1. Find the co-ordinates of the mid-point of the line segments joining the following pairs of points:
(i) (2, – 3), (– 6, 7)
(ii) (5, – 11), (4, 3)
(iii) (a + 3, 5b), (2a – 1, 3b + 4)

Solution:
Co-ordinates of midpoint of line joining the points (x_1,y_1) and (x_2,y_2) = $\{(x_1+x_2)/2, (y_1+y_2)/2\}$
(i) ∴ Co-ordinates of midpoint of line joining the points (2, -3) and (-6,7) = {(2+-6)/2, (-3+7)/2}
= (-4/2, 4/2)
= (-2, 2)
Hence the co-ordinates of midpoint of line joining the points (2, -3) and (-6,7) is (-2, 2).

(ii) Co-ordinates of midpoint of line joining the points (x_1,y_1) and (x_2,y_2) = $\{(x_1+x_2)/2, (y_1+y_2)/2\}$
∴ Co-ordinates of midpoint of line joining the points (5, -11) and (4,3) = {(5+4)/2, (-11+3)/2}
= (9/2, -8/2)
= (9/2, -4)
Hence the co-ordinates of midpoint of line joining the points (5, -11) and (4,3) is (9/2, -4).

(iii) Co-ordinates of midpoint of line joining the points (x_1,y_1) and (x_2,y_2) = $\{(x_1+x_2)/2, (y_1+y_2)/2\}$
∴ Co-ordinates of midpoint of line joining the points (a+3, 5b) and (2a-1,3b+4) = {(a+3+2a-1)/2, (5b+3b+4)/2}
= {(3a+2)/2, (8b+4)/2}
= {(3a+2)/2, (4b+2)}
Hence the co-ordinates of midpoint of line joining the points (a+3, 5b) and (2a-1,3b+4) are {(3a+2)/2, (4b+2)}.

2. The co-ordinates of two points A and B are (– 3, 3) and (12, – 7) respectively. P is a point on the line segment AB such that AP : PB = 2 : 3. Find the co-ordinates of P.

Solution:
Let the co-ordinates of P(x, y) divides AB in the ratio m:n.
A(-3,3) and B(12,-7) are the given points.
Given m:n = 2:3
$x_1 = -3$, $y_1 = 3$, $x_2 = 12$, $y_2 = -7$, m = 2 and n = 3
∴By Section formula $x = (mx_2+nx_1)/(m+n)$
∴x = (2×12+3×-3)/(2+3)
∴x = (24-9)/5
x = 15/5
⇒ x = 3

By Section formula $y = (my_2+ny_1)/(m+n)$
∴y = (2×-7+3×3)/5
∴y = (-14+9)/5
∴y = -5/5
⇒ y = -1
Hence the co-ordinate of point P are (3,-1).

3. P divides the distance between A (– 2, 1) and B (1, 4) in the ratio of 2 : 1. Calculate the co-ordinates of the point P.

Solution:

Let the co-ordinates of P(x, y) divides AB in the ratio m:n.
A(-2,1) and B(1,4) are the given points.
Given m:n = 2:1
$x_1 = -2$, $y_1 = 1$, $x_2 = 1$, $y_2 = 4$, m = 2 and n = 1
∴By Section formula $x = (mx_2+nx_1)/(m+n)$
∴$x = (2×1+1×-2)/(2+1)$
∴$x = (2-2)/3$
$x = 0/3$
⇒ $x = 0$

By Section formula $y = (my_2+ny_1)/(m+n)$
∴$y = (2×4+1×1)/(2+1)$
∴$y = (8+1)/3$
$y = 9/3$
⇒ $y = 3$
Hence the co-ordinate of point P are (0,3).

4. (i) Find the co-ordinates of the points of trisection of the line segment joining the point (3, – 3) and (6, 9).
(ii) The line segment joining the points (3, – 4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are (p, – 2) and (5/3, q) respectively, find the values of p and q.

Solution:

Let P and Q be the points of trisection of AB
i.e., AP = PQ = QB
Given A(3,-3) and B(6,9)
∴$x_1 = 3$, $y_1 = -3$, $x_2 = 6$, $y_2 = 9$
∴P(x, y) divides AB internally in the ratio 1 : 2.
∴m:n = 1:2
∴ By applying the section formula, the coordinates of P are as follows.

By Section formula $x = (mx_2+nx_1)/(m+n)$
$x = (1×6+2×3)/(1+2)$

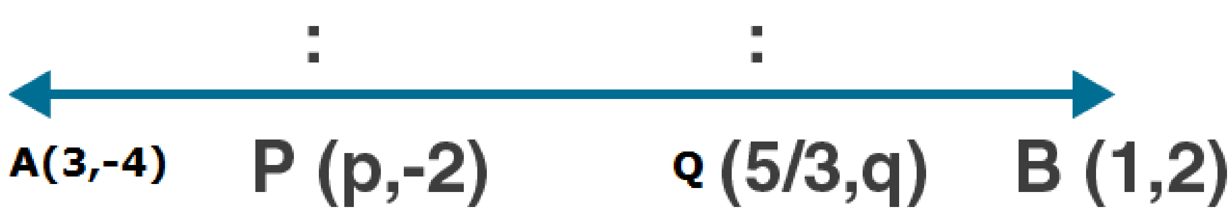
x = (6+6)/3
x = 12/3
x = 4

By Section formula $y = (my_2+ny_1)/(m+n)$
$\therefore y = (1\times9+2\times-3)/(2+1)$
$\therefore y = (9-6)/3$
y = 3/3
$\Rightarrow y = 1$
Hence the co-ordinate of point P are (4,1).

Now, Q also divides AB internally in the ratio 2 : 1.
$\therefore m:n = 2:1$
$\therefore$ By applying the section formula, the coordinates of P are as follows.
By Section formula $x = (mx_2+nx_1)/(m+n)$
$\therefore x = (2\times6+1\times3)/(1+2)$
x = (12+3)/3
x = 15/3
x = 5

By Section formula $y = (my_2+ny_1)/(m+n)$
$\therefore y = (2\times9+1\times-3)/(2+1)$
$\therefore y = (18-3)/3$
y = 15/3
$\Rightarrow y = 5$
Hence the co-ordinate of point Q are (5,5).

(ii) Let P(p,-2) and Q(5/3, q) be the points of trisection of AB



i.e., AP = PQ = QB
Given A(3,-4) and B(1,2)
$\therefore x_1 = 3, y_1 = -4, x_2 = 1, y_2 = 2$
$\therefore$ P(p, -2) divides AB internally in the ratio 1 : 2.
By Section formula $x = (mx_2+nx_1)/(m+n)$
$p = (1\times1+2\times3)/(1+2)$
p = (1+6)/3
p = 7/3

Now, Q also divides AB internally in the ratio 2 : 1.
∴ m:n = 2:1
∴ Q(5/3, q) divides AB internally in the ratio 2 : 1.
By Section formula $y = (my_2+ny_1)/(m+n)$
∴ q = (2×2+1×-4)/(2+1)
∴ q = (4-4)/3
q = 0/3
⇒ q = 0
Hence the value of p and q are 7/3 and 0 respectively.

5. (i) The line segment joining the points A (3, 2) and B (5, 1) is divided at the point P in the ratio 1 : 2 and it lies on the line 3x – 18y + k = 0. Find the value of k.
(ii) A point P divides the line segment joining the points A (3, – 5) and B (– 4, 8) such that AP/PB = k/1 If P lies on the line x + y = 0, then find the value of k.

Solution:
(i) Let the co-ordinates of P(x, y) divides AB in the ratio m:n.
A(3,2) and B(5,1) are the given points.
Given m:n = 1:2
$x_1 = 3$, $y_1 = 2$, $x_2 = 5$, $y_2 = 1$, m = 1 and n = 2
∴ By Section formula $x = (mx_2+nx_1)/(m+n)$
∴ x = (1×5+2×3)/(1+2)
∴ x = (5+6)/3
⇒ x = 11/3

By Section formula $y = (my_2+ny_1)/(m+n)$
∴ y = (1×1+2×2)/(1+2)
∴ y = (1+4)/3
⇒ y = 5/3

Given P lies on the line 3x-18y+k = 0
Substitute x and y in above equation
3×(11/3)-18×(5/3)+k = 0
⇒ 11-30+k = 0
⇒ -19+k = 0
⇒ k = 19
Hence the value of k is 19.

(ii) Let the co-ordinates of P(x, y) divides AB in the ratio m:n.
A(3,-5) and B(-4,8) are the given points.
Given AP/PB = k/1
∴ m:n = k:1
$x_1 = 3$, $y_1 = -5$, $x_2 = -4$, $y_2 = 8$, m = k and n = 1
∴ By Section formula $x = (mx_2+nx_1)/(m+n)$
∴ x = (k×-4+1×3)/(k+1)
∴ x = (-4k+3)/(k+1)
⇒ x = (-4k+3)/(k+1)

By Section formula $y = (my_2+ny_1)/(m+n)$
$\therefore y = (k\times8+1\times-5)/(k+1)$
$\therefore y = (-4k+3)/(k+1)$

Co-ordinate of P is $((-4k+3)/(k+1), (8k-5)/(k+1))$

Given P lies on line $x+y = 0$
Substitute value of x and y in above equation
$(-4k+3)/(k+1) + (8k-5)/(k+1) = 0$
$\Rightarrow (-4k+3) + (8k-5) = 0$
$\Rightarrow 4k-2 = 0$
$\Rightarrow 4k = 2$
$k = 2/4 = ½$
Hence the value of k is ½ .

6. Find the coordinates of the point which is three-fourth of the way from A (3, 1) to B (– 2, 5).

Solution:

Let P be the point which is three-fourth of the way from A(3,1) to B(-2,5).
$\therefore AP/AB = 3/ 4$
$AB = AP+PB$
$\therefore AP/AB = AP/(AP+PB) = 3/4$
$\Rightarrow 4AP = 3AP+3PB$
$\Rightarrow 4AP-3AP = 3PB$
$\Rightarrow AP = 3PB$
$\Rightarrow AP/PB = 3/1$
$\therefore$ The ratio $m:n = 3:1$
$x_1 = 3$, $y_1 = 1$, $x_2 = -2$, $y_2 = 5$
$\therefore$ By Section formula $x = (mx_2+nx_1)/(m+n)$
$\therefore x = (3\times-2+1\times3)/(3+1)$
$\therefore x = (-6+3)/4$
$\Rightarrow x = -3/4$

By Section formula $y = (my_2+ny_1)/(m+n)$
$\therefore y = (3\times5+1\times1)/(3+1)$

$\therefore y = (15+1)/4$
$\Rightarrow y = 16/4$
$\Rightarrow y = 4$
Hence the co-ordinates of P are (-3/4, 4).

7. Point P (3, – 5) is reflected to P' in the x- axis. Also P on reflection in the y-axis is mapped as P".
(i) Find the co-ordinates of P' and P".
(ii) Compute the distance P' P".
(iii) Find the middle point of the line segment P' P".
(iv) On which co-ordinate axis does the middle point of the line segment P P" lie ?

Solution:
(i) The image of P(3,-5) when reflected in X-axis will be (3,5).
When you reflect a point across the X-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite (its sign is changed).
$\therefore$ Co-ordinates of P' = (3,5)

Image of P(3,-5) when reflected in Y axis will be (-3,-5).
When you reflect a point across the Y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into its opposite (its sign is changed)
$\therefore$ Co-ordinates of P'' = (-3,-5)

(ii)Let P'(x_1, y_1) and P''(x_2 , y_2) be the given points
By distance formula d(P',P'') = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
Co-ordinates of P' = (3,5)
Co-ordinates of P'' = (-3,-5)
Here $x_1 = 3$, $y_1 = 5$, $x_2 = -3$, $y_2 = -5$
$\therefore$ d(P',P'') = $\sqrt{[(-3-3)^2+(-5-5)^2]}$
$= \sqrt{[(-6)^2+(-10)^2]}$
$= \sqrt{(36+100)}$
$= \sqrt{136}$
$= \sqrt{(4\times34)}$
$= 2\sqrt{34}$
Hence the distance between P' and P'' is $2\sqrt{34}$ units.

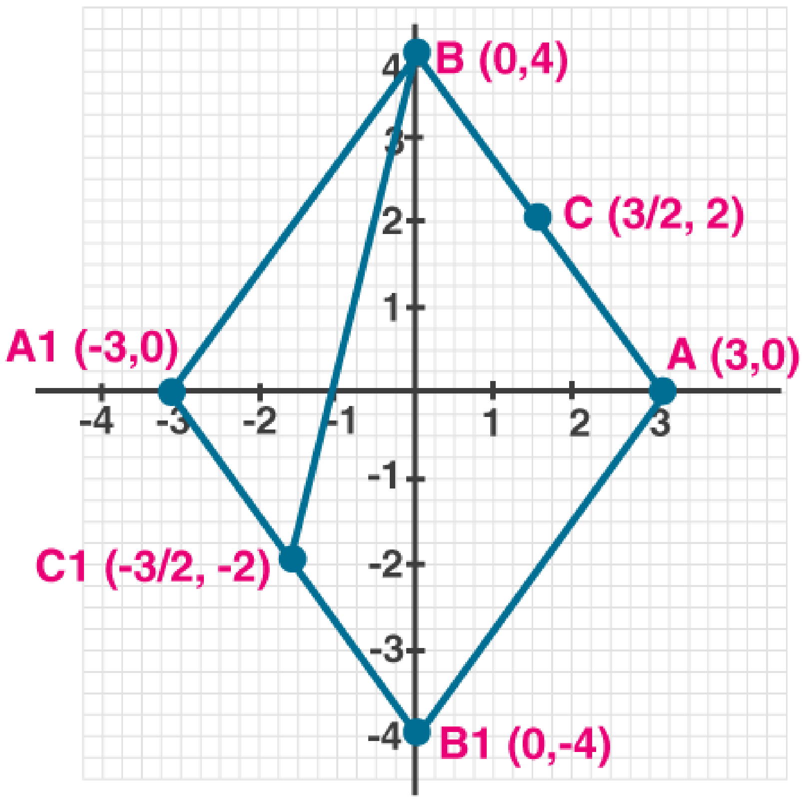
(iii) Co-ordinates of P' = (3,5)
Co-ordinates of P'' = (-3,-5)
Here $x_1 = 3$, $y_1 = 5$, $x_2 = -3$, $y_2 = -5$
Let Q(x,y) be the midpoint of P'P''
By midpoint formula,
$x = (x_1+x_2)/2$
$y = (y_1+y_2)/2$
$\therefore$ x = (3+-3)/2 = 0/2 = 0
y = (5+-5)/2 = 0/2 = 0
Hence the co-ordinate of midpoint of P'P'' is (0,0) .

(iv) Co-ordinates of P = (3,-5)
Co-ordinates of P'' = (-3,-5)
Here $x_1 = 3$, $y_1 = -5$, $x_2 = -3$, $y_2 = -5$
Let R(x,y) be the midpoint of PP''

By midpoint formula,
$x = (x_1+x_2)/2$
$y = (y_1+y_2)/2$
$\therefore$ x = (3+-3)/2 = 0/2 = 0
y = (-5+-5)/2 = -10/2 = -5
So the co-ordinate of midpoint of PP'' is (0,-5) .
Here x co-ordinate is zero.
Hence the point lies on Y-axis.

8. Use graph paper for this question. Take 1 cm = 1 unit on both axes. Plot the points A(3, 0) and B(0, 4).
(i) Write down the co-ordinates of A1, the reflection of A in the y-axis.
(ii) Write down the co-ordinates of B1, the reflection of B in the x-axis.
(iii) Assign the special name to the quadrilateral ABA1B1.
(iv) If C is the midpoint is AB. Write down the co-ordinates of the point C1, the reflection of C in the origin.
(v) Assign the special name to quadrilateral ABC1B1.

Solution:



(i) Co-ordinates of point A are (3,0).
When you reflect a point across the Y-axis, the y-coordinate remains the same,
but the x-coordinate is transformed into its opposite (its sign is changed)
Hence the reflection of A in the Y axis is (-3,0).

(ii) Co-ordinates of point B are (0,4).
When you reflect a point across the X-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite (its sign is changed).
Hence the reflection of B in the X-axis is (0,-4)

(iii) The quadrilateral ABA1B1 will be a rhombus.

(iv) Let C be midpoint of AB.
Co-ordinate of C = ((3+0)/2 , (0+4)/2) = (3/2, 2) [Midpoint formula]
In a point reflection in the origin, the image of the point (x,y) is the point (-x,-y).
Hence the reflection of C in the origin is (-3/2, -2)

(v) In quadrilateral ABC1B1, AB ∥ B1C1
Hence the quadrilateral ABC1B1 is a trapezium.

9. The line segment joining A (– 3, 1) and B (5, – 4) is a diameter of a circle whose centre is C. find the co-ordinates of the point C. (1990)

Solution:
Given Co-ordinates of A = (-3,1)
Co-ordinates of B = (5,-4)
Here $x_1 = -3$, $y_1 = 1$, $x_2 = 5$, $y_2 = -4$
Let C(x,y) be the midpoint of AB
By midpoint formula,
$x = (x_1+x_2)/2$
$y = (y_1+y_2)/2$
$\therefore$ x = (-3+5)/2 = 2/2 = 1
y = (1+-4)/2 = -3/2
Hence the co-ordinate of midpoint of AB is C(1,-3/2) .

10. The mid-point of the line segment joining the points (3m, 6) and (– 4, 3n) is (1, 2m – 1). Find the values of m and n.

Solution:
Let the midpoint of line joining the points A(3m,6) and B(-4,3n) be C(1,2m-1).
Here $x_1 = 3m$, $y_1 = 6$, $x_2 = -4$, $y_2 = 3n$
x = 1 , y = 2m-1
By Midpoint formula,
$x = (x_1+x_2)/2$
$\therefore$ 1 = (3m+-4)/2
$\Rightarrow$ 3m-4 = 2
$\Rightarrow$ 3m = 2+4
$\Rightarrow$ 3m = 6
$\Rightarrow$ m = 6/3 = 2
By Midpoint formula,
$y = (y_1+y_2)/2$
$\therefore$ 2m-1 = (6+3n)/2
$\Rightarrow$ 4m-2 = 6+3n
Put m = 2 in above equation

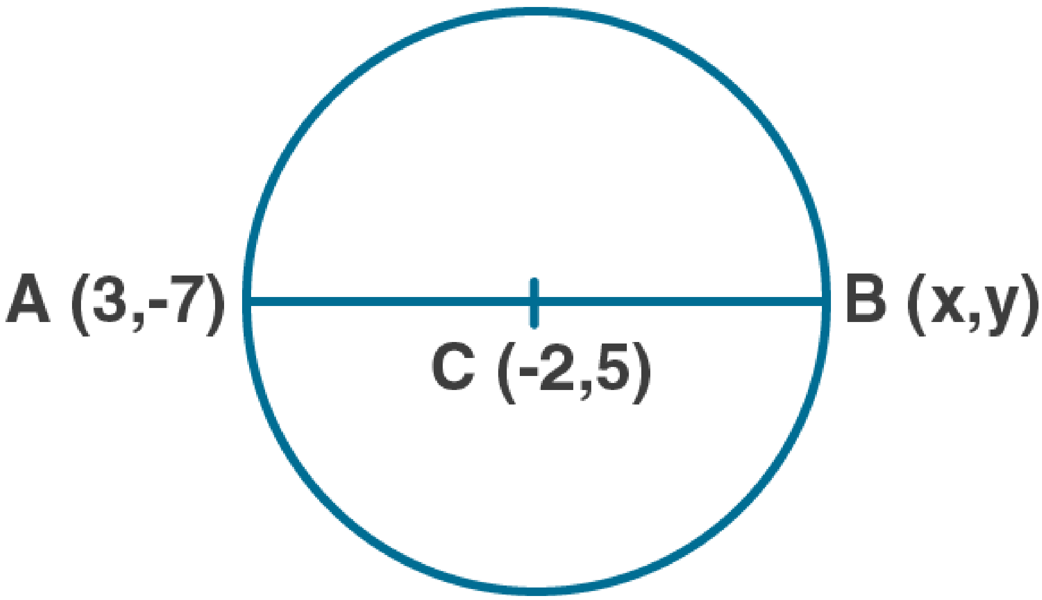
$4 \times 2 - 2 = 6 + 3n$
$8 - 2 - 6 = 3n$
$\Rightarrow 3n = 0$
$\Rightarrow n = 0$
Hence the value of m and n are 2 and 0 respectively.

11. The co-ordinates of the mid-point of the line segment PQ are (1, – 2). The co-ordinates of P are (– 3, 2). Find the co-ordinates of Q.(1992)

Solution:
Let the co-ordinates of Q be (x_2, y_2).
Given co-ordinates of P = (-3,2)
Co-ordinates of midpoint = (1,-2)
Here $x_1 = -3$, $y_1 = 2$, $x = 1$, $y = -2$
By Midpoint formula,
$x = (x_1+x_2)/2$
$\therefore 1 = (-3+x_2)/2$
$\Rightarrow 2 = -3+x_2$
$\Rightarrow x_2 = 2+3 = 5$
By Midpoint formula,
$y = (y_1+y_2)/2$
$\therefore -2 = (2+y_2)/2$
$\Rightarrow -4 = 2+y_2$
$\Rightarrow y_2 = -4-2$
$\Rightarrow y_2 = -6$
Hence the co-ordinates of Q are (5,-6).

12. AB is a diameter of a circle with centre C (– 2, 5). If point A is (3, – 7). Find:
(i) the length of radius AC.
(ii) the coordinates of B.

Solution:



(i) Length of radius AC = d(A,C)
Co-ordinates of A = (3,-7)
Co-ordinates of C = (-2,5)
Here $x_1 = 3$, $y_1 = -7$, $x_2 = -2$, $y_2 = 5$
By distance formula, d(A,C) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
= $\sqrt{[(-2-3)^2+(5-(-7))^2]}$
= $\sqrt{[(-5)^2+(12)^2]}$
= $\sqrt{[25+144]}$
= $\sqrt{169}$
= 13
Hence the radius is 13 units.

(ii)Given AB is the diameter and C is the centre of the circle.
∴By midpoint formula, -2 = (x+3)/2
⇒ -4 = x+3
⇒ x = -4-3 = -7
∴By midpoint formula, 5 = (-7+y)/2
⇒ 10 = -7+y
⇒y = 10+7 = 17
Hence the co-ordinates of B are (-7,17).

13. Find the reflection (image) of the point (5, – 3) in the point (– 1, 3).

Solution:
Let the co-ordinates of the image of the point P(5,-3) be
P1(x, y) in the point (-1, 3) then the point (-1, 3) will be the midpoint of PP1.

By midpoint formula, $x = (x_1+x_2)/2$
$\therefore -1 = (5+x_2)/2$ [$x = -1, x_1 = 5$]
$-2 = 5+x_2$
$x_2 = -2-5 = -7$
By midpoint formula, $y = (y_1+y_2)/2$
$3 = (-3+y_2)/2$ [$y = 3, y_1 = -3$]
$6 = -3+y_2$
$y_2 = 6+3 = 9$
Hence the co-ordinates of the image of P is (-7,9).

14. The line segment joining A(-1,5/3) the points B (a, 5) is divided in the ratio 1 : 3 at P, the point where the line segment AB intersects y-axis. Calculate
(i) the value of a
(ii) the co-ordinates of P. (1994)

Solution:
(i) Let P(x,y) divides the line segment joining the points A(-1,5/3), B(a,5) in the ratio 1:3,
Here m:n = 1:3
$x_1 = -1$, $y_1 = 5/3$, $x_2 = a$, $y_2 = 5$
$\therefore$By Section formula $x = (mx_2+nx_1)/(m+n)$
$\therefore x = (1\times a+3\times -1)/(1+3)$
$\therefore x = (a-3)/4$
$\Rightarrow x = (a-3)/4$ …..(i)

By Section formula $y = (my_2+ny_1)/(m+n)$
$\therefore y = (1\times 5+3\times 5/3)/(3+1)$
$\therefore y = (5+5)/4$
$\Rightarrow y = 10/4$
$\Rightarrow y = 5/2$ ….(ii)

Given P meets Y axis. So its x co-ordinate will be zero.
i.e, $(a-3)/4 = 0$
$\Rightarrow a-3 = 0$
$\Rightarrow a = 3$

(ii) $x = (a-3)/4$ [From (i)]
Substitute a = 3 in above equation.
$x = (3-3)/4 = 0$
$y = 5/2$ [From (ii)]
Hence the co-ordinates of P are (0,5/2).

15. The point P (– 4, 1) divides the line segment joining the points A (2, – 2) and B in the ratio of 3 : 5. Find the point B.

Solution:
Let the co-ordinates of B be (x_2,y_2).
Given co-ordinates of A = (2,-2)
Co-ordinates of P = (-4,1)
Ratio m:n = 3:5

$x_1 = 2$, $y_1 = -2$, $x = -4$, $y = 1$
P divides AB in the ratio 3:5
$\therefore$ By section formula, $x = (mx_2+nx_1)/(m+n)$
$\therefore -4 = (3\times x_2+5\times 2)/(3+5)$
$\therefore -4 = (3x_2+10)/8$
$-32 = 3x_2+10$
$\Rightarrow 3x_2 = -32-10 = -42$
$\Rightarrow x_2 = -42/3 = -14$

By Section formula $y = (my_2+ny_1)/(m+n)$
$\therefore 1= (3\times y_2+5\times -2)/(3+5)$
$\therefore 1 = (3y_2-10)/8$
$\Rightarrow 8 = 3y_2-10$
$\Rightarrow 3y_2 = 8+10 = 18$
$\Rightarrow y = 18/3 = 6$
Hence the co-ordinates of B are (-14,6).

16. (i) In what ratio does the point (5, 4) divide the line segment joining the points (2, 1) and (7 ,6) ?
(ii) In what ratio does the point (– 4, b) divide the line segment joining the points P (2, – 2), Q (– 14, 6) ? Hence find the value of b.

Solution:
(i) Let the ratio that the point (5,4) divide the line segment joining the points (2,1) and (7,6) be m:n,
Here $x_1 = 2$, $y_1 = 1$, $x_2 = 7$, $y_2 = 6$, $x = 5$, $y = 4$
$\therefore$ By section formula, $x = (mx_2+nx_1)/(m+n)$
$\therefore 5 = (m\times 7+n\times 2)/(m+n)$
$\therefore 5 = (7m+2n)/(m+n)$
$\therefore 5(m+n) = 7m+2n$
$\therefore 5m+5n = 7m+2n$
$\therefore 5m-7m = 2n-5n$
$\therefore -2m = -3n$
$\therefore m/n = -3/-2 = 3/2$
Hence the ratio m:n is 3:2.

(ii) Let the ratio that the point (-4,b) divide the line segment joining the points (2,-2) and (-14,6) be m:n,
Here $x_1 = 2$, $y_1 = -2$, $x_2 = -14$, $y_2 = 6$, $x = -4$, $y = b$
$\therefore$ By section formula, $x = (mx_2+nx_1)/(m+n)$
$\therefore -4 = (m\times -14+n\times 2)/(m+n)$
$\therefore -4= (-14m+2n)/(m+n)$
$\therefore -4(m+n) = -14m+2n$
$\therefore -4m-4n = -14m+2n$
$\therefore -4m+14m = 2n+4n$
$\therefore 10m = 6n$
$\therefore m/n = 6/10 = 3/5$
Hence the ratio m:n is 3:5.

By Section formula $y = (my_2+ny_1)/(m+n)$
$\therefore b= (3\times 6+5\times -2)/(3+5)$

$\therefore$ b = (18-10)/8
$\Rightarrow$ b = 8/8
$\Rightarrow$ b = 1
Hence the value of b is 1 and the ratio m:n is 3:5.

17. The line segment joining A (2, 3) and B (6, – 5) is intercepted by the x-axis at the point K. Write the ordinate of the point k. Hence, find the ratio in which K divides AB. Also, find the coordinates of the point K.

Solution:
Since the point K is on X axis, its y co-ordinate is zero.
Let the point K be (x,0).
Let the point K divides the line segment joining A(2,3) and B(6,-5) in the ratio m:n.
Here $x_1 = 2$, $y_1 = 3$, $x_2 = 6$, $y_2 = -5$, y = 0
By Section formula $y = (my_2+ny_1)/(m+n)$
$\therefore$ 0 = (m×-5+n×3)/(m+n)
$\therefore$ 0 = (-5m+3n)/m+n
$\Rightarrow$ -5m+3n = 0
$\Rightarrow$ -5m = -3n
$\Rightarrow$ m/n = -3/-5 = 3/5
Hence the point K divides the line segment in the ratio 3:5.

$\therefore$ By section formula, $x = (mx_2+nx_1)/(m+n)$
$\therefore$ x = (3×6+5×2)/(3+5)
$\therefore$ x = (18+10)/8
$\therefore$ x = 28/8 = 7/2
Hence the co-ordinates of K are (7/2, 0).

18. If A (– 4, 3) and B (8, – 6), (i) find the length of AB.
(ii) in what ratio is the line joining AB, divided by the x-axis? (2008)

Solution:
(i) Given points are A(-4,3) and B(8,-6).
Here $x_1 = -4$, $y_1 = 3$
$x_2 = 8$, $y_2 = -6$
By distance formula, $d(AB) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ $d(AB) = \sqrt{[(8-(-4))^2+(-6-3)^2]}$
$\therefore$ $d(AB) = \sqrt{[(12)^2+(-9)^2]}$
$\therefore$ $d(AB) = \sqrt{(144+81)}$
$\therefore$ $d(AB) = \sqrt{225}$
$\therefore$ d(AB) = 15
Hence the length of AB is 15 units.

(ii)Let m:n be the ratio in which the line AB is divided by the X axis.
Since the line meets X axis, its y co-ordinate is zero.
By Section formula $y = (my_2+ny_1)/(m+n)$
$\therefore$ 0 = (m×-6+n×3)/(m+n)
$\therefore$ 0 = (-6m+3n)/m+n
$\Rightarrow$ -6m+3n = 0

$\Rightarrow$ -6m = -3n
$\Rightarrow$ m/n = -3/-6 = 3/6 = 1/2
Hence the ratio is 1:2.

19. (i) Calculate the ratio in which the line segment joining (3, 4) and(– 2, 1) is divided by the y-axis.
(ii) In what ratio does the line x – y – 2 = 0 divide the line segment joining the points (3, – 1) and (8, 9)? Also, find the coordinates of the point of division.

Solution:
(i) Let m:n be the ratio in which the line segment joining (3,4) and (-2,1) is divided by the Y axis.
Since the line meets Y axis, its x co-ordinate is zero.
Here $x_1 = 3$, $y_1 = 4$
$x_2 = -2$, $y_2 = 1$
$\therefore$ By section formula, x = $(mx_2+nx_1)/(m+n)$
$\therefore$ 0 = (m×-2+n×3)/(m+n)
$\therefore$ 0 = (-2m+3n)/(m+n)
$\therefore$ 0 = -2m+3n
$\therefore$ 2m = 3n
$\therefore$ m/n = 3/2
Hence the ration m:n is 3:2.

(ii)Let the line x-y-2 = 0 divide the line segment joining the points (3,-1) and (8,9) in the ratio m:n at the point P(x,y)
Here $x_1 = 3$, $y_1 = -1$
$x_2 = 8$ $y_2 = 9$
$\therefore$ By section formula, x = $(mx_2+nx_1)/(m+n)$
$\therefore$ x = (m×8+n×3)/(m+n)
$\therefore$ x = (8m+3n)/(m+n) …. (i)
By Section formula y = $(my_2+ny_1)/(m+n)$
$\therefore$ y = (m×9+n×-1)/(m+n)
$\therefore$ y = (9m-n)/(m+n) …. (ii)

Since the point P(x,y) lies on the line x-y-2 = 0,
eqn (i) and (ii) will satify the equation x-y-2 = 0 …(iii)
Substitute (i) and (ii) in (iii)
[(8m+3n)/(m+n)] - [(9m-n)/(m+n)]-2 = 0
[(8m+3n)/(m+n)] - [(9m-n)/(m+n)]-[2(m+n)/(m+n)] = 0
8m+3n-(9m-n)-2(m+n) = 0
8m+3n-9m+n-2m-2n = 0
-3m+2n = 0
-3m = -2n
$\therefore$ m/n = -2/-3 = 2/3
Hence the ratio m:n is 2:3.
Substitute m and n in (i)
x = (8m+3n)/(m+n)
$\therefore$ x = (8×2+3×3)/(2+3)
$\therefore$ x = (16+9)/5
$\therefore$ x = 25/5 = 5
Substitute m and n in (ii)

$y = (9m-n)/(m+n)$
$\therefore y = (9\times2-3)/(2+3)$
$\therefore y = (18-3)/5$
$\therefore y = 15/5 = 3$
Hence the co-ordinates of P are (5,3).

20. Given a line segment AB joining the points A (– 4, 6) and B (8, – 3). Find:
(i) the ratio in which AB is divided by the y-axis.
(ii) find the coordinates of the point of intersection.
(iii)the length of AB.

Solution:
(i) Let m:n be the ratio in which the line segment joining A (-4,6) and B(8,-3) is divided by the Y axis.
Since the line meets Y axis, its x co-ordinate is zero.
Here $x_1 = -4$, $y_1 = 6$
$x_2 = 8$, $y_2 = -3$

$\therefore$ By section formula, $x = (mx_2+nx_1)/(m+n)$
$\therefore 0 = (m\times8+n\times-4)/(m+n)$
$\therefore 0 = (8m+-4n)/(m+n)$
$\therefore 0 = 8m+-4n$
$\therefore$ 8m = 4n
$\therefore m/n = 4/8 = 1/2$
Hence the ration m:n is 1:2.

(ii) By Section formula $y = (my_2+ny_1)/(m+n)$
Substitute m and n in above equation
$\therefore y = (1\times-3+2\times6)/(1+2)$
$\therefore y = (-3+12)/3$
$\therefore y = 9/3 = 3$
So the co-ordinates of the point of intersection are (0,3).

(iii) By distance formula, $d(AB) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ $d(AB) = \sqrt{[(8-(-4))^2+(-3-6)^2]}$
$\therefore$ $d(AB) = \sqrt{[(12)^2+(-9)^2]}$
$\therefore$ $d(AB) = \sqrt{(144+81)}$
$\therefore$ $d(AB) = \sqrt{225}$
$\therefore$ $d(AB) = 15$
Hence the length of AB is 15 units.

21. (i) Write down the co-ordinates of the point P that divides the line joining A (– 4, 1) and B (17,10) in the ratio 1 : 2.
(ii)Calculate the distance OP where O is the origin.
(iii)In what ratio does the y-axis divide the line AB ?

Solution:
(i)Let P(x,y) divides the line segment joining the points A(-4,1), B(17,10) in the ratio 1:2,
Here $x_1 = -4$, $y_1 = 1$
$x_2 = 17$, $y_2 = 10$

m:n = 1:2
∴By section formula, $x = (mx_2+nx_1)/(m+n)$
∴$x = (1\times17+2\times-4)/(1+2)$
∴$x = (17+-8)/3$
∴$x = 9/3$
∴$x = 3$

By Section formula $y = (my_2+ny_1)/(m+n)$
∴$y = (1\times10+2\times1)/(1+2)$
∴$y = (10+2)/3$
∴$y = 12/3 = 4$
Hence the co-ordinates of the point P are (3,4).

(ii)Since O is the origin, the co-ordinates of O are (0,0).
By distance formula, $d(OP) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
∴$d(OP) = \sqrt{[(0-3)^2+(0-4)^2]}$
∴$d(OP) = \sqrt{[(3)^2+(4)^2]}$
∴$d(OP) = \sqrt{(9+16)}$
∴$d(OP) = \sqrt{25} = 5$
Hence the distance OP is 5 units.

(iii)Let m:n be the ratio in which Y axis divide line AB.
Since AB touches Y axis, its x co-ordinate will be zero.
Here $x_1 = -4$, $y_1 = 1$
$x_2 = 17$, $y_2 = 10$
∴By section formula, $x = (mx_2+nx_1)/(m+n)$
∴$0 = (m\times17+n\times-4)/(m+n)$
∴$0 = (17m-4n)/(m+n)$
⇒ $17m-4n = 0$
⇒ $17m = 4n$
⇒$m/n = 4/17$
∴m:n = 4:17
Hence the ratio in which Y axis divide line AB is 4:17.

22. Calculate the length of the median through the vertex A of the triangle ABC with vertices A (7, – 3), B (5, 3) and C (3, – 1).

Solution:
Let M(x,y) be the median of ΔABC through A to BC.
M will be the midpoint of BC.
$x_1 = 5$, $y_1 = 3$
$x_2 = 3$, $y_2 = -1$
By midpoint formula, $x = (x_1+x_2)/2$
∴ $x = (5+3)/2 = 8/2 = 4$

By midpoint formula, $y = (y_1+y_2)/2$
∴ $y = (3+-1)/2 = 2/2 = 1$
Hence the co-ordinates of M are (4,1).
By distance formula, $d(AM) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

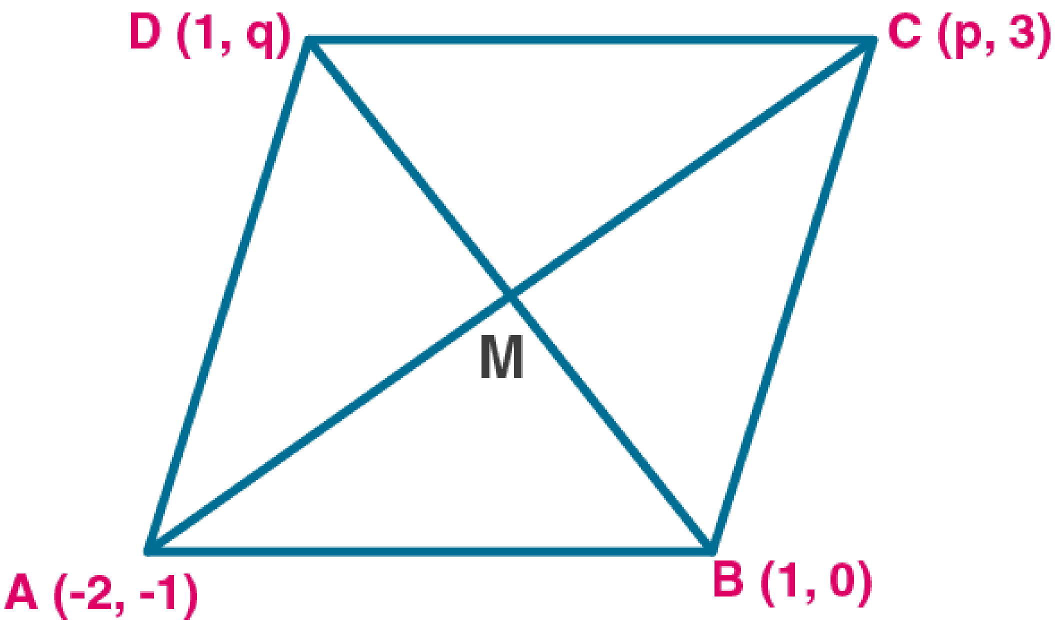
$x_1 = 7$, $y_1 = -3$
$x_2 = 4$, $y_2 = 1$
$\therefore d(AM) = \sqrt{[(4-7)^2+(1-(-3))^2]}$
$\therefore d(AM) = \sqrt{[(-3)^2+(4)^2]}$
$\therefore d(AM) = \sqrt{(9+16)}$
$\therefore d(AM) = \sqrt{25} = 5$
Hence the length of the median AM is 5 units.

23. Three consecutive vertices of a parallelogram ABCD are A (1, 2), B (1, 0) and C (4, 0). Find the fourth vertex D.

Solution:
Let M be the midpoint of the diagonals of the parallelogram ABCD.
Co-ordinate of M will be the midpoint of diagonal AC.
Given points are A(1,2), B(1,0) and C(4,0).
Consider line AC.
$x_1 = 1$, $y_1 = 2$
$x_2 = 4$, $y_2 = 0$
By midpoint formula, $x = (x_1+x_2)/2$
$\therefore x = (1+4)/2 = 5/2$
By midpoint formula, $y = (y_1+y_2)/2$
$\therefore y = (2+0)/2 = 2/2 = 1$
Hence the co-ordinates of M are (5/2,1).
M is also the midpoint of diagonal BD.
Consider line BD and M as midpoint.
$x_1 = 1$, $y_1 = 0$
$x = 5/2$, $y = 1$
By midpoint formula, $x = (x_1+x_2)/2$
$\therefore 5/2 = (1+x_2)/2$
$\therefore 5 = 1+x_2$
$\therefore x_2 = 5-1 = 4$
By midpoint formula, $y = (y_1+y_2)/2$
$\therefore 1 = (0+y_2)/2$
$\therefore 1 = y_2/2$
$\Rightarrow y_2 = 2$
Hence the co-ordinates of D are (4,2).

24. If the points A (– 2, – 1), B (1, 0), C (p, 3) and D (1, q) form a parallelogram ABCD, find the values of p and q.

Solution:



Given vertices of the parallelogram are A(-2,-1), B(1, 0), C(p,3) and D(1,q).
Let M(x,y) be the midpoint of the diagonals of the parallelogram ABCD.
Diagonals AC and BD bisect each other at M.
When M is the midpoint of AC
By midpoint formula,
x = (-2+p)/2 = (p-2)/2 ..(i)
y = (-1+3)/2 = 2/2 = 1 ..(ii)
When M is the midpoint of BD
By midpoint formula,
x = (1+1)/2 = 2/2 = 1 ..(iii)
y = (q+0)/2 = q/2 ..(iv)
Equating (i) and (iii), we get
(p-2)/2 = 1
⇒ p-2 = 2
⇒ p = 2+2 = 4
Equating (i2) and (iv), we get
q/2 = 1
⇒q = 2
Hence the value of p and q are 4 and 2 respectively.

25. If two vertices of a parallelogram are (3, 2) (– 1, 0) and its diagonals meet at (2, – 5), find the other two vertices of the parallelogram.

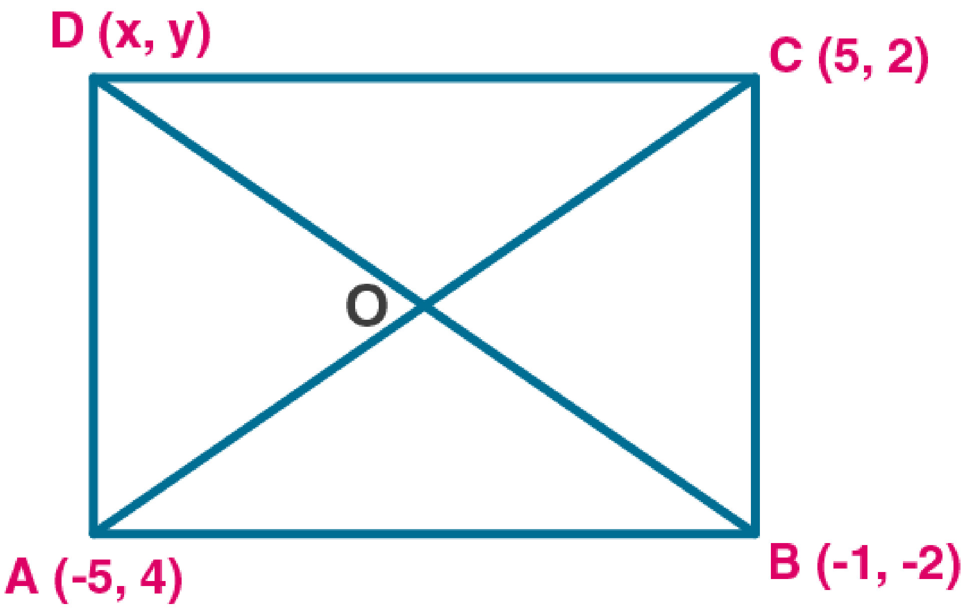
Solution:
Let A(3,2) and B(-1,0) be the two vertices of the parallelogram ABCD.
Let M(2,-5) be the point where diagonals meet.

Since the diagonals of the parallelogram bisect each other, M is the midpoint of AC and BD.
Consider A-M-C
Let co-ordinate of C be (x_2,y_2)
$x_1 = 3$, $y_1 = 2$
$x = 2$, $y = -5$
By midpoint formula, $x = (x_1+x_2)/2$
$2 = (3+x_2)/2$
$\Rightarrow 3+x_2 = 4$
$\Rightarrow x_2 = 4-3 = 1$
By midpoint formula, $y = (x_1+x_2)/2$
$-5 = (2+y_2)/2$
$-10 = 2+y_2$
$\Rightarrow y_2 = -10-2 = -12$
Hence the co-ordinates of the point C are (1,-12).

Consider B-M-D
Let co-ordinate of D be (x_2,y_2)
$x_1 = -1$, $y_1 = 0$
$x = 2$, $y = -5$
By midpoint formula, $x = (x_1+x_2)/2$
$2 = (-1+x_2)/2$
$\Rightarrow -1+x_2 = 4$
$\Rightarrow x_2 = 4+1 = 5$
By midpoint formula, $y = (x_1+x_2)/2$
$-5 = (0+y_2)/2$
$-10 = 0+y_2$
$\Rightarrow y_2 = -10$
Hence the co-ordinates of the point D are (5,-10).

26. Prove that the points A (– 5, 4), B (– 1, – 2) and C (5, 2) are the vertices of an isosceles right angled triangle. Find the co-ordinates of D so that ABCD is a square.

Solution:



Given points are A(-5,4), B(-1,-2) and C(5,2) are given.
Since these are vertices of an isosceles triangle ABC then AB = BC.
By distance formula, d(AB) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
Here $x_1 = -5$, $y_1 = 4$
$x_2 = -1$, $y_2 = -2$
$\therefore$ d(AB) = $\sqrt{[(-1-(-5))^2+(-2-4)^2]}$
$\therefore$ d(AB) = $\sqrt{[(4)^2+(6)^2]}$
$\therefore$ d(AB) = $\sqrt{(16+36)}$
$\therefore$ d(AB) = $\sqrt{52}$ …(i)
By distance formula, d(BC) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
Here $x_1 = -1$, $y_1 = -2$
$x_2 = 5$, $y_2 = 2$
$\therefore$ d(BC) = $\sqrt{[(5-(-1))^2+(2-(-2))^2]}$
$\therefore$ d(BC) = $\sqrt{[(6)^2+(4)^2]}$
$\therefore$ d(BC) = $\sqrt{(36+16)}$
$\therefore$ d(BC) = $\sqrt{52}$ …(ii)
From (i) and (ii) AB = BC
So given points are the vertices of isosceles triangle.

By distance formula, d(AC) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
Here $x_1 = -5$, $y_1 = 4$
$x_2 = 5$, $y_2 = 2$
$\therefore$ d(AC) = $\sqrt{[(5-(-5))^2+(2-4)^2]}$
$\therefore$ d(AC) = $\sqrt{[(10)^2+(-2)^2]}$

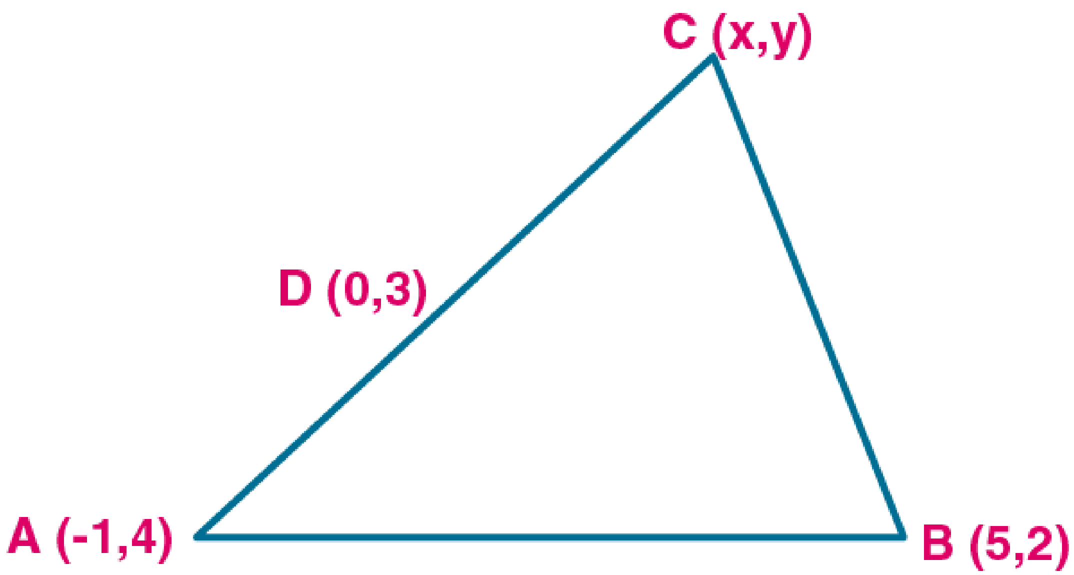
$\therefore$ d(AC) = $\sqrt{(100+4)}$
$\therefore$ d(AC) = $\sqrt{104}$ ….(iii)
Apply Pythagoras theorem to triangle ABC
$AB^2+BC^2 = (\sqrt{52})^2+(\sqrt{52})^2$
= 52+52
= 104 ….(iv)
$AC^2 = (\sqrt{104})^2 = 104$…(v)
From (iv) and (v) we got
$AB^2+BC^2 = AC^2$
So Pythagoras theorem is satisfied.
So the triangle is an isosceles right angled triangle.
Hence proved.

If ABCD is a square, let the diagonals meet at O.
Diagonals of a square bisect each other. So, O is the midpoint of AC and BD.
Consider A-O-C
$x_1 = -5$, $y_1 = 4$
$x_2 = 5$, $y_2 = 2$
By midpoint formula, $x = (x_1+x_2)/2$
$x = (-5+5)/2 = 0/2 = 0$
By midpoint formula, $y = (y_1+y_2)/2$
$y = (4+2)/2 = 6/2 = 3$
So co-ordinate of O is (0,3).

Consider B-O-D
Let co-ordinate of D be (x_2,y_2)
$x_1 = -1$, $y_1 = -2$
$x = 0$, $y = 3$
By midpoint formula, $x = (x_1+x_2)/2$
$0 = (-1+x_2)/2$
$\Rightarrow -1+x_2 = 0$
$\Rightarrow x_2 = 1$
By midpoint formula, $y = (x_1+x_2)/2$
$3 = (-2+y_2)/2$
$6 = -2+y_2$
$\Rightarrow y_2 = 6+2 = 8$
Hence the co-ordinates of the point D are (1,8).

27. Find the third vertex of a triangle if its two vertices are (– 1, 4) and (5, 2) and midpoint of one sides is (0, 3).

Solution:

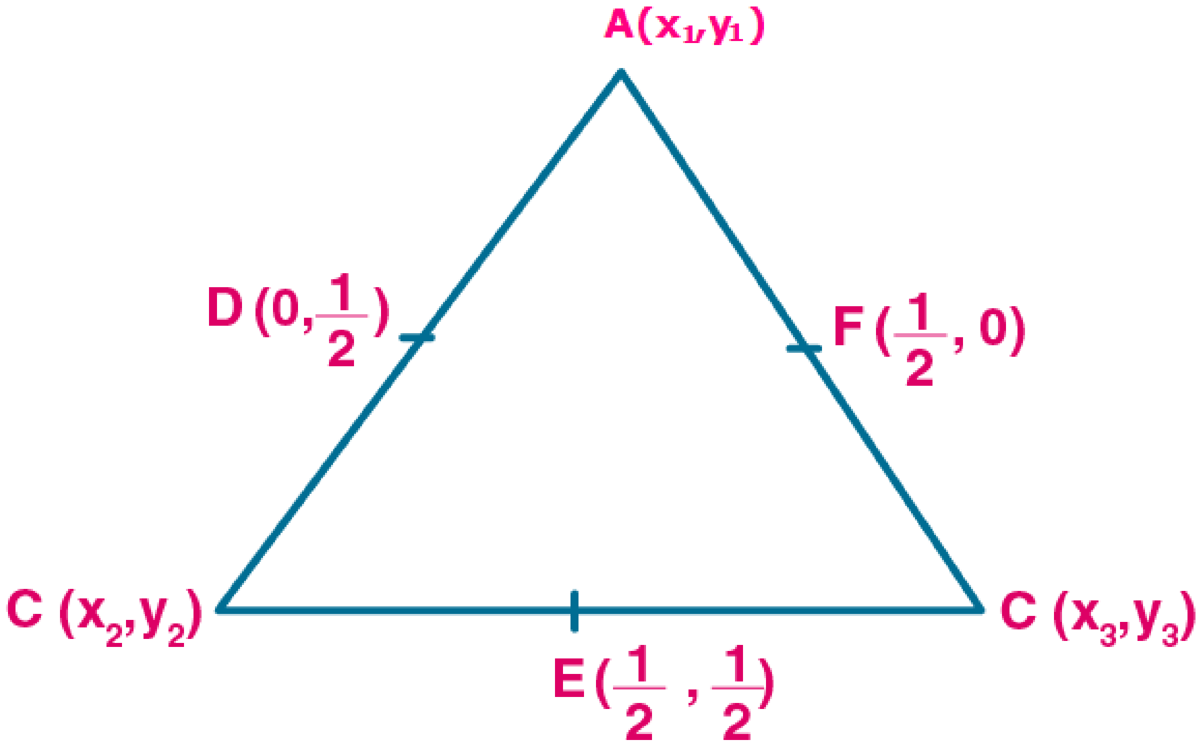


Let A (-1,4) and B(5,2) are the vertices of the triangle and let D(0,3) is the midpoint of side AC.
Let co-ordinate of C be (x,y).
Consider D(0,3) as midpoint of AC
By midpoint formula,
$(-1+x)/2 = 0$
$\therefore -1 + x = 0$
$\therefore x = 1$
By midpoint formula,
$(4+y)/2 = 3$
$\therefore 4+y = 6$
$\therefore y = 6-4 = 2$
So the co-ordinates of C are (1,2).

Consider D(0,3) as midpoint of BC
By midpoint formula,
$(5+x)/2 = 0$
$\therefore 5 + x = 0$
$\therefore x = -5$
By midpoint formula,
$(2+y)/2 = 3$
$\therefore 2+y = 6$
$\therefore y = 6-2 = 4$
So the co-ordinates of C are (-5,4).
Hence the co-ordinates of the point C will be (1,2) or (-5,4).

28. Find the coordinates of the vertices of the triangle the middle points of whose sides are (0, ½) , (½ , ½) and (½ , 0).

Solution:



Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ be the vertices of the triangle ABC.
Consider AB
By midpoint formula, $(x_1+x_2)/2 = 0$
$\therefore\ x_1+x_2 = 0$
$\therefore\ x_1 = -x_2$ ..(i)
By midpoint formula, $(y_1+y_2)/2 = ½$
$\therefore\ y_1+y_2 = 1$ …(ii)

Consider AC
By midpoint formula, $(x_1+x_3)/2 = ½$
$\therefore\ x_1+x_3 = 1$ …(iii)
By midpoint formula, $(y_1+y_3)/2 = 0$
$\therefore\ y_1+y_3 = 0$
$\therefore\ y_1 = -y_3$ …(iv)

Consider BC
By midpoint formula, $(x_2+x_3)/2 = ½$
$\therefore\ x_2+x_3 = 1$ …(v)
By midpoint formula, $(y_2+y_3)/2 = ½$
$\therefore\ y_2+y_3 = 1$ …(vi)

Substitute (i) in (iii)

Then (iii) becomes $-x_2+x_3 = 1$
Equation (v)$\Rightarrow$ $x_2+x_3 = 1$
Adding above two equations, we get
$2x_3 = 2$
$\therefore x_3 = 2/2 = 1$
Substitute $x_3 = 1$ in (iii), we get $x_1 = 0$
$\therefore x_2 = 0$ [From (i)]
So $x_1 = 0$, $x_2 = 0$, $x_3 = 1$

Substitute (iv) in (ii)
Then (ii) becomes $-y_3+y_2 = 1$
Equation (vi)$\Rightarrow$ $y_2+y_3 = 1$
Adding above two equations, we get
$2y_2 = 2$
$\therefore y_2 = 2/2 = 1$
Substitute $y_2 = 1$ in (i), we get $y_1 = 0$
$\therefore y_3 = 0$
So $y_1 = 0$, $y_2 = 1$, $y_3 = 0$
Hence the Co-ordinates of vertices are A(0,0), B(0,1) and C(1,0).

29. Show by section formula that the points (3, – 2), (5, 2) and (8, 8) are collinear.

Solution:
Let the point B(5,2) divides the line joining A(3,-2) and C(8,8) in the ratio m:n.
Then by section formula, $x = (mx_2+nx_1)/(m+n)$
$\therefore 5 = (m\times8+n\times3)/(m+n)$
$\therefore 5 = (8m+3n)/(m+n)$
$5m+5n = 8m+3n$
$2n = 3m$
$\therefore m/n = 2/3$ …(i)
By section formula, $y = (my_2+ny_1)/(m+n)$
$\therefore 2 = (m\times8+n\times-2)/(m+n)$
$\therefore 2 = (8m-2n)/(m+n)$
$2m+2n = 8m-2n$
$6m = 4n$
$m/n = 4/6 = 2/3$ …(ii)
Here ratios are same.
So the points are collinear.

30. Find the value of p for which the points (– 5, 1), (1, p) and (4, – 2) are collinear.

Solution:
Let A(-5,1) divides the line joining (1,p) and (4,-2) in the ratio m:n
Then by section formula, $x = (mx_2+nx_1)/(m+n)$
$\therefore -5 = (m\times4+n\times1)/(m+n)$
$\therefore -5 = (4m+n)/(m+n)$
$-5m-5n = 4m+n$
$-9m = 6n$
$\therefore m/n = -9/6 = -2/3$ …(i)

By section formula, $y = (my_2+ny_1)/(m+n)$
$\therefore$ $1 = (m\times-2+n\times p)/(m+n)$
$\therefore$ $1 = (-2m+pn)/(m+n)$
$m+n = -2m+pn$
$3m = (p-1)n$
$m/n = (p-1)/3$ ….(ii)
Equating (i) and (ii)
$(p-1)/3 = -2/3$
$p-1 = -2$
$p = -2+1 = -1$
Hence the value of p is -1.

31. A (10, 5), B (6, – 3) and C (2, 1) are the vertices of triangle ABC. L is the mid point of AB, M is the mid-point of AC. Write down the co-ordinates of L and M. Show that LM = ½ BC.

Solution:
Given points are A(10,5), B(6,-3) and C(2,1).
Let L(x,y) be the midpoint of AB.
Here $x_1= 10$, $y_1 = 5$
$x_2 = 6$, $y_2 = -3$
By midpoint formula, $x = (x_1+x_2)/2$
$\therefore x = (10+6)/2 = 16/2 = 8$
By midpoint formula, $y = (y_1+y_2)/2$
$\therefore y = (5-3)/2 = 2/2 = 1$
So co-ordinates of L are (8,1).

Let M(x,y) be the midpoint of AC.
Here $x_1= 10$, $y_1 = 5$
$x_2 = 2$, $y_2 = 1$
By midpoint formula, $x = (x_1+x_2)/2$
$\therefore x = (10+2)/2 = 12/2 = 6$
By midpoint formula, $y = (y_1+y_2)/2$
$\therefore y = (5+1)/2 = 6/2 = 3$
So co-ordinates of M are (6,3).

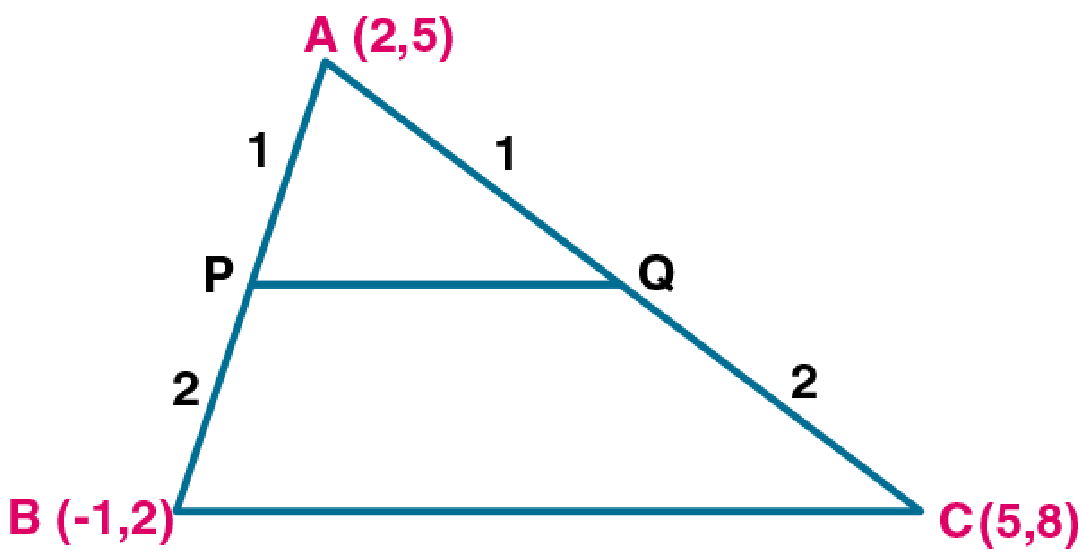
By distance formula, $d(LM) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
The points are L(8,1) and M(6,3)
So $x_1= 8$, $y_1 = 1$
$x_2 = 6$, $y_2 = 3$
$\therefore$ $d(LM) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ $d(LM) = \sqrt{[(6-8)^2+(3-1)^2]}$
$\therefore$ $d(LM) = \sqrt{[(-2)^2+(2)^2]}$
$\therefore$ $d(LM) = \sqrt{(4+4)}$
$\therefore$ $d(LM) = \sqrt{8} = 2\sqrt{2}$ …(i)

By distance formula, $d(BC) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
The points are B(6,-3) and C(2,1).
So $x_1= 6$, $y_1 = -3$
$x_2 = 2$, $y_2 = 1$

$\therefore$ d(BC) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ d(BC) = $\sqrt{[(2-6)^2+(1-(-3))^2]}$
$\therefore$ d(BC) = $\sqrt{[(-4)^2+(4)^2]}$
$\therefore$ d(BC) = $\sqrt{(16+16)}$
$\therefore$ d(BC) = $\sqrt{32} = 4\sqrt{2}$ …(ii)
From (i) and (ii), LM = ½ BC

32. A (2, 5), B (– 1, 2) and C (5, 8) are the vertices of a triangle ABC. P and Q are points on AB and AC respectively such that AP : PB = AQ : QC = 1 : 2.
(i) Find the co-ordinates of P and Q.
(ii) Show that PQ = 1/3 BC

Solution:



(i) Given vertices of the △ABC are A(2,5), B(-1,2) and C(5,8).
P and Q are points on AB and AC respectively such that AP:PB = AQ :QC = 1:2.
P(x,y) divides AB in the ratio 1:2.
x_1= 2, y_1 = 5
x_2 = -1, y_2 = 2
m:n = 1:2
By section formula, x = $(mx_2+nx_1)/(m+n)$
$\therefore$ x = (1×-1+2×2)/(1+2)
$\therefore$ x = (-1+4)/(3)
$\therefore$ x = 3/3 = 1
By section formula, y = $(my_2+ny_1)/(m+n)$
$\therefore$ y = (1×2+2×5)/(1+2)
$\therefore$ y = (2+10)/(3)

$\therefore$ y = 12/3 = 4
$\therefore$ Co-ordinates of P are (1,4).

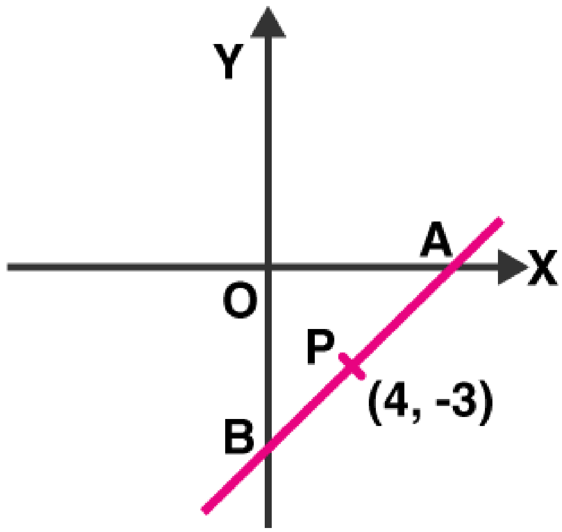
Q(x,y) divides AC in the ratio 1:2.
x_1= 2, y_1 = 5
x_2 = 5, y_2 = 8
m:n = 1:2
By section formula, x = $(mx_2+nx_1)/(m+n)$
$\therefore$ x = (1×5+2×2)/(1+2)
$\therefore$ x = (5+4)/(3)
$\therefore$ x = 9/3 = 3
By section formula, y = $(my_2+ny_1)/(m+n)$
$\therefore$ y = (1×8+2×5)/(1+2)
$\therefore$ y = (8+10)/(3)
$\therefore$ y = 18/3 = 6
$\therefore$ Co-ordinates of Q are (3,6).

(ii) By distance formula, d(PQ) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
Points are P(1,4) and Q(3,6).
So x_1= 1, y_1 = 4
x_2 = 3, y_2 = 6
$\therefore$ d(PQ) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ d(PQ) = $\sqrt{[(3-1)^2+(6-4)^2]}$
$\therefore$ d(PQ) = $\sqrt{[(2)^2+(2)^2]}$
$\therefore$ d(PQ) = $\sqrt{(4+4)}$
$\therefore$ d(PQ) = $\sqrt{8} = 2\sqrt{2}$ ..(i)

By distance formula, d(BC) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
Points are B(-1,2) and C(5,8).
So x_1= -1, y_1 = 2
x_2 = 5, y_2 = 8
$\therefore$ d(BC) = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$\therefore$ d(BC) = $\sqrt{[(5-(-1))^2+(8-2)^2]}$
$\therefore$ d(BC) = $\sqrt{[(6)^2+(6)^2]}$
$\therefore$ d(BC) = $\sqrt{(36+36)}$
$\therefore$ d(BC) = $\sqrt{72} = \sqrt{(36\times2)} = 6\sqrt{2}$ ..(ii)

BC/3 = $6\sqrt{2}/3 = 2\sqrt{2}$ = PQ
$\therefore$ PQ = 1/3 BC.
Hence proved.

33. The mid-point of the line segment AB shown in the adjoining diagram is (4, – 3). Write down the co-ordinates of A and B.



Solution:
Let P(4,-3) be the midpoint of line joining the points A and B.
Since A lies on X axis, its co-ordinates are $(x_2,0)$
Since B lies on Y axis, its co-ordinates are $(0,y_1)$
By midpoint formula, $x = (x_1+x_2)/2$
$\therefore 4 = (0+x_2)/2$
$\therefore\ x_2 = 4\times2 = 8$
By midpoint formula, $y = (y_1+y_2)/2$
$\therefore -3 = (y_1+0)/2$
$\therefore\ y_1 = -3\times2 = -6$
Hence the co-ordinates of A and B are (8,0)and (0,-6) respectively.

34. Find the co-ordinates of the centroid of a triangle whose vertices are A (– 1, 3), B(1, – 1) and C (5, 1) (2006)

Solution:
Given vertices of the triangle are A(-1,3), B(1,-1) and C(5,1)
Co-ordinates of the centroid of a triangle, whose vertices are (x_1,y_1), (x_2,y_2) and (x_3,y_3) are
$[(x_1 + x_2+ x_3)/3, (y_1 + y_2+ y_3)/3]$
$(x_1,y_1) = (-1,3)$
$(x_2,y_2) = (1,-1)$
$(x_3,y_3) = (5,1)$
$\therefore (x_1 + x_2+ x_3)/3 = (-1+1+5)/3 = 5/3$
$\therefore (y_1 + y_2+ y_3)/3 = (3-1+1)/3 = 3/3 = 1$
Hence the co-ordinates of centroid are (5/3, 1).

35. Two vertices of a triangle are (3, – 5) and (– 7, 4). Find the third vertex given that the centroid is (2, – 1).

Solution:

Let third vertex be $C(x_3,y_3)$.
Given $(x_1,y_1) = (3,-5)$
$(x_2,y_2) = (-7,4)$
Co-ordinates of centroid are (2,-1)
Co-ordinates of the centroid of a triangle, whose vertices are (x_1,y_1), (x_2,y_2) and (x_3,y_3) are
$[(x_1 + x_2+ x_3)/3, (y_1 + y_2+ y_3)/3]$
$\therefore (x_1 + x_2+ x_3)/3 = (3+-7+x_3)/3 = 2$ [x co-ordinate of centroid]
$\therefore -4+x_3 = 2\times3$
$\therefore -4+x_3 = 6$
$\therefore\ x_3 = 6+4$
$\therefore\ x_3 = 10$

$(y_1 + y_2+ y_3)/3 = -1$ [y co-ordinate of centroid]
$-5+4+y_3 = -1\times3$
$-1+y_3 = -3$
$\therefore y_3 = -3+1$
$\therefore y_3 = -2$
Hence the third vertex is (10,-2).

36. The vertices of a triangle are A (– 5, 3), B (p, – 1) and C (6, q). Find the values of p and q if the centroid of the triangle ABC is the point (1, – 1).

Solution:
Given vertices of the triangle are A(-5,3), B(p,-1) and C(6,q).
Co-ordinates of centroid are (1,-1).
Co-ordinates of the centroid of a triangle, whose vertices are (x_1,y_1), (x_2,y_2) and (x_3,y_3) are
$[(x_1 + x_2+ x_3)/3, (y_1 + y_2+ y_3)/3]$
$(x_1,y_1) = (-5,3)$
$(x_2,y_2) = (p,-1)$
$(x_3,y_3) = (6,q)$
$\therefore$ x co-ordinate of centroid, $(x_1 + x_2+ x_3)/3 = (-5+p+6)/3 = 1$
$\therefore p+1 = 3$
$\therefore p = 3-1$
$\therefore p = 2$
$\therefore$ y co-ordinate of centroid, $(y_1 + y_2+ y_3)/3 = (3-1+q)/3 = -1$
$\therefore 2+q = 3\times-1$
$\therefore 2+q = -3$
$\therefore q = -3-2$
$\therefore q = -5$
Hence the value of p and q are 2 and -5 respectively.