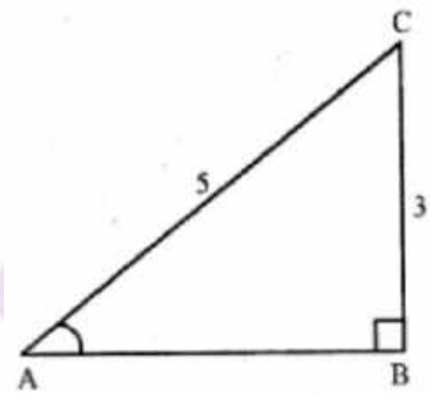
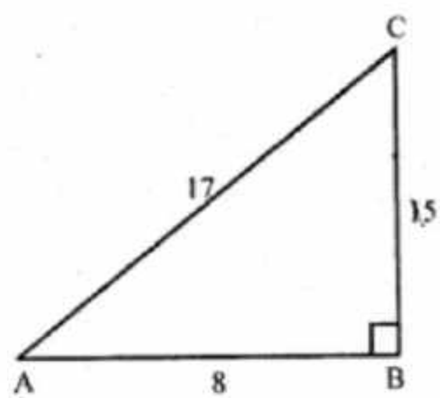
Exercise 18

1. If A is an acute angle and sin A = 3/5, find all other trigonometric ratios of angle A (using trigonometric identities).
Solution:



Given,
sin A = 3/5 and A is an acute angle
So, in ΔABC we have ∠B = 90°
And,
AC = 5 and BC = 3
By Pythagoras theorem,
AB = $\sqrt{(AC^2 - BC^2)}$
= $\sqrt{(5^2 - 3^2)} = \sqrt{(25 - 9)} = \sqrt{16}$
= 4
Now,
cos A = AB/AC = 4/5
tan A = BC/AB = 3/4
cot A = 1/tan θ = 4/3
sec A = 1/cos θ = 5/4
cosec A = 1/sin θ = 5/3

2. If A is an acute angle and sec A = 17/8, find all other trigonometric ratios of angle A (using trigonometric identities).
Solution:



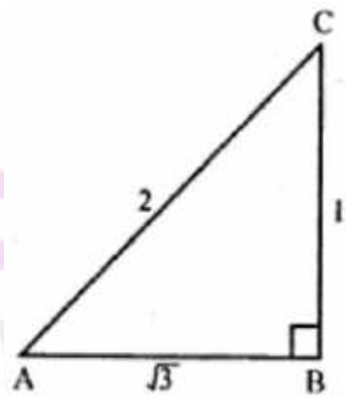
Given,
sec A = 17/8 and A is an acute angle
So, in Δ ABC we have ∠B = 90°
And,
AC = 17 and AB = 8
By Pythagoras theorem,
BC = $\sqrt{(AC^2 - AB^2)}$
= $\sqrt{(17^2 - 8^2)} = \sqrt{(289 - 64)} = \sqrt{225}$
= 15
Now,
sin A = BC/AC = 15/17
cos A = 1/sec A = 8/17
tan A = BC/AB = 15/8
cot A = 1/tan A= 8/15
cosec A = 1/sin A = 17/15

3. Express the ratios cos A, tan A and sec A in terms of sin A.
Solution:

We know that,

$\sin^2 A + \cos^2 A = 1$
So,
$\cos A = \sqrt{(1 - \sin^2 A)}$
$\tan A = \sin A/\cos A = \sin A/ \sqrt{(1 - \sin^2 A)}$
$\sec A = 1/\cos A = 1/ (\sqrt{1 - \sin^2 A})$

4. If tan A = 1/√3, find all other trigonometric ratios of angle A.
Solution:



Given, $\tan A = 1/\sqrt{3}$
In right Δ ABC,
$\tan A = BC/AB = 1/\sqrt{3}$
So,
$BC = 1$ and $AB = \sqrt{3}$
By Pythagoras theorem,
$AC = \sqrt{(AB^2 + BC^2)} = \sqrt{[(\sqrt{3})^2 + (1)^2]}$
$= \sqrt{(3 + 1)} = \sqrt{4} = 2$
Hence,
$\sin A = BC/AC = ½$
$\cos A = AB/AC = \sqrt{3}/2$
$\cot A = 1/\tan A = \sqrt{3}$
$\sec A = 1/\cos A = 2/\sqrt{3}$
$\text{cosec } A = 1/\sin A = 2/1 = 2$

5. If 12 cosec θ = 13, find the value of (2 sin θ – 3 cos θ)/ (4 sin θ – 9 cos θ)
Solution:

Given,
$12 \text{ cosec } \theta = 13$
$\Rightarrow \text{cosec } \theta = 13/12$
In right Δ ABC,
$\angle A = \theta$
So, $\text{cosec } \theta = AC/BC = 13/12$
$AC = 13$ and $BC = 12$
By Pythagoras theorem,
$AB = \sqrt{(AC^2 - BC^2)}$
$= \sqrt{[(13)^2 - (12)^2]}$
$= \sqrt{(169 - 144)}$
$= \sqrt{25}$
$= 5$
Now,
$\sin \theta = BC/AC = 12/13$
$\cos \theta = AB/AC = 5/13$
Hence,

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}}$$

$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} = \frac{\frac{9}{13}}{\frac{3}{13}} = \frac{9}{13} \times \frac{13}{3} = 3$$

Without using trigonometric tables, evaluate the following (6 to 10):
6. (i) $\cos^2 26^o + \cos 64^o \sin 26^o + (\tan 36^o/ \cot 54^o)$
(ii) $(\sec 17^o/ \text{cosec } 73^o) + (\tan 68^o/ \cot 22^o) + \cos^2 44^o + \cos^2 46^o$
Solution:

Given,
(i) $\cos^2 26^o + \cos 64^o \sin 26^o + (\tan 36^o/ \cot 54^o)$
$= \cos^2 26^o + \cos (90^o - 16^o) \sin 26^o + [\tan 36^o/ \cot (90^o - 54^o)]$
$= [\cos^2 26^o + \sin^2 26^o] + (\tan 36^o/ \tan 36^o)$
$= 1 + 1 = 2$

(ii) $(\sec 17^o/ \text{cosec } 73^o) + (\tan 68^o/ \cot 22^o) + \cos^2 44^o + \cos^2 46^o$
$= [\sec 17^o/ \text{cosec } (90^o - 73^o)] + [(\tan 90^o – 22^o)/ \cot 22^o] + \cos^2 (90^o - 44^o) + \cos^2 46^o$
$= [\sec 17^o/ \sec 17^o] + [\cot 22^o/ \cot 22^o] + [\sin^2 46^o + \cos^2 46^o]$
$= 1 + 1 + 1$
$= 3$

7. (i) $(\sin 65^o/ \cos 25^o) + (\cos 32^o/\sin 58^o) – \sin 28^o \sec 62^o + \text{cosec}^2 30^o$
(ii) $(\sin 29^o/ \text{cosec } 61^o) + 2 \cot 8° \cot 17° \cot 45° \cot 73° \cot 82° – 3(\sin^2 38° + \sin^2 52°)$.
Solution:

Given,
(i) $(\sin 65^o/ \cos 25^o) + (\cos 32^o/\sin 58^o) – \sin 28^o \sec 62^o + \text{cosec}^2 30^o$
$= (\sin 65^o/ \cos (90^o - 65^o)) + (\cos 32^o/\sin (90^o – 32^o)) – \sin 28^o \sec (90^o – 28^o) + 2^2$
$= (\sin 65^o/\sin 65^o) + (\cos 32^o/\cos 32^o) – [\sin 28^o \text{ x cosec } 28^o] + 4$
$= 1 + 1 – 1 + 4$
$= 5$

(ii) $(\sin 29^o/ \text{cosec } 61^o) + 2 \cot 8° \cot 17° \cot 45° \cot 73° \cot 82° – 3(\sin^2 38° + \sin^2 52°)$.
$= (\sin 29^o/ \text{cosec } (90^o - 29^o)) + [2 \cot 8° \cot 17° \cot 45° \cot (90° - 17^o) \cot (90^o - 8^o)] – 3(\sin^2 38° + \sin^2 (90° - 38^o))$
$= (\sin 29^o/ \sin 29^o) + [2 \cot 8° \cot 17° \cot 45° \tan 17^o \tan 8^o] – 3(\sin^2 38° + \cos^2 38°)$
$= 1 + 2[(\cot 8° \tan 8^o) (\cot 17° \tan 17^o) \cot 45°] – 3(1)$
$= 1 + 2[1 \text{ x } 1 \text{ x } 1] – 3$

= 1 + 2 – 3
= 0

8. (i) (sin 35º cos 55º + cos 35º sin 55 º)/ (cosec² 10º – tan² 80 º)
(ii) sin² 34º + sin² 56º + 2 tan18º tan 72º – cot² 30º
Solution:

Given,
(i) (sin 35º cos 55º + cos 35º sin 55 º)/ (cosec² 10º – tan² 80 º)

$$= \frac{\sin 35° \cos(90° - 35°) + \cos 35° \sin(90° - 35°)}{\text{cosec}^2 10° - \tan^2(90° - 10°)}$$

$$= \frac{\sin 35° \sin 35° + \cos 35° \cos 35°}{\text{cosec}^2 10° - \cot^2 10°}$$

$$= \frac{\sin^2 35° + \cos^2 35°}{\text{cosec}^2 10° - \cot^2 10°} = \frac{1}{1} = 1$$

(ii) $\sin^2 34° + \sin^2 56° + 2\tan 18° \tan 72° – \cot^2 30°$
$= \sin^2 34° + \sin^2 (90° - 34°) + 2\tan 18° \tan (90° – 18°) – \cot^2 30°$
$= [\sin^2 34° + \cos^2 34°] + 2\tan 18° \cot 18° – \cot^2 30°$
$= 1 + 2 \times 1 - (\sqrt{3})^2$
= 1 + 2 – 3
= 0

9. (i) (tan 25º/ cosec 65º)² + (cot 25º/ sec 65º)² + 2 tan 18º tan 45º tan 75º
(ii) (cos² 25º + cos² 65º) + cosec θ sec (90º – θ) – cot θ tan (90º - θ)
Solution:

Given,
(i) $(\tan 25°/ \text{cosec } 65°)^2 + (\cot 25°/ \sec 65°)^2 + 2 \tan 18° \tan 45° \tan 75°$

$$= \left(\frac{\tan 25°}{\text{cosec}(90° - 25°)}\right)^2 + \left(\frac{\cot 25°}{\sec(90° - 25°)}\right)^2 + 2\tan 18° \tan(90° - 18°) \tan 45°$$

$$= \left(\frac{\tan 25°}{\sec 25°}\right)^2 + \left(\frac{\cot 25°}{\text{cosec } 25°}\right)^2 + 2\tan 18° \cot 18° \tan 45°$$

$$= \left(\frac{\sin 25° \times \cos 25°}{\cos 25° \times 1}\right)^2 + \left(\frac{\cos 25° \times \sin 25°}{\sin 25° \times 1}\right) + 2 \times 1 \times 1$$

$$= \sin^2 25° + \cos^2 25° + 2 \qquad \left\{\begin{array}{l} \because \sin^2\theta + \cos^2\theta = 1 \\ \tan\theta \cot\theta = 1 \end{array}\right\}$$

$$= 1 + 2 = 3$$

(ii) $(\cos^2 25^o + \cos^2 65^o) + \text{cosec}\,\theta \sec(90^o - \theta) - \cot\theta \tan(90^o - \theta)$
$= \cos^2 25^o + \cos^2(90^o - 25^o) + \text{cosec}\,\theta \sec(90^o - \theta) - \cot\theta.\cot\theta$
$= (\cos^2 25^o + \sin^2 25^o) + (\text{cosec}^2\theta - \cot^2\theta)$
$= 1 + 1 = 2$

10. (i) $2(\sec^2 35° - \cot^2 55°) - \frac{\cos 28^O \text{cosec} 62^O}{\tan 18^O \tan 36^O \tan 30^O \tan 54^O \tan 72^O}$

(ii) $\frac{\text{cosec}^2(90-\theta) - \tan^2\theta}{2(\cos^2 48^O + \cos^2 42^O)} - \frac{2\tan^2 30^O \sec^2 52^O \sin^2 38^O}{\text{cosec}^2 70^O - \tan^2 20^O}$

Solution:

Given,

(i) $2(\sec^2 35° - \cot^2 55°) - \frac{\cos 28^O \text{cosec} 62^O}{\tan 18^O \tan 36^O \tan 30^O \tan 54^O \tan 72^O}$

$$= 2[\sec^2 35° - \cot^2(90° - 35°)] - \frac{\cos 28° \text{cosec}(90° - 28°)}{\tan 18° \tan(90° - 18°) \tan 36° \tan(90° - 36°) \tan 30°}$$

$$= 2[\sec^2 35° - \tan^2 35°] - \frac{\cos 28° \sec 28°}{\tan 18° \cot 18° \tan 36° \cot 36° \tan 30°}$$

$$\left\{\begin{array}{l} \because \sec^2\theta - \tan^2\theta = 1 \\ \tan\theta \cot\theta = 1 \\ \cos\theta \sec\theta = 1 \end{array}\right\}$$

$$= 2(1) - \frac{1}{1 \times 1 \times \frac{1}{\sqrt{3}}} = 2 - \frac{\sqrt{3}}{1} = 2 - \sqrt{3}$$

(ii) $$\frac{\text{cosec}^2(90° - \theta) - \tan^2\theta}{2(\cos^2 48° + \cos^2 42°)} - \frac{2(\tan^2 30° \sec^2 52° \sin^2 38°)}{\text{cosec}^2 70° - \tan^2 20°}$$

$$= \frac{\sec^2\theta - \tan^2\theta}{2(\cos^2 48° + \cos^2(90° - 48°)} - \frac{2[\tan^2 30° \sec^2 52° \sin^2(90° - 52°)}{\text{cosec}^2 70° - \tan^2(90° - 70°)}$$

$$= \frac{1}{2[\cos^2 48° + \sin^2 48°]} - \frac{2\left[\left(\frac{1}{\sqrt{3}}\right)^2 \sec^2 52° \cos^2 52°\right]}{\text{cosec}^2 70° - \cot^2 70}$$

$$\left\{\begin{array}{l} \because \sin^2\theta + \cos^2\theta = 1 \\ \sec^2\theta - \tan^2\theta = 1 \\ \text{cosec}^2\theta - \cot^2\theta = 1 \end{array}\right\}$$

$$= \frac{1}{2 \times 1} - \frac{2\left[\frac{1}{3} \times 1\right]}{1}$$

$$= \frac{1}{2} - \frac{2}{3} = \frac{3-4}{6} = \frac{-1}{6}$$

11. Prove that following:
(i) cos θ sin (90° – θ) + sin θ cos (90° – θ) = 1
(ii) tan θ/ tan (90º - θ) + sin (90º - θ)/ cos θ = $\sec^2$ θ
(iii) (cos (90º - θ) cos θ)/ tan θ + $\cos^2$ (90º - θ) = 1
(iv) sin (90º - θ) cos (90º - θ) = tan θ/ (1 + $\tan^2$ θ)
Solution:

(i) L.H.S. = cos θ sin (90° – θ) + sin θ cos (90° – θ)
= cos θ x cos θ + sin θ x sin θ
= $\cos^2$ θ + $\sin^2$ θ
= 1 = R.H.S.

(ii) L.H.S = tan θ/ tan (90º - θ) + sin (90º - θ)/ cos θ
= tan θ/ cot θ + cos θ/ cos θ
= tan θ/ (1/tan θ) + 1
= $\tan^2$ θ + 1 = $\sec^2$ θ = R.H.S.

(iii) L.H.S. = (cos (90º - θ) cos θ)/ tan θ + $\cos^2$ (90º - θ)
= (sin θ cos θ)/ tan θ + $\sin^2$ θ
= (sin θ cos θ)/ (sin θ/ cos θ) + $\sin^2$ θ
= $\cos^2$ θ + $\sin^2$ θ
= 1 = R.H.S.

(iv) $\sin(90° - \theta)\cos(90° - \theta) = \dfrac{\tan\theta}{1 + \tan^2\theta}$

L.H.S. $= \sin(90° - \theta)\cos(90° - \theta) = \cos\theta \sin\theta$ $\left\{\begin{array}{l}\because \sin(90° - \theta) - \cos\theta \\ \sin^2\theta + \cos^2\theta = 1\end{array}\right\}$

$$\text{R.H.S.} = \frac{\tan\theta}{1+\tan^2\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}} = \frac{\sin\theta}{\cos\theta} \times \cos^2\theta = \sin\theta\cos\theta$$

∴ L.H.S. = R.H.S.

Prove that following (12 to 30) identities, where the angles involved are acute angles for which the trigonometric ratios as defined:

12. (i) (sec A + tan A) (1 – sin A) = cos A

(ii) $(1 + \tan^2 A)$ (1 – sin A) (1 + sin A) = 1.

Solution:

Given,

(i) L.H.S = (sec A + tan A) (1 – sin A)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1+\sin A)\times(1-\sin A)}{\cos A} = \frac{1-\sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A} = \cos A = \text{R.H.S.}$$

$$(1 - \sin^2 A = \cos^2 A)$$

(ii) L.H.S. = $(1 + \tan^2 A)$ (1 – sin A) (1 + sin A)

$$= \left(1 + \frac{\sin^2 A}{\cos^2 A}\right)(1 - \sin^2 A)$$

$$= \frac{\cos^2 A + \sin^2 A}{\cos^2 A} \times \cos^2 A \quad \left\{\begin{matrix} \because 1 - \sin^2 A = \cos^2 A \\ \sin^2 A + \cos^2 A = 1 \end{matrix}\right\}$$

$$= \frac{1}{\cos^2 A} \times \cos^2 A = 1 = \text{R.H.S.}$$

13. (i) tan A + cot A = sec A cosec A

(ii) (1 – cos A) (1 + sec A) = tan A sin A.

Solution:

(i) L.H.S. = tan A + cot A

= sin A/cos A + cos A/sin A

= $(\sin^2 A + \cos^2 A)$/ (sin A cos A)

= 1/ (sin A cos A)

= sec A cosec A

= R.H.S

(ii) L.H.S. = (1 – cos A) (1 + sec A)

$= (1 - \cos A)\left(1 + \frac{1}{\cos A}\right)$

$= (1 - \cos A)\frac{(\cos A + 1)}{\cos A}$

$= \frac{(1 - \cos A)(1 + \cos A)}{\cos A} = \frac{1 - \cos^2 A}{\cos A} = \frac{\sin^2 A}{\cos A}$

$= \frac{\sin^2 A}{\cos A} = \sin A \times \frac{\sin A}{\cos A}$

$\{1 - \cos^2 A = \sin^2 A\}$

$= \tan A \sin A =$ R.H.S.

14. (i) 1/ (1 + cos A) + 1/ (1 – cos A) = 2 cosec^2 A
(ii) 1/(sec A + tan A) + 1/(sec A – tan A) = 2 sec A
Solution:

(i) L.H.S. = 1/ (1 + cos A) + 1/ (1 – cos A)

$= \frac{1 - \cos A + 1 + \cos A}{(1 + \cos A)(1 - \cos A)}$

$= \frac{2}{1 - \cos^2 A} = \frac{2}{\sin^2 A}$

$(\because 1 - \cos^2 A = \sin^2 A)$

$= 2\,\text{cosec}^2 A =$ R.H.S.

(ii)

L.H.S. $= \frac{1}{\sec A + \tan A} + \frac{1}{\sec A - \tan A}$

$= \frac{\sec A - \tan A + \sec A + \tan A}{(\sec A + \tan A)(\sec A - \tan A)}$

$= \frac{2 \sec A}{\sec^2 A - \tan^2 A} = \frac{2 \sec A}{1}$

$(\because \sec^2 A - \tan^2 A = 1)$

$= 2 \sec A =$ R.H.S.

15. (i) sin A/ (1 + cos A) = (1 – cos A)/ sin A
(ii) (1 – tan^2 A)/ (cot^2 A - 1) = tan^2 A

(iii) sin A/ (1 + cos A) = cosec A – cot A
Solution:

(i) L.H.S. = sin A/ (1 + cos A)
On multiplying and dividing by (1 – cos A), we have

$$= \frac{\sin A(1-\cos A)}{1-\cos^2 A} = \frac{\sin A(1-\cos A)}{\sin^2 A}$$

$$(\because 1-\cos^2 A = \sin^2 A)$$

$$= \frac{1-\cos A}{\sin A} = \text{R.H.S.}$$

$$(ii)\ \text{L.H.S.} = \frac{1-\tan^2 A}{\cot^2 A - 1} = \frac{1-\frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A}-1}$$

$$= \frac{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A - \sin^2 A}{\sin^2 A}} = \frac{\cos^2 A - \sin^2 A}{\cos^2 A} \times \frac{\sin^2 A}{\cos^2 A - \sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \text{R.H.S.}$$

$$(iii)\ \text{R.H.S.} = \text{cosec}A - \cot A$$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A} = \frac{1-\cos A}{\sin A}$$

$$= \frac{(1-\cos A)(1+\cos A)}{\sin A(1+\cos A)} \quad \{\text{Multiplying and dividing by } 1+\cos A\}$$

$$= \frac{1-\cos^2 A}{\sin A(1+\cos A)} = \frac{\sin^2 A}{\sin A(1+\cos A)} \quad \{\because 1-\cos^2 A = \sin^2 A\}$$

$$= \frac{\sin A}{1+\cos A} = \text{L.H.S.}$$

16. (i) (sec A – 1)/(sec A + 1) = (1 – cos A)/(1 + cos A)
(ii) $\tan^2 \theta$/ $(\sec \theta - 1)^2$ = (1 + cos θ)/ (1 – cos θ)
(iii) $(1 + \tan A)^2 + (1 - \tan A)^2 = 2 \sec^2 A$
(iv) $\sec^2 A + \text{cosec}^2 A = \sec^2 A. \text{cosec}^2 A$
Solution:

$$= \frac{\frac{1-\cos A}{\cos A}}{\frac{1+\cos A}{\cos A}} = \frac{1-\cos A}{\cos A} \times \frac{\cos A}{1+\cos A}$$

$$= \frac{1-\cos A}{1+\cos A} = \text{R.H.S.}$$

(ii) $\text{LHS} = \frac{\tan^2\theta}{(\sec\theta-1)^2} = \frac{\tan^2\theta}{\sec^2\theta+1-2\sec\theta}$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}+1-\frac{2}{\cos\theta}} = \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1+\cos^2\theta-2\cos\theta}$$

$$= \frac{\sin^2\theta}{(1-\cos\theta)^2} = \frac{(1-\cos^2\theta)}{(1-\cos\theta)^2} = \frac{(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)^2}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = \text{R.H.S. Proved.}$$

(iii) L.H.S. = $(1 + \tan A)^2 + (1 - \tan A)^2$
= $1 + 2 \tan A + \tan^2 A + 1 - 2 \tan A + \tan^2 A$
= $2 + 2 \tan^2 A$
= $2(1 + \tan^2 A)$ [As $1 + \tan^2 A = \sec^2 A$]
= $2 \sec^2 A$
= R.H.S.

(iv) L.H.S = $\sec^2 A + \text{cosec}^2 A$
= $1/\cos^2 A + 1/\sin^2 A$
= $(\sin^2 A + \cos^2 A)/ (\sin^2 A \cos^2 A)$
= $1/ (\sin^2 A \cos^2 A)$
= $\sec^2 A \text{ cosec}^2 A$ = R.H.S

17. (i) (1 + sin A)/ cos A + cos A/ (1 + sin A) = 2 sec A
(ii) tan A/ (sec A - 1) + tan A/ (sec A + 1) = 2cosec A
Solution:

(i) L.H.S. = (1 + sin A)/ cos A + cos A/ (1 + sin A)

$$= \frac{(1+\sin A)(1+\sin A)+\cos^2 A}{\cos A(1+\sin A)}$$

$$= \frac{1+\sin A+\sin A+\sin^2 A+\cos^2 A}{\cos A(1+\sin A)}$$

$$= \frac{1+2\sin A+1}{\cos A(1+\sin A)} = \frac{2+2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2(1+\sin A)}{\cos A(1+\sin A)} = \frac{2}{\cos A} = 2\sec A$$

= R.H.S.

(ii) $\frac{\tan A}{\sec A-1} + \frac{\tan A}{\sec A+1} = 2\operatorname{cosec} A$

$$\text{L.H.S.} = \frac{\tan A}{\sec A-1} + \frac{\tan A}{\sec A+1}$$

$$= \tan A\left(\frac{1}{\sec A-1} + \frac{1}{\sec A+1}\right)$$

$$= \tan A\left(\frac{\sec A+1+\sec A-1}{(\sec A-1)(\sec A+1)}\right)$$

$$= \frac{\tan A\times 2\sec A}{\sec^2 A-1} = \frac{2\sec A\tan A}{\tan^2 A} = \frac{2\sec A}{\tan A}$$

$$= \frac{2\times 1\times\cos A}{\cos A\times\sin A}$$

$$= \frac{2}{\sin A} = 2\operatorname{cosec} A = \text{R.H.S.}$$

18. (i) cosec A/ (cosec A - 1) + cosec A/ (cosec A + 1) = 2 sec^2 A
(ii) cot A – tan A = (2cos^2 A - 1)/ (sin A – cos A)
(iii) (cot A – 1)/ (2 – sec^2 A) = cot A/ (1 + tan A)
Solution:

(i) L.H.S. = $\frac{cosecA}{cosecA-1} + \frac{cosecA}{cosecA+1}$

$$= \operatorname{cosec} A\left[\frac{1}{\operatorname{cosec} A - 1} + \frac{1}{\operatorname{cosec} A + 1}\right]$$

$$= \operatorname{cosec} A\left[\frac{\operatorname{cosec} A + 1 + \operatorname{cosec} A - 1}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}\right]$$

$$= \frac{\operatorname{cosec} A \times 2\operatorname{cosec} A}{\operatorname{cosec}^2 A - 1} = \frac{2\operatorname{cosec}^2 A}{\cot^2 A}$$

$$= \frac{2 \times \sin^2 A}{\sin^2 A \times \cos^2 A} = \frac{2}{\cos^2 A}$$

$= 2\sec^2 A =$ R.H.S.

(*ii*) L.H.S. $= \cot A - \tan A$

$$= \frac{\cos A}{\sin A} - \frac{\sin A}{\cos A} = \frac{\cos^2 A - \sin^2 A}{\sin A \cos A}$$

$$= \frac{\cos^2 A - (1 - \cos^2 A)}{\sin A \cos A}$$

$$= \frac{\cos^2 A - 1 + \cos^2 A}{\sin A \cos A}$$

$$= \frac{2\cos^2 A - 1}{\sin A \cos A} = \text{R.H.S.}$$

(*iii*) L.H.S. $= \dfrac{\cot A - 1}{2 - \sec^2 A} = \dfrac{\dfrac{\cos A}{\sin A} - 1}{2 - \dfrac{1}{\cos^2 A}}$

$$= \frac{\dfrac{\cos A - \sin A}{\sin A}}{\dfrac{2\cos^2 A - 1}{\cos^2 A}}$$

$$= \frac{\cos A - \sin A}{\sin A} \times \frac{\cos^2 A}{2\cos^2 A - 1}$$

$$= \frac{\cos^2 A(\cos A - \sin A)}{\sin A(2\cos^2 A - 1)}$$

$$= \frac{\cos^2 A(\cos A - \sin A)}{\sin A[2\cos^2 A - (\sin^2 A + \cos^2 A)]}$$

$$= \frac{\cos^2 A(\cos A - \sin A)}{\sin A[2\cos^2 A - \sin^2 A - \cos^2 A]}$$

$$= \frac{\cos^2 A(\cos A - \sin A)}{\sin A(\cos^2 A - \sin^2 A)}$$

$$= \frac{\cos^2 A(\cos A - \sin A)}{\sin A(\cos A + \sin A)(\cos A - \sin A)}$$

$$= \frac{\cos^2 A}{\sin A(\cos A + \sin A)}$$

$$\text{R.H.S.} = \frac{\cot A}{1+\tan A} = \frac{\frac{\cos A}{\sin A}}{1+\frac{\sin A}{\cos A}}$$

$$= \frac{\frac{\cos A}{\sin A}}{\frac{\cos A + \sin A}{\cos A}}$$

$$= \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A + \sin A}$$

$$= \frac{\cos^2 A}{\sin A(\cos A + \sin A)}$$

$\therefore$ L.H.S. = R.H.S.

19. (i) $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$
(ii) $\cos\theta/(1 - \tan\theta) - \sin^2\theta/(\cos\theta - \sin\theta) = \cos\theta + \sin\theta$
Solution:

(i) L.H.S = $\tan^2\theta - \sin^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \sin^2\theta \times \frac{\sin^2\theta}{\cos^2\theta}$$

$\sin^2\theta \times \tan^2\theta = \tan^2\theta \sin^2\theta$ = R.H.S.

(ii) $$\frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta$$

$$\text{L.H.S.} = \frac{\cos\theta}{1-\tan\theta} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}} - \frac{\sin^2\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\cos^2 \theta}{\cos \theta - \sin \theta} - \frac{\sin^2 \theta}{\cos \theta - \sin \theta}$$

$$= \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta - \sin \theta}$$

$$= \frac{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)}{\cos \theta - \sin \theta}$$

$= \cos \theta + \sin \theta =$ R.H.S.

20. (i) $\text{cosec}^4 \theta - \text{cosec}^2 \theta = \cot^4 \theta + \cot^2 \theta$
(ii) $2 \sec^2 \theta - \sec^4 \theta - 2 \text{cosec}^2 \theta + \text{cosec}^4 \theta = \cot^4 \theta - \tan^4 \theta$.
Solution:

(i) L.H.S. = $\text{cosec}^4 \theta - \text{cosec}^2 \theta$
$= \text{cosec}^2 \theta (\text{cosec}^2 \theta - 1)$
$= \text{cosec}^2 \theta \cot^2 \theta$ [$\text{cosec}^2 \theta - 1 = \cot^2 \theta$]
$= (\cot^2 \theta + 1) \cot^2 \theta$
$= \cot^4 \theta + \cot^2 \theta$
= R.H.S.

(ii) L.H.S. = $2 \sec^2 \theta - \sec^4 \theta - 2 \text{cosec}^2 \theta + \text{cosec}^4 \theta$
$= 2 (\tan^2 \theta + 1) - (\tan^2 \theta + 1)^2 - 2 (1 + \cot^2 \theta) + (1 + \cot^2 \theta)^2$

$$\left\{ \begin{array}{l} \because \sec^2 \theta = \tan^2 \theta + 1 \\ \text{cosec}^2 \theta = 1 + \cot^2 \theta \end{array} \right\}$$

$= 2 \tan^2 \theta + 2 - (\tan^4 \theta + 2 \tan^2 \theta + 1) - 2 - 2 \cot^2 \theta + (1 + 2 \cot^2 \theta + \cot^4 \theta)$
$= 2 \tan^2 \theta + 2 - \tan^4 \theta - 2 \tan^2 \theta - 1 - 2 - 2 \cot^2 \theta + 1 + 2 \cot^2 \theta + \cot^4 \theta$
$= \cot^4 \theta - \tan^4 \theta$ = R.H.S.

21. (i) $\frac{1+cos\theta-sin^2\theta}{sin\theta(1+cos\theta)}$ = cot θ
(ii) $(\tan^3 \theta - 1)/ (\tan \theta - 1) = \sec^2 \theta + \tan \theta$
Solution:

(i) L.H.S. $= \frac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)} = \frac{\cos\theta + \cos^2\theta}{\sin\theta(1 + \cos\theta)}$ $\{\because 1 - \sin^2 \theta = \cos^2 \theta\}$

$$= \frac{\cos\theta(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)} = \frac{\cos\theta}{\sin\theta} = \cot \theta$$

= R.H.S.

(ii) $\frac{\tan^3\theta - 1}{\tan\theta - 1} = \sec^2\theta + \tan\theta$

L.H.S. = $\frac{(\tan\theta - 1)}{\tan\theta - 1}(\tan^2\theta + \tan\theta + 1)$ $\{\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\}$

$= \tan^2\theta + \tan\theta + 1 = \tan^2\theta + 1 + \tan\theta$

$= \sec^2\theta + \tan\theta$ $\{\because \sec^2\theta = \tan^2\theta + 1\}$

= R.H.S.

22. (i) (1 + cosec A)/ cosec A = $\cos^2$ A/ (1 – sin A)

(ii) $\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}$

Solution:

(i) $\frac{1+\operatorname{cosec} A}{\operatorname{cosec} A} = \frac{\cos^2 A}{1-\sin A}$

L.H.S = $\frac{1+\operatorname{cosec} A}{\operatorname{cosec} A}$

L.H.S. = $\frac{1+\operatorname{cosec} A}{\operatorname{cosec} A} = \frac{1+\frac{1}{\sin A}}{\frac{1}{\sin A}} = \frac{\sin A + 1}{\sin A} \times \frac{\sin A}{1}$

$= \sin A + 1$

R.H.S. = $\frac{\cos^2 A}{1-\sin A} = \frac{1-\sin^2 A}{1-\sin A}$

$= \frac{(1+\sin A)(1-\sin A)}{1-\sin A} = 1 + \sin A = \sin A + 1$

$\therefore$ L.H.S. = R.H.S.

(ii) $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

L.H.S. = $\sqrt{\frac{1-\cos A}{1+\cos A}}$

Rationalising the denominator

$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}$

$$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$

$$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= \operatorname{cosec} A - \cot A = \text{R.H.S.}$$

23. (i) $\sqrt{\frac{1+sinA}{1-sinA}}$ **= tan A + sec A**

(ii) $\sqrt{\frac{1-cosA}{1+cosA}}$ **= cosec A – cot A**

Solution:

(i) $\sqrt{\frac{1+sinA}{1-sinA}} = tanA + secA$

L.H.S = $\sqrt{\frac{1+sinA}{1-sinA}}$

L.H.S. = $\sqrt{\frac{1+\sin A}{1-\sin A}}$

Rationalising the denominator,

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A = \tan A + \sec A = \text{R.H.S.}$$

(*ii*) $\sqrt{\frac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$

L.H.S. = $\sqrt{\frac{1-\cos A}{1+\cos A}}$

Rationalising the denominator,

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} = \frac{1-\cos A}{\sin A}$$

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A} = \operatorname{cosec} A - \cot A = \text{R.H.S.}$$

24. (i) $\sqrt{\frac{secA-1}{secA+1}} + \sqrt{\frac{secA+1}{secA-1}}$ **= 2 cosec A**
(ii) cos A cot A/ (1 – sin A) = 1 + cosec A
Solution:

(i) $\sqrt{\frac{secA-1}{secA+1}} + \sqrt{\frac{secA+1}{secA-1}} = 2cosecA$

L.H.S $= \sqrt{\frac{secA-1}{secA+1}} + \sqrt{\frac{secA+1}{secA-1}}$

$$\text{L.H.S.} = \sqrt{\frac{\sec A-1}{\sec A+1}} + \sqrt{\frac{\sec A+1}{\sec A-1}}$$

$$= \frac{\sqrt{\sec A-1}}{\sqrt{\sec A+1}} + \frac{\sqrt{\sec A+1}}{\sqrt{\sec A-1}}$$

$$= \frac{\sec A-1+\sec A+1}{\sqrt{(\sec A+1)(\sec A-1)}} = \frac{2\sec A}{\sqrt{\sec^2 A-1}}$$

$\{\because \sec^2 A - 1 = \tan^2 A\}$

$$= \frac{2\sec A}{\sqrt{\tan^2 A}} = \frac{2\sec A}{\tan A}$$

$$= \frac{2\times\cos A}{\cos A\times\sin A} = \frac{2}{\sin A}$$

$= 2 \operatorname{cosec} A = \text{R.H.S.}$

(ii) $\frac{\cos A \cot A}{1-\sin A} = 1 + \operatorname{cosec} A$

$$\text{L.H.S.} = \frac{\cos A \cot A}{1 - \sin A} = \frac{\cos A \cos A}{\sin A(1 - \sin A)} \quad \left\{\cos A = \frac{\cos A}{\sin A}\right\}$$

$$= \frac{\cos^2 A}{\sin A(1 - \sin A)} = \frac{1 - \sin^2 A}{\sin A(1 - \sin A)}$$

$$\{\because \cos^2 A = 1 - \sin^2 A\}$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\sin A(1 - \sin A)} = \frac{1 + \sin A}{\sin A}$$

$$= \frac{1}{\sin A} + \frac{\sin A}{\sin A} = \text{cosec } A + 1$$

$$= 1 + \text{cosec } A = \text{R.H.S.}$$

25. (i) (1 + tan A)/ sin A + (1 + cot A)/ cos A = 2 (sec A + cosec A)
(ii) $\sec^4 A - \tan^4 A = 1 + 2\tan^2 A$
Solution:

(i) $\frac{1+tanA}{sinA} + \frac{1+cotA}{cosA} = 2(secA + cosecA)$

$$\text{L.H.S} = \frac{1+tanA}{sinA} + \frac{1+cotA}{cosA}$$

$$= \frac{1 + \frac{\sin A}{\cos A}}{\sin A} + \frac{1 + \frac{\cos A}{\sin A}}{\cos A}$$

$$= \frac{\cos A + \sin A}{\cos A \times \sin A} + \frac{\sin A + \cos A}{\cos A \times \sin A}$$

$$= 2\left[\frac{\cos A + \sin A}{\cos A \sin A}\right]$$

$$= 2\left[\frac{\cos A}{\cos A \sin A} + \frac{\sin A}{\cos A \sin A}\right]$$

$$= 2\left[\frac{1}{\sin A} + \frac{1}{\cos A}\right]$$

$$= 2(\text{cosec } A + \sec A)$$

$$= 2(\sec A + \text{cosec } A) = \text{R.H.S.}$$

(ii) $\sec^4 A - \tan^4 A = 1 + 2\tan^2 A$

L.H.S. = $\sec^4 A - \tan^4 A$

$= (\sec^2 A - \tan^2 A)(\sec^2 A + \tan^2 A)$

$= (1 + \tan^4 A - \tan^4 A)(1 + \tan^4 A + \tan^4 A)$ [As $\sec^2 A = \tan^4 A + 1$]

$= 1(1 + 2\tan^2 A)$

$= 1 + 2\tan^2 A$ = R.H.S.

26. (i) $\text{cosec}^6 A - \cot^6 A = 3\cot^2 A \text{ cosec}^2 A + 1$

(ii) $\sec^6 A - \tan^6 A = 1 + 3\tan^2 A + 3\tan^4 A$

Solution:

(i) $\text{cosec}^6 A - \cot^6 A = 3\cot^2 A \text{ cosec}^2 A + 1$

L.H.S = $\text{cosec}^6 A - \cot^6 A$

$= (\text{cosec}^2 A)^3 - (\cot^2 A)^3$

$= (\text{cosec}^2 \theta - \cot^2 A)^3 + 3\text{ cosec}^2 A \cot^2 A(\text{cosec}^2 A - \cot^2 A)$

$= (1)^3 + 3\text{ cosec}^2 A \cot^2 A \times 1$

$= 1 + 3\cot^2 A \text{ cosec}^2 A$

$= 3\cot^2 A \text{ cosec}^2 A + 1$ = R.H.S.

(ii) $\sec^6 A - \tan^6 A = 1 + 3\tan^2 A + 3\tan^4 A$

L.H.S. = $\sec^6 A - \tan^6 A$

$= (\sec^2 A)^3 - (\tan^2 A)^3$

$= (\sec^2 A - \tan^2 A)^3 + 3\sec^2 A \tan^2 A(\sec^2 A - \tan^2 A)$

$= (1)^3 + 3\sec^2 A \tan^2 A \times 1$

$= 1 + 3\sec^2 A \tan^2 A$

$= 1 + 3[(1 + \tan^2 A)(\tan^2 A)]$

$= 1 + 3[\tan^2 A + \tan^4 A]$

$= 1 + 3\tan^2 A + 3\tan^4 A$ = R.H.S.

27.

(i) $\frac{\cot\theta - \text{cosec}\theta - 1}{\cot\theta - \text{cosec}\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}$

(ii) $\frac{\sin\theta}{\cot\theta + \text{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \text{cosec}\theta}$

Solution:

(i)

$$\text{L.H.S} = \frac{\cot\theta - \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} + 1}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\sin\theta} \times \frac{\sin\theta}{\cos\theta - 1 + \sin\theta}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\cos\theta - 1 + \sin\theta}$$

$$= \frac{\cos\theta + (1 - \sin\theta)}{\cos\theta - (1 - \sin\theta)}$$

$$= \frac{[\cos\theta + (1 - \sin\theta)][\cos\theta + (1 - \sin\theta)]}{[\cos\theta - (1 - \sin\theta)][\cos\theta + (1 - \sin\theta)]}$$

$$= \frac{[\cos\theta + (1 - \sin\theta)]^2}{\cos^2\theta - (1 - \sin\theta)^2}$$

$$= \frac{(\cos\theta + 1 - \sin\theta)^2}{\cos^2\theta - (1 + \sin^2\theta - 2\sin\theta)}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 1 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{\cos^2\theta - 1 - \sin^2\theta + 2\sin\theta}$$

$$= \frac{1 + 1 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{1 - \sin^2\theta - 1 - \sin^2\theta + 2\sin\theta}$$

$$= \frac{2 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta}{2\sin\theta - 2\sin^2\theta}$$

$$= \frac{2(1 + \cos\theta) - 2\sin\theta(1 + \cos\theta)}{2\sin\theta(1 - \sin\theta)}$$

$$= \frac{(1 + \cos\theta)2(1 - \sin\theta)}{2\sin\theta(1 - \sin\theta)}$$

$$= \frac{1 + \cos\theta}{\sin\theta} = \text{R.H.S.}$$

(ii) $\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$

L.H.S. $= \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} + \sin\theta} = \frac{\sin\theta}{\frac{\cos\theta + 1}{\sin\theta}}$

$= \frac{\sin^2\theta}{1+\cos\theta}$

$= \frac{1-\cos^2\theta}{1+\cos\theta} = \frac{(1+\cos\theta)(1-\cos\theta)}{1+\cos\theta}$

$= 1 - \cos\theta$

R.H.S. $= 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$

$= 2 + \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}}$

$= 2 + \frac{\sin^2\theta}{\cos\theta - 1}$

$= \frac{2\cos\theta - 2 + \sin^2\theta}{\cos\theta - 1}$

$= \frac{2\cos\theta - 2 + (1-\cos^2\theta)}{\cos\theta - 1}$

$= \frac{2(\cos\theta - 1) + (1+\cos\theta)(1-\cos\theta)}{\cos\theta - 1}$

= (cos θ - 1) (2 – 1 – cos θ)/ (cos θ - 1)

= 1 – cos θ

Hence, L.H.S = R.H.S.

28. (i) (sin θ + cos θ) (sec θ + cosec θ) = 2 + sec θ cosec θ
(ii) (cosec A – sin A) (sec A – cos A) $\sec^2 A$ = tan A
Solution:

(i) (sin θ + cos θ) (sec θ + cosec θ) = 2 + sec θ cosec θ
L.H.S. = (sin θ + cos θ) (sec θ + cosec θ)

$= (\sin\theta + \cos\theta)\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)$

$= \frac{(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1 + 2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} + \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \text{cosec}\,\theta\sec\theta + 2$

$= 2 + \sec\theta\,\text{cosec}\,\theta.$

= R.H.S.

(ii)

L.H.S = $(\text{cosec}\,A - \sin A)(\sec A - \cos A)\sec^2 A$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\frac{1}{\cos^2 A}$

$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)\frac{1}{\cos^2 A}$

$= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \cdot \frac{1}{\cos^2 A} = \frac{\sin A}{\cos A} = \tan A$

= R.H.S.

29.

(i) $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

(ii) $\frac{\tan^2 A}{1 + \tan^2 A} + \frac{\cot^2 A}{1 + \cot^2 A} = 1$

Solution:

(i) $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2$

L.H.S = $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A}$

$= \frac{(\sin A + \cos A)(\sin^2 A - \sin A\cos A + \cos^2 A)}{(\sin A + \cos A)} + \frac{(\sin A - \cos A)(\sin^2 A + \sin A\cos A + \cos^2 A)}{(\sin A - \cos A)}$

$= (1 - \sin A\cos A) + (1 + \sin A\cos A)$

$[\because \sin^2 A + \cos^2 A = 1]$

$= 1 - \sin A\cos A + 1 + \sin A\cos A = 2 =$ R.H.S.

(ii) $\frac{\tan^2 A}{1+\tan^2 A} + \frac{\cot^2 A}{1+\cot^2 A} = 1$

$$\text{L.H.S.} = \frac{\tan^2 A}{1+\tan^2 A} + \frac{\frac{1}{\tan^2 A}}{1+\frac{1}{\tan^2 A}}$$

$$= \frac{\tan^2 A}{1+\tan^2 A} + \frac{\frac{1}{\tan^2 A}}{\frac{\tan^2 A + 1}{\tan^2 A}}$$

$$= \frac{\tan^2 A}{1+\tan^2 A} + \frac{\tan^2 A}{\tan^2 A(\tan^2 A + 1)}$$

$$= \frac{\tan^2 A}{1+\tan^2 A} + \frac{1}{1+\tan^2 A}$$

$$= \frac{1+\tan^2 A}{1+\tan^2 A} = 1 = \text{R.H.S.}$$

30. (i) 1/ (sec A + tan A) – 1/cos A = 1/cos A – 1/(sec A – tan A)
(ii) (sin A + sec A)^2 + (cos A + cosec A)^2 = (1 + sec A cosec A)^2
(iii) (tan A + sin A)/ (tan A – sin A) = (sec A + 1)/ (sec A – 1)
Solution:

(i) $\frac{1}{secA+tanA} - \frac{1}{cosA} = \frac{1}{cosA} - \frac{1}{secA-tanA}$

$$\text{L.H.S} = \frac{1}{secA+tanA} - \frac{1}{cosA}$$

$$= \frac{1}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}} - \frac{1}{\cos A}$$

$$= \frac{\cos A}{1+\sin A} - \frac{1}{\cos A}$$

$$= \frac{\cos^2 A - 1 - \sin A}{\cos A(1+\sin A)} = \frac{-\sin^2 - \sin A}{\cos A(1+\sin A)}$$

$$= \frac{-\sin A(1+\sin A)}{\cos A(1+\sin A)} = -\tan A$$

$$\text{R.H.S.} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$$

$$= \frac{1}{\cos A} - \frac{1}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}$$

$$= \frac{1}{\cos A} - \frac{\cos A}{1 - \sin A}$$

$$= \frac{1 - \sin A - \cos^2 A}{\cos A(1 - \sin A)} = \frac{\sin^2 A - \sin A}{\cos A(1 - \sin A)}$$

$$= \frac{-\sin A + \sin^2 A}{\cos A(1 - \sin A)} = \frac{-\sin A(1 - \sin A)}{\cos A(1 - \sin A)}$$

$$= \frac{-\sin A}{\cos A} = -\tan A$$

$\therefore$ L.H.S. = R.H.S.

(*ii*) $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A \operatorname{cosec} A)^2$

L.H.S. = $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$

$$= \sin^2 A + \sec^2 A + 2\sin A \sec A + \cos^2 A + \operatorname{cosec}^2 A + 2\cos A \operatorname{cosec} A$$

$$= (\sin^2 A + \cos^2 A) + (\sec^2 A + \operatorname{cosec}^2 A) + 2\sin A \times \frac{1}{\cos A} + 2 \times \cos A \times \frac{1}{\sin A}$$

$$= 1 + \left[\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}\right] + \frac{2\sin^2 A + 2\cos^2 A}{\sin A \cos A}$$

$$= 1 + \left[\frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A}\right] + \frac{2[\sin^2 A + \cos^2 A]}{\sin A \cos A}$$

$$= 1 + \frac{1}{\cos^2 A \sin^2 A} + \frac{2}{\sin A \cos A} \quad [\because \sin^2\theta + \cos^2\theta + 1]$$

$$= \left(1 + \frac{1}{\cos A \sin A}\right)^2 \quad [\because (a+b)^2 = a^2 + (b)^2 + 2ab)]$$

$$= (1 + \operatorname{cosec} A \sec A)^2$$

= R.H.S.

(*iii*) $\frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\sec A + 1}{\sec A - 1}$

$$\text{L.H.S.} = \frac{\tan A + \sin A}{\tan A - \sin A} = \frac{\frac{\sin A}{\cos A} + \sin A}{\frac{\sin A}{\cos A} - \sin A} = \frac{\frac{\sin A + \sin A \cos A}{\cos A}}{\frac{\sin A - \sin A \cos A}{\cos A}}$$

$$= \frac{\sin A(1 + \cos A)}{\sin A(1 - \cos A)} = \frac{1 + \cos A}{1 - \cos A}$$

On dividing each term by cos A, we have

$$= \frac{\frac{1}{\cos A} + 1}{\frac{1}{\cos A} - 1} = \frac{\sec A + 1}{\sec A - 1} = \text{R.H.S.}$$

31. If sin θ + cos θ = √2 sin (90° – θ), show that cot θ = √2 + 1
Solution:

Given, sin θ + cos θ = √2 sin (90° – θ)
sin θ + cos θ = √2 cos θ
On dividing by sin θ, we have
1 + cot θ = √2 cot θ
1 = √2 cot θ - cot θ
(√2 - 1) cot θ = 1
cot θ = 1/ (√2 – 1)

$$= \frac{1\times(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} \quad \text{(Rationalising the denominator)}$$

$$= \frac{(\sqrt{2}+1)}{(\sqrt{2})^2-(1)^2} = \frac{\sqrt{2}+1}{2-1} = \frac{\sqrt{2}+1}{1}$$

$$= \sqrt{2} + 1 = \text{R.H.S.}$$

Hence, cot θ = √2 + 1

32. If 7 $\sin^2$ θ + 3 $\cos^2$ θ = 4, 0° ≤ θ ≤ 90°, then find the value of θ.
Solution:

Given,
7 $\sin^2 \theta + 3 \cos^2 \theta = 4$, 0° ≤ θ ≤ 90°
3 $\sin^2 \theta + 3 \cos^2 \theta + 4 \sin^2 \theta = 4$
3 $(\sin^2 \theta + 3 \cos^2 \theta) + 4 \sin^2 \theta = 4$
3 (1) + 4 $\sin^2 \theta = 4$
4 $\sin^2 \theta = 4 – 3$
$\sin^2 \theta = ¼$
Taking square-root on both sides, we get
sin θ = ½
Thus, θ = 30°

33. If sec θ + tan θ = m and sec θ – tan θ = n, prove that mn = 1.
Solution:

Given,
sec θ + tan θ = m
sec θ – tan θ = n
Now,
mn = (sec θ + tan θ) (sec θ – tan θ)
= $\sec^2 \theta – \tan^2 \theta = 1$
Thus, mn = 1

34. If x – a sec θ + b tan θ and y = a tan θ + b sec θ, prove that $x^2 – y^2 = a^2 – b^2$.
Solution:

Given,
x = a sec θ + b tan θ,
y = a tan θ + b sec θ
Now,
$x^2 – y^2 = (a \sec \theta + b \tan \theta)^2 – (a \tan \theta + b \sec \theta)^2$
$= (a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta) - (a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \sec \theta \tan \theta)$
$= a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta - a^2 \tan^2 \theta - b^2 \sec^2 \theta - 2ab \sec \theta \tan \theta$
$= a^2 (\sec^2 \theta - \tan^2 \theta) - b^2 (\sec^2 \theta - \tan^2 \theta)$
$= a^2 \times 1 - b^2 \times 1$ $\{\sec^2 \theta - \tan^2 \theta = 1\}$
$= a^2 - b^2$
- Hence proved.

35. If x = h + a cos θ and y = k + a sin θ, prove that $(x – h)^2 + (y – k)^2 = a^2$.
Solution:

Given,
x = h + a cos θ
y = k + a sin θ
Now,
x – h = a cos θ
y – k = a sin θ
On squaring and adding we get
$(x – h)^2 + (y – k)^2 = a^2 \cos^2 \theta + a^2 \sin^2 \theta$
$= a^2 (\sin^2 \theta + \cos^2 \theta)$
$= a^2 (1)$ [Since, $\sin^2 \theta + \cos^2 \theta = 1$]
- Hence proved

Chapter Test

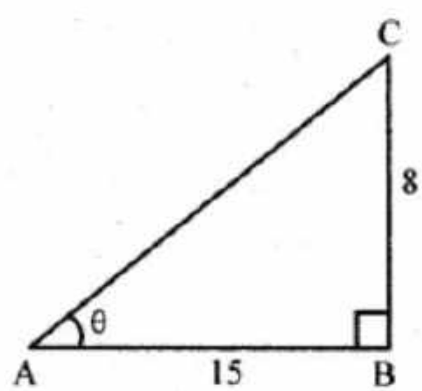
1. (i) If θ is an acute angle and cosec θ = √5, find the value of cot θ – cos θ.
(ii) If θ is an acute angle and tan θ = 8/15, find the value of sec θ + cosec θ.
Solution:

Given, θ is an acute angle and cosec θ = √5
So,
sin θ = 1/√5
And, cos θ = √(1 – $\sin^2$ θ)
cos θ = √(1 – $(1/\sqrt{5})^2$)
= √(1 – (1/5))
= √(4/5)
cos θ = 2/√5
Now,
cot θ – cos θ = (cos θ/sin θ) – cos θ

$$= \sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{1-\frac{1}{5}} = \sqrt{\frac{5-1}{5}}$$

$$= \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

(ii) Given, θ is an acute angle and tan θ = 8/15
In fig. we have
tan θ = BC/AB = 8/15
So, BC = 8 and AB = 15
By Pythagoras theorem, we have
AC = √($AB^2 + BC^2$) = √($5^2 + 8^2$) = √(25 + 64) = √289
⇒ AC = 17



Now,
sec θ = AC/AB = 17/15
cosec θ = AC/BC = 17/8
So,
sec θ + cosec θ = 17/15 + 17/8
= (136 + 255)/ 120
= 391/120
= $3\frac{31}{120}$

2. Evaluate the following:

(i) 2 x ($\frac{\cos^2 20^O + \cos^2 70^O}{\sin^2 25^O + \sin^2 65^O}$) – tan 45º + tan 13º tan 23º tan 30º tan 67º tan 77º

(ii) $\frac{sin^2 22^O + sin^2 68^O}{cos^2 22^O + cos^2 68^O}$ **+ sin² 63º + cos 63º sin 27º**

Solution:

(i) $2 \times \left(\frac{cos^2 20^O + cos^2 70^O}{sin^2 25^O + sin^2 65^O}\right) - \tan 45° + \tan 13° \tan 23° \tan 30° \tan 67° \tan 77°$

$= 2\left[\frac{\cos^2(90° - 70°) + \cos^2 70°}{\sin^2 25° + \sin^2(90° - 25°)}\right] - \tan 45° + \tan 13° \tan 77° \tan 23° \tan 67° \tan 30°$

$= 2\left[\frac{\sin^2 70° + \cos^2 70}{\sin^2 25° + \cos^2 25°}\right] - 1 + \tan 13° \tan(90° - 13°) \tan 23° \tan(90° - 23°) \times \frac{1}{\sqrt{3}}$

$= 2\left(\frac{1}{1}\right) - 1 + \tan 13 \cot 13° \tan 23° \cot 23° \times \frac{1}{\sqrt{3}}$

$= 2 - 1 + 1 \times 1 \times \frac{1}{\sqrt{3}}$

$= 2 - 1 + \frac{1}{\sqrt{3}} = 1 + \frac{1}{\sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{3}}$

$= \frac{(\sqrt{3}+1)\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{3+\sqrt{3}}{3}$

(ii) $\frac{\sec 29°}{\text{cosec} 61°} + 2 \cot 8° \cot 17° \cot 45° \cot 73° \cot 82° - 3(\sin^2 38° + \sin^2 52°)$

$= \frac{\sec 29°}{\text{cosec}(90° - 29°)} + 2 \cot 8° \times \cot(90° - 8°) \times \cot 17° \times \cot(90° - 17°) \cot 45° - 3[\sin^2 38° + \sin^2(90° - 38°)]$

$= \frac{\sec 29°}{\sec 29°} + 2 \cot 8° \tan 8° \times \cot 17° \tan 17° \times 1 - 3(\sin^2 38° + \cos^2 38°)$

$= 1 + 2 \times 1 \times 1 \times 1 - 3 \times 1 = 1 + 2 - 3 = 0$

(iii) $\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$

$= \frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2 22° + \cos^2(90° - 22°)} + \sin^2 63° + \cos 63° \sin(90° - 63°)$

$= \frac{\sin^2 22° + \cos^2 22°}{\cos^2 22° + \sin^2 22°} + \sin^2 63° + \cos 63° \times \cos 63°$

$= \frac{1}{1} + (\sin^2 63° + \cos^2 63°) = 1 + 1 = 2$

3. If 4/3 (sec^2 59º – cot^2 31º) – 2/3 sin 90º + 3 tan^2 56º tan^2 34º = x/2, then find the value of x.
Solution:

Given,
4/3 (sec^2 59º – cot^2 31º) – 2/3 sin 90º + 3 tan^2 56º tan^2 34º = x/2

$$\Rightarrow \frac{4}{3}[\sec^2 59^\circ - \cot^2(90^\circ - 59^\circ)] - \frac{2}{3}\sin 90^\circ + 3\tan^2 56^\circ \tan^2(90^\circ - 56^\circ) = \frac{x}{3}$$

$$\frac{4}{3}[\sec^2 59^\circ - \tan^2 59^\circ] - \frac{2}{3} \times 1 + 3\tan^2 56^\circ \cot^2 56^\circ = \frac{x}{3}$$

$$\frac{4}{3} \times 1 - \frac{2}{3} + 3 \times 1 = \frac{x}{3}$$

$$\frac{4}{3} - \frac{2}{3} + 3 = \frac{x}{3}$$

$$\frac{4-2+9}{3} = \frac{x}{3}$$

$$\frac{11}{3} = \frac{x}{3} \Rightarrow x = \frac{11 \times 3}{3} = 11$$

$\therefore\ x = 11$

4. (i) cos A/ (1 – sin A) + cos A/ (1 + sin A) = 2 sec A
(ii) cos A/ (cosec A + 1) + cos A/ (cosec A - 1) = 2 tan A
Solution:

(i) $\frac{\cos A}{1-\sin A} + \frac{\cos A}{1+\sin A} = 2\sec A$

L.H.S = $\frac{\cos A}{1-\sin A} + \frac{\cos A}{1+\sin A}$

$$= \cos A\left[\frac{1}{1-\sin A} + \frac{1}{1+\sin A}\right]$$

$$= \cos A\left[\frac{1+\sin A+1-\sin A}{(1-\sin A)(1+\sin A)}\right]$$

$$= \cos A\left[\frac{2}{1-\sin^2 A}\right] = \frac{2\cos A}{\cos^2 A}$$

$$= \frac{2}{\cos A} = 2\sec A = \text{R.H.S.}$$

(ii) $\frac{\cos A}{\operatorname{cosec} A+1}+\frac{\cos A}{\operatorname{cosec} A-1}=2\tan A$

L.H.S.= $\frac{\cos A}{\operatorname{cosec} A+1}+\frac{\cos A}{\operatorname{cosec} A-1}$

$=\cos A\left[\frac{1}{\operatorname{cosec} A+1}+\frac{1}{\operatorname{cosec} A-1}\right]$

$=\cos A\left[\frac{\operatorname{cosec} A-1+\operatorname{cosec} A+1}{(\operatorname{cosec} A+1)(\operatorname{cosec} A-1)}\right]$

$=\frac{\cos A\,[2\operatorname{cosec} A]}{\operatorname{cosec}^2 A-1}=\frac{2\cos A}{\sin A\,(\cot^2 A)}$

$=\frac{2\cot A}{\cot^2 A}=\frac{2}{\cot A}=2\tan A=$ R.H.S.

5. (i) $\frac{(cos\theta-sin\theta)(1+tan\theta)}{2cos^2\theta-1}=sec\theta$

(ii) (cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ) = 1.

Solution:

(i) $\frac{(cos\theta-sin\theta)(1+tan\theta)}{2cos^2\theta-1}=sec\theta$

L.H.S = $\frac{(cos\theta-sin\theta)(1+tan\theta)}{2cos^2\theta-1}$

$=\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\cos\theta\,(2\cos^2\theta-1)}$

$=\frac{\cos^2\theta-\sin^2\theta}{\cos\theta\,(2\cos^2\theta-1)}=\frac{\cos^2\theta-(1-\cos^2\theta)}{\cos\theta\,(2\cos^2\theta-1)}$

$=\frac{\cos^2\theta-1+\cos^2\theta}{\cos\theta\,(2\cos^2\theta-1)}=\frac{2\cos^2\theta-1}{\cos\theta\,(2\cos^2\theta-1)}$

$=\frac{1}{\cos\theta}=\sec\theta=$ R.H.S.

(ii) (cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ) = 1.

L.H.S. = (cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ)

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{(\cos^2\theta \times \sin^2\theta)}{\sin\theta\cos\theta} \times \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\cos^2\theta}{\sin^2\theta\cos^2\theta} = 1 = \text{R.H.S.}$$

6. (i) $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$

(ii) $\frac{cot\theta}{cosec\theta+1} + \frac{cosec\theta+1}{cot\theta} = 2sec\theta$

Solution:

Given,

(i) $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$

L.H.S. $= \sin^2\theta + \cos^4\theta$

$= (1 - \cos^2\theta) + \cos^4\theta$

$= \cos^4\theta - \cos^2\theta + 1$

$= \cos^2\theta(\cos^2\theta - 1) + 1$

$= \cos^2\theta(-\sin^2\theta) + 1$

$= 1 - \sin^2\theta\cos^2\theta$

Now,

R.H.S. $= \cos^2\theta + \sin^4\theta$

$= (1 - \sin^2\theta) + \sin^4\theta$

$= \sin^4\theta - \sin^2\theta + 1$

$= \sin^2\theta(\sin^2\theta - 1) + 1$

$= \sin^2\theta(-\cos^2\theta) + 1$

$= 1 - \sin^2\theta\cos^2\theta$

Hence, L.H.S. = R.H.S.

(ii) $\frac{\cot\theta}{\operatorname{cosec}\theta+1} + \frac{\operatorname{cosec}\theta+1}{\cot\theta} = 2\sec\theta$

$$\text{L.H.S.} = \frac{\cot\theta}{\operatorname{cosec}\theta+1} + \frac{\operatorname{cosec}\theta+1}{\cot\theta}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}+1} + \frac{\frac{1}{\sin\theta}+1}{\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1+\sin\theta}{\sin\theta}} + \frac{\frac{1+\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\cos\theta}{\sin\theta} \times \frac{\sin\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\sin\theta} \times \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + 1 + \sin^2\theta + 2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{1+1+2\sin\theta}{(1+\sin\theta)\cos\theta} = \frac{2+2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \frac{2}{\cos\theta}$$

$$= 2\sec\theta = \text{R.H.S.}$$

7. (i) $\sec^4 A(1 - \sin^4 A) - 2\tan^2 A = 1$

(ii) $\frac{1}{\sin A+\cos A+1} + \frac{1}{\sin A+\cos A-1}$ = sec A + cosec A

Solution:

(i) $\sec^4 A(1 - \sin^4 A) - 2\tan^2 A = 1$

L.H.S. = $\sec^4 A(1 - \sin^4 A) - 2\tan^2 A$

$$= \frac{1}{\cos^4 A}(1+\sin^2 A)(1-\sin^2 A) - 2\tan^2 A \quad [\because a^2 - b^2 = (a+b)(a-b)]$$

$$= \frac{(1+\sin^2 A)\cos^2 A}{\cos^4 A} - 2\frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1+\sin^2 A}{\cos^2 A} - \frac{2\sin^2 A}{\cos^2 A}$$

$$= \frac{1+\sin^2 A - 2\sin^2 A}{\cos^2 A}$$

$$= \frac{1-\sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$$

$$(\because 1 - \sin^2 A = \cos^2 A)$$

$= \text{R.H.S.}$

(ii) $\dfrac{1}{\sin A + \cos A + 1} + \dfrac{1}{\sin A + \cos A - 1} = \sec A + \operatorname{cosec} A$

$$\text{L.H.S.} = \frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1}$$

$$= \frac{\sin A + \cos A - 1 + \sin A + \cos A + 1}{(\sin A + \cos A + 1)(\sin A + \cos A - 1)}$$

$$= \frac{2(\sin A + \cos A)}{(\sin A + \cos A)^2 - (1)^2}$$

$$= \frac{2(\sin A + \cos A)}{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}$$

$$= \frac{2(\sin A + \cos A)}{1 + 2\sin A \cos A - 1}$$

$$= \frac{2(\sin A + \cos A)}{2\sin A \cos A}$$

$$= \frac{\sin A + \cos A}{\sin A \cos A} = \frac{\sin A}{\sin A \cos A} + \frac{\cos A}{\sin A \cos A}$$

$$= \frac{1}{\cos A} + \frac{1}{\sin A}$$

$$= \sec A + \operatorname{cosec} A = \text{R.H.S.}$$

8. (i) $\dfrac{sin^3\theta + cos^3\theta}{sin\theta cos\theta}$ **+ sin θ cos θ = 1**
(ii) (sec A – tan A)^2 (1 + sin A) = 1 – sin A.
Solution:

(i) $\dfrac{sin^3\theta+cos^3\theta}{sin\theta cos\theta} + sin\theta cos\theta = 1$

$$\text{LHS} = \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta$$

$$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}{(\sin\theta + \cos\theta)} + \sin\theta\cos\theta$$

$= \sin^2\theta + \cos^2\theta - \sin\theta\cos\theta + \sin\theta\cos\theta$

$= 1 = \text{R.H.S.}$

(ii) $(\sec A - \tan A)^2 (1 + \sin A) = 1 - \sin A$

$$\text{L.H.S.} = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2 (1 + \sin A)$$

$$= \left(\frac{1-\sin A}{\cos A}\right)^2 (1 + \sin A)$$

$$= \frac{(1-\sin A)^2}{\cos^2 A}(1 + \sin A)$$

$$= \frac{(1-\sin A)^2(1+\sin A)}{1-\sin^2 A}$$

$$= \frac{(1-\sin A)^2(1+\sin A)}{(1-\sin A)(1+\sin A)}$$

$= 1 - \sin A = \text{R.H.S.}$

9. (i) $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A-\sin A}$ **= sin A + cos A**

(ii) (sec A – cosec A) (1 + tan A + cot A) = tan A sec A – cot A cosec A

(iii) $\frac{\tan^2\theta}{\tan^2\theta-1} - \frac{\csc^2\theta}{\sec^2\theta-\csc^2\theta} = \frac{1}{\sin^2\theta-\cos^2\theta}$

Solution:

(i) $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A-\sin A} = \sin A + \cos A$

$$\text{L.H.S} = \frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A-\sin A}$$

$$= \frac{\cos A}{1-\frac{\sin A}{\cos A}} - \frac{\sin^2 A}{\cos A-\sin A}$$

$$= \frac{\cos A \times \cos A}{\cos A-\sin A} - \frac{\sin^2 A}{\cos A-\sin A}$$

$$= \frac{\cos^2 A}{\cos A-\sin A} - \frac{\sin^2 A}{\cos A-\sin A}$$

$$= \frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$$

$$= \frac{(\cos A+\sin A)(\cos A-\sin A)}{(\cos A-\sin A)} = \cos A + \sin A = \text{R.H.S.}$$

(*ii*) (sec A – cosec A) (1 + tan A + cot A) = tan A sec A – cot A cosec A

L.H.S. = (sec A – cosec A) (1 + tan A + cot A)

$$= \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \frac{\sin A - \cos A}{\sin A \cos A} \times \frac{\sin A \cos A + \sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{\sin A - \cos A}{\sin A \cos A} \times \frac{\sin A \cos A + 1}{\sin A \cos A}$$

$$= \frac{(\sin A - \cos A)(\sin A \cos A + 1)}{\sin^2 A \cos^2 A}$$

R.H.S. = tan A sec A – cot A cosec A

$$= \frac{\sin A}{\cos A} \cdot \frac{1}{\cos A} - \frac{\cos A}{\sin A} \cdot \frac{1}{\sin A}$$

$$= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A} = \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\sin^2 A \cos^2 A}$$

$$= \frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\sin^2 A \cos^2 A}$$

$$= \frac{(\sin A - \cos A)(\sin A \cos A + 1)}{\sin^2 A \cos^2 A}$$

∴ L.H.S. = R.H.S.

(*iii*) $$\frac{\tan^2 \theta}{\tan^2 \theta - 1} + \frac{\operatorname{cosec}^2 \theta}{\sec^2 \theta - \operatorname{cosec}^2 \theta} = \frac{1}{\sin^2 \theta - \cos^2 \theta}$$

$$\text{L.H.S.} = \frac{\tan^2 \theta}{\tan^2 \theta - 1} + \frac{\operatorname{cosec}^2 \theta}{\sec^2 \theta - \operatorname{cosec}^2 \theta}$$

$$= \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta} - 1} + \frac{\frac{1}{\sin^2 \theta}}{\frac{1}{\cos^2 \theta} - \frac{1}{\sin^2 \theta}} = \frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} + \frac{\frac{1}{\sin^2 \theta}}{\frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}}$$

$$= \frac{\sin^2 \theta}{\sin^2 \theta - \cos^2 \theta} + \frac{\sin^2 \theta \cos^2 \theta}{\sin^2 \theta (\sin^2 \theta - \cos^2 \theta)}$$

$$= \frac{\sin^2 \theta}{\sin^2 \theta - \cos^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta - \cos^2 \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} = \frac{1}{\sin^2 \theta - \cos^2 \theta} = \text{R.H.S.}$$

10. $\frac{sinA+cosA}{sinA-cosA}+\frac{sinA-cosA}{sinA+cosA}=\frac{2}{sin^2A-cos^2A}=\frac{2}{1-2cos^2A}=\frac{2sec^2A}{tan^2A-1}$

Solution:

L.H.S $=\frac{sinA+cosA}{sinA-cosA}+\frac{sinA-cosA}{sinA+cosA}$

$=\frac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{\sin^2 A-\cos^2 A}$

$=\frac{2(\sin^2 A+\cos^2 A)}{\sin^2 A-\cos^2 A}$

$=\frac{2\times 1}{\sin^2 A-\cos^2 A}=\frac{2}{\sin^2 A-\cos^2 A}$

= R.H.S.

Now, we have

$=\frac{2}{\sin^2 A-\cos^2 A}$

$=\frac{2}{1-\cos^2 A-\cos^2 A}=\frac{2}{1-2\cos^2 A}$

= R.H.S.

Now, we have

$=\frac{2}{\sin^2 A-\cos^2 A}$

$=\frac{2}{\sin^2 A-(1-\sin^2 A)}$

$=\frac{2}{\sin^2 A-1+\sin^2 A}$

$=\frac{2}{\sin^2 A-\cos^2 A}=\frac{\frac{2}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A}-\frac{\cos^2 A}{\cos^2 A}}$

$=\frac{2\sec^2 A}{\tan^2 A-1}$ = R.H.S.

Hence,

$\frac{sinA+cosA}{sinA-cosA}+\frac{sinA-cosA}{sinA+cosA}=\frac{2}{sin^2A-cos^2A}=\frac{2}{1-2cos^2A}=\frac{2sec^2A}{tan^2A-1}$

11. $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$
Solution:

Given,
$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$
L.H.S. $= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$
$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$
$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)] - 3(\sin^4\theta + \cos^4\theta) + 1$
$= 2(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$
$= 2\sin^4\theta + 2\cos^4\theta - 2\sin^2\theta\cos^2\theta - 3\sin^4\theta - 3\cos^4\theta + 1$
$= 1 - \sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta$
$= 1 - [\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta]$
$= 1 - 1$
$= 0 =$ R.H.S.

12. If $\cot\theta + \cos\theta = m$, $\cot\theta - \cos\theta = n$, then prove that $(m^2 - n^2)^2 = 16$.
Solution:

Given,
$\cot\theta + \cos\theta = m$ … (i)
$\cot\theta - \cos\theta = n$ … (ii)
Adding (i) and (ii), we get

$$2\cot\theta = m + n \Rightarrow \cot\theta = \frac{m+n}{2}$$

$$\therefore \tan\theta = \frac{2}{m+n} \quad \text{... (iii)}$$

Subtracting (*ii*) from (*i*),

$$2\cos\theta = m - n \Rightarrow \cos\theta = \frac{m-n}{2}$$

$$\therefore \sec\theta = \frac{2}{m-n} \quad \text{... (iv)}$$

Now, squaring and subracting (iii) from (iv), we have

$$\sec^2\theta - \tan^2\theta = \left(\frac{2}{m-n}\right)^2 - \left(\frac{2}{m+n}\right)^2$$

$$1 = \frac{4}{(m-n)^2} - \frac{4}{(m+n)^2}$$

$$\Rightarrow 4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$$

$$4\left[\frac{(m+n)^2-(m-n)^2}{(m+n)^2(m-n)^2}\right]=1$$

$$\frac{4(4\,mn)}{(m^2-n^2)^2}=1$$

$$\frac{16mn}{(m^2-n^2)^2}=1$$

$$\therefore\ (m^2-n^2)^2=16\,mn.$$

13. If sec θ + tan θ = p, prove that sin θ = $(p^2 - 1)/(p^2 + 1)$
Solution:

Given, sec θ + tan θ = p

$$\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}=p$$

$$\frac{1+\sin\theta}{\cos\theta}=p$$

Squaring on both sides,

$$\frac{(1+\sin\theta)^2}{\cos^2\theta}=p^2\Rightarrow\frac{(1+\sin\theta)^2}{1-\sin^2\theta}=p^2$$

$$\frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}=p^2$$

$$\Rightarrow\frac{1+\sin\theta}{1-\sin\theta}=\frac{p^2}{1}$$

Applying componendo and dividendo,

$$\frac{1+\sin\theta+1-\sin\theta}{1+\sin\theta-1+\sin\theta}=\frac{p^2+1}{p^2-1}$$

$$\frac{2}{2\sin\theta}=\frac{p^2+1}{p^2-1}\Rightarrow\frac{1}{\sin\theta}=\frac{p^2+1}{p^2-1}$$

$$\therefore\ \sin\theta=\frac{p^2-1}{p^2+1}$$

14. If tan A = n tan B and sin A = m sin B, prove that $\cos^2 A = (m^2 - 1)/(n^2 - 1)$
Solution:

Given,
tan A = n tan B and sin A = m sin B
n = tan A/ tan B
m = sin A/ sin B

$$\frac{1}{\sin B} = \frac{m}{\sin A} \Rightarrow \text{cosec } B = \frac{m}{\sin A}$$

$$\frac{1}{\tan B} = \frac{n}{\tan A} \Rightarrow \cot B = \frac{n}{\tan A}$$

Now, $\text{cosec}^2 B - \cot^2 B = 1$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$$

$$m^2 - n^2 \cos^2 A = \sin^2 A$$

$$m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$m^2 - 1 = \cos^2 A + n^2 \cos^2 A$$

$$m^2 - 1 = (n^2 - 1) \cos^2 A$$

$$\Rightarrow \cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

15. If sec A = x + 1/4x, then prove that sec A + tan A = 2x or 1/2x
Solution:

Given, sec A = x + 1/4x
We know that,

$$\tan A = \pm\sqrt{\sec^2 A - 1}$$

$$= \pm\sqrt{\left(x + \frac{1}{4x}\right)^2 - 1}$$

$$= \pm\sqrt{x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1}$$

$$= \pm\sqrt{x^2 + \frac{1}{16x^2} - \frac{1}{2}}$$

$$= \pm\left(x - \frac{1}{4x}\right)$$

$\therefore$ $\sec A + \tan A = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$

or $x + \frac{1}{4x} - x + \frac{1}{4x} = \frac{1}{2x}$

16. When 0° < θ < 90°, solve the following equations:
(i) $2\cos^2\theta + \sin\theta - 2 = 0$
(ii) $3\cos\theta = 2\sin^2\theta$
(iii) $\sec^2\theta - 2\tan\theta = 0$
(iv) $\tan^2\theta = 3(\sec\theta - 1)$.
Solution:

Given, $0° < \theta < 90°$
(i) $2\cos^2\theta + \sin\theta - 2 = 0$
$2(1 - \sin^2\theta) + \sin\theta - 2 = 0$
$2 - 2\sin^2\theta + \sin\theta - 2 = 0$
$-2\sin^2\theta + \sin\theta = 0$
$\sin\theta(1 - 2\sin\theta) = 0$
So, either $\sin\theta = 0$ or $1 - 2\sin\theta = 0$
If $\sin\theta = 0$
$\Rightarrow \theta = 0^o$
And, if $1 - 2\sin\theta = 0$
$\sin\theta = ½$
$\Rightarrow \theta = 30^o$
Thus, $\theta = 0^o$ or 30^o

(ii) $3\cos\theta = 2\sin^2\theta$
$3\cos\theta = 2(1 - \cos^2\theta)$
$3\cos\theta = 2 - 2\cos^2\theta$
$2\cos^2\theta + 3\cos\theta - 2 = 0$
$2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$
$2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2)$
$(2\cos\theta - 1)(\cos\theta + 2) = 0$
So, either $2\cos\theta - 1 = 0$ or $\cos\theta + 2 = 0$
If $2\cos\theta - 1 = 0$
$\cos\theta = ½$
$\Rightarrow \theta = 60^o$
And, for $\cos\theta + 2 = 0$
$\Rightarrow \cos\theta = -2$ which is not possible being out of range.
Thus, $\theta = 60^o$

(iii) $\sec^2\theta - 2\tan\theta = 0$
$(1 + \tan^2\theta) - 2\tan\theta = 0$
$\tan^2\theta - 2\tan\theta + 1 = 0$

$(\tan\theta - 1)^2 = 0$
$\tan\theta - 1 = 0$
$\Rightarrow \tan\theta = 1$
Thus, $\theta = 45^o$

(iv) $\tan^2\theta = 3(\sec\theta - 1)$
$(\sec^2\theta - 1) = 3\sec\theta - 3$
$\sec^2\theta - 1 - 3\sec\theta + 3 = 0$
$\sec^2\theta - 3\sec\theta + 2 = 0$
$\sec^2\theta - 2\sec\theta - \sec\theta + 2 = 0$
$\sec\theta(\sec\theta - 2) - 1(\sec\theta = 2) = 0$
$(\sec\theta - 1)(\sec\theta - 2) = 0$
So, either $\sec\theta - 1 = 0$ or $\sec\theta - 2 = 0$
If $\sec\theta - 1 = 0$
$\sec\theta = 1$
$\Rightarrow \theta = 0^o$
And, if $\sec\theta - 2 = 0$
$\sec\theta = 2$
$\Rightarrow \theta = 60^o$
Thus, $\theta = 0^o$ or 60^o