Exercise 4

1. Solve the inequation, 3x – 11 < 3 where x ∈ {1, 2, 3,……, 10}. Also, represent its solution on a number line.
Solution:

Given inequation, $3x – 11 < 3$
$3x < 3 + 11$
$3x < 14$
⇒ $x < 14/3$
But, x ∈ {1, 2, 3,……, 10}
Hence, the solution set is {1, 2, 3, 4}.
Representing the solution on a number line:

2. Solve 2(x – 3) < 1, x ∈ {1, 2, 3, …. 10}
Solution:

Given inequation, $2(x – 3) < 1$
$2x – 6 < 1$
$2x < 7$
⇒ $x < 7/2$
But, x ∈ {1, 2, 3, …. 10}
Hence, the solution set is {1, 2, 3}

3. Solve 5 – 4x > 2 – 3x, x ∈ W. Also represent its solution on the number line.
Solution:

Given inequation, $5 – 4x > 2 – 3x$
$- 4x + 3x > 2 – 5$
$-x > -3$
On multiplying both sides by -1, the inequality reverses
⇒ $x < 3$
Since, x ∈ W
The solution set is {0, 1, 2}
Representing the solution on a number line:

4. List the solution set of 30 – 4 (2x – 1) < 30, given that x is a positive integer.
Solution:

Given inequation, $30 – 4 (2x – 1) < 30$
$30 – 8x + 4 < 30$

34 – 8x < 30
-8x < 30 – 34
-8x < -4   [On multiplying both sides by -1, the inequality reverses]
8x > 4
x > 4/8
⇒ x > 1/2
As x is a positive integer
The solution set is {1, 2, 3, … }

5. Solve: 2 (x – 2) < 3x – 2, x ∈ {– 3, – 2, – 1, 0, 1, 2, 3}.
Solution:

Given inequation, 2 (x – 2) < 3x – 2
2x - 4 < 3x – 2
2x – 3x < -2 + 4
-x < 2
⇒ x > -2
But, x ∈ {– 3, – 2, – 1, 0, 1, 2, 3}
Hence, the solution set is {– 1, 0, 1, 2, 3}.

6. If x is a negative integer, find the solution set of 2/3 + 1/3 (x + 1) > 0.
Solution:

Given inequation, 2/3 + 1/3 (x + 1) > 0.
2/3 + x/3 + 1/3 > 0
x/3 + 1 > 0
x/3 > -1
⇒ x > -3
As x is a negative integer
The solution set is {-1, -2}.

7. Solve: (2x - 3)/4 ≥ ½ , x ∈ {0, 1, 2,…,8}
Solution:

Given inequation, (2x - 3)/4 ≥ ½
2x – 3 ≥ 4 x ½
2x – 3 ≥ 2
2x ≥ 2 + 3
2x ≥ 5
⇒ x ≥ 5/2
But, x ∈ {0, 1, 2, …,8}
Hence, the solution set is {3, 4, 5, 6, 7, 8}.

8. Solve x – 3 (2 + x) > 2 (3x – 1), x ∈ { – 3, – 2, – 1, 0, 1, 2, 3}. Also represent its solution on the number line.
Solution:

Given inequation, x – 3 (2 + x) > 2 (3x – 1)
x – 6 – 3x > 6x – 2
-2x – 6 > 6x – 2
-6x – 2x > -2 + 6
-8x > 4
x < -4/8
⇒ x < -1/2
But, x ∈ { – 3, – 2, – 1, 0, 1, 2, 3}
Hence, the solution set is {-3, -2, -1}

9. Given x ∈ {1, 2, 3, 4, 5, 6, 7, 9} solve x – 3 < 2x – 1.
Solution:

Given inequation, x – 3 < 2x – 1
x – 2x < – 1 + 3
-x < 2
⇒ x > -2
But, x ∈ {1, 2, 3, 4, 5, 6, 7, 9}
Hence, the solution set is {1, 2, 3, 4, 5, 6, 7, 9}.

10. Given A = {x: x ∈ I, – 4 ≤ x ≤ 4}, solve 2x – 3 < 3 where x has the domain A. Graph the solution set on the number line.
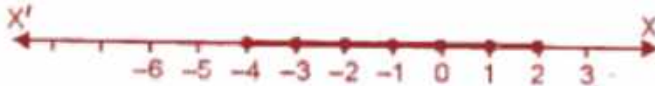**Solution:**

Given equation, 2x – 3 < 3
2x < 6
⇒ x < 3
But x has a the domain A = {x: x ∈ I, – 4 ≤ x ≤ 4}
A = {-4, -3, -2, -1, 0, 1, 2, 3, 4}
Hence, the solution set is {-4, -3, -2, -1, 0, 1, 2}.
Representing the solution on a number line:

11. List the solution set of the inequation
½ + 8x > 5x – 3/2, x ∈ Z
Solution:

Given inequation, ½ + 8x > 5x – 3/2
8x – 5x > -3/2 – ½
3x > -4/2
⇒ x > -2/3
As x ∈ Z
The solution set is {0, 1, 2, 3, 4, 5, …}

12. List the solution set of (11 – 2x)/5 ≥ (9 – 3x)/8 + 3/4, x ∈ N
Solution:

Given inequation, (11 – 2x)/5 ≥ (9 – 3x)/8 + ¾
(11 – 2x)/5 ≥ (9 – 3x + 6)/8
8 (11 – 2x) ≥ 5 (15 – 3x)
88 – 16x ≥ 75 – 15x
15x – 16x ≥ 75 – 88
-x ≥ -13
⇒ x ≤ 13
As x ∈ N
Hence, the solution set is {1, 2, 3, 4, …, 13}.

13. Find the values of x, which satisfy the inequation : $-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}$ **, x ∈ N. Graph the solution set on the number line.**
Solution:

Given inequation, $-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}$
-2 ≤ (3 – 4x)/ 6 ≤ 11/6
-12 ≤ 3 – 4x ≤ 11
-12 – 3 ≤ -4x ≤ 11 – 3
-15 ≤ -4x ≤ 8
-15/4 ≤ -x ≤ 8/4
⇒ 15/4 ≥ x ≥ -2
As x ∈ N,
The solution set is {1, 2, 3}.
Representing the solution on a number line:
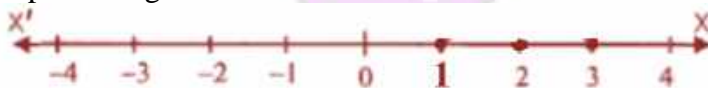
14. If x ∈ W, find the solution set of 3/5 x – (2x - 1)/3 > 1. Also graph the solution set on the number line, if possible.
Solution:

Given inequation, 3/5 x – (2x - 1)/3 > 1
9/15 x – 5(2x - 1)/15 > 1   [Taking L.C.M]
9x – 5(2x - 1) > 15   [Multiplying by 15 on both sides]
9x – 10x + 5 > 15
-x > 15 – 5
-x > 10
⇒ x < -10
But, x ∈ W

Hence, the solution set is a null set.
Thus, it can't be represented on number line.

15. Solve:
(i) x/2 + 5 ≤ x/3 + 6 where x is a positive odd integer.
(ii) (2x + 3)/3 ≥ (3x - 1)/4 where x is positive even integer.
Solution:

(i) Given inequation, x/2 + 5 ≤ x/3 + 6
(x + 10)/2 ≤ (x + 18)/3   [Taking L.C.M on both sides]
3 (x + 10) ≤ 2 (x + 18)   [On cross-multiplying]
3x + 30 ≤ 2x + 36
3x – 2x ≤ 36 – 30
⇒ x ≤ 6
As x is a positive odd integer.
Hence, the solution set is {1, 3, 5}.

(ii) Given inequation, (2x + 3)/3 ≥ (3x - 1)/4
4 (2x + 3) ≥ 3 (3x - 1)   [On cross-multiplying]
8x + 12 ≥ 9x – 3
-9x + 8x ≥ -12 – 3
-x ≥ -15
⇒ x ≤ 15
As x is positive even integer.
Hence, the solution set is {2, 4, 6, 8, 10, 12, 14}.

16. Given that x ∈ I, solve the inequation and graph the solution on the number line:
3 ≥ (x - 4)/2 + x/3 ≥ 2
Solution:

Given inequation, 3 ≥ (x - 4)/2 + x/3 ≥ 2
Now, let's take
3 ≥ (x - 4)/2 + x/3, we have
3 ≥ (3x – 12 + 2x)/6   [Taking L.C.M]
18 ≥ 5x – 12
30 ≥ 5x
⇒ x ≤ 6 …. (i)
Next,
(x - 4)/2 + x/3 ≥ 2
(3x – 12 + 2x)/6 ≥ 2
5x – 12 ≥ 12
5x ≥ 24
x ≥ 24/5 ⇒ x ≥ 4.8 … (ii)
Hence, from (i) and (ii) we have
Solution of x = {5, 6}
Representing the solution on a number line:

17. Given x ∈ {1, 2, 3, 4, 5, 6, 7, 9}, find the values of x for which -3 < 2x – 1 < x + 4.
Solution:

Given inequation, -3 < 2x – 1 < x + 4
So, we have
-3 < 2x – 1   and   2x – 1 < x + 4
-2x < 3 – 1   and   2x – x < 4 + 1
-2x < 2   and   x < 5
x > -1   and   x < 5
As x ∈ {1, 2, 3, 4, 5, 6, 7, 9}
The solution set is {1, 2, 3, 4}.

18. Solve: 1 ≥ 15 – 7x > 2x – 27, x ∈ N
Solution:

Given inequation, 1 ≥ 15 – 7x > 2x – 27,
So, we have
1 ≥ 15 – 7x   and   15 – 7x > 2x – 27
7x ≥ 15 – 1   and   -2x – 7x > -27 – 15
7x ≥ 14   and   -9x > -42
x ≥ 2   and   -x > -42/9
x ≥ 2   and   x < 14/3
⇒ 2 ≤ x < 14/3
But as x ∈ N
The solution set is {2, 3, 4}.

19. If x ∈ Z, solve 2 + 4x < 2x – 5 ≤ 3x. Also represent its solution on the number line.
Solution

Given inequation, 2 + 4x < 2x – 5 ≤ 3x
So, we have
2 + 4x < 2x – 5   and   2x – 5 ≤ 3x
4x – 2x < -5 – 2   and   2x – 3x ≤ 5
2x < -7   and   -x ≤ 5
x < -7/2   and   x ≥ -5
⇒ -5 ≤ x < -7/2
As x ∈ Z
The solution set is {-5, -4}.
Representing the solution on a number line:
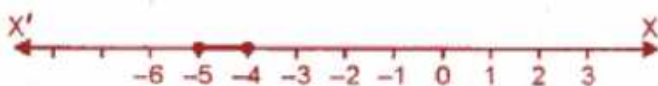
20. Solve the inequation $12 + 1\frac{5}{6}x \le 5 + 3x$, x ∈ R. Represent the solution on a number line.
Solution:

Given inequation, $12 + 1\frac{5}{6}x \le 5 + 3x$
12 + 11/6 x ≤ 5 + 3x
72 + 11x ≤ 30 + 18x   [Multiplying by 6 on both sides]
11x – 18x ≤ 30 – 72
-7x ≤ -42
-x ≤ -6
x ≥ 6
As x ∈ R
The solution set is {x : x ∈ R, x ≥ 6}
Representing the solution on a number line:
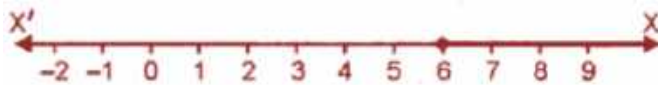
21. Solve: (4x - 10)/3 ≤ (5x - 7)/2, x ∈ R and represent the solution set on the number line.
Solution:

Given inequation, (4x - 10)/3 ≤ (5x - 7)/2
2 (4x - 10) ≤ 3 (5x - 7)   [On cross-multiplying]
8x – 20 ≤ 15x – 21
8x – 15x ≤ -21 + 20
-7x ≤ -1
-x ≤ -1/7
x ≥ 1/7
As x ∈ R
Hence, the solution set is {x: x ∈ R, x ≥ 1/7}
Representing the solution on a number line:

22. Solve 3x/5 – (2x - 1)/3 > 1, x ∈ R and represent the solution set on the number line.
Solution:

Given inequation, 3x/5 – (2x - 1)/3 > 1
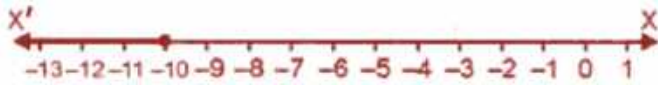(9x – 10x + 5)/15 > 1   [Taking L.C.M]
-x + 5 > 15
-x > 15 – 5
-x > 10
x < -10
As x ∈ R
Hence, the solution set is {x: x ∈ R, x < -10}
Representing the solution on a number line:

23. Solve the inequation – 3 ≤ 3 – 2x < 9, x ∈ R. Represent your solution on a number line.
Solution:

Given inequation, – 3 ≤ 3 – 2x < 9
– 3 – 3 ≤ – 2x < 9 – 3
-6 ≤ -2x < 6
-3 ≤ -x < 3
-3 < x ≤ 3
As x ∈ R
The solution set is {x: x ∈ R, -3 < x ≤ 3}
Representing the solution on a number line:

24. Solve 2 ≤ 2x – 3 ≤ 5, x ∈ R and mark it on a number line.
Solution:

Given inequation, 2 ≤ 2x – 3 ≤ 5
2 + 3 ≤ 2x ≤ 5 + 3
5 ≤ 2x ≤ 8
5/2 ≤ x ≤ 4
Hence, the solution set is {x: x ∈ R, 5/2 ≤ x ≤ 4}
Representing the solution on a number line:
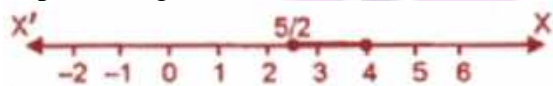
25. Given that x ∈ R, solve the following inequation and graph the solution on the number line: -1 ≤ 3 + 4x < 23.
Solution:

Given inequation, -1 ≤ 3 + 4x < 23
-1 – 3 ≤ 4x < 23 - 3
-4 ≤ 4x < 20
-4/4 ≤ x < 20/4
-1 ≤ x < 5
Hence, the solution set is {-1 ≤ x < 5; x ∈ R}
Representing the solution on a number line:

26. Solve the following inequation and graph the solution on the number line.

$-2\frac{2}{3} \le x + \frac{1}{3} < 3 + \frac{1}{3}$, x ∈ R

Solution:

Given inequation,

$-2\frac{2}{3} \le x + \frac{1}{3} < 3 + \frac{1}{3}$

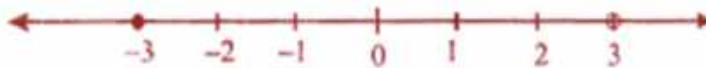-8/3 ≤ (3x + 1)/3 < 10/3

-8 ≤ 3x + 1 < 10   [Multiplying by 3]

-8 – 1 ≤ 3x < 10 – 1

-9 ≤ 3x < 9

-3 ≤ x < 3   [Dividing by 5]

Thus, the solution set is {x: x ∈ R, -3 ≤ x < 3}

Representing the solution on a number line:

27. Solve the following inequation and represent the solution set on the number line:

$-3 < -\frac{1}{2} - \frac{2x}{3} \le \frac{5}{6}, x \in R$

Solution:

Given in equation,

$-3 < -\frac{1}{2} - \frac{2x}{3} \le \frac{5}{6}, x \in R$

-3 < -(3 + 4x)/6 ≤ 5/6   [Taking L.C.M]

-18 < -3 – 4x ≤ 5   [Multiplying by 6]

-18 + 3 < -4x ≤ 5 + 3

-15 < -4x ≤ 8

-15/4 < -x ≤ 8/4

-2 ≤ x < 15/4

Hence, the solution set is {x : x ∈ R, -2 ≤ x < 15/4}

Representing the solution on a number line:

28. Solve $\frac{2x+1}{2} + 2(3 - x) \ge 7, x \in R$ **. Also graph the solution set on the number line**

Solution:

Given inequation,

$\frac{2x+1}{2} + 2(3 - x) \ge 7, x \in R$

[2x + 1 + 4(3 - x)]/2 ≥ 7   [Taking L.C.M]

2x + 1 + 12 – 4x ≥ 14

-2x + 13 ≥ 14

-2x ≥ 14 – 13

-2x ≥ 1
-x ≥ ½
x ≤ -½
Hence, the solution set is {x : x ∈ R, x ≤ -½}
Representing the solution on a number line:
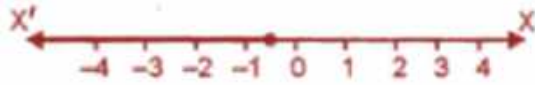
29. Solving the following inequation, write the solution set and represent it on the number line –

$3(x – 7) ≥ 15 – 7x > \dfrac{x+1}{3}$, x ∈ R

Solution:

Given inequation, $–3(x – 7) ≥ 15 – 7x > \dfrac{x+1}{3}$

–3x + 21 ≥ 15 – 7x > (x + 1)/3
So,
-3x + 21 ≥ 15 – 7x
7x – 3x ≥ 15 – 21
4x ≥ -6
x ≥ -6/4
x ≥ -3/2
And,
15 – 7x > (x + 1)/3
3(15 – 7x) > x + 1
45 – 21x > x + 1
-21x – x > 1 – 45
-22x > -44
-x > -44/22
x < 2
Hence, the solution set is {x : x ∈ R , -3/2 ≤ x < 2}
Representing the solution on a number line:

30. Solve the inequation :
$-2\frac{1}{2} + 2x \leq \frac{4x}{3} \leq \frac{4}{3} + 2x, \quad x \in W$. **Graph the solution set on the number line.**

Solution:

Given inequation,
$-2\frac{1}{2} + 2x \leq \frac{4x}{3} \leq \frac{4}{3} + 2x, \quad x \in W$
-5/2 + 2x ≤ 4x/3 ≤ 4/3 + 2x
So, we have
-5/2 + 2x ≤ 4x/3   and   4x/3 ≤ 4/3 + 2x

2x – 4x/3 ≤ 5/2   and   4x/3 – 2x ≤ 4/3
(6x – 4x)/3 ≤ 5/2   and   (4x – 6x)/3 ≤ 4/3   [Taking L.C.M]
2x ≤ 15/2   and   -2x ≤ 4
x ≤ 15/4   and   -x ≤ 2   [Dividing by 2]
x ≤ 15/4   and   x ≥ -2
-2 ≤ x ≤ 15/4
For x ∈ W
Thus, the solution set is {0, 1, 2, 3}
Representing the solution on a number line:

31. Solve the inequation 2x – 5 ≤ 5x + 4 < 11, where x ∈ I. Also represent the solution set on the number line.
Solution:

Given inequation, 2x – 5 ≤ 5x + 4 < 11
So, we have
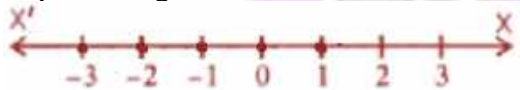2x – 5 ≤ 5x + 4   and   5x + 4 < 11
2x – 5x ≤ 4 + 5   and   5x < 11 – 4
-3x ≤ 9   and   5x < 7
-x ≤ 9/3   and   x < 7/5
x ≥ -3   and   x < 7/5
-3 ≤ x < 7/5
As x ∈ I
Thus, the solutions set is {-3, -2, -1, 0, 1}
Representing the solution on a number line:

32. If x ∈ I, A is the solution set of 2 (x – 1) < 3x – 1 and B is the solution set of 4x – 3 ≤ 8 + x, find A ∩ B.
Solution:

Given inequations,
2 (x – 1) < 3x – 1   and   4x – 3 ≤ 8 + x for x ∈ I
Solving for both, we have
2x – 3x < 2 – 1   and   4x – x ≤ 8 + 3
-x < 1   and   3x ≤ 11
x > -1   and   x ≤ 11/3
Hence,
Solution set A = {0, 1, 2, 3, …}
Solution set B = {3, 2, 1, 0, -1, … }
Thus, A ∩ B = {0, 1, 2, 3}

33. If P is the solution set of -3x + 4 < 2x – 3, x ∈ N and Q is the solution set of 4x – 5 < 12, x ∈ W, find
(i) P ∩ Q
(ii) Q – P.
Solution:

Given inequations,
-3x + 4 < 2x – 3 where x ∈ N and 4x – 5 < 12 where x ∈ W
So, solving
-3x + 4 < 2x – 3 where x ∈ N
-3x – 2x < -3 – 4
-5x < -7
x > 7/5
Hence, the solution set P is {2, 3, 4, 5, …}
And, solving
4x – 5 < 12 where x ∈ W
4x < 12 + 5
4x < 17
x < 17/4
Hence, the solution set Q is {0, 1, 2, 3, 4}
Therefore,
(i) P ∩ Q = {2, 3, 4}
(ii) Q – P = {0, 1}

34. A = {x : 11x – 5 > 7x + 3, x ∈ R} and B = {x : 18x – 9 ≥ 15 + 12x, x ∈ R}
Find the range of set A ∩ B and represent it on a number line
Solution:

Given, A = {x : 11x – 5 > 7x + 3, x ∈ R} and B = {x : 18x – 9 ≥ 15 + 12x, x ∈ R}
Solving for A,
11x – 5 > 7x + 3
11x – 7x > 3 + 5
4x > 8
x > 2
Hence, A = {x : x > 2, x ∈ R}
Next, solving for B
18x – 9 ≥ 15 + 12x
18x – 12x ≥ 15 + 9
6x ≥ 24
x ≥ 4
Hence, B = {x : x ≥ 4, x ∈ R}
Thus, A ∩ B = x ≥ 4
Representing the solution on a number line:
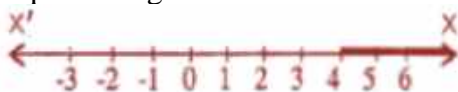
35. Given: P {x : 5 < 2x – 1 ≤ 11, x ∈ R}
Q {x : – 1 ≤ 3 + 4x < 23, x ∈ I} where
R = (real numbers), I = (integers)
Represent P and Q on number line. Write down the elements of P ∩ Q.
Solution:

Given, P {x : 5 < 2x – 1 ≤ 11, x ∈ R} and Q {x : – 1 ≤ 3 + 4x < 23, x ∈ I}
Solving for P,
5 < 2x – 1 ≤ 11
5 + 1 < 2x ≤ 11 + 1
6 < 2x ≤ 12
3 < x ≤ 6
Hence, P = P {x : 3 < x ≤ 6, x ∈ R}
Representing the solution on a number line:
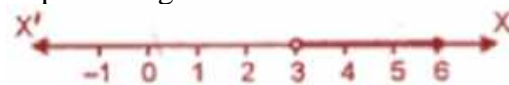
Next, solving for Q
– 1 ≤ 3 + 4x < 23
-1 – 3 ≤ 4x < 23 - 3
-4 ≤ 4x < 20
-1 ≤ x < 5
Hence, solution Q = {-1, 0, 1, 2, 3, 4}
Representing the solution on a number line:

Therefore, P ∩ Q = {4}

36. If x ∈ I, find the smallest value of x which satisfies the inequation $2x + \frac{5}{2} > \frac{5x}{3} + 2$
Solution:

Given inequation, $2x + \frac{5}{2} > \frac{5x}{3} + 2$
(4x + 5)/2 > (5x + 6)/3   [Taking L.C.M]
3 (4x + 5) > 2 (5x + 6)   [On cross-multiplication]
12x + 15 > 10x + 12
12x – 10x > 12 – 15
2x > -3
x > -3/2
Hence, for x ∈ I the smallest value of x is -1.

37. Given 20 – 5 x < 5 (x + 8), find the smallest value of x, when
(i) x ∈ I
(ii) x ∈ W

(iii) x ∈ N.
Solution:

Given inequation, 20 – 5 x < 5 (x + 8)
20 – 5x < 5x + 40
-5x – 5x < 40 – 20
-10x < 20
-x < 20/10
x > -2
Thus,
(i) For x ∈ I, the smallest value = -1
(ii) For x ∈ W, the smallest value = 0
(iii) For x ∈ N, the smallest value = 1

38. Solve the following inequation and represent the solution set on the number line:
$$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x, x \in R$$
Solution:

Given inequation,
$$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x, x \in R$$
So, we have

4x – 19 < 3x/5 – 2	and	3x/5 – 2 ≤ -2/5 + x
4x – 3x/5 < 19 – 2	and	3x/5 – x ≤ 2 – 2/5
(20x – 3x)/5 < 17	and	(3x – 5x)/5 ≤ (10 - 2)/5
17x < 35	and	-2x ≤ 8 [Multiplying by 5]
x < 5	and	-x ≤ 4
x < 5	and	x ≥ 4

-4 ≤ x < 5, x ∈ R
Hence, the solution set is {x : -4 ≤ x < 5, x ∈ R}
Representing the solution on a number line:
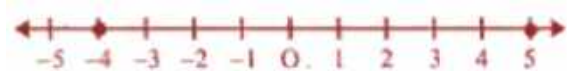
39. Solve the given inequation and graph the solution on the number line:
2y – 3 < y + 1 ≤ 4y + 7; y ∈ R.
Solution:

Given inequation, 2y – 3 < y + 1 ≤ 4y + 7
So, we have

2y – 3 < y + 1	and	y + 1 ≤ 4y + 7
2y – y < 1 + 3	and	y – 4y ≤ 7 – 1
y < 4	and	-3y ≤ 6
y < 4	and	-y ≤ 2 ⇒ y ≥ -2

Thus, -2 ≤ y < 4

The solution set is {y : -2 ≤ y < 4, y ∈ R}
Representing the solution on a number line:
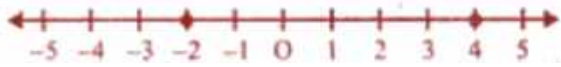
40. Solve the inequation and represent the solution set on the number line.

$$-3 + x \le \frac{8x}{3} + 2 \le \frac{14}{3} + 2x, \; Where \quad x \in I$$

Solution:

Given inequation,

$$-3 + x \le \frac{8x}{3} + 2 \le \frac{14}{3} + 2x, \; Where \quad x \in I$$

So, we have

-3 + x ≤ 8x/3 + 2	and	8x/3 + 2 ≤ 14/3 + 2x
x – 8x/3 ≤ 2 + 3	and	8x/3 – 2x ≤ 14/3 – 2
(3x - 8x)/3 ≤ 5	and	(8x – 6x)/3 ≤ (14 - 6)/3 [Taking L.C.M]
-5x/3 ≤ 5	and	2x ≤ 8
-5x ≤ 15	and	x ≤ 8/2
-x ≤ 3	and	x ≤ 4
x ≥ -3	and	x ≤ 4

⇒ -3 ≤ x ≤ 4
Thus, the solution set is {-3, -2, -1, 0, 1, 2, 3, 4}
Representing the solution on a number line:

41. Find the greatest integer which is such that if 7 is added to its double, the resulting number becomes greater than three times the integer.
Solution:

Let's consider the greatest integer to be x
Then according to the given condition, we have
2x + 7 > 3x
2x – 3x > -7
-x > -7
x < 7   , x ∈ R
Hence, the greatest integer value is 6.

42. One-third of a bamboo pole is buried in mud, one-sixth of it is in water and the part above the water is greater than or equal to 3 metres. Find the length of the shortest pole.
Solution:

Let's assume the length of the shortest pole = x metre
Now,
Length of the pole which is buried in mud = x/3
Length of the pole which is in the water = x/6

Then according to the given condition, we have

x – [x/3 + x/6] ≥ 3

x – [(2x + x)/6] ≥ 3

x – 3x/6 ≥ 3

x – x/2 ≥ 3

x/2 ≥ 3

x ≥ 6   [Multiplying by 6]

Therefore, the length of the shortest pole is 6 metres.

Chapter Test

1. Solve the inequation: 5x – 2 ≤ 3 (3 - x) where x ∈ {-2, -1, 0, 1, 2, 3, 4}. Also represent its solution on the number line.
Solution:

Given inequation, 5x – 2 ≤ 3 (3 - x)
5x – 2 ≤ 9 - 3x
5x + 3x ≤ 9 + 2
8x ≤ 11
x ≤ 11/8
As x ∈ {-2, -1, 0, 1, 2, 3, 4}
The solution set is {-2, -1, 0, 1}
Representing the solution on a number line:

2. Solve the inequation: 6x – 5 < 3x + 4, x ∈ I
Solution:

Given inequation, 6x – 5 < 3x + 4
6x – 3x < 4 + 5
3x < 9
x < 9/3
x < 3
As x ∈ I
The solution set is {2, 1, 0, -1, -2, …}

3. Find the solution set of the inequation x + 5 ≤ 2x + 3; x ∈ R
Graph the solution set on the number line.
Solution:

Given inequation, x + 5 ≤ 2x + 3
x – 2x ≤ 3 – 5
-x ≤ -2
x ≥ 2
As x ∈ R
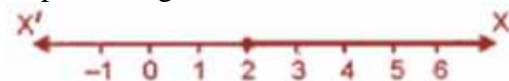Thus, the solution set is {2, 3, 4, 5, …}
Representing the solution on a number line:

4. If x ∈ R (real numbers) and -1 < 3 – 2x ≤ 7, find solution set and present it on a number line.
Solution:

Given inequation, -1 < 3 – 2x ≤ 7

-1 – 3 < -2x ≤ 7 - 3
-4 < -2x ≤ 4
-4/2 < -x ≤ 4/2
-2 < -x ≤ 2
Thus, -2 ≤ x < 2
The solution set is {x : x ∈ R, -2 ≤ x < 2}
Representing the solution on a number line:

5. Solve the inequation:

$$\frac{5x+1}{7} - 4\left(\frac{x}{7} + \frac{2}{5}\right) \le 1\frac{3}{5} + \frac{3x-1}{7}, x \in R$$

Solution:

Given inequation,

$$\frac{5x+1}{7} - 4\left(\frac{x}{7} + \frac{2}{5}\right) \le 1\frac{3}{5} + \frac{3x-1}{7}, x \in R$$

(5x + 1)/7 – 4 (5x + 14)/35 ≤ 8/5 + (3x - 1)/7
[5(5x + 1) - 4 (5x + 14)]/ 35 ≤ [56 + 5 (3x – 1)]/ 35   [Taking L.C.M]
(25x + 5 – 20x – 56) ≤ 56 + 15x – 5
5x – 51 ≤ 51 + 15x
5x – 15x ≤ 51 + 51
-10x ≤ 102
-x ≤ 102/10
x ≥ - 51/5
Hence, the solution set is {x : x ∈ R , x ≥ - 51/5}