligned}$$

Think

Suppose we take the second piece to be x and the first piece to be $(200 - x)$, how will the steps vary? Will the answer be different?

Thus the length of the first piece is 120*cm* and the length of second piece is 200*cm* − 120*cm* = 80*cm*, which happens to be the smaller.

Example: 11

A mother is five times as old as her daughter. After 2 years, the mother will be four times as old as her daughter. What are their present ages?

Solution:

Age / Person	Now	After 2 years
Daughter	x	$x+2$
Mother	$5x$	$5x+2$

Given condition: After two years, Mother's age = 4 times of Daughter's age

$$5x+2 = 4(x+2)$$
$$5x+2 = 4x+8$$
$$5x - 4x = 8-2$$
$$x = 6$$

Hence daughter's present age = 6 years;

and mother's present age = $5x = 5 \times 6 = 30$ years

Example: 12

The denominator of a fraction is 3 more than its numerator. If 2 is added to the numerator and 9 is added to the denominator, the fraction becomes $\frac{5}{6}$. Find the original fraction.

Solution:

Let the original fraction be $\frac{x}{y}$.

Given that $y = x + 3$. (Denominator = Numerator + 3).

Therefore, the fraction can be written as $\frac{x}{x+3}$. As per the given condition, $\frac{x+2}{(x+3)+9} = \frac{5}{6}$

By cross multiplication, $6(x+2) = 5(x+3+9)$

$$6x+12 = 5(x+12)$$
$$6x+12 = 5x+60$$
$$6x - 5x = 60 - 12$$
$$x = 60 - 12$$
$$x = 48.$$

Therefore, the original fraction is $\frac{x}{x+3} = \frac{48}{48+3} = \frac{48}{51}$.

Example: 13

The sum of the digits of a two-digit number is 8. If 18 is added to the value of the number, its digits get reversed. Find the number.

Solution:

Let the two digit number be xy (i.e., ten's digit is x, ones digit is y)

Its value can be expressed as $10x + y$.

Given, $x + y = 8$ which gives $y = 8 - x$

Therefore its value is $10x + y$

$= 10x + 8 - x$

$= 9x + 8.$

The new number is yx with value is $10y + x$

$= 10(8 - x) + x$

$= 80 - 9x$

Given, when 18 is added to the given number (xy) gives new number (yx)

$(9x + 8) + 18 = 80 - 9x$

This simplifies to $9x + 9x = 80 - 8 - 18$

$18x = 54$

$x = 3 \Rightarrow y = 8 - 3 = 5$

The two digit number is $xy = 35$

Example: 14

From home, Rajan rides on his motorbike at 35 km/hr and reaches his office 5 minutes late. If he had ridden at 50 km/hr, he would have reached his office 4 minutes earlier. How far is his office from his home?

Solution:

Speed 1 = 35 km/hr
Speed 2 = 50 km/hr

Let the distance be 'x' km. (Recall that time $= \frac{\text{Distance}}{\text{Speed}}$)

Time taken to cover 'x' km at 35 km/hr: $T_1 = \frac{x}{35}$ hr

Time taken to cover 'x' km at 50 km/hr: $T_2 = \frac{x}{50}$ hr

According to the problem, the difference between two timings

$= 4-(-5)$

$= 4+5 = 9$ minutes

$= \frac{9}{60}$ hour (changing minutes to hour)

Given, $T_1 - T_2 = \frac{9}{60}$

$$\frac{x}{35} - \frac{x}{50} = \frac{9}{60}$$

$$\frac{10x - 7x}{350} = \frac{9}{60}$$

$$\frac{3x}{350} = \frac{9}{60}$$

$$x = \frac{9}{60} \times \frac{350}{3}$$

The distance to his office $x = 17\frac{1}{2}$ km.

Exercise 2.2

1. **Fill in the blanks:**
   (i) The solution of the equation ax+b=0 is________.
   (ii) If a and b are positive integers then the solution of the equation ax=b has to be always________.
   (iii) One-sixth of a number when subtracted from the number itself gives 25. The number is________.
   (iv) If the angles of a triangle are in the ratio 2:3:4 then the difference between the greatest and the smallest angle is ________.
   (v) In an equation a + b = 23. The value of a is 14 then the value of b is________.

2. **Say True or False**
   (i) "Sum of a number and two times that number is 48" can be written as y+2y=48
   (ii) 5(3x+2) = 3(5x−7) is a linear equation in one variable.
   (iii) $x = 25$ is the solution of one third of a number is less than 10 the original number.

3. One number is seven times another. If their difference is 18, find the numbers.
4. The sum of three consecutive odd numbers is 75. Which is the largest among them?
5. The length of a rectangle is $\frac{1}{3}$ of its breadth. If its perimeter is 64m, then find the length and breadth of the rectangle.
6. A total of 90 currency notes, consisting only of ₹5 and ₹10 denominations, amount to ₹500. Find the number of notes in each denomination.
7. At present, Thenmozhi's age is 5 years more than that of Murali's age. Five years ago, the ratio of Thenmozhi's age to Murali's age was 3:2. Find their present ages.
8. A number consists of two digits whose sum is 9. If 27 is subtracted from the original number, its digits are interchanged. Find the original number.
9. The denominator of a fraction exceeds its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, we get $\frac{3}{2}$. Find the original fraction.
10. If a train runs at 60 km/hr it reaches its destination late by 15 minutes. But, if it runs at 85 kmph it is late by only 4 minutes . Find the distance to be covered by the train.

Objective Type Questions

11) Sum of a number and its half is 30 then the number is_______.
    (a) 15 (b) 20 (c) 25 (d) 40

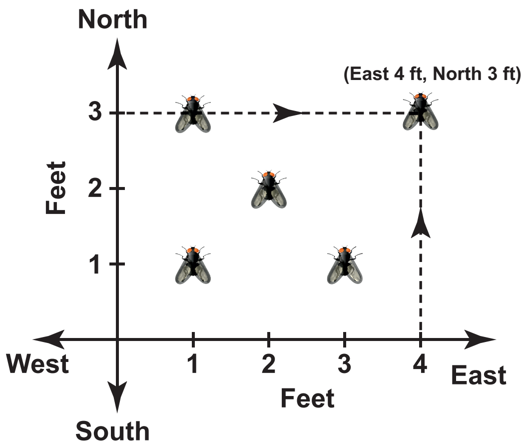
12) The exterior angle of a triangle is 120° and one of its interior opposite angle 58°, then the other opposite interior angle is_________.
    (a) 62° (b) 72° (c) 78° (d) 68°

13) What sum of money will earn ₹500 as simple interest in 1 year at 5% per annum?
    (a) 50000 b) 30000 (c) 10000 (d) 5000

14) The product of LCM and HCF of two numbers is 24. If one of the number is 6, then the other number is _________.
    (a) 6 (b) 2 (c) 4 (d) 8

2.5 Graph

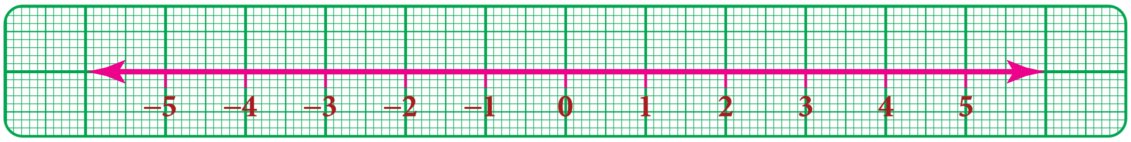
2.5.1 Introduction:

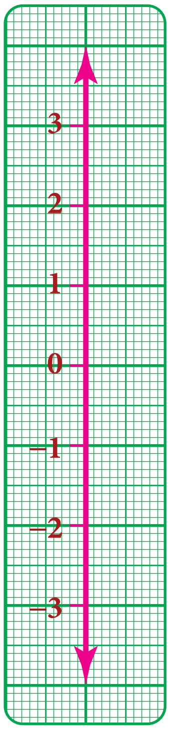


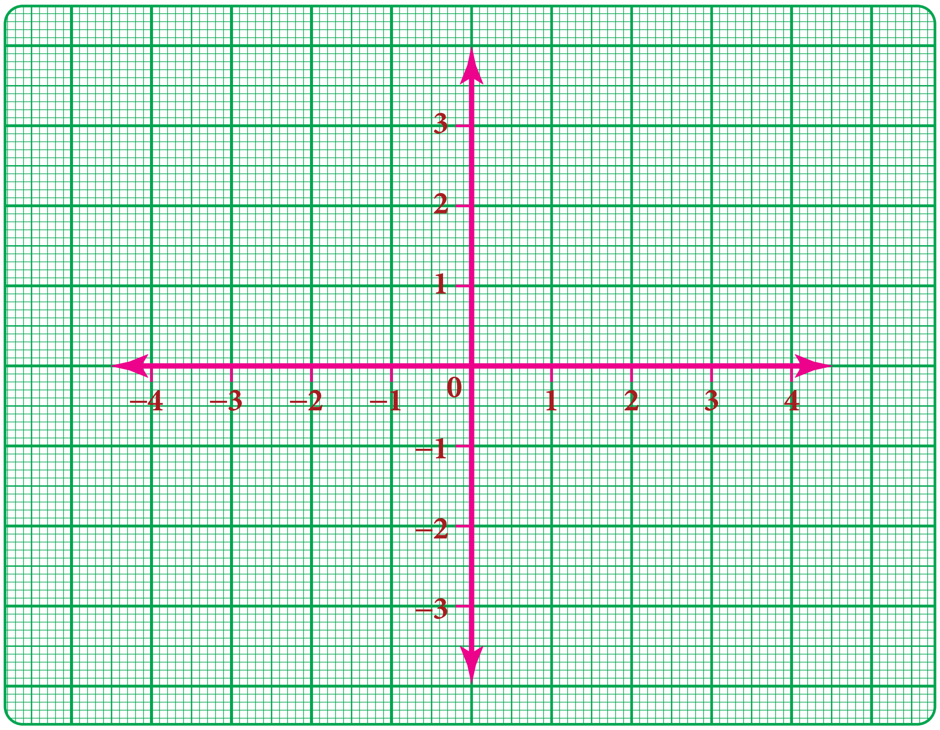
There was an instance in the 17th century when Rene Descartes, a famous mathematician became ill and from his bed, noticed an insect hovering over a corner and sitting at various places on the ceiling. He wanted to identify all the places where the insect sat on the ceiling. Immediately, he drew the top plane of the room in a paper, creating the horizontal and vertical lines. Based on these perpendicular lines, he used the directions and understood that the places of the insect can be spotted by the movement of the insect in the east, west, north and south directions. He called that place as (x,y) in the plane which indicates two values , one (x) in the horizontal direction and the other (y) in the vertical direction (say east and north in this case). This is how the concept of graphs came into existence.

2.5.2 Graph sheets:

Graph is just a visual method for showing relationships between numbers. In the previous class, we studied how to represent integers on a number line horizontally. Now take one more number line – vertically. We take the graph sheet keeping both number lines mutually perpendicular to each other at '0'zero as given in the figure. The number lines and the marking integers should be placed along the dark lines of the graph sheet.







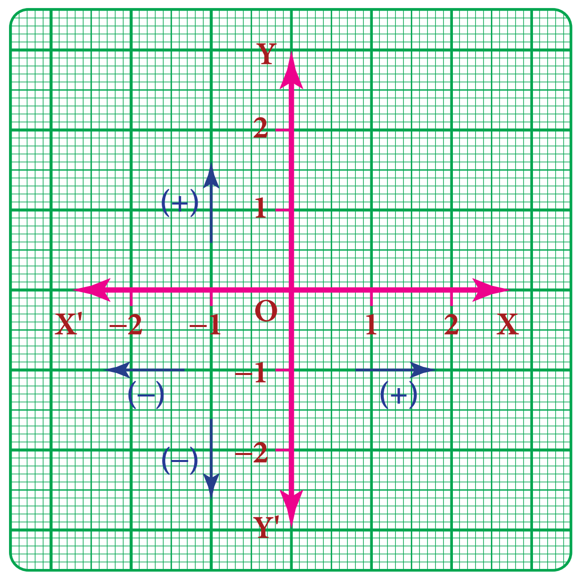
The intersecting point of the perpendicular lines 'O' represent the origin (0, 0).

Do you know?

Rene Descartes: The French mathematician and philosopher was born in 1596. Descartes presented his results in the book "Discourse on the method". His major contribution lies in bringing forth coordinate system that also bears his name. This Cartesian coordinate system tends to explain the algebraic equations through geometrical shapes. As his famous dictum is "I think, therefore I am".

Cartesian system

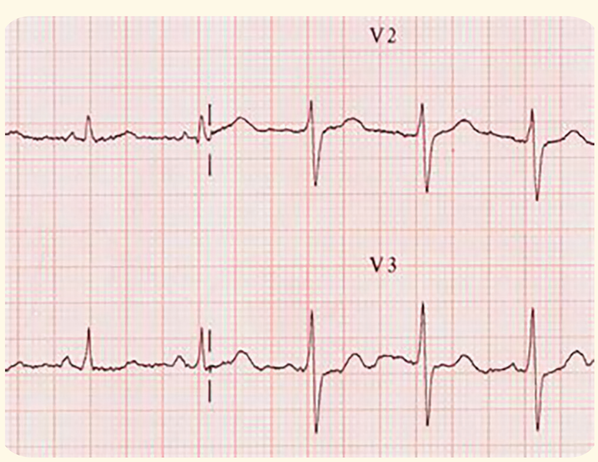
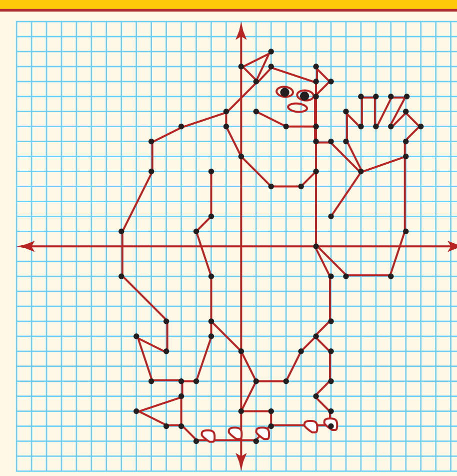
Rene Descartes system of fixing a point with the help of two measurements, horizontal and vertical, is known as **Cartesian system,** in his honour. The horizontal line is named as XOX', called the X-axis. The vertical line is named as YOY', called the Y- axis. Both the axes are called **coordinate axes**. The plane containing the X axis and the Y axis is known as the coordinate plane or the **Cartesian plane.**



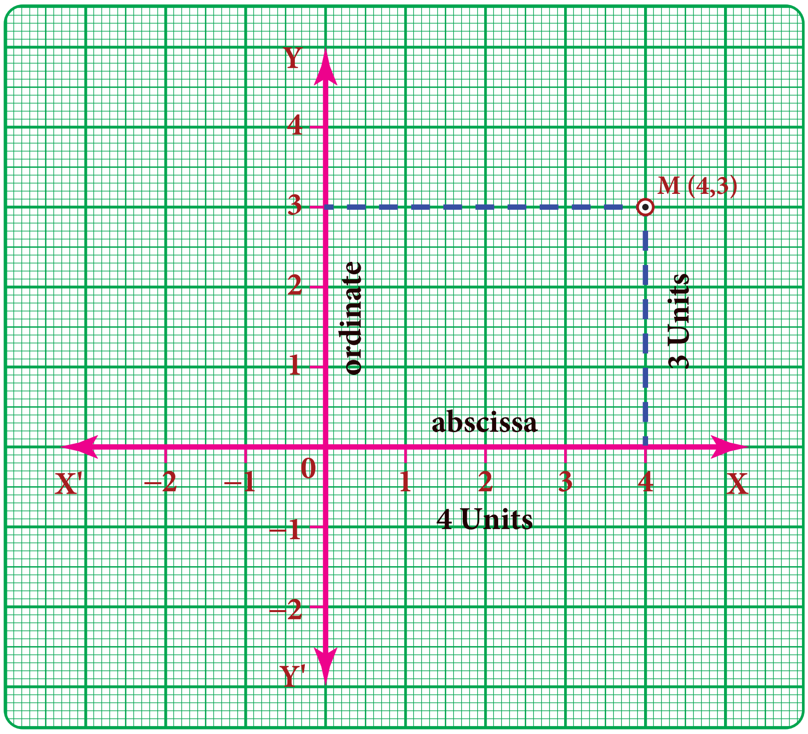
2.5.3 Signs in the graphs:

1. X-coordinate of a point is positive along OX and negative along OX'
2. Y-coordinate of a point is positive along OY and negative along OY'

Mathematics Alive – Graph in Real Life

	
Graphs are used in the medical field for various purposes like in ECG, EEG etc.,	Graphs are used for drawing

2.5.4 Ordered pairs:



A point represents a position in a plane. A point is denoted by a pair (a,b) of two numbers 'a' and 'b' listed in a specific order in which 'a' represents the distance along the X-axis and 'b' represents the distance along the Y-axis. It is called an ordered pair (a,b). It helps us to locate *precisely* a point in the plane. Each point can be exactly identified by a pair of numbers. It is also clear that the point (b,a) is not same as (a,b) as they both indicate different orders.

We have, XOX' and YOY' as the co-ordinate axes and let 'M' be (4,3) in the plane. To locate 'M'

(i) you (always) start at O, a *fixed* point (which we have, agreed to call as *origin*),

(ii) *first*, move 4 units along the horizontal direction (that is, the direction of x-axis)

(iii) *and then* trek along the y- direction by 3 units.

To understand how we have travelled to reach M, we denote by (4,3).

4 is called the x-coordinate of M and 3 is called the y-coordinate of M.

It is also habitual to name the x-coordinate as abscissa and the y-coordinate as ordinate. (4,3) is as an *ordered pair*.

Think

If instead of (4,3), we write (3,4) and try to mark it, will it represent 'M' again?

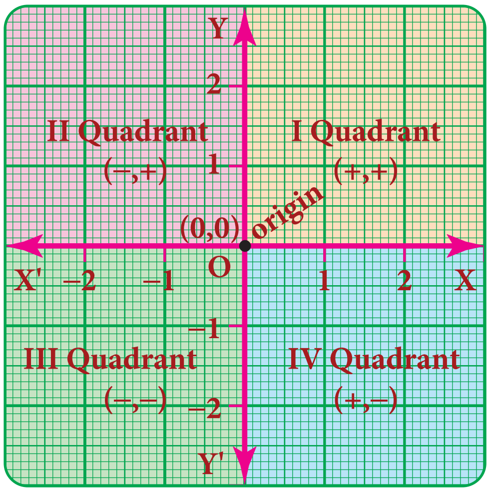
Note

As we studied in class VI, a place is represented by a point which is marked by English capital letters A, B, C…..Z. Places are represented by points in maps.
.A .F .K

2.5.5 Quadrants:

The coordinate axes divide the plane of the graph into four regions called quadrants. It is a convention that the quadrants are named in the anti clock wise sense starting from the positive side of the X axis.

Quadrant	Sign
I the region XOY	x>0, y>0, then the coordinates are (+,+) Examples: (5,7) (2,9) (10,15)
II the region X'OY	x<0, y>0 then the coordinates are (–,+) Examples (–2,8) (–1,10) (–5,3)
III the region X'OY'	x<0, y<0 then the coordinates are (–,–) Examples: (–2,–3) (–7,–1) (–5,–7)
IV the region XOY'	x>0, y<0 then the coordinates are (+,–) Examples : (1,–7) (4,–2) (9,–3)



Coordinate of a point on the axes:

- If y=0 then the coordinate (x, 0) lies on the 'x'-axis.
  For example (2, 0) (–5, 0) (7,0) are points on the 'x'-axis.
- If x=0 then the coordinate (0,y) lies on the 'y'-axis.
  For example (0, 3) (0,–4) (0, 9) are points on the 'y'-axis.

2.6.6 Plotting the given points on a graph:

Consider the following points (4,3), (–4,5), (–3,–6), (5,–2), (6,0), (0,–5)

(i) To locate (4,3).

Start from origin O, move 4 units along OX and from 4, move 3 units parallel to OY to reach M(4,3).

(ii) To locate (–4,5)

From the origin, move 4 units along OX' and from –4, move 5 units parallel to OY to reach N(–4,5).

(iii) To locate (– 3, – 6)

From the origin move 3units along OX' and from –3, move 6 units parallel to OY' to reach P(-3,-6).

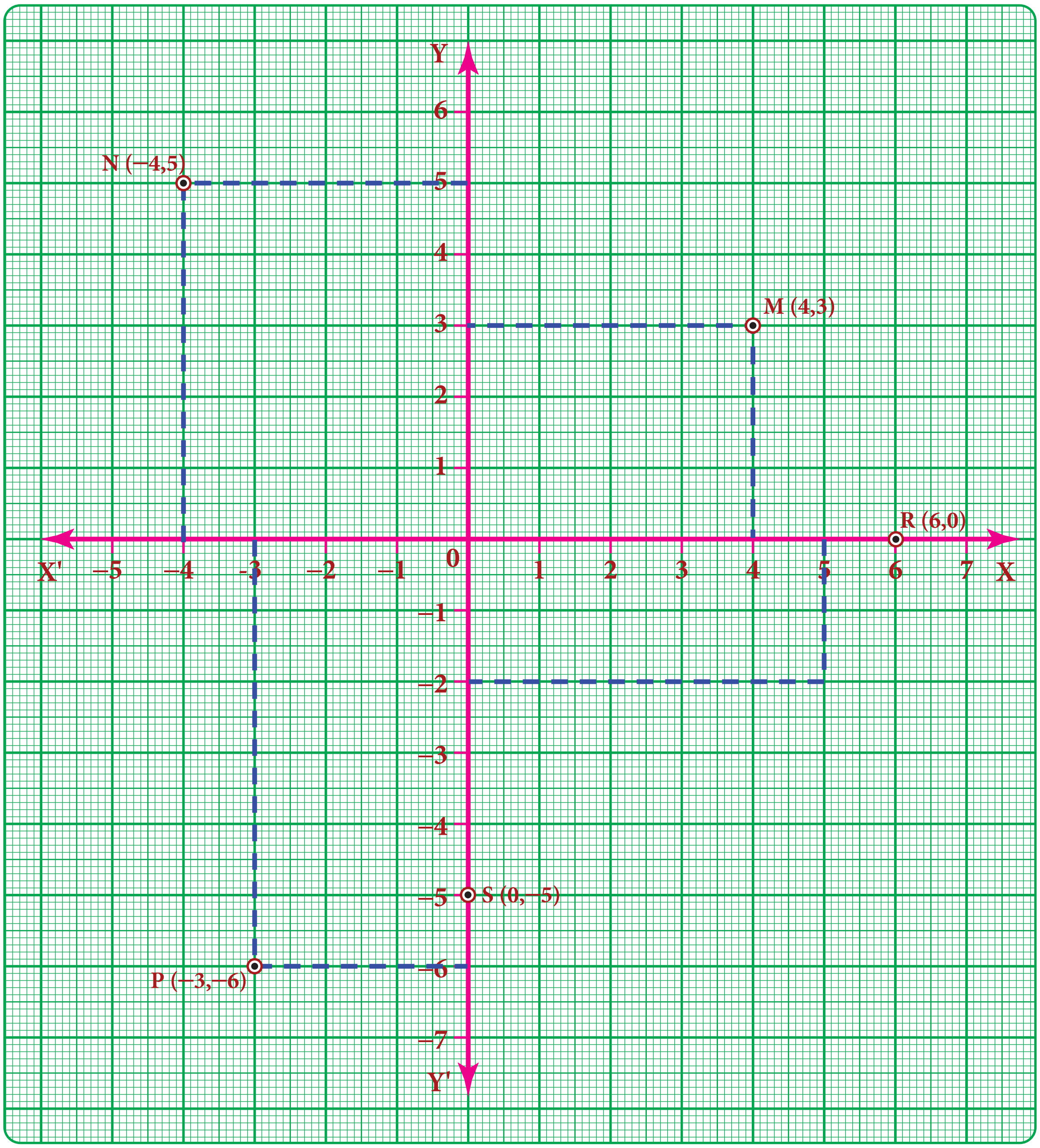
(iv) To locate (5,–2)

From the origin move 5 points along OX and from 5, move 2 units parallel to OY' to reach Q(5,–2).

(v) To locate (6,0) and (0, – 5)

In the given point (6, 0), X-coordinate is 6 and Y-coordinate is zero. So the point lies on the x-axis. Move 6 units on OX from the origin; to reach R(6, 0).

In the given point (0,–5) X-coordinate is zero and Y- coordinate is (–5). So, the point lies on Y-axis, move 5 units on OY' from the origin to reach S (0,–5).

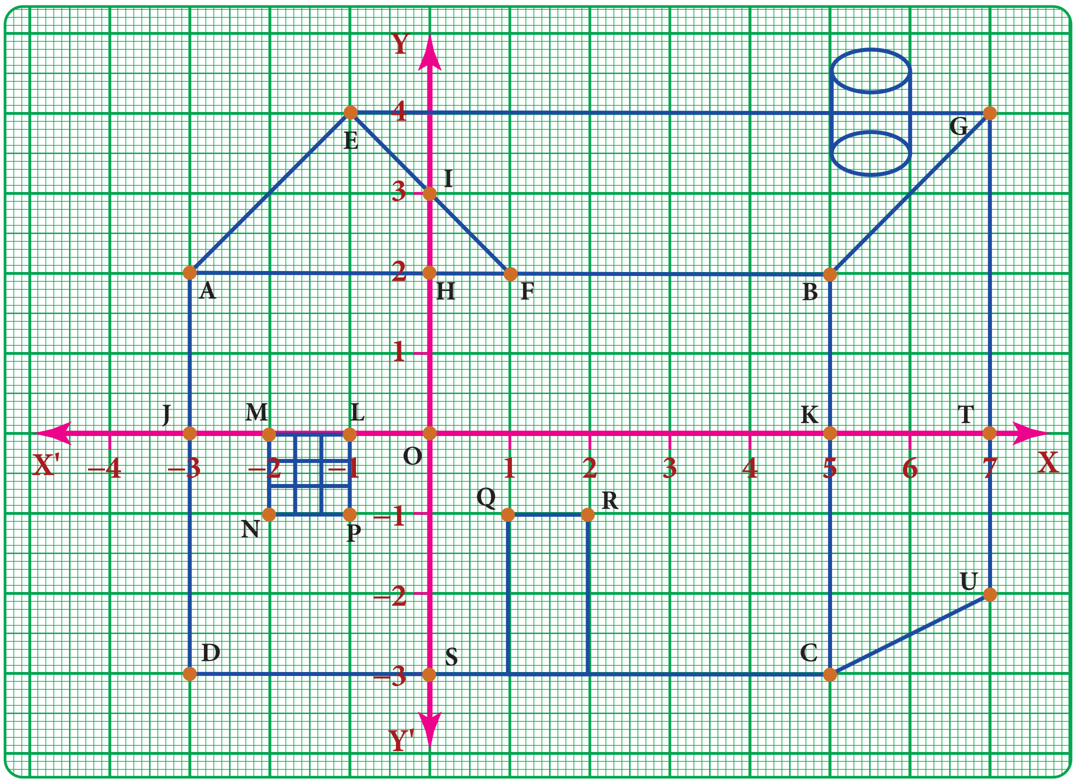


Try these

1. Complete the table given below.

S.No	Point	Sign of X-coordinate	Sign of Y-coordinate	Quadrant
1	(−7,2)			
2	(10,−2)			
3	(−3,−7)			
4.	(3,1)			
5.	(7,0)			
6.	(0,−4)			

2. **Write the coordinates of the points marked in the following figure**



Exercise 2.3

1 **Fill in the blanks:**

(i) X- axis and Y-axis intersect at ___________.

(ii) The coordinates of the point in third quadrant are always___________.

(iii) (0, −5) point lies on _________axis.

(iv) The x- coordinate is always _______on the y-axis.

(v) __________ coordinates are the same for a line parallel to Y-axis.

2. **Say True or False:**

(i) (−10,20) lies in the second quadrant.

(ii) (−9, 0) lies on the x-axis.

(iii) The coordinates of the origin are (1,1).

Z8S7Q9

3 Find the quadrants without plotting the points on a graph sheet.

(3, −4), (5,7), (2,0), (−3, −5), (4, −3), (−7,2), (−8,0), (0,10), (−9,50).

4 Plot the following points in a graph sheet.

A(5,2), B(−7, −3), C(−2,4), D(−1,−1), E(0, −5), F(2,0), G(7, −4), H(−4,0), I(2,3), J(8,−4), K(0,7).

5 Use the graph to determine the coordinates where each figure is located.

a) Star ________

b) Bird ________

c) Red Circle ________

d) Diamond ________

e) Triangle ________

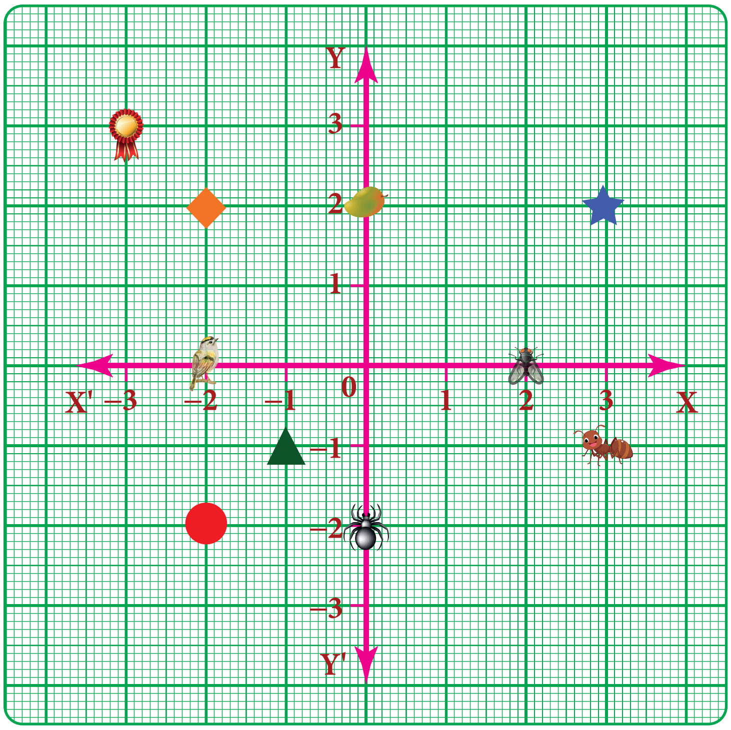
f) Ant ________

g) Mango ________

h) Housefly ________

i) Medal ________

j) Spider ________



2.6.7 To obtain a straight line:

Now, we know how to plot the points on the graph. The points may lie on the graph in different order. If we join any two points we will get a straight line.

(i) Draw a straight line by joining the points A (−2,6) and B (4,−3)

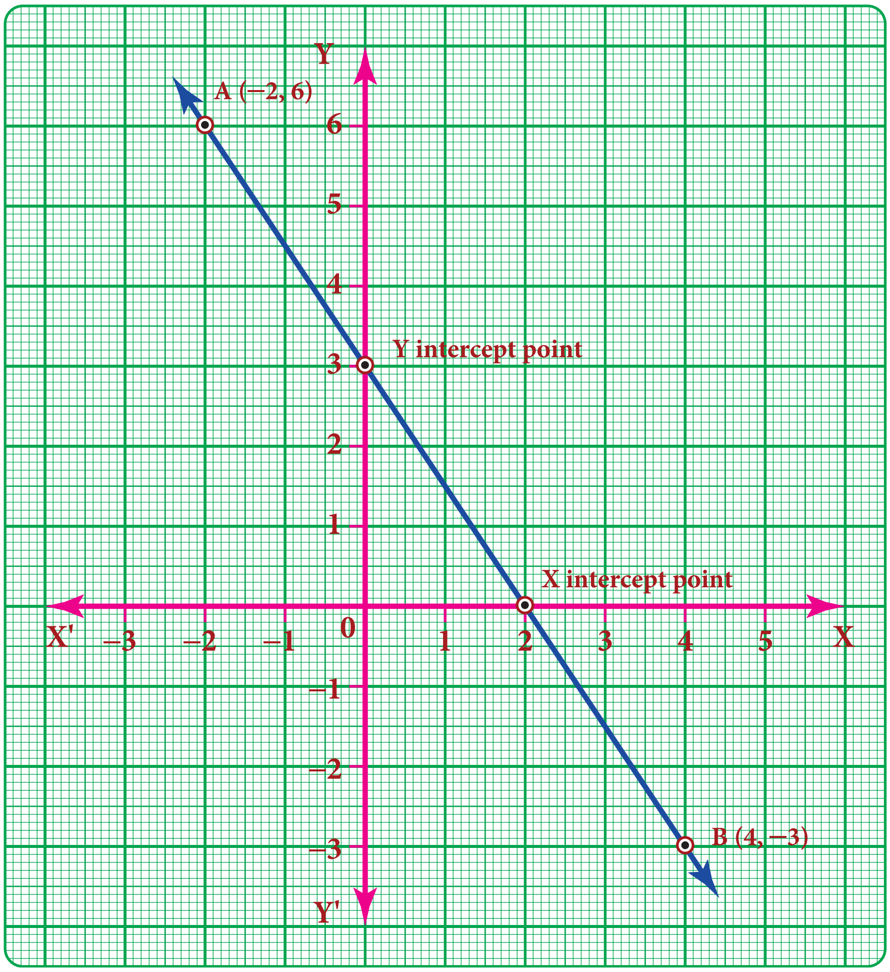
Solution:

The given first point A (−2,6) lies in the II quadrant and plot it. Second point B (4,−3) lies in the IV quadrant and plot it.

Now join the point A and point B using scale and extend it. We get a straight line.

Note:

The straight line intersects X axis at (2,0) and Y axis at (0,3).



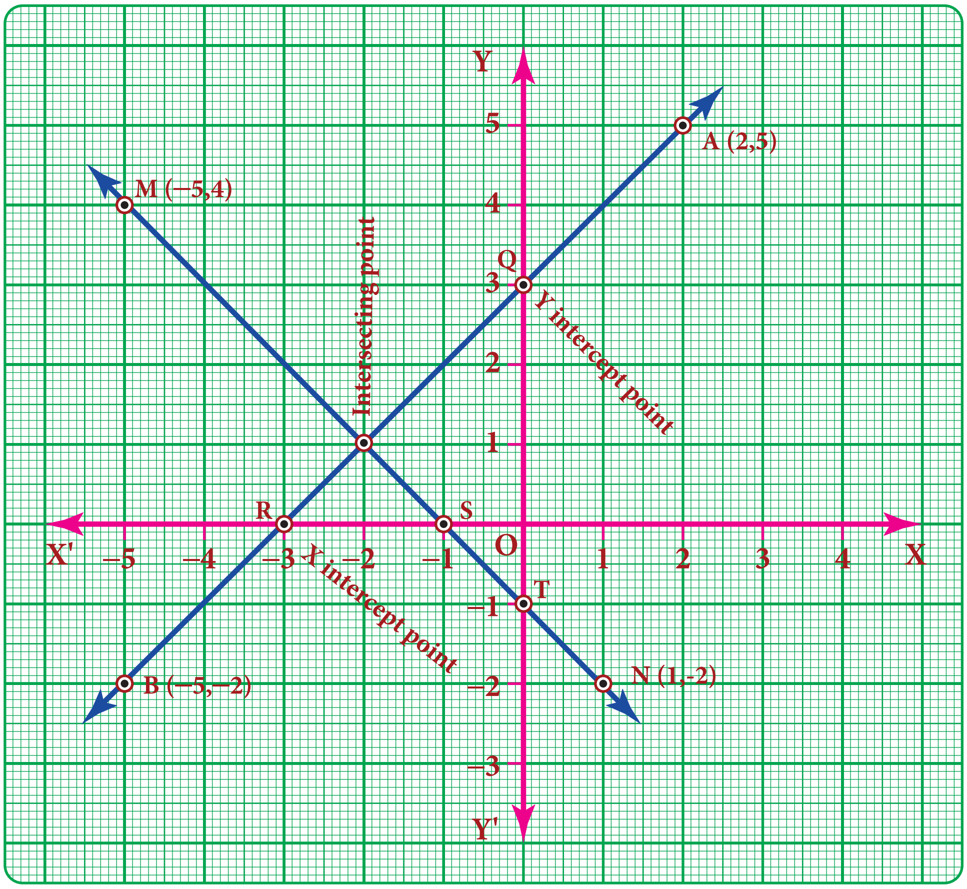
(ii) Draw straight lines by joining the points A(2, 5) B(−5,−2) M(−5, 4) N(1,−2) also find the point of intersection

Plot the first pair of points A and B in I and III quadrants. Join the points and extend it to get AB straight line. Plot the second pair of points M and N in II and IV quadrants. Join the points and extend it to get MN straight line.

Now, both lines are intersect at P(−2,1)

(i) The line AB interest the coordinate axis, ie) x -axis at R(−3,0) and y-axis at Q(0,3)

(ii) The line MN interest the c`oordinate axis, ie) x -axis at S(−1,0) and y-axis at T(0,−1)



2.6.8 Line parallel to the coordinate axes

- If a line is parallel to the X-axis, its distance from X axis is the same and it is represented as y = c .
- If a line is parallel to the Y-axis, its distance from Y axis is the same and it is represented as x = k. (Here c and k are constants)

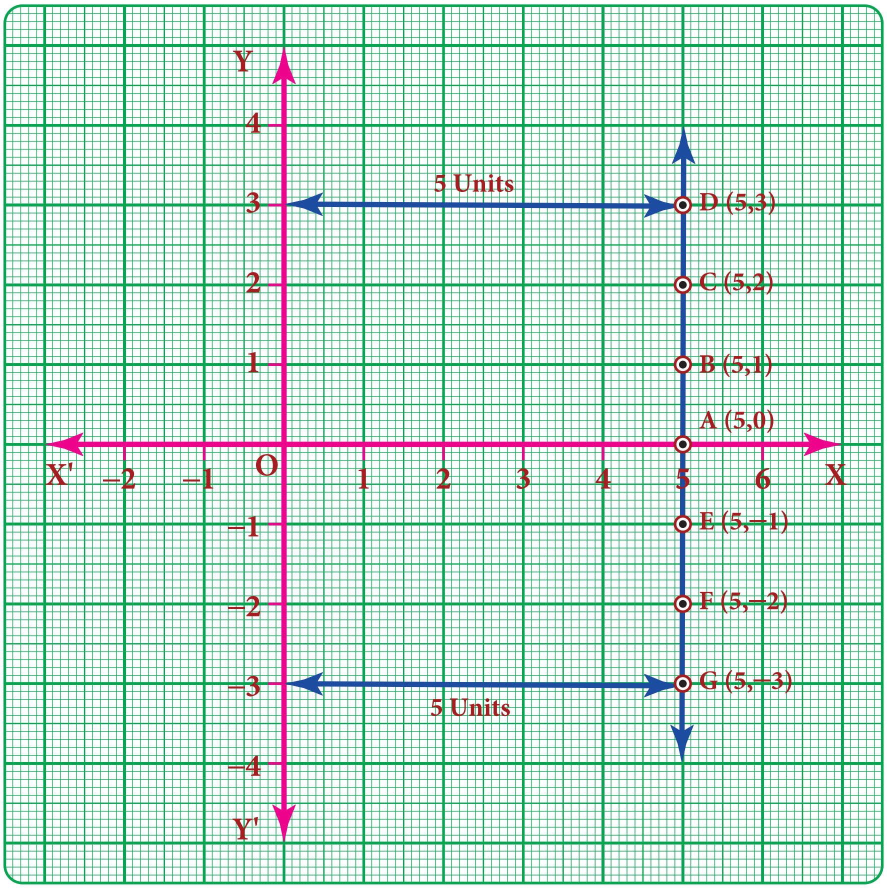
(i) Draw the graph of $x = 5$

Solution:

x=5 means that x-coordinate is always 5 for whatever value of y-coordinate. So we may give any value for y-coordinate and this is tabulated as follows.

X	5	5	5	5	5	x =5 is given (fixed)
Y	−2	−1	0	2	3	Take any value for y (Why?)

The points are (5,–2) (5,–2) (5,0) (5,2) (5,3). Plot the points in the graph and join them.We get a straight line parallel to Y axis at a distance of 5 units from the Y axis



Note

X=0 represents the - Y axis

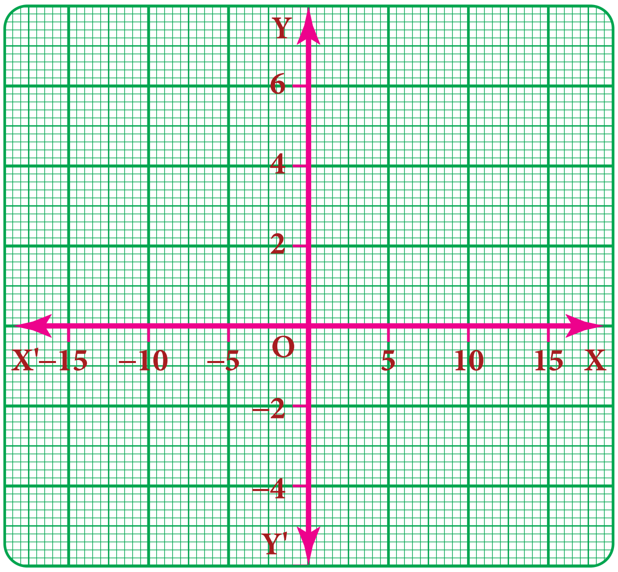
Y=0 represents the - X axis

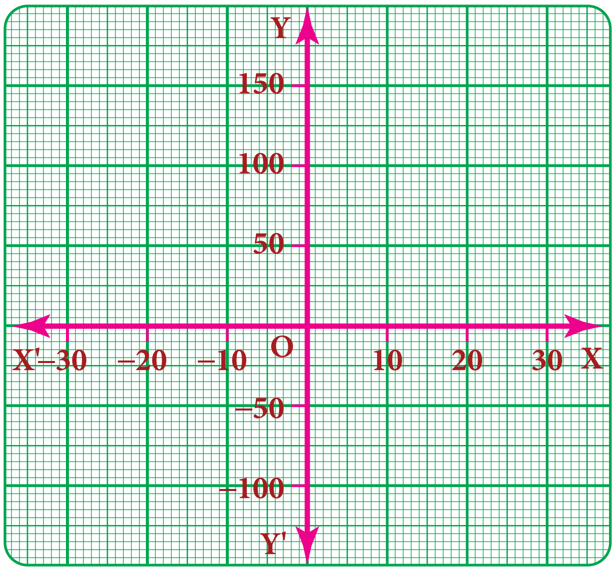
Think

Which of the points (5, –10) (0, 5) (5, 20) lie on the straight line X = 5?

2.6.9 Scale in a graph:

There will be situations in drawing a graph where 'y' is a big multiple of 'x' and the usual graph in units may not be enough to locate the 'y' coordinate and vice-versa. In this situation, we use the concept of scale in both the axes as per the need. Represent a convenient scale at the right side corner of the graph. A few examples are given below.





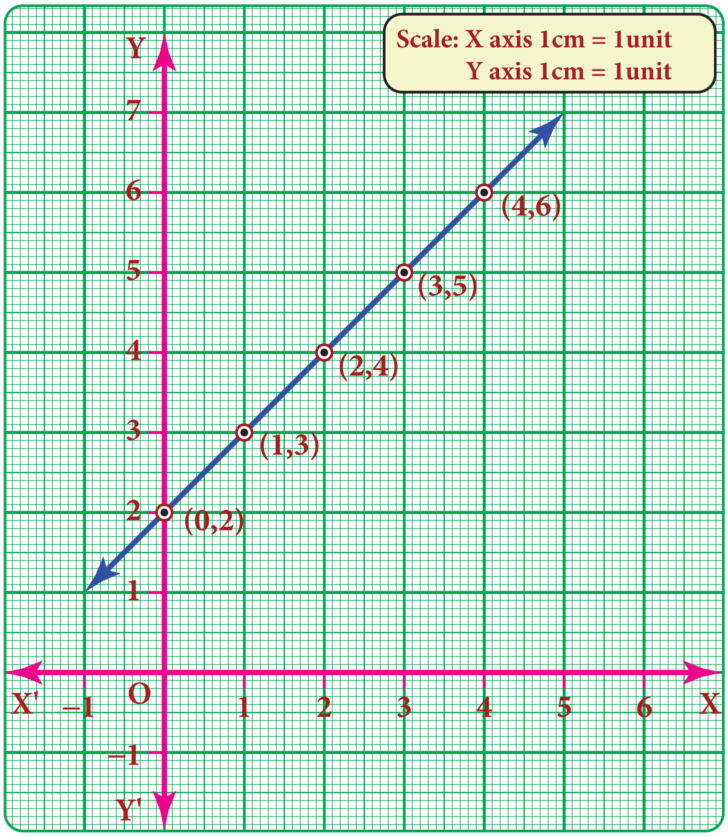
Example 1 On the X axis 1 cm = 5 units. On the Y axis 1 cm = 2 units.	Example 2 On the X axis 1 cm = 10 units. On the Y axis 1 cm = 50 units.

2.7 Linear graph

2.7.1 Linear pattern:

Plot the following points on a coordinate plane: (0,2), (1,3), (2,4), (3,5), (4, 6). What do you find? They all lie on a line! There is some pattern in them. Look at the y- coordinate in each ordered pair: 2 = 0+2; 3 = 1+2; 4 = 2+2; 5 = 3+2; 6 = 4+2 In each pair, the y- coordinate is 2 more than the x- coordinate. The coordinates of each point have the same relationship between them. All the points plotted lie on a line!

In such a case, when all the points plotted lie on a line, we say '*a linear pattern*' exists.



In this example, we found that in each ordered pair y value = x value +2. Therefore the linear pattern above can be denoted by the algebraic equation $y = x + 2$. Such an equation is called a *linear equation* and the line graph for linear equation is called a *linear graph*.

Linear equations use one (or more) variables where one variable is dependent on the other(s).

The longer the distance we travel by a taxi, the more we have to pay. The distance travelled is an example of an *independent variable*. Being dependent on the distance, the taxi fare is called the *dependent variable*.

The more one uses electricity, greater will be the amount of electricity bill. The amount of electricity consumed is an example for independent variable and the bill amount is naturally the dependent variable.

2.7.2 Graph of a linear function in two variables:

We have talked about parallel lines, intersecting lines etc., in geometry but never actually looked at how far apart they were, or where they were .Drawing graphs helps us place lines. A linear equation is an equation with two variables (like x and y) whose graph is a line. To graph a linear equation, we need to have at least two points, but it is usually safe to use more than two points. (Why?) When choosing points, it would be nice to include both positive and negative values as well as zero. There is a **unique line** passing through any pair of points.

Example:1

Draw the graph of y = 5 x

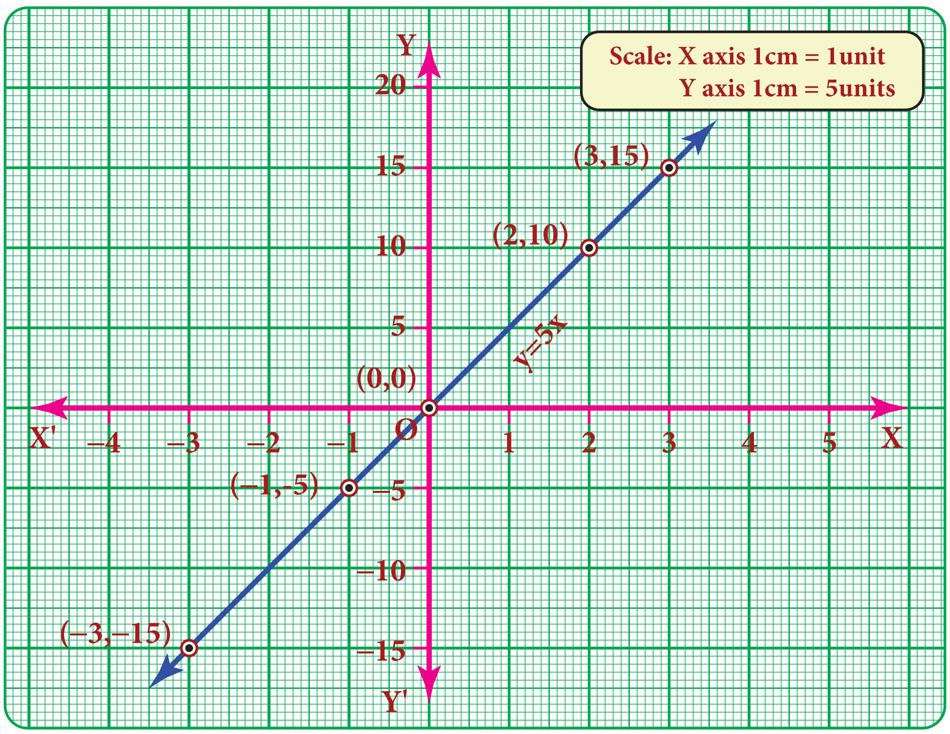
x	−3	−1	0	2	3
y	−15	−5	0	10	15

x	$y = 5^{x+1}$
−3	5 × (−3) = −15
−1	5 × (−1) = −5
0	5× (0) = 0
2	5× (2) = 10
3	5× (3) = 15

Solution:

The given equation y = 5 x means that for any value of x, y takes five times of x value.

Plot the point (–3,–15) (–1,–5) (0,0) (2,10) (3,15)



Example: 2

Graph the equation $y = x + 1$.

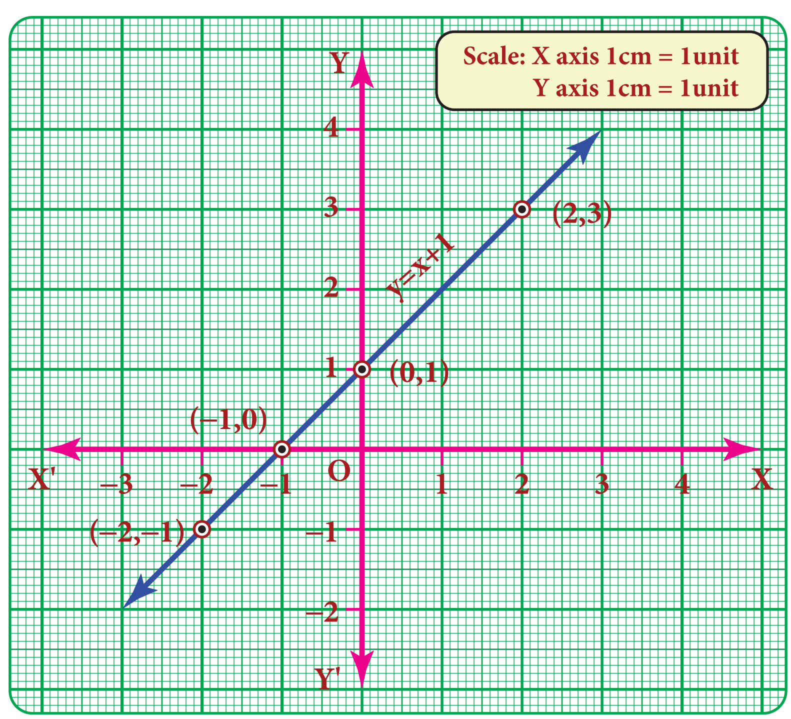
Begin by choosing a couple of values for x and y. It will firstly help to see

(i) what happens to y when x is zero and

(ii) what happens to x when y is zero.

After this we can go on to find one or two more values.

Let us find at least two more ordered pairs. For easy graphing, let us avoid fractional answers. We shall make suitable guesses.



x	−2	−1	0	1	2
y	−1	0	1	2	3

x	y = x+1
−2	−2+1 = -1
−1	−1+1 = 0
0	0+1 = 1
1	1+1 = 2
2	2+1 = 3

We have now five points of the graph: **(–2,–1) (–1, 0) (0, 1) (1, 2) and (2, 3).**

2.7.3 Applications:

Let us look into some real-life examples and their linear nature.

Example: 3

Relation between Quantity and Cost

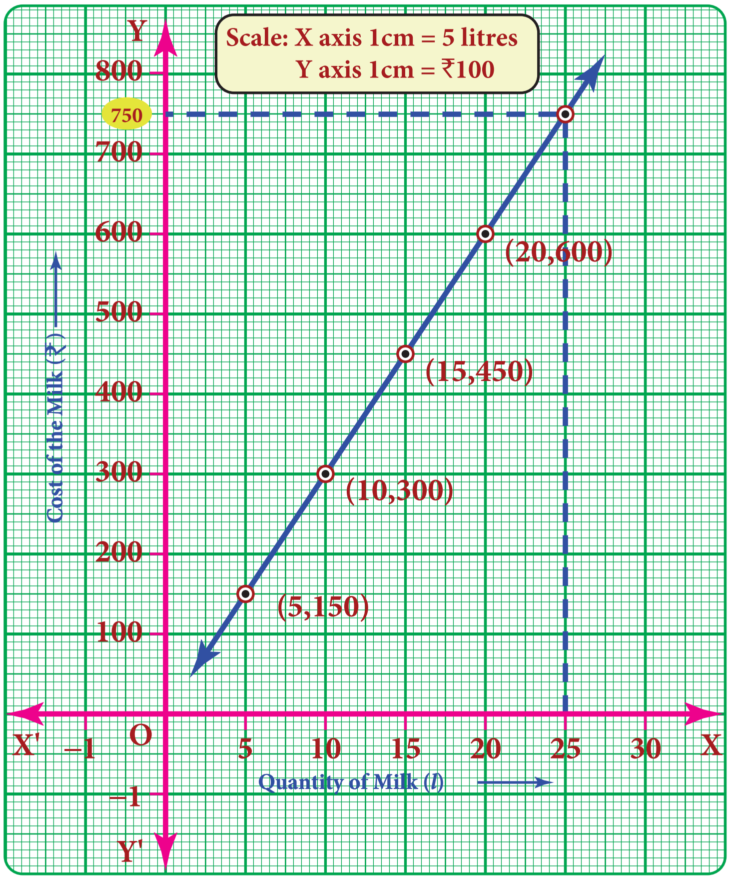
The following table gives the quantity of milk and its cost.

Quantity of milk	5	10	15	20
Cost of milk in ₹	150	300	450	600

Plot the graph.

Solution

- Take a suitable scale on both the axes Here, we take on the x axis 1cm = 5 litres on the, y axis 1cm = 100 rupees.
- Mark number of litres of milk along the x -axis.
- Mark the cost of milk along the y-axis.
- Plot the points (5,150) (10,300) (15,450) (20,600).
- Join the points.



This graph can help us to estimate few more things also. Suppose we like to find the cost of 25 litres of milk. Mark 25 on the x-axis, follow the line parallel to y-axis through 25 till we meet the drawn line at P. From P we take a horizontal line to meet the y-axis. This meeting point of y-axis is the required answer.

Thus, the cost of 25 litres of milk is ₹750. This is the graph of linear equation in two quantities, and hence they are in direct variation.

Note

The orientation of the graph will be different according to the scale chosen.

Example: 4

Relation between Principal and Simple Interest:

A bank gives 10% simple interest on deposits made by Senior citizens. Illustrate by a graph the relation between the deposit and the interest gained. Use the graph to compute

(i) The annual interest obtainable for investment of ₹450;

(ii) The amount a Senior citizen has to invest to get an annual simple interest of ₹80.

Solution:

Using the formula for calculating the simple interest, the following table of values is prepared.

Deposit (in. ₹)	100	200	300	500	1000
Annual S.I. (in. ₹)	10	20	30	50	100

Deposit	*Interest*
₹100	$\frac{100\times1\times10}{100}$ = ₹10
₹200	$\frac{200\times1\times10}{100}$ = ₹20
₹300	$\frac{300\times1\times10}{100}$ = ₹30
₹500	$\frac{500\times1\times10}{100}$ = ₹50
₹1000	$\frac{1000\times1\times10}{100}$ = ₹100

These are the points which are to be plotted in the graph sheet. Let us take the deposits along x-axis and annual simple interest along y-axis.

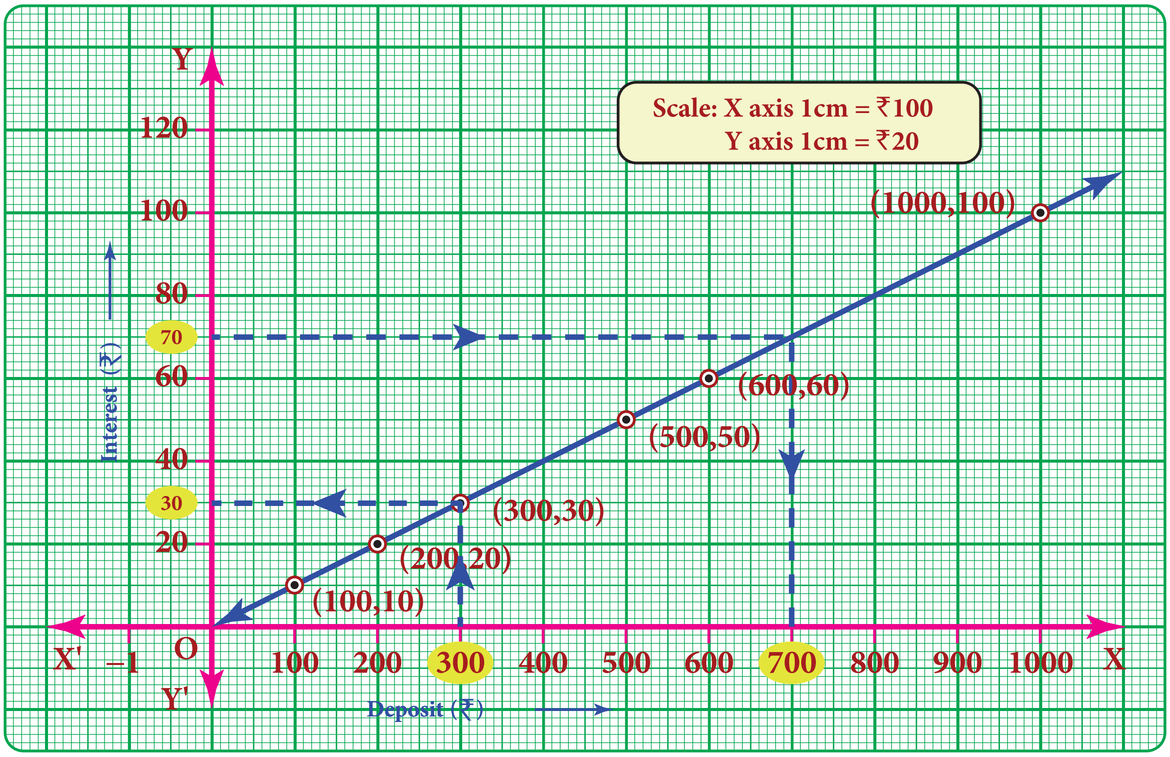
We choose the scale as follows:

Then we plot the points and draw the straight line.

From the graph we find:

(i) Corresponding to ₹300 on the x-axis, we get the interest as ₹30 on the y-axis.

(ii) Corresponding to ₹70 on the y-axis, we get the deposit as ₹700 on the x-axis.



Example: 5

Relation between Time and Distance:

A train runs constantly at a speed of 80km/hr. Draw a time – distance graph for this situation. Also find the

(i) time – taken to cover 240 km.

(ii) distance covered in 5 ½ hours.

Solution:

Given, the train runs constantly at a speed of 80 km/hr.

i.e For 1 hour = 80 km

2 hours = 2×80 = 160 km

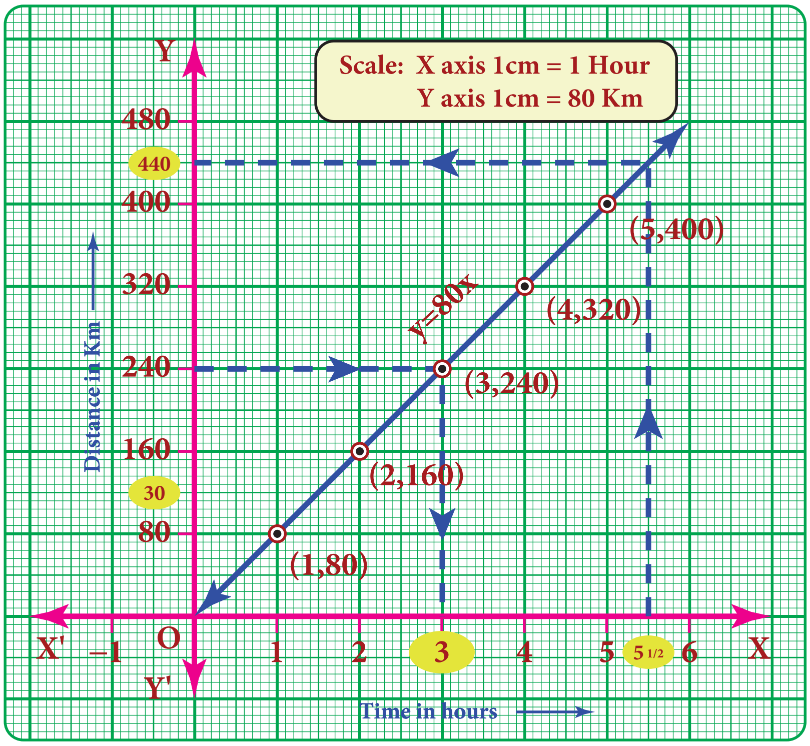
3 hours = 3×80 = 240 km

Hour	1	2	3	4	5
Distance	80	160	240	320	400

We can tabulate as above,

Take a suitable scale

1) Mark the number of hours on the x -axis.
2) Mark the distance inKm on the y-axis.
3) Plot the points (1,80) (2,160) (3,240) (4,320) and (5,400).
4) Join the points and get a straight line3.
5) From the graph.
   (i) Time taken to cover 240 km is 3 hrs.
   (ii) The distance covered in 5 ½ hrs 440 km.



Think

Why is it given that the speed is 'constant'?
If the speed is not constant, will the graph be the same?
The graph is named as $y = 80x$ algebraically. Why?

Exercise 2.4

1. **Fill in the blanks:**

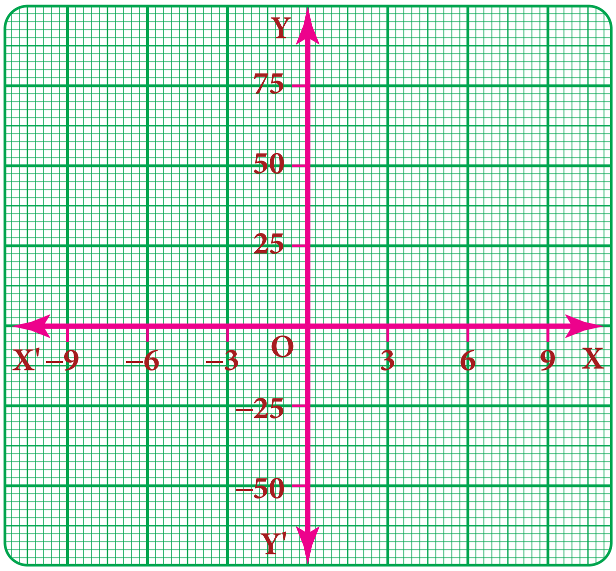
   (i) $y = px$ where $p \in z$ always passes through the__________.

   (ii) The intersecting point of the line $x = 4$ and y = – 4 is__________.

   (iii) Scale for the given graph ,

   On the x-axis 1 cm = ------- units

   y-axis 1 cm = ------- units



2. **Say True or False.**

   (i) The points (1,1) (2,2) (3,3) lie on a same straight line.

   (ii) $y = -9x$ not passes through the origin.

3. Will a line pass through (2, 2) if it intersects the axes at (2, 0) and (0, 2).

4. A line passing through (4, –2) and intersects the Y-axis at (0, 2). Find a point on the line in the second quadrant.

5. If the points P(5, 3) Q(–3, 3) R(–3, –4) and S form a rectangle, then find the coordinate of S.

6. A line passes through (6, 0) and (0, 6) and an another line passes through (–3,0) and (0, –3). What are the points to be joined to get a trapezium?

7. Find the point of intersection of the line joining points (–3, 7) (2, –4) and (4,6) (–5,–7). Also find the point of intersection of these lines and also their intersection with the axis.

8. Draw the graph of the following equations

   (i) $x = -7$ (ii) y = 6

9. Draw the graph of (i) $y = -3x$ ii) y = x–4

10. Find the values.

Let $y = x + 3$	Let $2x + y - 6 = 0$
(i) If x = 0, find y.	(i) If x = 0, find y.
(ii) If y = 0, find x.	(ii) If y = 0, find x.
(iii) If x = –2, find y.	(iii) If x = –2, find y.
(iv) If y = –3, find x.	(iv) If y = –3, find x.

11. The following is a table of values connecting the radii of a few circles and their circumferences (Taking $\pi = \frac{22}{7}$)

Illustrate the relation with a graph and find

(i) The radius when the circumference is 242 units.

(ii) The circumference when the radius is 24.5 units.

Radius (r)	Circumference ($2\pi r$)
7	$2 \times \frac{22}{7} \times 7 = 44$
14	$2 \times \frac{22}{7} \times 14 = 88$
21	$2 \times \frac{22}{7} \times 21 = 132$
28	$2 \times \frac{22}{7} \times 28 = 176$
35	$2 \times \frac{22}{7} \times 35 = 220$
42	$2 \times \frac{22}{7} \times 42 = 264$
49	$2 \times \frac{22}{7} \times 49 = 308$

12. An over-head tank is full with water. Water leaks out from it, at a constant rate of 10 litres per hour. Draw a "time-wastage" graph for this situation and find

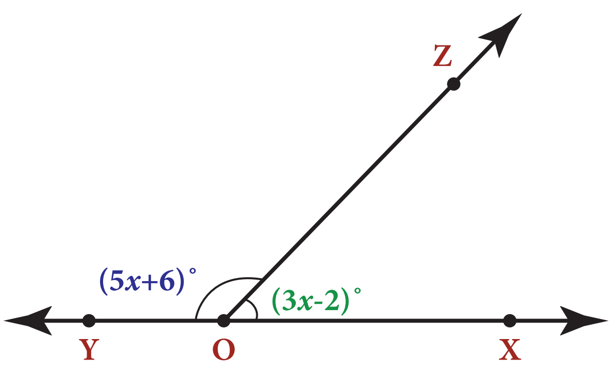
(i) The water wasted in 150 minutes

(ii) The time at which 75 litres of water is wasted.

Exercise 2.5

Miscellaneous Practice Problems

1. The sum of three numbers is 58. The second number is three times of two-fifth of the first number and the third number is 6 less than the first number. Find the three numbers.
2. In triangle ABC, the measure of ∠B is two-third of the measure of ∠A. The measure of ∠C is 20° more than the measure of ∠A. Find the measures of the three angles.
3. Two equal sides of an isosceles triangle are 5y–2 and 4y+9 units. The third side is 2y+5 units. Find 'y' and the perimeter of the triangle.
4. In the given figure, angle XOZ and angle ZOY form a linear pair. Find the value of x.



5. Draw a graph for the following data:

Side of a square (cm)	2	3	4	5	6
Area (cm^2)	4	9	16	25	36

Does the graph represent a linear relation?

Challenging Problems

6. Three consecutive integers, when taken in increasing order and multiplied by 2, 3 and 4 respectively, total up to 74. Find the three numbers.

7. 331 students went on a field trip. Six buses were filled to capacity and 7 students had to travel in a van. How many students were there in each bus?

8. A mobile vendor has 22 items, some which are pencils and others are ball pens. On a particular day, he is able to sell the pencils and ball pens. Pencils are sold for ₹15 each and ball pens are sold at ₹20 each. If the total sale amount with the vendor is ₹380, how many pencils did he sell?

9. Draw the graph of the lines $y = x$, $y = 2x$, $y = 3x$ and $y = 5x$ on the same graph sheet. Is there anything special that you find in these graphs?

10. Consider the number of angles of a convex polygon and the number of sides of that polygon. Tabulate as follows:

Name of Polygon	*No. of angles*	*No. of sides*

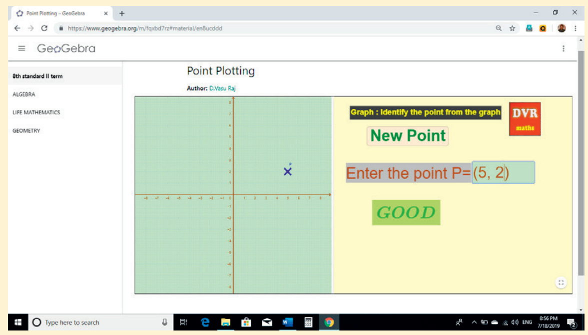
Use this to draw a graph illustrating the relationship between the number of angles and the number of sides of a polygon.

Summary

- An equation containing only one variable with its highest power as one is called a linear equation in one variable.
- This linear equation in one variable is also known as **simple equation**.
- The value which replaces a variable in an equation so as to make the two sides of the equation equal is called a solution or root of the equation.
- The shifting of a number from one side of an equation to other is called transposition.
- Graphing is just a visual method for showing relationships between numbers.
- The horizontal line is named as XOX', called the X-axis. The vertical line is named as YOY', called the Y-axis. Both the axes are called **coordinate axes**. The plane containing the x axis and the y axis is known as the coordinate plane or the **Cartesian plane**.
- A point is denoted by a pair (a,b) of two numbers 'a' and 'b' listed in a specific order in which 'a' represents the distance along the X-axis and 'b' represents the distance along the Y axis. It is called an ordered pair (a,b).
- The coordinate axes divide the plane of the graph into four regions called quadrants.
- The line graph for the linear equation is called a **linear graph**.

ICT CORNER

Expected Outcome

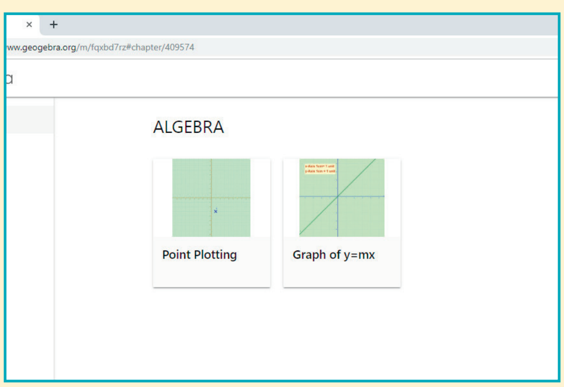


Step – 1

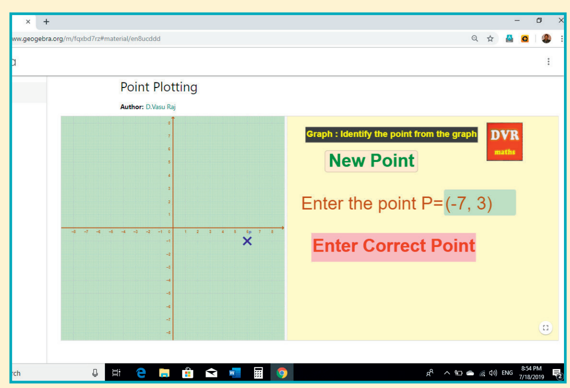
Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named "ALGEBRA" will open. Click on the worksheet named "Point Plotting".

Step - 2

In the given worksheet you can get new point by clicking on "New point". Enter the correct point in the input box and press enter.



Step – 1



Step – 2

Go through the remaining worksheets given for this chapter

Browse in the link

Algebra:

https://www.geogebra.org/m/fqxbd7rz#chapter/409574 or Scan the QR Code.

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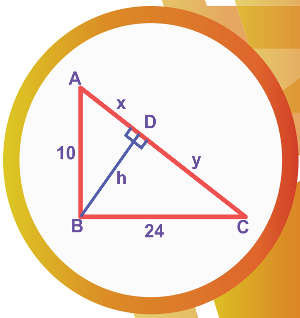
3 GEOMETRY



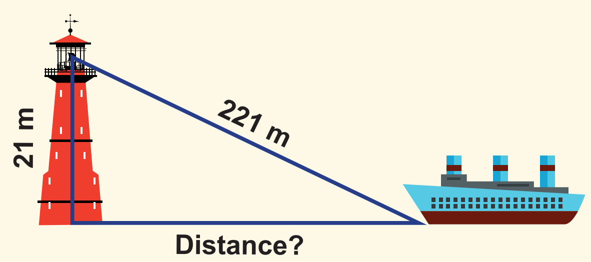
Learning Objectives:

- To understand the Pythagoras theorem and solve problems using it.
- To know how to construct Trapeziums and Parallelograms.

D1R6R1

3.1 Introduction

In the first term, we have learnt about similar and congruent triangles and the construction of quadrilaterals. Now, we will learn about the Pythagorean theorem and the construction of trapeziums and parallelograms.

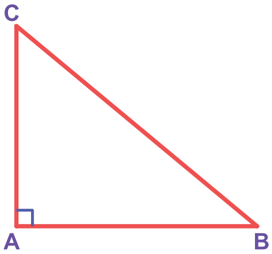
Mathematics alive – Geometry in Real life	
	
The Pythagoras theorem is useful in finding the distance and the heights of objects.	We can see the shape of a trapezium in building.

3.2 The Pythagorean Theorem

The Pythagorean theorem or simply Pythagoras theorem, named after the ancient Greek Mathematician Pythagoras (570-495 BC (BCE)) is definitely one of the most famous and celebrated theorems in the whole of mathematics. People have proved this theorem in numerous ways possibly the most for any mathematical theorem. They are very diverse which include both geometric and algebric proofs dating back to thousands of years.

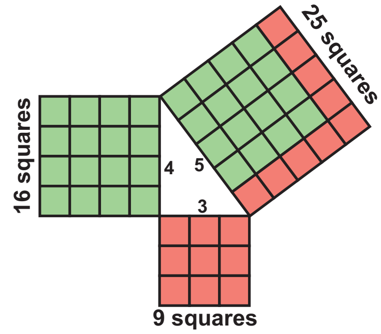
Statement of the theorem

In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



In ΔABC, $BC^2 = AB^2 + AC^2$

Visual Illustraton:



The given figure contains a triangle of sides of measures 3 units, 4 units and 5 units. From this well known $3-4-5$ triangle, one can easily visualise and understand the meaning of the Pythagorean theorem.

In the figure, the sides of measure 3 units and 4 units are called the legs or sides of the right angled triangle. The side of measure 5 units is called the hypotenuse. Recall that the hypotenuse is the greatest side in a right angled triangle

Now, it is easily seen that a square formed with side 5 units (hypotenuse) has $5 \times 5 = 25$ unit squares (small squares) and the squares formed with side 3 units and 4 units have $3 \times 3 = 9$ unit squares and $4 \times 4 = 16$ unit squares respectively. As per the statement of the theorem, the number of unit squares on the hypotenuse is exactly the sum of the unit squares on the other two legs (sides) of the right angled triangle. Isn't this amazing?

Yes, we find that

$5\times5 = 3\times3+4\times4$

i.e. $25 = 9+16$ (True)

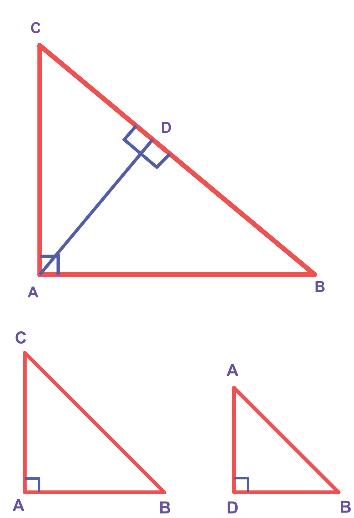
M3J7F1

Proof of the theorem using similarity of triangles

Given: $\angle BAC = 90^\circ$

To prove: $BC^2 = AB^2 + AC^2$

Construction: Draw $AD \perp BC$



Proof:

In ΔABC and ΔDBA,

$\angle B$ is common and $\angle BAC = \angle ADB = 90^\circ$

Therefore, $\Delta ABC \sim \Delta DBA$ (AA similarity)

Hence, the ratio of corresponding sides are equal.

$$\Rightarrow \frac{AB}{DB} = \frac{BC}{BA}$$

$$\Rightarrow AB^2 = BC \times DB \text{ ------------(1)}$$

Similarly $\Delta ABC \sim \Delta DAC$,

$$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$$

$$\Rightarrow AC^2 = BC \times DC \text{ --------------(2)}$$

Adding (1) and (2), we get

Try this

How will you prove $\Delta ABC \sim \Delta DAC$?

$$AB^2 + AC^2 = BC \times DB + BC \times DC$$
$$= BC \times (DB + DC)$$
$$= BC \times BC$$

$\therefore \Rightarrow AB^2 + AC^2 = BC^2$ and

Hence the proof of the theorem.

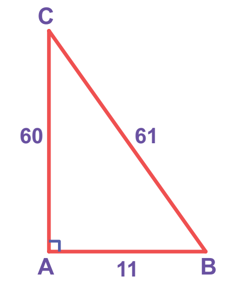
Note

If $AB^2 + AC^2 = BC^2$ then,

$AB^2 = BC^2 - AC^2$ and $AC^2 = BC^2 - AB^2$

3.3 Converse of Pythagoras theorem

If in a triangle, the square on the greatest side is equal to the sum of squares on the other two sides, then the triangle is right angled triangle.



Example:

In the triangle ABC,

$AB^2 + AC^2 = 11^2 + 60^2 = 3721 = 61^2 = BC^2$

Hence, $\Delta\, ABC$ is a right angled triangle.

DO YOU KNOW?

(i) There are special sets of numbers a, b and c that makes the Pythagorean relationship true and these sets of special numbers are called Pythagorean triplets. **Example:** (3, 4, 5) is a Pythagorean triplet.

(ii) Let **k** be any positive integer greater than 1 and (a, b, c) be a Pythagorean triplet, then (ka, kb, kc) is also a Pythagorean triplet.

Examples:

k	**(3,4,5)**	**(5,12,13)**
2k	(6,8,10)	(10,24,26)
3k	(9,12,15)	(15,36,39)
4k	(12,16,20)	(20,48,52)

So, it is clear that we can have infinitely many Pythagorean triplets just by multiplying any Pythagorean triplet by **k.**

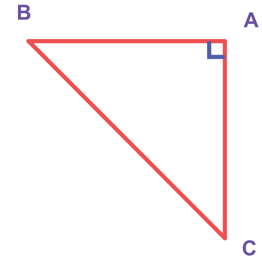
Try these

Check whether the following are Pythagorean triplets.

i) 57,176,185 ii) 264, 265, 23 iii) 8, 41,40

We shall now see a few examples on the use of Pythagoras theorem in problems.

Example: 1



In the figure, $AB \perp AC$

a) What type of Δ is ABC?

b) What are AB and AC of the $\Delta\, ABC$?

c) What is CB called as ?

d) If $AC = AB$ then, what is the measure of $\angle B$ and $\angle C$?

Solution

a) ΔABC is right angled as $AB \perp AC$ at A.

b) AB and AC are legs of ΔABC.

c) CB is called as the hypotenuse.

d) $\angle B + \angle C = 90^\circ$ and equal angles are opposite to equal sides. Hence, $\angle B = \angle C = \frac{90^\circ}{2} = 45^\circ$

Example: 2

Can a right triangle have sides that measure 5*cm*, 12*cm* and 13*cm?*

Solution:

Take $a = 5$, $b = 12$ and $c = 13$

Now, $a^2 + b^2 = 5^2 + 12^2 = 25 + 144 = 169 = 13^2 = c^2$

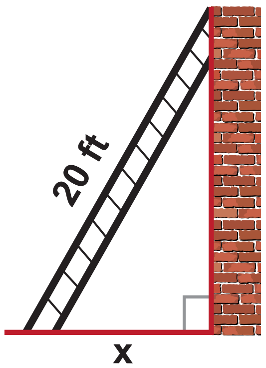
By the converse of Pythagoras theorem, the triangle with given measures is a right angled triangle.

Example: 3

A 20-feet ladder leans against a wall at height of 16 feet from the ground. How far is the base of the ladder from the wall?

Solution:

The ladder, wall and the ground form a right triangle with the ladder as the hypotenuse. From the figure, by Pythagoras theorem,



$$20^2 = 16^2 + x^2$$

$$\Rightarrow 400 = 256 + x^2$$

$$\Rightarrow x^2 = 400 - 256 = 144 = 12^2$$

$$\Rightarrow x = 12 \text{ feet}$$

Therefore, the base (foot) of the ladder is 12 feet away from the wall.

Activity-1

We can construct sets of Pythagorean triplets as follows.

Let **m** and **n** be any two positive integers (m > n):

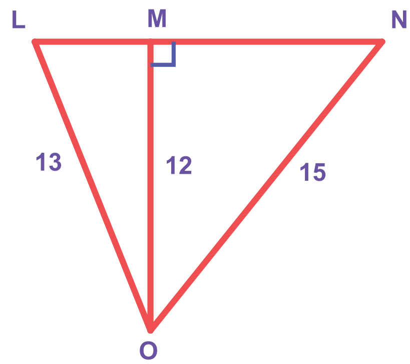
(a, b, c) is a Pythagorean triple if $a = m^2 - n^2, b = 2mn$ and $c = m^2 + n^2$ (Think, why?)

Complete the table.

m	n	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$	Pythagorean triplet
2	1	___	___	___	______
3	2	___	___	___	______
4	1	15	8	17	(15, 8, 17)
7	2	45	28	53	(45, 28, 53)

Example: 4

Find LM, MN, LN and also the area of $\Delta\ LON$.



Solution:

From ΔLMO, by Pythagoras theorem,

$$LM^2 = OL^2 - OM^2$$

$$\Rightarrow LM^2 = 13^2 - 12^2 = 169 - 144 = 25 = 5^2$$

$$\therefore\ LM = 5\ units$$

From ΔNMO, by Pythagoras theorem,

$$MN^2 = ON^2 - OM^2$$

$$= 15^2 - 12^2 = 225 - 144 = 81 = 9^2$$

$$\therefore\ MN = 9\ units$$

Hence, $LN = LM + MN = 5 + 9 = 14\ units$

$$\text{Area of } \Delta\ LON = \frac{1}{2} \times base \times height$$

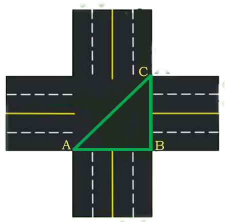
$$= \frac{1}{2} \times LN \times OM$$

$$= \frac{1}{2} \times 14 \times 12$$

$$= 84\ square\ units.$$

Example: 5

A junction where two roads intersect at right angles is as shown in the figure. Find AC if AB = 8 m and BC = 15 m.



Solution:

Now ΔABC is right angled.

Therefore, by Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 8^2 + 15^2 = 64 + 225 = 289$$

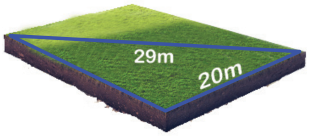
$$AC^2 = 17^2$$

$$\Rightarrow\ AC = 17m.$$

Therefore, the length of the diagonal of the two intersecting roads is 17 m.

Example: 6

Find the area of a rectangular plot of land shown in the figure.



Solution:

Here, the hypotenuse is 29 m. One side of the right triangle is 20 m. let the other side be 'l' m

Therefore, by Pythagoras theorem,

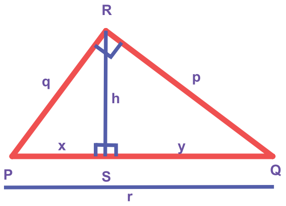
$$l^2 = 29^2 - 20^2 = 841 - 400 = 441 = 21^2$$

$\therefore\ l = 21m$

Therefore, the area rectangular plot of land $= l \times b$ square units.

$= 20 \times 21 = 210\ m^2$.

3.4 The Altitude –on –Hypotenuse theorem (without proof)



Here, the hypotenuse of a right angle triangle is taken as its base. Draw an altitude to it (as given in the figure). We will have two more, smaller right triangles. Isn't it? Now, all these three triangles are similar! Aren't they? (check!) We now state the theorem.

Theorem:

If an altitude is drawn to the hypotenuse of an right angled triangle, then

(i) The two triangles are similar to the given triangle and also to each other.

That is, $\Delta PRQ \sim \Delta PSR \sim \Delta RSQ$

(ii) $h^2 = xy$

(iii) $p^2 = yr$ and $q^2 = xr$, where $r = x+y$

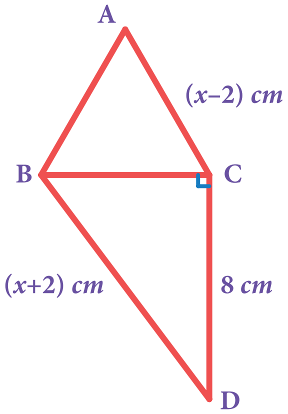
Note

It is to be noted that iii) is just one formula or relationship and not two formulae. It can be remembered as

(side of big triangle)^2 = (part of hypotenuse next to side) × (whole of the hypotenuse)

Example: 7

Δ ABC is equilateral and CD of the right triangle BCD is 8*cm*.
Find the side of the equilateral Δ ABC and also BD.



Solution:

As Δ ABC is equilateral from the figure, AB=BC=AC= $(x-2)cm$.

$\therefore$ From ΔBCD, by Pythagoras theorem

$$BD^2 = BC^2 + CD^2$$

$$\Rightarrow (x+2)^2 = (x-2)^2 + 8^2$$

$$x^2 + 4x + 4 = x^2 - 4x + 4 + 8^2$$

$$\Rightarrow 8x = 8^2$$

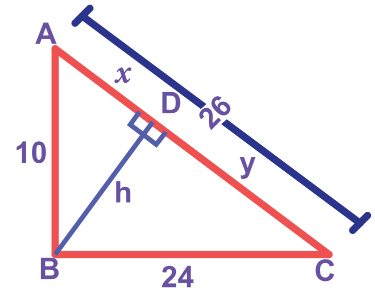
$$\Rightarrow x = 8\,cm$$

$\therefore$ The side of the equilateral $\Delta ABC = 6\,cm$ and BD =10*cm*.

Example: 8

From the figure, find x and y and verify $\Delta\ ABC$ is a right angled triangle.

Solution:



Now, by altitude-on-hypotenuse theorem,

$AB^2 = AD \times AC$ *gives*,

$10^2 = x \times 26$

$\Rightarrow x = \frac{100}{26} = \frac{50}{13}$ *units* and

$BC^2 = CD \times AC$ *gives*,

$24^2 = y \times 26$

$\Rightarrow y = \frac{576}{26} = \frac{288}{13}$ *units* and

In $\Delta\ ABC,\ AB^2 + BC^2 = 10^2 + 24^2 = 676 = 26^2 = AC^2$.

Therefore, $\Delta\ ABC$ is a right angled triangle.

Activity-2

Find all integer-sided right angled triangles with hypotenuse 85.

M5M9T7

Exercise 3.1

1. **Fill in the blanks:**

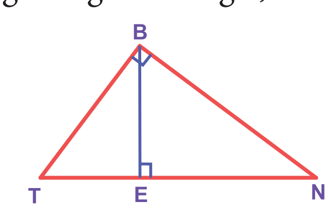
(i) If in a $\Delta PQR, PR^2 = PQ^2 + QR^2$, then the right angle of Δ PQR is at the vertex ________.

(ii) If 'l' and 'm' are the legs and 'n' is the hypotenuse of a right angled triangle then, $l^2 =$ ________.

(iii) If the sides of a triangle are in the ratio 5:12:13 then, it is ________.

(iv) If a perpendicular is drawn to the hypotenuse of a right angled triangle, then each of the three pairs of triangles formed are ________.

(v) In the figure, $BE^2 =$ TE $\times$ _____.



2. **Say True or False.**

(i) 8, 15, 17 is a Pythagorean triplet.

(ii) In a right angled triangle, the hypotenuse is the greatest side.

(iii) One of the legs of a right angled triangle PQR having $\angle R = 90^\circ$ is PQ.

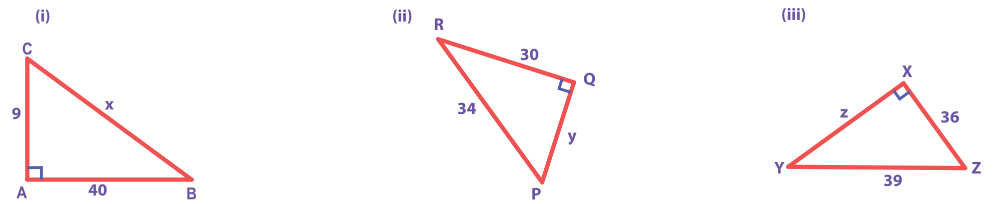
(iv) The hypotenuse of a right angled triangle whose sides are 9 and 40 is 49.

(v) Pythagoras theorem is true for all types of triangles.

3. Check whether given sides are the sides of right-angled triangles, using Pythagoras theorem.

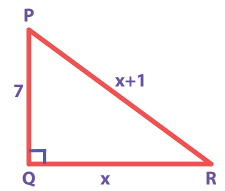
(i) 8,15,17 (ii) 12,13,15 (iii) 30,40,50 (iv) 9,40,41 (v) 24,45,51

4. Find the unknown side in the following triangles.

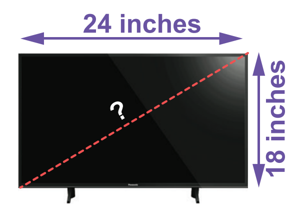


5. An isosceles triangle has equal sides each 13*cm* and a base 24*cm* in length. Find its height.

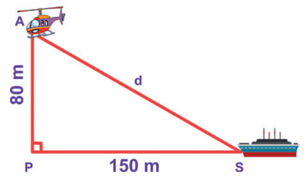
6. In the figure, find PR and QR.



7. The length and breadth of the screen of an LED-TV are 24 inches and 18 inches. Find the length of its diagonal.

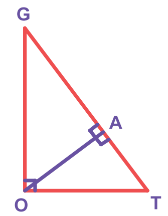


8. Find the distance between the helicopter and the ship.

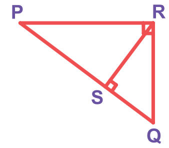


9. From the figure,

(i) If TA=3*cm* and OT =6*cm*, find TG.

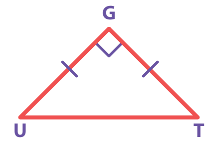


10. If RQ = 15*cm* and RP = 20*cm*, find PQ, PS and SQ.



Objective Type Questions

11. If Δ GUT is isosceles and right angled, then $\angle TUG$ is ________.
(a) 30° (b) 40° (c) 45° (d) 55°



12. The hypotenuse of a right angled triangle of sides 12*cm* and 16*cm* is __________.
(a) 28*cm* (b) 20*cm* (c) 24*cm* (d) 21*cm*

13. The area of a rectangle of length 21*cm* and diagonal 29*cm* is __________cm^2.
(a) 609 (b) 580 (c) 420 (d) 210

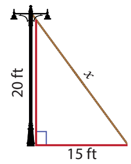
14. If the square of the hypotenuse of an isosceles right triangle is 50cm^2, the length of each side is ____________.
(a) 25*cm* (b) 5*cm* (c) 10*cm* (d) 20*cm*

15. The sides of a right angled triangle are in the ratio 5:12:13 and its perimeter is 120 units then, the sides are ______________.
(a) 25, 36, 59 (b) 10,24,26 (c) 36, 39, 45 (d) 20,48,52

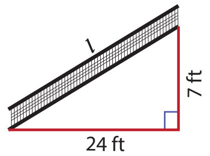
Exercise 3.2

Miscellaneous Practice Problems

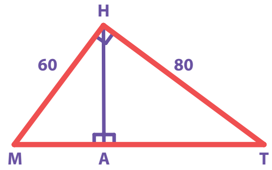
1. The sides of a triangle are $1.2cm$, $3.5cm$ and $3.7cm$. Is this triangle a right triangle? If so, which side is the hypotenuse?
2. Rithika buys an LED TV which has a 25 inches screen. If its height is 7 inches, how wide is the screen? Her TV cabinet is 20 inches wide. Will the TV fit into the cabinet? Why?
3. Find the length of the support cable required to support the tower with the floor.



4. A ramp is constructed in a hospital as shown. Find the length of the ramp.

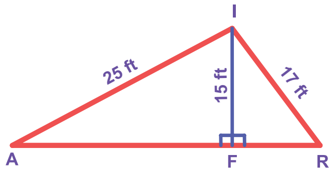


5. In the figure, find MT and AH.

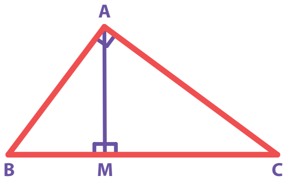


Challenging problems

6. Mayan travelled 28 km due north and then 21 km due east. What is the least distance that he could have travelled from his starting point?
7. If ΔAPK is an isosceles right angled triangle, right angled at K. Prove that $AP^2 = 2AK^2$.
8. The diagonals of the rhombus is $12cm$ and $16cm$. Find its perimeter. (Hint: the diagonals of rhombus bisect each other at right angles).
9. In the figure, find AR.



10. ΔABC is a right angled triangle in which $\angle A = 90°$ and AM $\perp$ BC. Prove that $AM = \dfrac{AB \times AC}{BC}$. Also if AB=30$cm$ and AC=40cm, find AM.

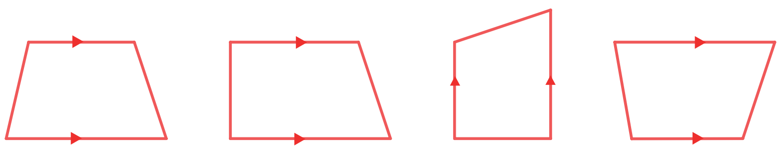


3.5 Construction of Trapeziums

In the first term, we have learnt how to construct the quadrilaterals. To draw a quadrilateral, how many measurements do you need? 5 measurements. Isn't it? Let us see the special quadrilaterals which need less than 5 measurements. Based on the nature of sides and angles of a quadrilateral, it gets special names like trapezium, parallelogram, rhombus, rectangle, square and kite.

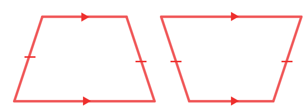
Now, you will learn how to construct trapeziums.

Trapezium is a quadrilateral in which a pair of opposite sides are parallel. To construct a trapezium, draw one of the parallel sides as a base and on that base construct a triangle with the 2 more measurements. Now, through the vertex of that triangle, construct the parallel line opposite to the base so that the triangle lies between the parallel sides. As the fourth vertex lies on this parallel line, mark it with the remaining measure. Hence, we need four independent measures to construct a trapezium. The given shapes are examples of trapeziums.



Note: The arrow marks in the above shapes represent parallel sides.

If the non-parallel sides of a trapezium are equal in length and form equal angles at one of its bases, then it is called an **isosceles trapezium**.



DO YOU KNOW?

In the US and UK, Trapezoid and Trapezium are differentiated as below.

	Trapezoid	Trapezium
US	a quadrilateral with a pair of parallel sides .	a quadrilateral with no parallel sides.
UK	a quadrilateral with no parallel sides.	a quadrilateral with a pair of parallel sides.

Try these

1. The area of the trapezium is __________.
2. The distance between the parallel sides of a trapezium is called as __________.
3. If the height and parallel sides of a trapezium are $5cm$, $7cm$ and $5cm$ respectively, then its area is __________.
4. In an isosceles trapezium, the non-parallel sides are __________ in length.
5. To construct a trapezium, __________ measurements are enough.
6. If the area and sum of the parallel sides are $60cm^2$ and $12cm$, its height is __________.

Let us construct a trapezium with the given measurements

1. Three sides and one diagonal.
2. Three sides and one angle.
3. Two sides and two angles.
4. Four sides.

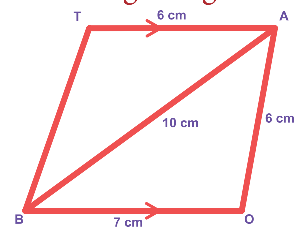
3.5.1 Constructing a trapezium when its three sides and one diagonal are given:

Example: 9

Construct a trapezium **BOAT** in which $\overline{BO}$ is parallel to $\overline{TA}$, BO=7*cm*, OA=6*cm*, BA=10*cm* and TA=6*cm*. Also find its area.

Solution :

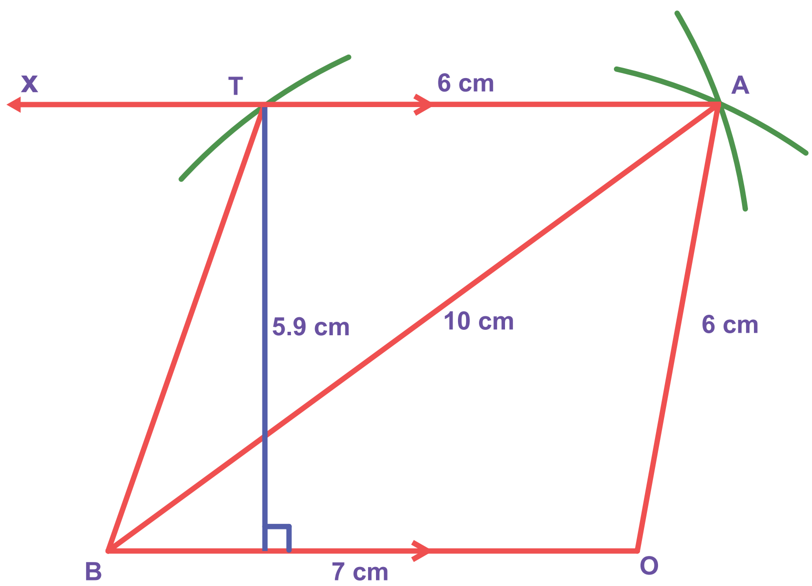
Rough diagram



Given:

BO=7*cm*, OA=6*cm*, BA=10*cm* ,

TA=6*cm* and $\overline{BO} \parallel \overline{TA}$



Steps:

1. Draw a line segment BO = 7*cm*.
2. With B and O as centres, draw arcs of radii 10*cm* and 6*cm* respectively and let them cut at A.
3. Join BA and OA.
4. Draw AX parallel to BO
5. With A as centre, draw an arc of radius 6*cm* cutting AX at T.
6. Join BT. BOAT is the required trapezium.

Calculation of area:

Area of the trapezium BOAT $= \frac{1}{2} \times h \times (a+b)\ sq.units$

$= \frac{1}{2} \times 5.9 \times (7+6) = 38.35\ sq.cm$

3.5.2 Constructing a trapezium when its three sides and one angle are given:

Example: 10

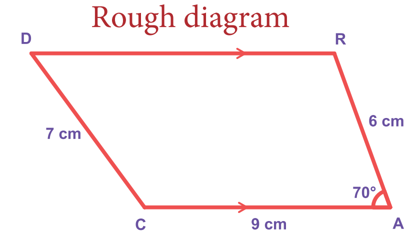
Construct a trapezium **CARD** in which $\overline{CA}$ is parallel to $\overline{DR}$, CA=9*cm*, $\angle CAR = 70$, AR=6*cm* and CD=7*cm*. Also find its area.

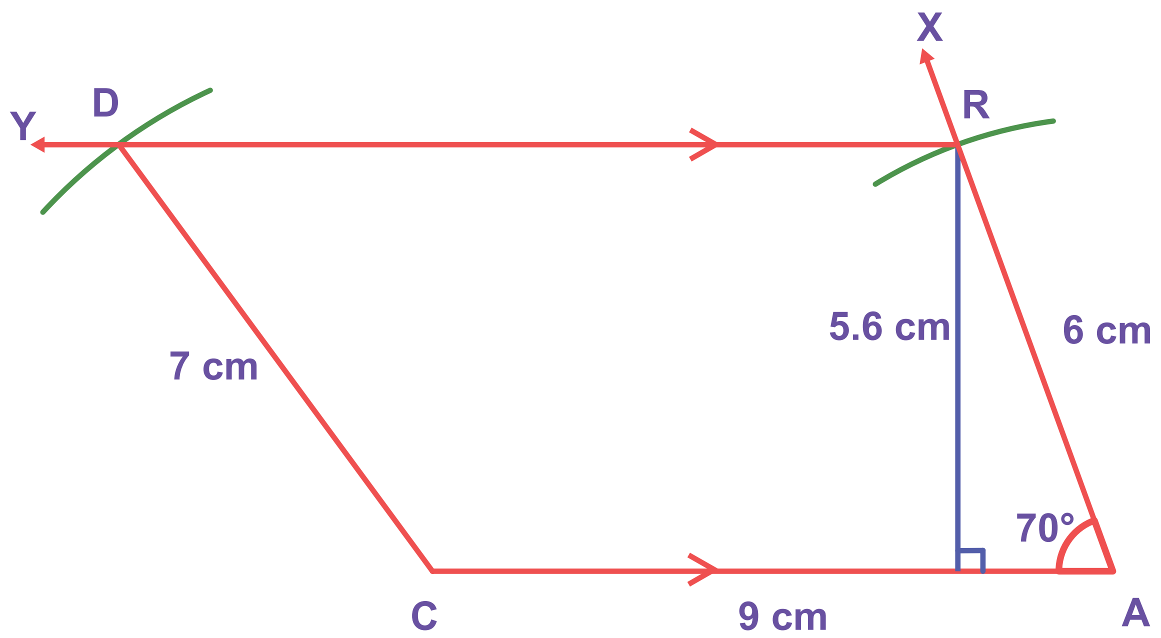
Solution:

Given:

CA=9*cm*, $\angle CAR = 70°$ AR=6*cm*, and CD=7*cm* and $\overline{CA} \parallel \overline{DR}$

Rough diagram





Steps:

1. Draw a line segment CA= 9*cm*.
2. Construct an angle $\angle CAX = 70°$ at A.
3. With A as centre, draw an arc of radius 6*cm* cutting AX at R.
4. Draw RY parallel to CA.
5. With C as centre, draw an arc of radius 7*cm* cutting RY at D.
6. Join CD. CARD is the required trapezium.

Calculation of area:

Area of the trapezium CARD $= \frac{1}{2} \times h \times (a+b)\, sq.units$

$= \frac{1}{2} \times 5.6 \times (9+11) = 56\, sq.cm$

3.5.3 Constructing a trapezium when its two sides and two angles are given:

Example: 11

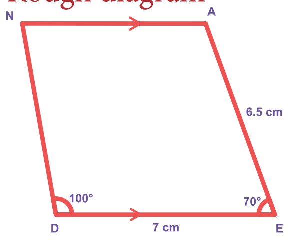
Construct a trapezium DEAN in which $\overline{DE}$ is parallel to $\overline{NA}$, DE=7cm, EA=6.5cm $\angle EDN = 100°$ and $\angle DEA = 70°$. Also findits area.

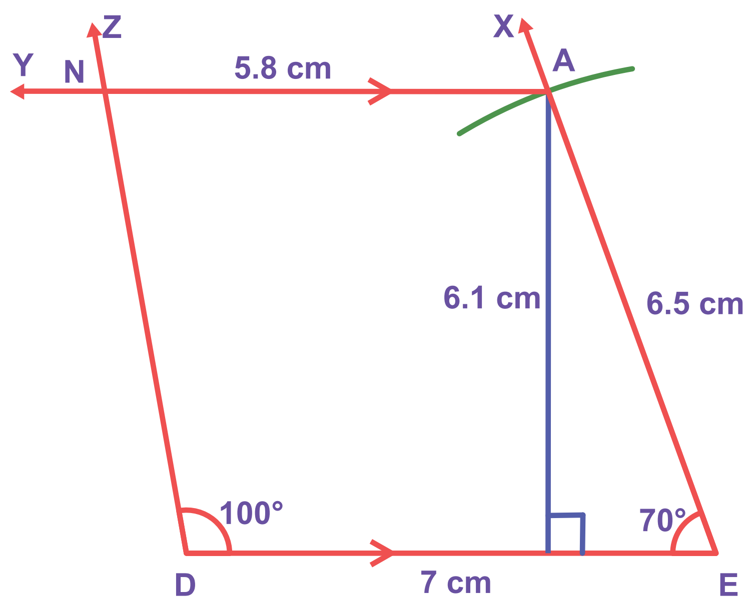
Solution :

Given:

DE=7cm, EA=6.5cm $\angle EDN = 100°$ and $\angle DEA = 70°$ and $\overline{DE} \,||\, \overline{NA}$

Rough diagram





Steps:

1. Draw a line segment DE= 7cm.
2. Construct an angle $\angle DEX = 70°$ at E.
3. With E as centre draw an arc of radius 6.5cm cutting EX at A.
4. Draw AY parallel to DE.
5. Construct an angle $\angle EDZ = 100°$ at D cutting AY at N.
6. DEAN is the required trapezium.

Calculation of area:

Area of the trapezium DEAN $= \frac{1}{2} \times h \times (a+b)\, sq.units$

$= \frac{1}{2} \times 6.1 \times (7+5.8) = 39.04\, sq.cm$

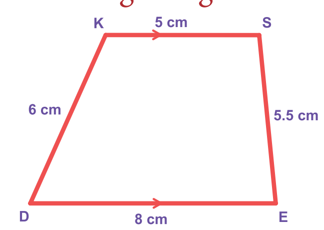
3.5.4 Constructing a trapezium when its four sides are given:

Example: 12

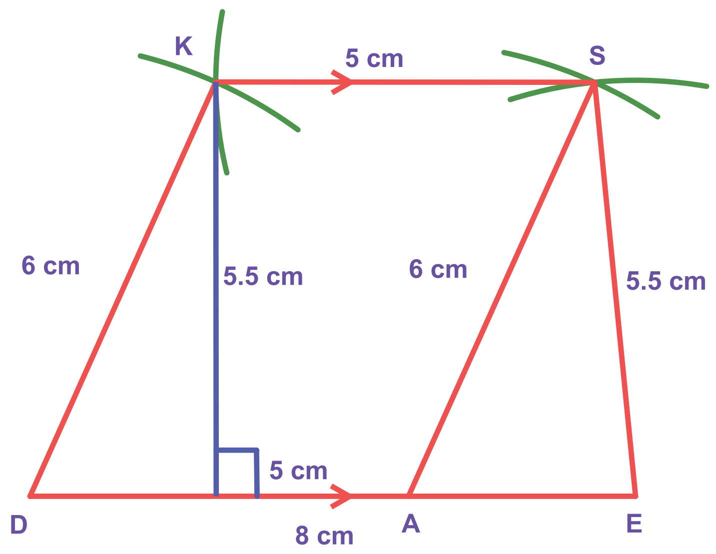
Construct a trapezium DESK in which $\overline{DE}$ is parallel to $\overline{KS}$, DE=8*cm*, ES=5.5*cm*, KS =5*cm* and KD=6*cm*. Find also its area.

Solution:

Rough diagram



Given:
DE=8*cm*, ES=5.5*cm*, KS =5*cm*, KD=6*cm* and $\overline{DE} \parallel \overline{KS}$



Steps:

1. Draw a line segment DE= 8*cm*.
2. Mark the point A on DE such that DA=5*cm*.
3. With A and E as centres, draw arcs of radii 6*cm* and 5.5*cm* respectively. Let them cut at S. Join AS and ES.
4. With D and S as centres, draw arcs of radii 6*cm* and 5*cm* respectively. Let them cut at K. Join DK and KS.
5. DESK is the required trapezium.

Calculation of area:

Area of the trapezium DESK $= \frac{1}{2} \times h \times (a+b)\, sq.units$

$= \frac{1}{2} \times 5.5 \times (8+5) = 35.75\, sq.cm$

Exercise 3.3

I. Construct the following trapeziums with the given measures and also find their area.

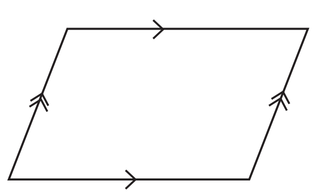
1. AIMS with $\overline{AI} \parallel \overline{SM}$, AI=6*cm*, IM=5*cm*, AM=9*cm* and MS=6.5*cm*.
2. BIKE with $\overline{BI} \parallel \overline{EK}$, BI=4*cm*, IK=3.5*cm*, BK=6*cm* and BE=3.5*cm*.
3. CUTE with $\overline{CD} \parallel \overline{ET}$, CU=7*cm*, $\angle UCE = 80°$ CE=6*cm* and TE=5*cm*.
4. DUTY with $\overline{DU} \parallel \overline{YT}$, DU= 8*cm*, $\angle DUT = 60°$,UT = 6*cm* and TY= 5*cm*.
5. ARMY with $\overline{AR} \parallel \overline{YM}$, AR=7*cm*, RM=6.5*cm* $\angle RAY = 100°$ and $\angle ARM = 60°$
6. BELT with $\overline{BE} \parallel \overline{TL}$, BE=10*cm*, BT=7*cm* $\angle EBT = 85°$ and $\angle BEL = 110°$.
7. CITY with $\overline{CI} \parallel \overline{YT}$, CI=7*cm*, IT=5.5*cm*, TY=4*cm* and YC=6*cm*.
8. DICE with $\overline{DI} \parallel \overline{EC}$, DI=6*cm*, IC= ED=5*cm* and CE=3*cm*.

3.6 Construction of Parallelograms

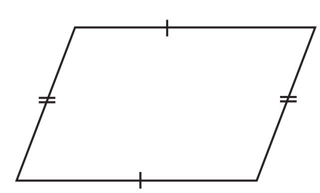
In the previous session, we have learnt how to construct trapeziums. In a trapezium, a pair of opposite sides are parallel, Isn't it? What about the other pair? They are non-parallel sides. Can you guess the quadrilaterals which have both pairs of opposite sides as parallel? Yes, you call them as parallelogram, rhombus, rectangle and square.

Here, we will come to know about parallelograms. **A parallelogram is a quadrilateral in which the opposite sides are parallel.** To construct a parallelogram, draw a triangle with the given measurements. Then, the fourth vertex is found by using the measurements as same as the adjacent sides. Hence, three independent measurements are enough to construct a parallelogram.

Note



Similar arrows indicates the parallel sides.



Similar lines indicates the congruent sides.

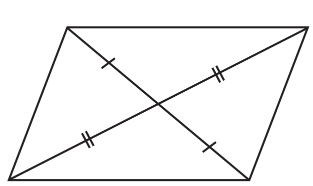


Figure shows that the diagonals bisect each other.

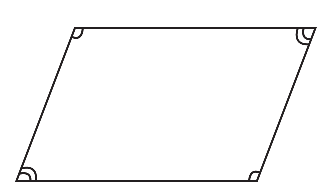


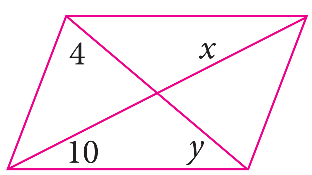
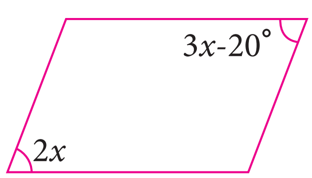
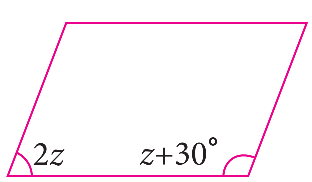
Figure shows that the opposite angles are congruent.

Think

Can a rhombus, a square or a rectangle be called as a parallelogram? Justify your answer.

Try these

1. In a parallelogram, the opposite sides are ___________ and ____________.
2. If $\angle A$ of a parallelogram ABCD is 100° then, find $\angle B, \angle C$ and $\angle D$.
3. Diagonals of a parallelogram ___________ each other.
4. If the base and height of the parallelogram are 20*cm* and 5*cm* then, its area is _____.
5. Find the unknown values in the given parallelograms and write the property used to find them.

	1. x = _________________. 2. Property used =________________.
	1. x = _________________. 2. y = _________________. 3. Property used =________________.
	1. x = _________________. 2. Property used =__________________.
	1. z = _________________. 2. Property used =________________.

Let us construct a parallelogram with the given measurements

1. Two adjacent sides and one angle.
2. Two adjacent sides and one diagonal.
3. Two diagonals and one included angle.
4. One side, one diagonal and one angle.

3.6.1 Constructing a parallelogram when its two adjacent sides and one angle are given:

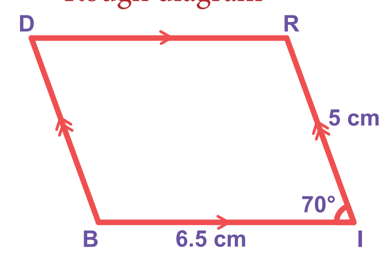
Example: 13

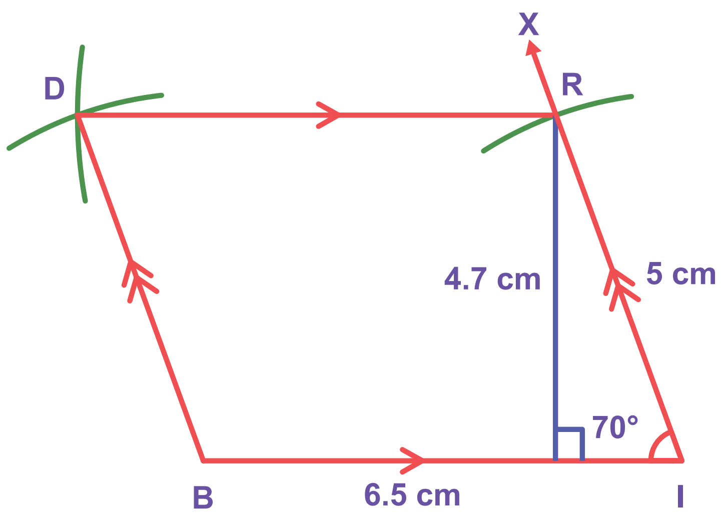
Construct a parallelogram BIRD with BI=6.5cm, IR=5cm and $\angle BIR = 70^\circ$. Also find its area.

Solution:

Given:

BI=6.5cm, IR=5cm and $\angle BIR = 70^\circ$





Steps:

1. Draw a line segment BI=6.5cm.
2. Make an angle $\angle BIX = 70^\circ$ at I on $\overline{BI}$.
3. With I as centre, draw an arc of radius 5cm cutting IX at R.
4. With B and R as centres, draw arcs of radii 5cm and 6.5cm respectively. Let them cut at D.
5. Join BD and RD.
6. BIRD is the required parallelogram.

Calculation of area:

Area of the parallelogram BIRD $= bh$ sq.units

$= 6.5 \times 4.7 = 30.55\, sq.cm$

3.6.2 Constructing a parallelogram when its two adjacent sides and one diagonal are given:

Example: 14

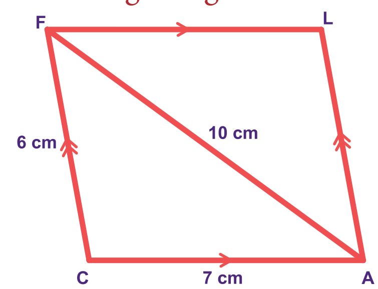
Construct a parallelogram CALF with CA=7*cm*, CF=6*cm* and AF=10*cm*. Also find its area.

Solution :

Given:

CA=7*cm*, CF=6*cm* and AF=10*cm*

Rough diagram



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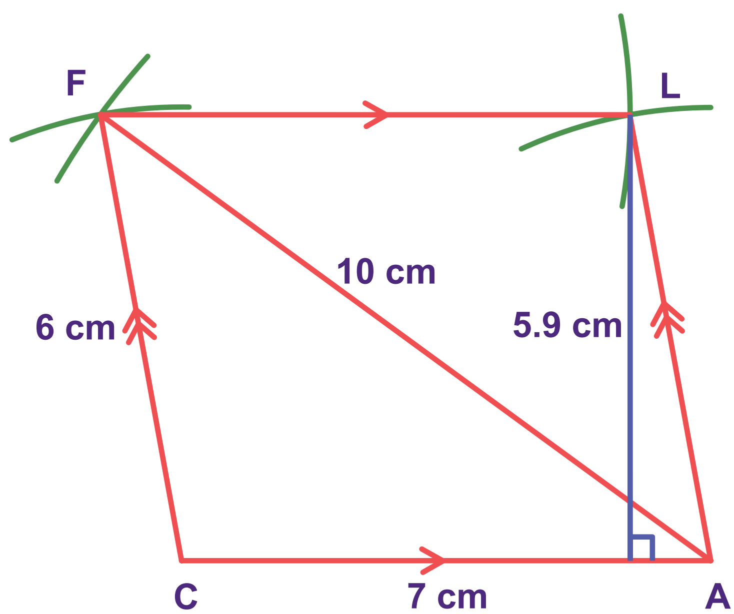


Steps:

1. Draw a line segment CA=7*cm*.
2. With C and A as centres, draw arcs of radii 7*cm* and 6*cm* respectively. Let them cut at F.
3. Join CF and AF.
4. With A and F as centres, draw arcs of radii 6*cm* and 7*cm* respectively. Let them cut at L.
5. Join AL and FL.
6. CALF is the required parallelogram.

Calculation of area:

Area of the parallelogram CALF = bh sq.units

$= 7\times 5.9 = 41.3\, sq.cm$

3.6.3 Constructing a parallelogram when its two diagonals and one included angle are given:

Example: 15

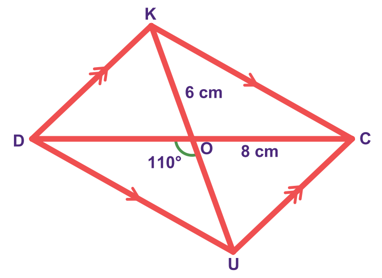
Construct a parallelogram DUCK with DC=8*cm*, UK=6*cm* and $\angle DOU = 110°$. Also find its area.

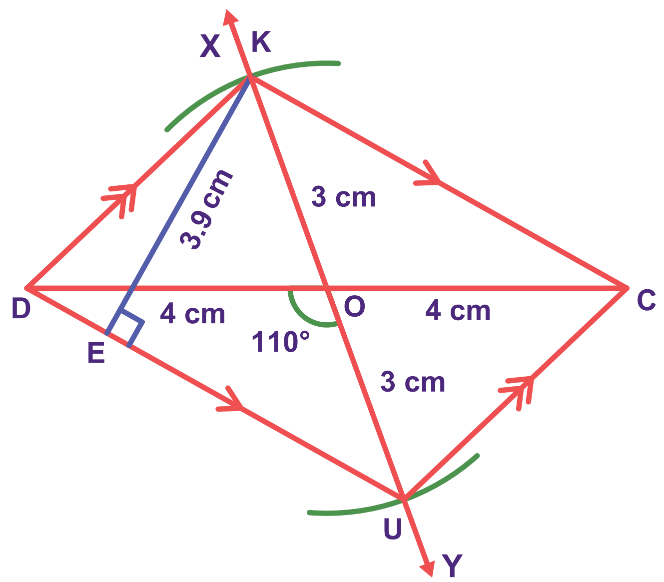
Solution :

Given:

DC=8*cm*, UK=6*cm* and $\angle DOU = 110°$

Rough diagram





Steps:

1. Draw a line segment DC=8*cm*.
2. Mark O the midpoint of $\overline{DC}$.
3. Draw a line $\overleftrightarrow{XY}$ through O which makes $\angle DOY = 110°$.
4. With O as centre and 3*cm* as radius draw two arcs on $\overleftrightarrow{XY}$ on either sides of $\overline{DC}$. Let the arcs cut $\overrightarrow{OX}$ at K and $\overrightarrow{OY}$ at U
5. Join $\overline{DU}$, $\overline{UC}$, $\overline{CK}$ and $\overline{KD}$.
6. DUCK is the required parallelogram.

Calculation of area:

Area of the parallelogram DUCK = bh sq.units

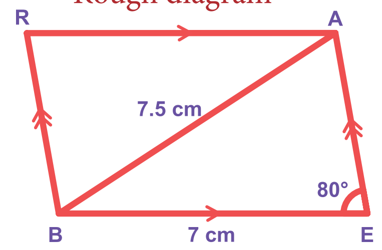
$= 5.8 \times 3.9 = 22.62\, sq.cm$

3.6.4 Constructing a parallelogram when its one side, one diagonal and one angle are given:

Example: 16

Construct a parallelogram BEAR with BE=7cm, BA=7.5cm and $\angle BEA = 80°$. Also find its area.

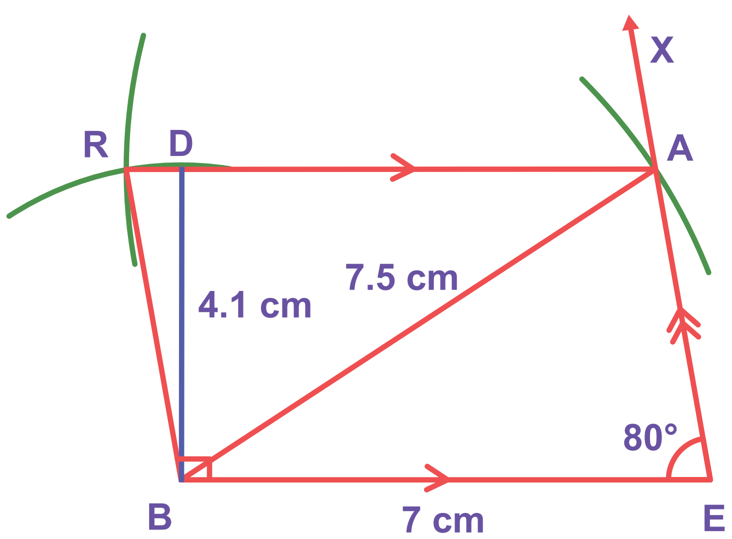
Rough diagram



Solution:

Given:

BE=7cm, BA=7.5cm and $\angle BEA = 80°$



Steps:

1. Draw a line segment BE=7cm.
2. Make an angle $\angle BEX = 80°$ at E on $\overline{BE}$.
3. With B as centre, draw an arc of radius 7.5cm cutting EX at A and Join BA.
4. With B as centre, draw an arc of radius equal to the length of $\overline{AE}$.
5. With A as centre, draw an arc of radius 7cm. Let both arcs cut at R.
6. Join BR and AR.
7. BEAR is the required parallelogram.

Calculation of area:

Area of the parallelogram BEAR = bh sq.units

$= 7 \times 4.1 = 28.7\ sq.cm$

Exercise 3.4

I. Construct the following parallelograms with the given measurements and find their area.

1. ARTS, AR=6cm, RT=5cm and $\angle ART = 70°$.
2. BANK, BA=7cm, BK=5.6cm and $\angle KBA = 85°$.
3. CAMP, CA=6cm, AP=8cm and CP=5.5cm.
4. DRUM, DR=7cm, RU=5.5cm and DU=8cm.
5. EARN, ER=10cm, AN=7cm and $\angle EOA = 110°$ where $\overline{ER}$ and $\overline{AN}$ intersect at O.
6. FAIR, FI=8cm, AR=6cm and $\angle IOR = 80°$ where $\overline{FI}$ and $\overline{AR}$ intersect at O.
7. GAIN, GA=7.5cm, GI=9cm and $\angle GAI = 100°$.
8. HERB, HE=6cm, $\angle EHB = 60°$ and EB=7cm.

Summary

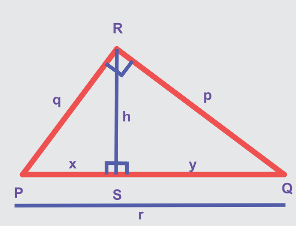
- In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. This is Pythagoras theorem. ΔABC, $BC^2 = AB^2 + AC^2$
- If in a triangle the square on the greatest side is equal to the sum of squares on the other two sides, then the triangle is right angled triangle. This is converse of Pythagoras theorem.
- If an altitude is drawn to the hypotenuse of an right angled triangle, then

  (i) The two triangles are similar to the given triangle and also to each other.

  That is, $\Delta PRQ \sim \Delta PSR \sim \Delta RSQ$

  (ii) $h^2 = xy$

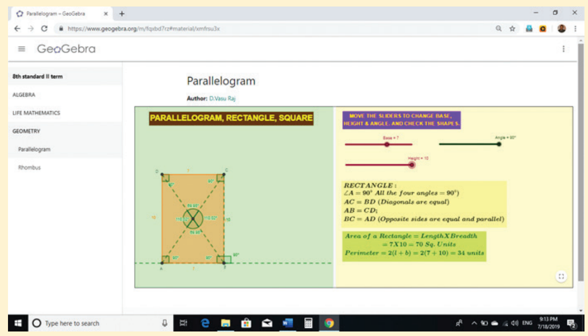
  (iii) $p^2 = yr$ and $q^2 = xr$, where $r = x+y$



- A trapezium is a quadrilateral in which a pair of opposite sides are parallel.
- A parallelogram is a quadrilateral in which the opposite sides are parallel.

ICT CORNER

Expected Outcome

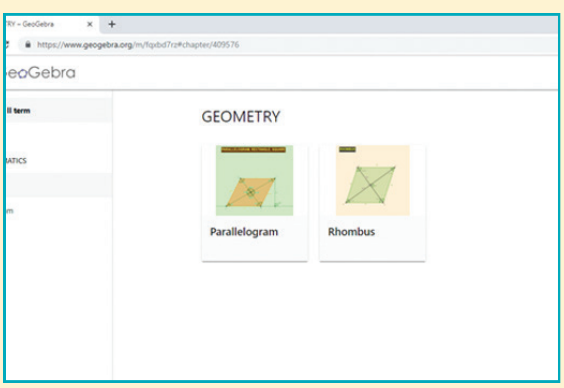


Step – 1

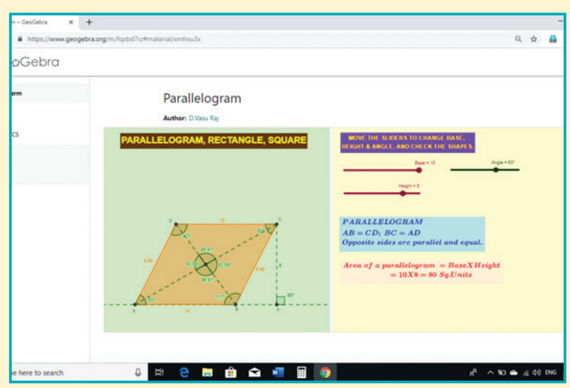
Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named “GEOMETRY” will open. Click on the worksheet named “Parallelogram”.

Step - 2

In the given worksheet you can move the sliders Base, Height and the angle. Check for what value(s) the parallelogram becomes rectangle and square. Study the properties.



Step – 1



Step – 2

Go through the remaining worksheets given for this chapter

Browse in the link

Geometry:

https://www.geogebra.org/m/fqxbd7rz#chapter/409576

or Scan the QR Code.

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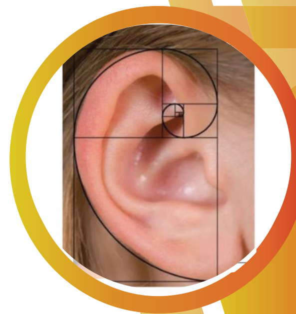
4 INFORMATION PROCESSING



Learning Objectives:

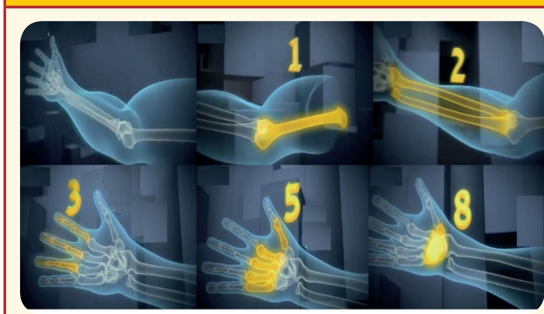
- To observe the Fibonacci pattern in physical and biological phenomena.
- To think critically and choose the best method in finding the HCF of numbers.
- To understand how information can be processed, encrypted and decrypted.

4.1 Introduction

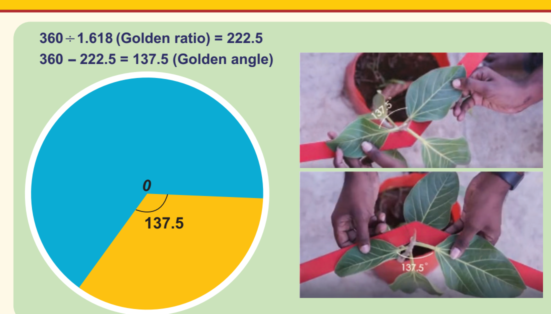
We have learnt in earlier classes, on how all beautiful things in nature as well as man made things are connected with Mathematics. Now, we just refresh everyone's memory and show how Math can be beautiful when seen in physical and biological things everywhere around us.

From each cell's nucleus to the infinitely winding spirals of galaxies that are immeasurable. There is one golden process that nearly all wonders of nature depend upon. What is the number sequence that we find in each of these instances, that is so instrumental in living things? Yes, that is the Fibonacci sequence. Let us learn more about it.

Mathematics Alive – Information Processing in our Real life



Biological example for the Fibonacci numbers



Verifying Golden angle values in nature

4.2 Fibonacci Numbers

Fibonacci (real name Leonardo Bonacci) was a Italy mathematician who developed the Fibonacci Sequence. It looks like this: 1, 1, 2, 3, 5, 8, 13, 21, 34... and it goes on. Remember the pattern of the Fibonacci sequence we already studied in standard VI.

Let us tabulate the Fibonacci sequence and find a rule.

Term (n)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	...
F (n)	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	...

F(1)	F(2)	F(3)	F(4)	F(5)	F(6)	F(7)	...
1	1	2	3	5	8	13	...

+ =

F(3) = F(2)+F(1)

We observe that the 3rd term of the Fibonacci sequence is the sum of **2nd term** and the **1st term**.

That is, **F(3) = F(2) + F(1)** and so we can extend and write the rule is **F(n)= F(n–1) + F(n–2)**

where **F(n)** is the **nth** term

F(n–1) is the previous term to the **nth** term

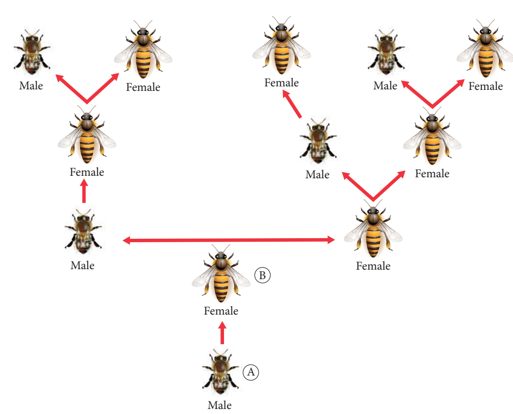
F(n–2) is the term before the **(n–1)th** term

This is how the Fibonacci Sequence is found. Let us learn more from the following real life examples .

Situation

Let us look at the family tree of a male drone bee and a female bee given below.

Here, female bees have 2 parents, a male (drone) and a female whereas male (drone) bees have just one parent, a female. (Males(drone) are produced by the queen's unfertilized eggs, so male(drone) bees only have a mother but no father!)



From the picture the following points are noted:

1. The male (A) has **1** parent, a female (B).
2. The male (A) has **2** grandparents, since his mother had two parents, a male (A) and a female (B).
3. The male has **3** great-grandparents: his grandmother had two parents but his grandfather had only one.

Now, answer, how many great-great-grandparents did the male (A) have?

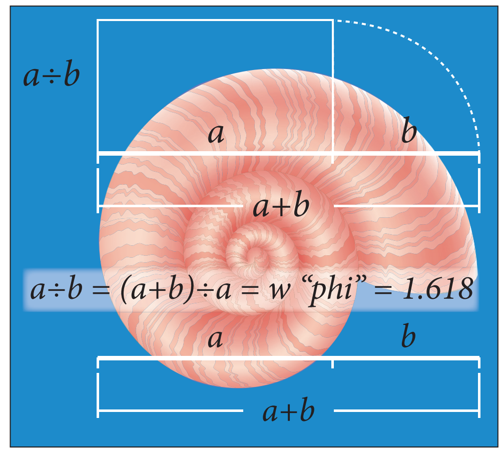
Let us try to find the relationship among the pattern of bees family by representing in the tabular form given below,

Number of	Parents	Grandparents	Great Grandparents	Great- Great Grandparents	Great- Great-Great Grandparents
a Male bee	1	2	3	5	8
a Female bee	2	3	5	8	13

We see the Fibonacci numbers 1, 1, 2, 3, 5,… in the above table.

Bee population aren't the only place in nature where Fibonacci numbers occur, they also appear in the beautiful shapes of trees, leaves, seeds, shells, storms and you… Yes! you are an example of the beauty of the Fibonacci Sequence. The human body has various representations of the Fibonacci Sequence proportions, from your face to your ear to your hands and beyond! You have now been proven to be mathematically gorgeous, so go forth and be beautiful! ...and maybe think math is a little bit better than you first thought? Let us, learn about some more forms of the Fibonacci sequence.

4.2.1. Patterns in the Fibonacci sequence:



When we take any two successive (one after the other) Fibonacci numbers and divide the larger by the smaller number the result of dividing the pairs of numbers gives us the approximate value of the **Golden ratio "phi" (φ) = 1.618. 1÷1=1,2÷1=2, 3÷2=1.5, 5÷3=1.666.., 8÷5=1.6... 233÷144=1.618 ....** and so on.

This diagram gives us an idea of the proportions of a Golden Rectangle. It is divided into two pieces, and the ratio of the two parts (a to b) is the **Golden Ratio**. If we investigate even further, we will find that the two parts together (a + b) is the same ratio to just the left part (a)for example, let us take 8^{th} and 9^{th} Fibonacci numbers where a = 34 and b = 21

(i) a÷b=34÷21=1.61

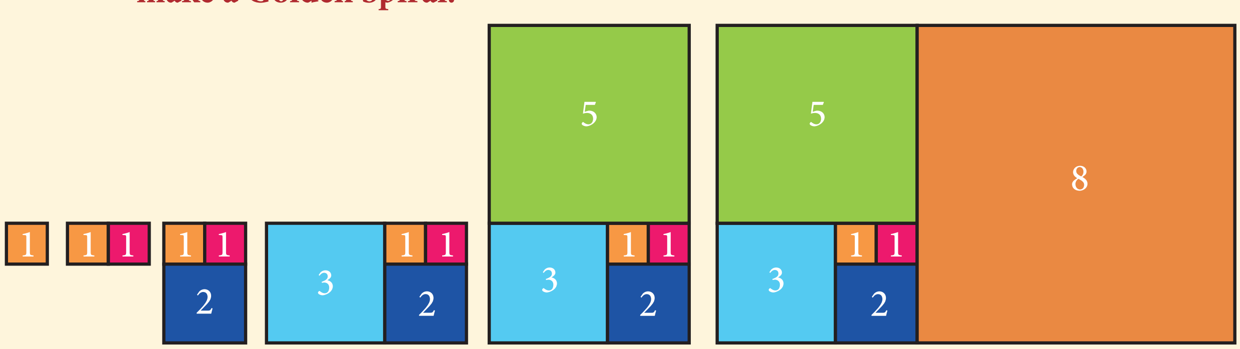
(ii) (a+b)÷b=(21+34)÷ 34=1.61

Therefore we conclude that **a÷b=(a+b)÷b= (φ) = 1.61**

The Fibonacci numbers will also create the **Golden Spiral** as shown in the snail picture. What does the Fibonacci sequence have to do with golden rectangles and the golden ratio? Do the following activity to know the connection.

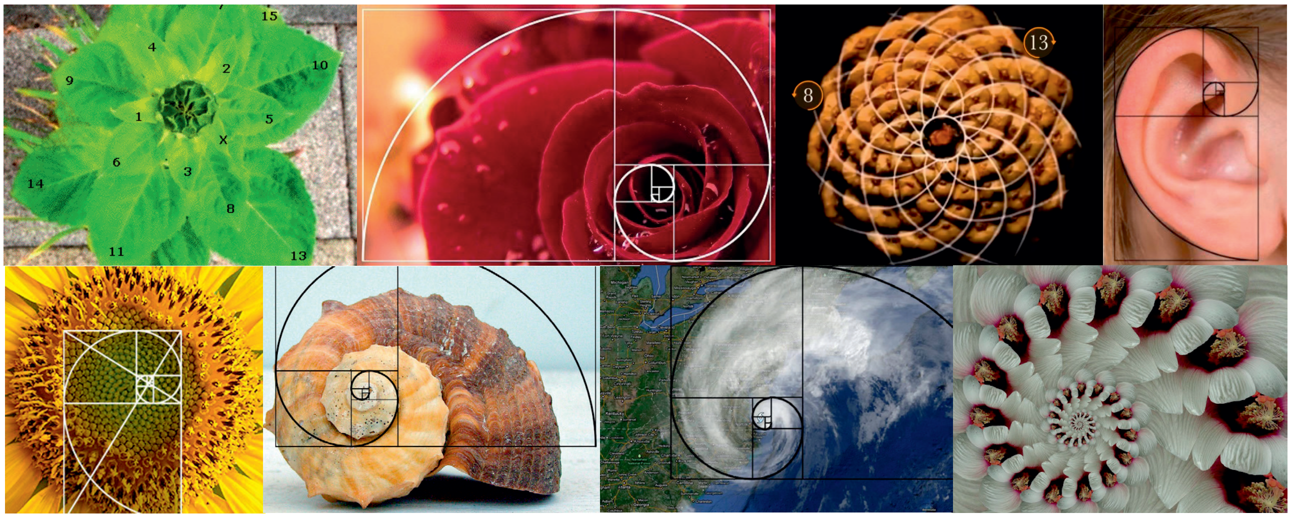
Activity-1

I. To draw a Golden Spiral using Fibonacci squares - Connect the squares through diagonals by curve from corner to corner across each square to make a Golden Spiral.



FIBONACCI SQUARES

Examples where Fibonacci sequence found in nature



Example: 1

Given that one pair of new born rabbits they produce a new pair each month and from the second month, each new pair can breed themselves. Find how many pairs of rabbits are bred from one pair in a year, and find the relationship between the number of months and the number of pairs of rabbits by tabulation (a pair means (a male and a female)).

Solution:

1st month
= 1(given new born pair)

2nd month
= 1(pair of adult)

3rd month
= 1(pair of adult)+1(new born pair) = 2

4th month
= 2(pairs of adult)+1(new born pair) = 3

5th month
= 3(pairs of adult)+2(new born pairs) = 5

6th month
= 5(pairs of adult)+3(new born pairs) = 8

The above figure clearly forms the sequence is **1,1,2,3,5,8...** Here, we find the pattern in which each number is in the Fibonacci sequence, obtained by adding together with previous two. Going on like this to find subsequent numbers at the **twelfth month,** we will get **144 pairs of rabbits.** In the other words, twelfth Fibonacci number is 144.

Number of months	1	2	3	4	5	6	7	8	9	10	11	12
Number of pairs of rabbits	1	1	2	3	5	8	13	21	34	55	89	144

Activity-2

Using the given Table I, find the pattern, Answer the following questions and colour the values in the given Table II. One is done for you.

Table I

Term(n)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	...
F(n)	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	

1. Where are the even Fibonacci Numbers?
   Colour both the term n and where **F(n) is even** in **yellow.**
   Do you find any pattern?
   Every **Third** Fibonacci number is **a multiple of 2(even).**
   i.e. **a multiple of F(3)** or **2=F(3).**
2. Where there are Fibonacci numbers which are **multiple of 3 ?**
   Colour both the term **n and** where **F(n) is multiple of 3** in **red.**
   Write down the pattern you find
   Every ....Fibonacci number is ....
   i.e. **a multiple of ....** or **......** =F(4).
3. What about the **multiple of 5**?
   Colour both the term **n** and where **F(n) is multiple of 5** in **blue.**
   Write down the pattern you find.
   Every ....i.e. ....
4. What about the **multiple of 8?**
   Colour both the term **n** where **F(n) is multiple of 8** in **green.**
   Write down the pattern you find.
   Every ....i.e. ....

Table II

Term(n)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	...
F(n)	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	

From the above activity, we conclude that

Every Fibonacci number is a factor of (a term number of) Fibonacci numbers in miltiples.

	n	3	4	5	6	7	8	9	10	11	12	13	14	15	...
	F(n)	2	3	5	8	13	21	34	55	89	144	233	377	610	...
Factors	2=F(3)	✓	×	×	✓	×	×	✓	×	×	✓	×	×	✓	Every 3rd Fib number
	3=F(4)	×	✓	×	×	×	✓	×	×	×	✓	×	×	×	Every 4th Fib number
	5=F(5)	×	×	✓	×	×	×	×	✓	×	×	×	×	✓	Every 5th Fib number
	8=F(6)	×	×	×	✓	×	×	×	×	×	✓	×	×	×	Every 6th Fib number
	F(k)														Every Kth Fib number

From the table, we find the general rule as **Every kth Fibonacci number is a multiple of F(k).**

Note

The difference between two consecutive numbers of the Fibonacci sequence increase very quickly. For example F(5) – F(4) = 5 – 3 = **2** ; F(10) – F(9) = 55 – 34 = **21** ; F(15) – F(14) = 610 – 377 = **233** ; Observe that the difference grow very fast.

Exercise 4.1

G8W5R5

1. Choose the correct answer:

(i) What is the eleventh Fibonacci number?

(a) 55 (b) 77 (c) 89 (d) 144

(ii) If F(n) is a Fibonacci number and n =8, which of the following is true?

(a) F(8) = F(9)+F(10) (b) F(8) = F(7)+F(6) (c) F(8) = F(10)×F(9) (d) F(8) = F(7)–F(6)

(iii) Every 3rd number of the Fibonacci sequence is a multiple of ________

(a) 2 (b) 3 (c) 5 (d) 8

(iv) Every ________ number of the Fibonacci sequence is a multiple of 8

(a) 2^{nd} (b) 4^{th} (c) 6^{th} (d) 8^{th}

(v) The difference between the 18^{th} and 17^{th} Fibonacci number is

(a) 233 (b) 377 (c) 610 (d) 987

2. In the given graph sheet draw and colour how the Fibonacci number pattern makes a spiral in the Golden Rectangle.

4.3 Highest Common Factor

We have learnt in class VI that iteration is a process wherein a set of instructions or structures are repeated in a sequence for a specified number of times or until a condition is met. Here, we are going to learn to find HCF by listing all factors and find the biggest, then to find HCF by repeated subtraction and see to how much faster the iteration goes (and how in fewer steps you get the HCF) and then how to improve further by repeated division and remainder and that both lead to the same solution but one is faster than the other.

We know that, HCF is used in simplifying or reducing fractions. To understand how this concept applies in real life, imagine the following situation.

Situation : 1

Let us, assume that you have 20 mangoes and 15 apples and you want to donate those together equally among the orphan children. How many maximum number of orphan children can you help?

Here, basically question demands finding HCF of two numbers. HCF is Highest Common Factor, also known as GCD (Greatest Common Divisor). HCF of two or more than two numbers is such that, it is the largest possible number which divides all the numbers completely.

Here, let us find the HCF of 20 mangoes and 15 apples.

Factors of 20 = 1,2,4,5,10,20

Factors of 15 = 1,3,5,15

So, the HCF of 20 and 15 is 5. That is you can help maximum 5 orphan children.

For **5 children** you can give **4 mangoes** ($20 \div 5 = 4$)  and **3 apples** ($15 \div 5 = 3$)  each. In this way you can distribute equally the mangoes and the apples to each child.

Now, let us learn some more methods to find HCF.

4.3.1 Methods To Find HCF (Highest Common Factor):

1. Repeated Division Method:

Situation: 2

Let us, suppose there are 18 students in Class VII and 27 in Class VIII and each class is divided into teams to prepare for an upcoming sports tournament, will the winning teams from each class play each other in the final. What would be the biggest possible team size that you could divide both these classes such that each team has exactly the same number of students and that no one is left behind.

Let us first find the HCF of 18 and 27 by the following methods that we have learnt in earlier classes.

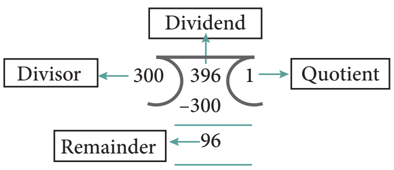
Factorisation Method	Prime Factorisation Method
Factors of 18 = 1, 2, 3, 6, 9, 18	Prime Factors of 18 = 2 x 3 x 3
Factors of 27 = 1, 3, 9, 27	Prime Factors of 27 = 3 x 3 x 3
HCF of 18 and 27 = **9**	HCF of 18 and 27 = 3 x 3 = **9**

We find that **the HCF of 18 and 27 is 9** by both the methods.

The above methods are easy to finding HCF, but for larger numbers these methods are tedious to find factors of the given numbers. In that case, alternatively we have some more methods to find HCF. Let us learn more about the other methods of finding HCF.

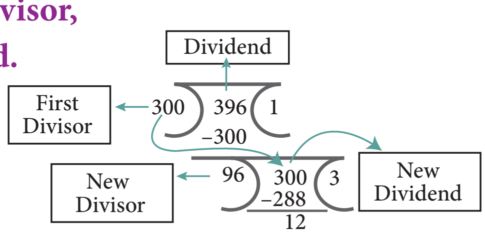
But what if the Class VII had 396 students and Class VIII had 300 students? Then what would be the biggest possible team size? Well, the above said two methods may not help here quickly. So, we can use continuous division method for finding the highest common factor.

STEP 1: Divide the larger number by the smaller number.



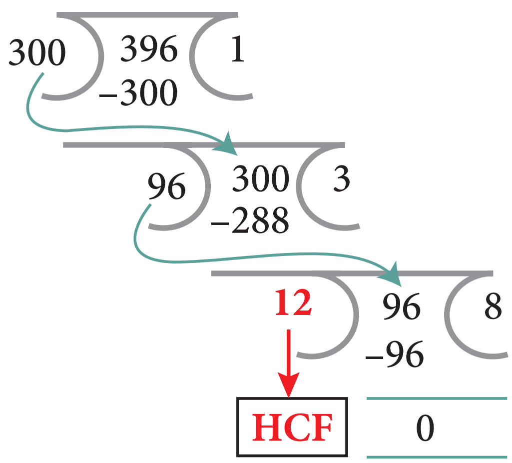
Here, 360 is the larger number. So, we divide 360(Dividend) by 300 (Divisor). We got the Remainder as 96.

STEP 2: The remainder from Step 1 becomes the new divisor, and divisor of Step 1 becomes the new dividend.



From the step 1, we got 96 as remainder. So, In the second step 96 becomes the new divisor and 300 becomes the new dividend.

STEP 3: Repeat this division process till remainder becomes zero. The divisor of the last division (when remainder is zero) is our HCF.



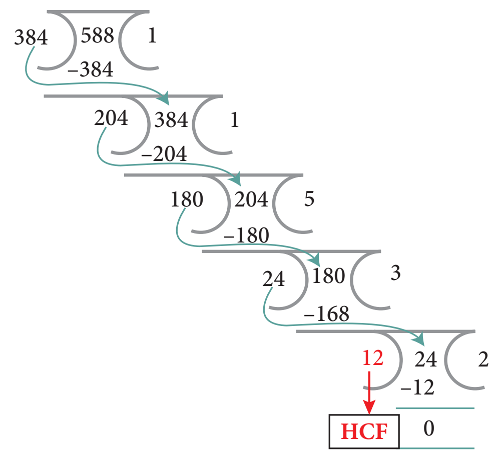
From step 2, we got 12 as the new remainder which will become the new divisor. By the Third step 12 becomes the new divisor and 96 becomes the new dividend. **Now, the remainder is zero. when, 12 is the last divisor of the division. Therefore,12 is our required HCF.**

Hence, the **HCF of 396 and 300 is 12** is got as shown in the picture given above. So the team would be 12 students.

Let us learn more about this from the following example.

Example: 2

There are 270 ginger chocolates, 384 milk chocolates and 588 coconut chocolates. What is the largest number of containers possible so that each container contains the same number of chocolates of each kind?



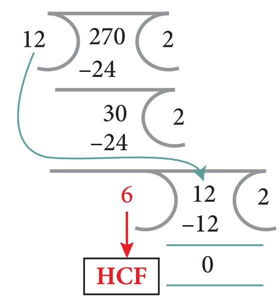
Solution:

Here, we have to find HCF of 270, 384 and 588.

STEP 1: First find the HCF of any two of the given numbers (follow the same step 1, 2 and 3 of the above example). Here, find HCF of (384, 588) first.

STEP 2: The HCF of the first two numbers which is 12 becomes the divisor and the third number 270 becomes the dividend.

STEP 3: Repeat this division process till the remainder becomes zero. The last divisor is the HCF. Here, 6 is the last divisor.



Hence, HCF of 270, 384 and 588 is 6. Therefore, we needs 6 containers so that each of them contains (270 ÷ 6 = 45) **45 ginger chocolates**, (384 ÷ 6 = 64) **64 milk chocolates** and (588 ÷ 6 = 98) **98 coconut chocolates**.

2. Repeated Subtraction Method:

STEP 1: To find the HCF for the given two numbers say **m** and **n** we do the subtraction continuously unless m and n becomes equal.

STEP 2: If m is greater than **n,** then we perform **m – n** and we assign the result (the difference) as **m.** Again we check whether **m and n are equal or not** and repeat the process. If **m** is less than **n,** then we perform **n – m** and we assign the result (the difference) as **n**. Again we check whether **m and n are equal or not** and repeat the process.

STEP 3: When **m** and **n** values are **equal** then that equal number will be the HCF (m, n).

Now, let us see the example that how we get the HCF of given any two numbers in this particular repeated subtraction method.

Example: 3

Find the HCF of 144 and 120

Solution:

STEP 1: Here , take m = 144 and n = 120

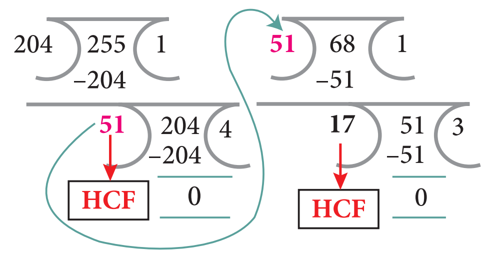
Check whether m = n or m > n or m < n. Here m > n.

STEP 2: Subtract the smaller number from the larger number till m = n.

First	144 – 120 = 24	Repeat	120 – 24 = 96	Repeat	96 – 24 = 72
Repeat	72 – 24 = 48	Repeat	48 – 24 = 24	Repeat	24 – 24 = 0

Now m = n , Hence, we conclude that the HCF of 144 and 120 is 24.

Example: 4



Do the given repeated division problem in repeated subtraction method and verify the HCF of 255, 204 and 68.

Solution:

STEP 1: Here, let p = 255, q = 204 and r = 68

Check whether p = q or p > q or p < q. Here p > q.

STEP 2: Let us find the HCF of 255 and 204 first.

Now, subtract smaller number from larger number till p = q.

First 255 – 204 = 51   Repeat 204 – 51 = 153   Repeat 153 – 51 = 102

Repeat 102 – 51 = 51   Repeat 51 – 51 = 0

Now p = q , Hence, we conclude that the HCF of 255 and 204 is 51.

STEP 3: Now repeated same procedure for r – HCF (p, q)

Now, subtract smaller number from larger number till HCF (p, q) = r.

First 68 – 51 = 17   Repeat 51 – 17 = 34

Repeat 34 – 17 = 17   Repeat 17 – 17 = 0

Now HCF (p, q) = r, Hence, we conclude that the HCF of 255, 204 and 68 is 17.

Comparing both the repeated division and repeated subtraction methods, in finding the HCF, we can conclude that the repeated subtraction, in one way is easier and gives the HCF faster that the repeated division and one would want to easely do subtraction rather than division. Isn't it ?

Exercise 4.2

1. Choose the correct answer:

(i) Common prime factors of 30 and 250 are

(a) 2 x 5   (b) 3 x 5   (c) 2 x 3 x 5   (d) 5 x 5

(ii) Common prime factors of 36, 60 and 72 are

(a) 2 x 2   (b) 2 x 3   (c) 3 x 3   (d) 3 x 2 x 2

(iii) Two numbers are said to be co-prime numbers if their HCF is

(a) 2   (b) 3   (c) 0   (d) 1

2. Using repeated division method find HCF of the following:

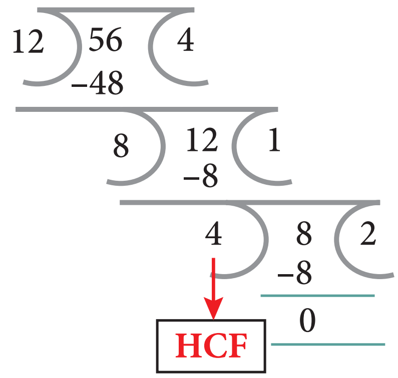
(i) 455 and 26   (ii) 392 and 256   (iii) 6765 and 610   (iv) 184, 230 and 276

3. Using repeated subtracting method find HCF of the following:

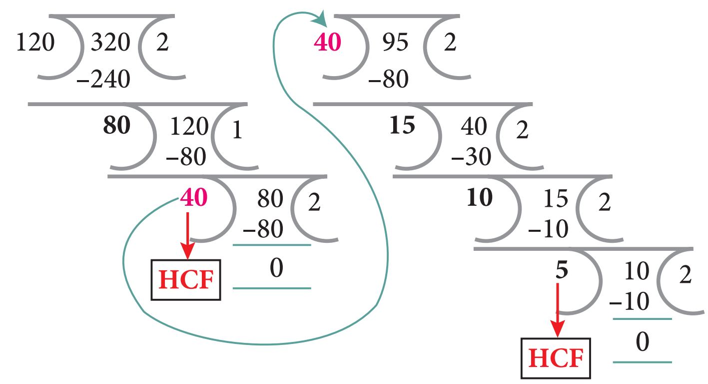
(i) 42 and 70 (ii) 36 and 80 (iii) 280 and 420 (iv) 1014 and 654

4. Do the given problems in repeated subtraction method

(i) 56 and 12



(ii) 320, 120 and 95



5. On a school trip, 56 girls and 98 boys went to Kanyakumari. They were divided into as many groups as possible so that there were equal numbers of girls and boys in each group. find the largest group possible? (To find the HCF using repeated division method)

6. Kalai wants to cut identical squares as big as she can, from a piece of paper measuring 168mm and by 196mm. What is the length of the side of the biggest square? (To find HCF using repeated subtraction method)

4.4 Cryptology

In today's world, security in information is a fundamental necessity not only for military and political departments but also for private communication. Today's world of communication has increased the importance of financial data exchange, image processing, biometrics and e-commerce transaction which in turn has made data security an important issue. Cryptology is defined as the science which is concerned with communication in secured form.

Cryptology – Some technical details

J6H2M8

Plain text: The original message is called plain text.

Cipher text or Cipher number: The encrypted output (converted message into code) is called Cipher text or Cipher number. Cipher text is written in capital letters, while plain text is usually written in lowercase. A secret key is to use something to generate the Cipher text from the plain text.

Encryption and Decryption: The process of converting the plain text to the Cipher text is called encryption and the vice versa is called decryption.

Let us try to create some Cipher text that we use in the form of coded message at some point in our real life.

4.4.1. Examples of Cipher Code:

1. Shifitng Cypher Text

Ceasar Cipher

The Ceasar Cipher is one of the earliest known and simplest ciphers. It is a type of substitution cipher in which each letter in the text is **"shifted"** a certain number of places down the alphabets.

To pass an encrypted message from one person to another, it is first necessary that both parties have the **'key'** for the cipher, so that the sender may encrypt it and the receiver may decrypt it. For the Caesar Cipher, the **"key"** is the number of characters to shift the cipher alphabet. So, we have to know how big the switch is to break the code.

Let us know about more from the following examples and situations.

Example: 5

Use Ceasar Cipher table set **+4** and to try to solve the given secret sentence.

fvieo mr gshiw ger fi xvmgoc

Solution:

Let us make Ceasar Cipher table first. Here, we have to set to **+4** table.

For that, we have to start letter **e** to set as **A**, **f** as **B** … likewise **d** as **Z.**

Now, the **+4** Ceasar Cipher table looks like

Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Cipher Text	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

The given plain text is

fvieo mr gshiw ger fi xvmgoc

To crack this secret code, follow the steps given below.

Step 1: Using Ceasar Cipher table, let us first match the most repeated letters. This will help us to progress faster.

fvieo mr gshiw ger fi xvmgoc

Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Cipher Text	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

f	v	i	e	o	m	r	g	s	h	i	w	g	e	r	f	i	x	v	m	g	o	c
B	R	E	A	K	I	N	C			E		C	A	N	B	E		R	I	C	K	

Step 2: Then, let us find remaining letters to complete the code.

Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Cipher Text	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

f	v	i	e	o	m	r	g	s	h	i	w	g	e	r	f	i	x	v	m	g	o	c
B	R	E	A	K	I	N	C	O	D	E	S	C	A	N	B	E	T	R	I	C	K	Y

Thus, the secret sentence is, **BREAK IN CODES CAN BE TRICKY**

2. Subtituting Cypher Text

Each letter in a text is "substituted" by certain pictures and symbols, A key message, group of words, letters, or a combination of these can be used to encode or decode the information. Let us learn more about from the following situation.

Situation

The teacher divides the class into two teams and display a worksheet given in the figure below for students to complete. Play a game, teams take turns suggesting letter substitutions and someone entering your suggestions in the table. Teams earn points for correct letter guesses.

Worksheet 1 :- Additive Cipher [key = 5]

Convert a given plain text into Additive Cipher text (number) code.

" mathematics is a unique symbolic language in which the whole world works and acts accordingly."

Tips for cracking additive ciphers:

- Especially for additive Ciphers, you only need **key number**. You can complete the cipher table by filling in the rest of the numbers in order.
- If you can, find and make **frequency table for alphabets**. It is help you to get started.
- Match the most **repeated letters** first and fill the Cipher number.
- Look for familiar one-letter words like **a** or **i**. Common two and three letter words like **of, to, in, it, at, the, and, for, you …..**
- Look for **consecutive numbers** in the Cipher text and match them with possible **consecutive letters** in the plain text.

Now, start to convert a plain text into Cipher number, first we have to make a cipher table as shown below. Here, key=5. As per key number we have to start and fill a = 05, b = 06 …….. z = 04 respectively.

Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Numbers	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Cipher Numbers	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	00	01	02	03	04

To start encoding the text, let us count the frequency of alphabets and frame frequency table as shown below.

Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Frequency of alphabets in the plain text	8	1	6	3	5	0	3	5	7	0	1	5	3	5	5	0	1	3	5	4	3	0	4	0	2	0

With the help of Cipher table and frequency table, let us fill the Cipher numbers for the plain text. Let us first match the most repeated letters (5 times and above), then 2 to 4 times repeated letters and finally remaining letters step by step as shown in given figure.

Step 1:- Cipher number for most repeated letters(5 times and above)

Plain text	m	a	t	h	e	m	a	t	i	c	s		i	s		a		u	n	i	q	u	e		s	y	m	b	o	l	i	c
Cipher Numbers		05		12	09		05		13	07	23		13	23		05			18	13			09		23				19	16	13	07

Plain text	l	a	n	g	u	a	g	e		i	n		w	h	i	c	h		t	h	e		w	h	o	l	e
Cipher Numbers	16	05	18			05		09		13	18			12	13	07	12			12	09			12	19	16	09

Plain text	w	o	r	l	d		w	o	r	k	s		a	n	d		a	c	t	s		a	c	c	o	r	d	i	n	g	l	y
Cipher Numbers		19		16				19			23		05	18			05	07		23		05	07	07	19			13	18		16	

Step 2: Cipher number for most repeated letters (2 times and above)

Plain text	m	a	t	h	e	m	a	t	i	c	s		i	s		a		u	n	i	q	u	e		s	y	m	b	o	l	i	c
Cipher Numbers	17	05	24	12	09	17	05	24	13	07	23		13	23		05		25	18	13		25	09		23	03	17		19	16	13	07

Plain text	l	a	n	g	u	a	g	e		i	n		w	h	i	c	h		t	h	e		w	h	o	l	e
Cipher Numbers	16	05	18	11	24	05	11	09		13	18		01	12	13	07	12		24	12	09		01	12	19	16	09

Plain text	w	o	r	l	d		w	o	r	k	s		a	n	d		a	c	t	s		a	c	c	o	r	d	i	n	g	l	y
Cipher Numbers	01	19	22	16	08		01	19	22		23		05	18	08		05	07	24	23		05	07	07	19	22	08	13	18	11	16	03

Step 3: Cipher numbers for non-repeated letters

Plain text	m	a	t	h	e	m	a	t	i	c	s		i	s		a		u	n	i	q	u	e		s	y	m	b	o	l	i	c
Cipher Numbers	17	05	24	12	09	17	05	24	13	07	23		13	23		05		25	18	13	21	25	09		23	03	17	06	19	16	13	07

Plain text	l	a	n	g	u	a	g	e		i	n		w	h	i	c	h		t	h	e		w	h	o	l	e
Cipher Numbers	16	05	18	11	25	05	11	09		13	18		01	12	13	07	12		24	12	09		01	12	19	16	09

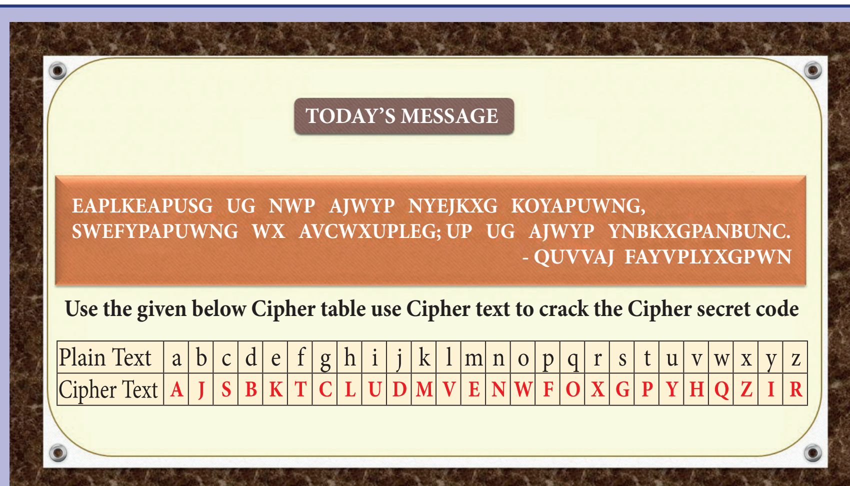
Plain text	w	o	r	l	d		w	o	r	k	s		a	n	d		a	c	t	s		a	c	c	o	r	d	i	n	g	l	y
Cipher Numbers	01	19	22	16	08		01	19	22	15	23		05	18	08		05	07	24	23		05	07	07	19	22	08	13	18	11	16	03

Thus, the Additive Cipher text for plain text ias follows:

"17 05 24 12 09 17 05 24 13 07 23 13 23 05 25 18 13 21 25 09 23 03 17 06 19 16 13 07
16 05 18 11 25 05 11 09 13 08 01 12 13 07 12 24 12 09 01 12 19 16 09 01 19 22 16 08
01 19 22 15 23 05 18 08 05 07 24 23 05 07 07 19 22 08 13 18 11 16 03"

Try this

Convert cipher code written in the school panel board as given in figure below.



Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Cipher Text	A	J	S	B	K	T	C	L	U	D	M	V	E	N	W	F	O	X	G	P	Y	H	Q	Z	I	R

Activity-3

TREASURE HUNT

Treasure in the Mathematics club room

The teacher divides the students into four groups and gives each group a code and a clue and then asks them to cracking the clues to find

(i) the identity of the treasure

(ii) the place of the treasure

(iii) the room in which the treasure took place.

You may take notes on this piece of paper as you proceed through the search

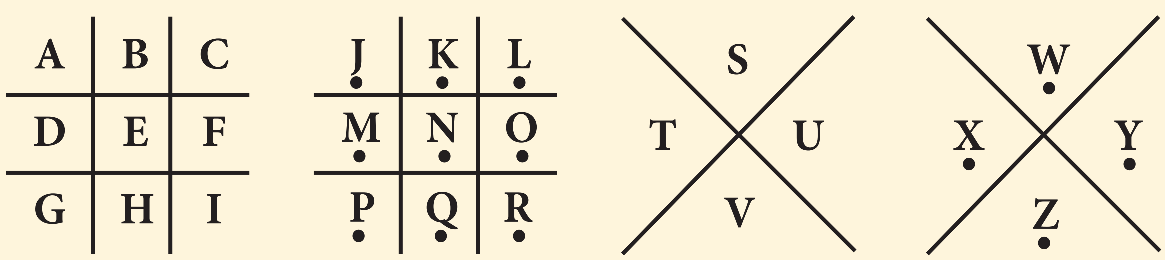
Code 1: Pigpen

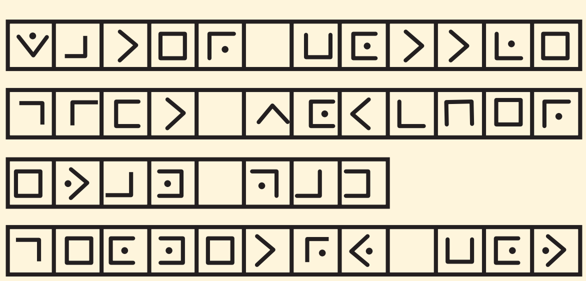
Message:- If you decode the clue here you can get the four expected treasure names

I. Fill in the blank boxes and decode

The **Pigpen code** looks like meaningless writing, but it is quite easy to catch on to. Each letter is represented by the part of the "Pigpen" that surrounds it.

The first code uses the following **key**. To complete the code, you need to work out how to use the key to decode the message.





=	W	A	T	E	R		B	O	T	T	L	E
=												
=												
=												

Code 2: Polybius Square Cipher

Message:- If you decode this you can get the clue to identify the name of the treasure name

II. Fill in the blanks

A Polybius Square is a table that allows someone to convert letters into numbers. Use the Polybius square rows and column values to find the code

Activity-3

5	A	B	C	D	E
4	F	G	H	I/J	K
3	L	M	N	O	P
2	Q	R	S	T	U
1	V	W	X	Y	Z
	1	2	3	4	5

(4,2) (3,4) (5,5)   (3,3) (1,5) (2,3) (5,5)   (4,3) (1,4)

T H E   ________   O F

(4,2) (2,2) (5,5) (1,5) (3,2) (5,2) (2,2) (5,5)   (4,5) (4,3) (5,5) (3,2)

________________   D O E S

(3,3) (4,3) (4,2)   (3,4) (1,5) (1,1) (5,5)   "(2,5)"   (1,5) (3,3) (4,5)   "(4,5)"

______   ________   ___   A N D   ______.

Code 3: Atbash Cipher

Message:- If you decode this you can get the clue room number where the treasure is present (It may be 24 or 25 or 26 or 27 or 28 or 29 or 30)

III. Find the code using the key as shown in given figure:

Atbash cipher is a substitution cipher with just one specific key where all the letters are reversed that is **A to Z** and **Z to A**.

Plain Text	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Cipher Text	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A

GSV ILLN MFNYVI RH Z NFOGRKOV LU ULFI ZMW HVEVM

THE ____ ________ ___ _ __________ ___ ____ ____ ______.

Code 4: Using a Key – Reflection Table

Message:- If you decode this clue you can get the possible places where the treasure is located (It used to sit)

Use the reflection table which is given below and find the correct word by using a reflected alphabet.

A	B	C	D	E	F	G	H	I	J	K	L	M
↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

J V A Q B J   -   ____________

P B Z C H G R E   G N O Y R   -   ____________

P U N V E   -   ____________

P H O O B N E Q   -   ____________

After finding the codes, the teacher then asks students to rearrange the clues one by one

Activity-3

CLUES

i. ____

ii. ____

iii. ____

iv. ____

RESULT

(i) The room in which the treasure took place :- ____________________.

(ii) The place of the treasure :- ______________________________________.

(iii) The identity of the treasure :- _____________________________.

(Hint:- If you answered the question number 6 in Exercise 4.3, you can compare and verify your results) The gift voucher contains (20 full marks awarded).

Try these

1. Use Pigpen Cipher code and write the code for your name ________and your chapter names

   (i) LIFE MATHEMATICS (ii) ALGEBRA

   (iii) GEOMETRY (iv) INFORMATION PROCESSING

2. Decode the following Shifting and Substituting secret codes given below. Which one is a easier for you?

   (i) Shifting method:- M N S G H M F H R H L O N R R H A K D

   (ii) Substituting method:- 

3. Write a short message to a friend and get them to decipher it.(use any one shifting and substituting code method)

Exercise 4.3

1. Choose the correct answer:

(i) In questions **A** and **B**, there are four groups of letters in each set. Three of these sets are **a** like in some way while one is different. Find the one which is different.

A. (a) C R D T (b) A P B Q (c) E U F V (d) G W H X

B. (a) H K N Q (b) I L O R (c) J M P S (d) A D G J

(ii) A group of letters are given. A numerical code has been given to each letter. These letters have to be unscrambled into a meaningful word. Find out the code for the word so formed from the 4 answers given.

L	I	N	C	P	E
1	2	3	4	5	6

(a) 2 3 4 1 5 6 (b) 5 6 3 4 2 1 (iii) 6 1 3 5 2 4 (iv) 4 2 1 3 5 6

(iii) In questions I and II, there are based on code language. Find the correct answer from the four alternatives given.

C In a certain code, '**M E D I C I N E**' is coded as '**E O J D J E F M**', then how is '**C O M P U T E R**' written in the same code ?

(a) C N P R V U F Q (b) C M N Q T U D R

(c) R F U V Q N P C (d) R N V F T U D Q

D If the word '**P H O N E**' is coded as '**S K R Q H**', how will '**R A D I O**' be coded ?

(a) S C G N H (b) V R G N G (c) U D G L R (d) S D H K Q

2. Fill in the blanks **(Use Atbash Cipher that is given in code 3)**

(i) G Z N R O = ____________________

(ii) V M T O R H S = ____________________

(iii) N Z G S V N Z G R X H = ____________________

(iv) H X R V M X V = ____________________

(v) H L X R Z O H X R V M X V = ____________________

3. Match the following **(a = 00................ Z= 25).**

(i)	mathematics	-	(a) 18 20 01 19 17 00 02 19 08 14 13
(ii)	addition	-	(b) 03 08 21 08 18 08 14 13
(iii)	subtraction	-	(c) 12 00 19 07 04 12 0019 08 02 18
(iv)	multiplication	-	(d) 00 03 03 08 19 08 14 13
(v)	division	-	(e) 12 20 11 19 08 15 11 15 02 00 19 08 14 13

4. Frame Additive cipher table **(key = 4).**

5. A message like **"Good Morning"** written in reverse would instead be **"Doog Gninrom".** In the same way decode the sentence given below:

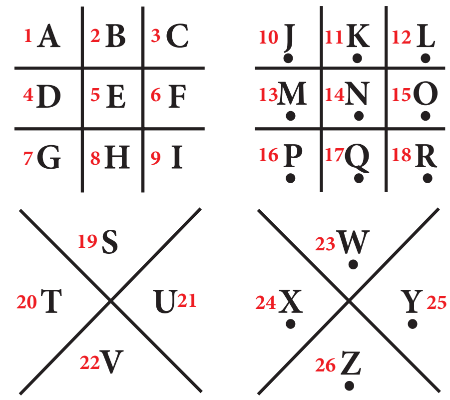
"Ot dnatsrednu taht scitamehtam nac eb decneirepxe erehwreve ni erutan dna laer efil."

6. Decode the given **Pigpen Cipher text** and compare your answer to get the **Activity 3 result**.

I. The room number in which the treasure took place

II. Place of the treasure :-

III. The name of the treasure :-



Exercise 4.4

Miscellaneous Practice Problems

1. The rule of Fibonacci Sequence is $F(n) = F(n-2) + F(n-1)$. Find the 11^{th} to 20^{th} Fibonacci numbers.

2. In a library, 385 Math books, 297 Science books and 143 Tamil books are bundled equally in numbers. What is the maximum numbers of books possible in a bundle, for all types of books? (Use repeated division method)

3. Find the length of the largest which piece of wood used to measure exactly the lengths 4m50cm and 6m 30cm woods.(Use repeated subtraction method)

4. Using both repeated division method and repeated subtraction method and find the greatest number that divides 167 and 95, leaving 5 as reminder.

5. Praveen recently got the registration number for his new two-wheeler. Here, the number is given in the form of mirror-image. Encode the image and find the correct registration number of praveen's two-wheeler.

T N 1 2 H 2 5 8 9

(a) 

(b) 

(c) 

(d) 9 8 5 2 H 2 1 N T

ANSWERS

LIFE MATHEMATICS

Exercise 1.1

1.	(i) $x = 500$	(ii) $3\frac{1}{3}\%$	(iii) $x = 50$	(iv) 70 %	(v) 52.52%
2.	(i) 50%	(ii) 75%	(iii) 100%	(iv) 96%	(v) $66\frac{2}{3}$ %
3.	(i) $x = 150$	4. R= 4%	5. (i) 500 ii) Cricket -45,Volleyball-42, Badminton -33		
6.	30	7. 110	8. $33\frac{1}{3}$ %	9. $x = 200$	10. $x = 100$
11.	No change	12. $\frac{44}{125}$	13. 400	14. 300	15. 87%
16	(c) 20%	17. (b) 49%	18. (a) 375	19. (d) 200	20. (d) 36

Exercise 1.2

1. (i) Cost price	(ii) ₹7000	(iii) ₹600	(iv) 8%	(iv) ₹945
2. ₹902	3. ₹670	4. $11\frac{1}{9}\%$	5. 50%	6. ₹1152
7. 4% loss	8. (i) ₹207 (ii) ₹12600 (iii) 18%		9. ₹4445	
10. 60%	11. Discount of 8% is better	12. ₹5600	13. (c) 25%	14. (b) 550
15. (b) 168	16. (d) ₹250	17. (a) 40%		

Exercise 1.3

1.	(i) ₹1272	(ii) ₹820	(iii) ₹20,000	(iv) $A = P\left(1+\frac{r}{400}\right)^{4n}$	(v) ₹32
2.	(i) True	(ii) False	(iii) True	(iv) False	(i) True
3. ₹162		4. ₹936.80	5. ₹5618	6. 3 years	7. ₹1875

8. 4% 9. $1\frac{1}{2}$ years 10. ₹10,875 11. 700cm 12. ₹0.50 13. ₹ 54

14. 5% 15. ₹ 16000 16. (c) 6 17. (b) 1 year 18. (b) ₹ 12500

19. (a) ₹2000 20. (d) ₹2500

Exercise 1.4
Miscellaneous Practice Problems

1. T3, $93\frac{2}{3}$ % 2. ₹38163 3. 20% 4. ₹15000 5. $2\frac{2}{9}$% loss

6. ₹8000 7. ₹ 104

Challenging Problems

8. 100 9. 20 % 10. 100% 11. 30% 12. ₹ 331

ALGEBRA

Exercise 2.1

1. (i) $x=7$ (ii) $y=11$ (iii) $m=7$ (iv) $p=15$ (v) One

2. (i) True (ii) False 3. (c) (iii),(i), (iv), (v), (ii)

4. (i) $x=11$ (ii) $y=\frac{1}{-4}$ (iii) $x=-1$ 5. (i) $x=-4$ (ii) $p=-1$ (iii) $x=-11$

6. (i) $x=-2$ (ii) $m=-4$

Exercise 2.2

1. (i). $x=-\frac{b}{a}$ (ii) Positive (iii) $x=30$ (iv) 40° (v) b=9

2. (i) True (ii) False (iii) False 3. 3, 21 4. 27

5. $l=8$ cm, $b=24$ cm 6. (80,10) notes 7. Murali's age is 15 years old, Thenmozhi's age is 20 years old 8. 63 9. $\frac{13}{21}$

10. 64.6km 11. (b) 20 12. (a) 62 13. (c) 10000 14. (c) 4

Exercise 2.3

1. (i) Origin (0,0) (ii) negative (iii) y-axis (iv) zero (v) X-Coordinate
2. (i) True (ii) True (iii) False

Exercise 2.4

1. (i) Origin (ii) (4,-4) (iii) x-axis 1*cm*=3 units, y-axis 1*cm*=25 units
2. (i) True (ii) False

Exercise 2.5

Miscellaneous Practice Problems

1. x = 20 2. $60°, 40°, 80°$ 3. y=11 units p=133 units 4. $116°, 64°$

Challenging Problems

6. 7,8,9 7. 54 8. 12 pencils

GEOMETRY

Exercise 3.1

1. (i) Q (ii) $n^2 - m^2$ (iii) a right angled triangle (iv)similar (v) EN
2. (i) True (ii) True (iii) False (iv) False (v) False
3. (i) Yes (ii) No (iii) Yes (iv) Yes (v) yes

4 (i) $x = 41$ (ii) $y = 16$ (iii) $z = 15$ 5. 5*cm* 6. 25, 24

7. 30 inches 8. 170*m* 9. (i) 12*cm* 10. 25cm, 16cm, 9cm

11. (c) $45°$ 12. (b) 20*cm* 13. (c) 420*cm* 14. (b) 5*cm* 15.(d) 20, 48, 52

Exercise 3.2

Miscellaneous Practice Problems

1. Yes, the side of length 3.7 cm is the hypotenuse.
2. 24 inches No. 3. 25 ft 4. 25 ft 5. 100, 48

Challenging Problems

6. 35 km 8. 40*cm* 9. 28 ft 10. 24*cm*

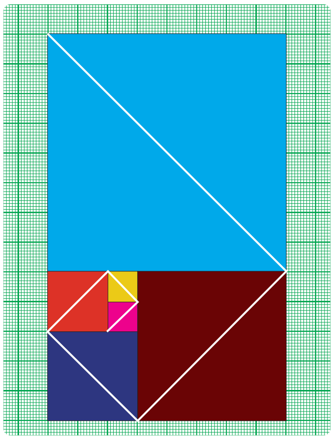
INFORMATION PROCESSING

Exercise 4.1

I (i) (c) 89 (ii) (b) F (8) = F(7) + F(6) (iii) (a) 2

(iv) (d) 6^{th} (v) (d) 9187

2



(more possible ways)

Exercise 4.2

1. (i) (a) 2×5	(ii) (b) 2×3	(i) (d) 1	
2. (i) 13	(ii) 8	(iii) 5	(i) 46
3. (i) 14	(ii) 4	(iii) 140	(i) 6
4. (i) 4	(ii) 5	5. 14	6. 28

Exercise 4.3

1 (i) A. (a) C R D T B. (d) A D G I (ii) (b) 5 6 3 4 2 1

(iii) (c) R F U Q N P C D. (c) U D G L R

2. (i) TAMIL (ii) ENGLISH (iii) MATHEMATICS (iv) SCIENCE (v) SOCIAL SCIENCE

3. (i) c,(ii) d (iii) a (iv)e (v) b

4

Plain Text	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Cipher Text	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	00	01	02	03

5. To understand that mathematics can be experienced everywhere in nature and real life.

6. (i) 28 (ii) C H A I R (iii) G I F T VOUCHER

Exercise 4.4

Miscellaneous Practice Problems

1. 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765

2. 11 3. 90 4. 18 5. (c) ୧ 8 ꙅ Ƨ H Ƨ I И T

Mathematical Terms	
Accrued	சேர்ந்த
Altitude on Hypotenuse	கர்ணத்தின் மீதமைந்த செங்குத்து
Amount	தொகை
Borrower	கடன் பெறுபவர்
Calculator	கணிப்பான்
Cartesian Plane	கார்டீசியன் தளம்
Cartesian System	கார்டீசியன் அமைப்பு
CGST	மத்திய சரக்கு மற்றும் சேவை வரி
Code	குறியீடு
Compound Interest	கூட்டு வட்டி
Construction	அமைப்பு / வரைமுறை
Consumer	நுகர்வோர்
Converse	மறுதலை
Conversion period	மாற்று காலம்
Co-ordinate axes	ஆய அச்சுகள்
Corresponding Sides	ஒத்த பக்கங்கள்
Cost price	அடக்க விலை (அ) வாங்கிய விலை
Cryptology	குறியாக்கவியல்
Decrease	குறைவு
Depreciation value	தேய்மான மதிப்பு
Diagonal	மூலைவிட்டம்
Difference	வித்தியாசம்
Discount	தள்ளுபடி
Duration	காலம்
Encryption	குறியாக்கம்
Equivalent	சமான
Graph Sheet	வரைபடத் தாள்
Greatest Side	மிக நீளமான பக்கம்
Hypotenuse	கர்ணம்
Increase	உயர்வு
Isosceles right angled triangle	இருசமபக்க செங்கோண முக்கோணம்

Mathematical Terms

Isosceles Trapezium	இருசமபக்க சரிவகம்
Legs or Sides	பக்கங்கள்
Legs (sides) of a right angled triangle	செங்கோணத்தைத் தாங்கும் பக்கங்கள்
Lender	கடன் வழங்குபவர்
Linear Equation	ஒருபடிச் சமன்பாடு/ நேரியச் சமன்பாடு
Linear Pattern	நேரிய அமைப்பு
Loan	கடன்
Loss	நட்டம்
Marked Price	குறித்த விலை
Ordered pairs	வரிசை சோடிகள்
Origin	ஆதிப்புள்ளி
Over head expenses	மேல்நிலை செலவுகள் இதர
Parallelogram	இணைகரம்
Per annum	ஓர் ஆண்டிற்கு
Percentage	சதவீதம்
Principal	அசல்
Profit	இலாபம்
Pythagoras Theorem	பிதாகரஸ் தோற்றம்
Pythagorean Triplets	பிதாகரஸ் மூன்றன் தொகுதி எண்கள்
Quadrant	கால்பகுதி
Quantity	அளவு
Rate of Growth	வளர்ச்சி வீதம்
Scale	அளவுத்திட்டம்
Selling Price	விற்ற விலை
SGST	மாநில சரக்கு மற்றும் சேவை வரி
Simple Equation	எளிய சமன்பாடு
Simple Interest	தனி வட்டி
Successive	தொடர்ச்சியான
Tax	வரி
Time	நேரம்
Transposition	இடமாற்று முறை

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