EXERCISE 7.1

1. An alloy consists of 27 ½ kg of copper and 2 ¾ kg of tin. Find the ratio by weight of tin to the alloy.
Solution:

It is given that
Copper = 27 ½ kg = 55/2 kg
Tin = 2 ¾ kg = 11/4 kg

We know that
Total alloy = 55/2 + 11/4
Taking LCM
= (110 + 11)/ 4
= 121/4 kg

Here
Ratio between tin and alloy = 11/4 kg: 121/4 kg
So we get
= 11: 121
= 1: 11

2. Find the compounded ratio of:
(i) 2: 3 and 4: 9
(ii) 4: 5, 5: 7 and 9: 11
(iii) (a – b): (a + b), $(a + b)^2$: $(a^2 + b^2)$ and $(a^4 – b^4)$: $(a^2 – b^2)^2$
Solution:

(i) 2: 3 and 4: 9
We know that
Compound ratio = 2/3 × 4/9
= 8/27
= 8: 27

(ii) 4: 5, 5: 7 and 9: 11
We know that
Compound ratio = 4/5 × 5/7 × 9/11
= 36/77
= 36: 77

(iii) (a – b): (a + b), $(a + b)^2$: $(a^2 + b^2)$ and $(a^4 – b^4)$: $(a^2 – b^2)^2$
We know that
Compound ratio = $(a – b)/ (a + b) \times (a + b)^2/ (a^2 + b^2) \times (a^4 – b^4)/ (a^2 – b^2)^2$
By further calculation
= $(a – b)/ (a + b) \times [(a + b) (a + b)]/ (a^2 + b^2) \times [(a^2 + b^2) (a + b) (a – b)]/ [(a + b)^2 (a – b)^2]$
So we get
= 1/1
= 1: 1

3. Find the duplicate ratio of
(i) 2: 3

(ii) √5: 7
(iii) 5a: 6b
Solution:

(i) 2: 3
We know that
Duplicate ratio of 2: 3 = 2^2: 3^2 = 4: 9

(ii) √5: 7
We know that
Duplicate ratio of √5: 7 = $\sqrt{5}^2$: 7^2 = 5: 49

(iii) 5a: 6b
We know that
Duplicate ratio of 5a: 6b = $(5a)^2$: $(6b)^2$ = $25a^2$: $36b^2$

4. Find the triplicate ratio of
(i) 3: 4
(ii) ½: 1/3
(iii) 1^3: 2^3
Solution:

(i) 3: 4
We know that
Triplicate ratio of 3: 4 = 3^3: 4^3 = 27: 64

(ii) ½: 1/3
We know that
Triplicate ratio of ½: 1/3 = $(1/2)^3$: $(1/3)^3$ = 1/8: 1/27 = 27: 8

(iii) 1^3: 2^3
We know that
Triplicate ratio of 1^3: 2^3 = $(1^3)^3$: $(2^3)^3$ = 1^3: 8^3 = 1: 512

5. Find the sub-duplicate ratio of
(i) 9: 16
(ii) ¼: 1/9
(iii) $9a^2$: $49b^2$
Solution:

(i) 9: 16
We know that
Sub-duplicate ratio of 9: 16 = √9: √16 = 3: 4

(ii) ¼: 1/9
We know that
Sub-duplicate ratio of ¼: 1/9 = √1/4: √1/9
So we get
= ½: 1/3
= 3: 2

(iii) $9a^2: 49b^2$
We know that
Sub-duplicate ratio of $9a^2: 49b^2 = \sqrt{9a^2}: \sqrt{49b^2} = 3a: 7b$

6. Find the sub-triplicate ratio of
(i) 1: 216
(ii) 1/8: 1/125
(iii) $27a^3: 64b^3$
Solution:

(i) 1: 216
We know that
Sub-triplicate ratio of $1: 216 = \sqrt[3]{1}: \sqrt[3]{216}$
By further calculation
$= (1^3)^{1/3}: (6^3)^{1/3}$
= 1: 6

(ii) 1/8: 1/125
We know that
Sub-triplicate ratio of $1/8: 1/125 = (1/8)^{1/3}: (1/125)^{1/3}$
It can be written as
$= [(1/2)^3]^{1/3}: [(1/5)^3]^{1/3}$
So we get
= ½: 1/5
= 5: 2

(iii) $27a^3: 64b^3$
We know that
Sub-triplicate ratio of $27a^3: 64b^3 = [(3a)^3]^{1/3}: [(4b)^3]^{1/3}$
So we get
= 3a: 4b

7. Find the reciprocal ratio of
(i) 4: 7
(ii) $3^2: 4^2$
(iii) 1/9: 2
Solution:

(i) 4: 7
We know that
Reciprocal ratio of 4: 7 = 7: 4

(ii) $3^2: 4^2$
We know that
Reciprocal ratio of $3^2: 4^2 = 4^2: 3^2 = 16: 9$

(iii) 1/9: 2
We know that
Reciprocal ratio of 1/9: 2 = 2: 1/9 = 18: 1

8. Arrange the following ratios in ascending order of magnitude:
2: 3, 17: 21, 11: 14 and 5: 7
Solution:

It is given that
2: 3, 17: 21, 11: 14 and 5: 7
We can write it in fractions as
2/3, 17/21, 11/14, 5/7
Here the LCM of 3, 21, 14 and 7 is 42
By converting the ratio as equivalent
$2/3 = (2 \times 14)/ (3 \times 14) = 28/42$
$17/21 = (17 \times 2)/ (21 \times 2) = 34/ 42$
$11/14 = (11 \times 3)/ (14 \times 3) = 33/42$
$5/7 = (5 \times 6)/ (7 \times 6) = 30/42$
Now writing it in ascending order
28/42, 30/42, 33/42, 34/42
By further simplification
2/3, 5/7, 11/14, 17/21
So we get
2: 3, 5: 7, 11: 14 and 17: 21

9. (i) If A: B = 2: 3, B: C = 4: 5 and C: D = 6: 7, find A: D.
(ii) If x: y = 2: 3 and y: z = 4: 7, find x: y: z.
Solution:

(i) It is given that
A: B = 2: 3, B: C = 4: 5 and C: D = 6: 7
We can write it as
A/ B = 2/3, B/C = 4/5, C/D = 6/7
By multiplication
$A/B \times B/C \times C/D = 2/3 \times 4/5 \times 6/7$
So we get
A/D = 16/35
A: D = 16: 35

(ii) We know that the LCM of y terms 3 and 4 is 12
Now making equals of y as 12
$x/y = 2/3 = (2 \times 4)/ (3 \times 4) = 8/12 = 8: 12$
$y/z = 4/7 \times 3/3 = 12/21 = 12: 21$
So x: y: z = 8: 12: 21

10. (i) If A: B = 1/4: 1/5 and B: C = 1/7: 1/6, find A: B: C.
(ii) If 3A = 4B = 6C, find A: B: C
Solution:

(i) We know that
$A: B = 1/4 \times 5/1 = 5/4$
$B: C = 1/7 \times 6/1 = 6/7$
Here the LCM of B terms 4 and 6 is 12
Now making terms of B as 12

A/B = (5 × 3)/ (4 × 3) = 15/12 = 15: 12
B/C = (6 × 2)/ (7 × 2) = 12/14 = 12: 14
So A: B: C = 15: 12: 14

(ii) It is given that
3A = 4B
We can write it as
A/B = 4/3
A: B = 4: 3
Similarly 4B = 6C
We can write it as
B/C = 6/4 = 3/2
B: C = 3: 2
So we get
A: B: C = 4: 3: 2

11. (i) If 3x + 5y/ 3x – 5y = 7/3, find x: y.
(ii) If a: b = 3: 11, find (15a – 3b): (9a + 5b).
Solution:

(i) 3x + 5y/ 3x – 5y = 7/3
By cross multiplication
9x + 15y = 21x – 35y
By further simplification
21x – 9x = 15y + 35y
12x = 50y
So we get
x/y = 50/12 = 25/6

Therefore, x: y = 25: 6

(ii) It is given that
a: b = 3: 11
a/b = 3/11
It is given that
(15a – 3b)/ (9a + 5b)
Now dividing both numerator and denominator by b
= [15a/b – 3b/b]/ [9a/b + 5b/b]
By further calculation
= [15a/b – 3]/ [9a/b + 5]
Substituting the value of a/ b
= [15 × 3/11 – 3]/ [9 × 3/11 + 5]
So we get
= [45/11 – 3]/ [27/11 + 5]
Taking LCM
= [(45 – 33)/ 11]/ [(27 + 55)/ 11]
= 12/11/ 82/11
We can write it as
= 12/11 × 11/82
= 12/82

= 6/41

Hence, (15a – 3b): (9a + 5b) = 6: 41.

12. (i) If $(4x^2 + xy): (3xy – y^2) = 12: 5$, find (x + 2y): (2x + y).
(ii) If y (3x – y): x (4x + y) = 5: 12. Find $(x^2 + y^2): (x + y)^2$.
Solution:

(i) $(4x^2 + xy): (3xy – y^2) = 12: 5$
We can write it as
$(4x^2 + xy)/ (3xy – y^2) = 12/ 5$
By cross multiplication
$20x^2 + 5xy = 36xy – 12y^2$
$20x^2 + 5xy – 36xy + 12y^2 = 0$
$20x^2 – 31xy + 12y^2 = 0$
Now divide the entire equation by y^2
$20x^2/y^2 – 31xy/y^2 + 12y^2/y^2 = 0$
So we get
$20 (x/y)^2 – 31 (x/y) + 12 = 0$
$20 (x/y)^2 – 15(x/y) – 16 (x/y) + 12 = 0$
Taking common terms
5 (x/y) [4 (x/y) – 3] – 4 [4 (x/y) – 3] = 0
[4 (x/y) – 3] [5 (x/y) – 4] = 0

Here 4 (x/y) – 3 = 0
4 (x/y) = 3
So we get x/y = ¾

Similarly 5 (x/y) – 4 = 0
5 (x/y) = 4
So we get x/y = 4/5

Now dividing by y
(x + 2y)/ (2x + y) = (x/y + 2)/ (2 x/y + 1)

(a) If x/y = 3/4, then
= (x/y + 2)/ (2 x/y + 1)
Substituting the values
= (3/4 + 2)/ (2 × 3/4 + 1)
By further calculation
= 11/4/ (3/2 + 1)
= 11/4/ 5/2
= 11/4 × 2/5
= 11/10
So we get
(x + 2y): (2x + y) = 11: 10

(b) If x/y = 4/5 then
(x + 2y)/ (2x + y) = [x/y + 2]/ [2 x/y + 1]
Substituting the value of x/y

= [4/5 + 2]/ [2 × 4/5 + 1]
So we get
= 14/5/ [8/5 + 1]
= 14/5/ 13/5
= 14/5 × 5/13
= 14/13

We get
(x + 2y)/ (2x + y) = 11/10 or 14/13
(x + 2y): (2x + y) = 11: 10 or 14: 13

(ii) y (3x – y): x (4x + y) = 5: 12
It can be written as
$(3xy – y^2)/ (4x^2 + xy) = 5/12$
By cross multiplication
$36xy – 12y^2 = 20x^2 + 5xy$
$20x^2 + 5xy – 36xy + 12y^2 = 0$
$20x^2 – 31xy + 12y^2 = 0$
Divide the entire equation by y^2
$20x^2/y^2 – 31\ xy/y^2 + 12y^2/y^2 = 0$
$20(x^2/y^2) – 31\ (xy/y^2) + 12 = 0$
We can write it as
$20(x^2/y^2) – 15\ (x/y) – 16\ (x/y) + 12 = 0$
Taking common terms
5 (x/y) [4 (x/y) – 3] – 4 [4 (x/y) – 3] = 0
[4 (x/y) – 3] [5 (x/y) – 4] = 0

Here
4 (x/y) – 3 = 0
So we get
4 (x/y) = 3
x/y = 3/4

Similarly
5 (x/y) – 4 = 0
So we get
5 (x/y) = 4
x/y = 4/5

(a) x/y = 3/4
We know that
$(x^2 + y^2): (x + y)^2 = (x^2 + y^2)/ (x + y)^2$
Dividing both numerator and denominator by y^2
$= (x^2/y^2 + y^2/y^2)/ [1/y^2\ (x + y)^2]$
$= (x^2/ y^2 + 1)\ (x/y + 1)^2$
Substituting the value of x/y
$= [(3/4)^2 + 1]/ [3/4 + 1]^2$
By further calculation
$= (9/16 + 1)/ (7/4)^2$
So we get

= 25/16/ 49/16
= 25/16 × 16/49
= 25/49
So we get
$(x^2 + y^2): (x + y)^2 = 25: 49$

(b) x/y = 4/5
We know that
$(x^2 + y^2): (x + y)^2 = (x^2 + y^2)/ (x + y)^2$
Dividing both numerator and denominator by y^2
$= (x^2/y^2 + y^2/y^2)/ [1/y^2 (x + y)^2]$
$= (x^2/ y^2 + 1) (x/y + 1)^2$
Substituting the value of x/y
$= [(4/5)^2 + 1]/ [4/5 + 1]^2$
By further calculation
$= (16/25 + 1)/ (9/5)^2$
So we get
= 41/25/ 81/25
= 41/25 × 25/81
= 41/81
So we get
$(x^2 + y^2): (x + y)^2 = 41: 81$

13. (i) If (x – 9): (3x + 6) is the duplicate ratio of 4: 9, find the value of x.
(ii) If (3x + 1): (5x + 3) is the triplicate ratio of 3: 4, find the value of x.
(iii) If (x + 2y): (2x – y) is equal to the duplicate ratio of 3: 2, find x: y.
Solution:

(i) $(x – 9)/ (3x + 6) = (4/9)^2$
So we get
(x – 9)/ (3x + 6) = 16/81
By cross multiplication
81x – 729 = 48x + 96
81x – 48x = 96 + 729
So we get
33x = 825
x = 825/33 = 25

(ii) $(3x + 1)/ (5x + 3) = 3^3/ 4^3$
So we get
(3x + 1)/ (5x + 3) = 27/64
By cross multiplication
64 (3x + 1) = 27 (5x + 3)
192x + 64 = 135x + 81
192x – 135x = 81 – 64
57x = 17
So we get
x = 17/57

(iii) $(x + 2y)/ (2x – y) = 3^2/ 2^2$

So we get
$(x + 2y)/ (2x – y) = 9/4$
By cross multiplication
$9 (2x – y) = 4 (x + 2y)$
$18x – 9y = 4x + 8y$
$18x = 4x = 8y + 9y$
So we get
$14x = 17y$
$x/y = 17/14$
$x: y = 17: 14$

14. (i) Find two numbers in the ratio of 8: 7 such that when each is decreased by 12 ½, they are in the ratio 11: 9.
(ii) The income of a man is increased in the ratio of 10: 11. If the increase in his income is Rs 600 per month, find his new income.
Solution:

(i) Ratio = 8: 7
Consider the numbers as 8x and 7x
Using the condition
$[8x – 25/2]/ [7x – 25/2] = 11/9$
Taking LCM
$[(16x – 25)/ 2]/ [(14x – 25)/ 2] = 11/9$
By further calculation
$[(16x – 25) \times 2]/ [2 (14x – 25)] = 11/9$
$(16x – 25)/ (14x – 25) = 11/9$
By cross multiplication
$154x – 275 = 144x – 225$
$154x – 144x = 275 – 225$
$10x = 50$
$x = 50/10 = 5$

So the numbers are
$8x = 8 \times 5 = 40$
$7x = 7 \times 5 = 35$

(ii) Consider the present income = 10x
Increased income = 11x
So the increase per month = 11x – 10x = x
Here x = Rs 600
New income = 11x = 11 × 600 = Rs 6600

15. (i) A woman reduces her weight in the ratio 7: 5. What does her weight become if originally it was 91 kg.
(ii) A school collected Rs 2100 for charity. It was decided to divide the money between an orphanage and a blind school in the ratio of 3: 4. How much money did each receive?
Solution:

(i) Ratio of original and reduced weight of woman = 7: 5
Consider original weight = 7x

Reduced weight = 5x
Here original weight = 91 kg
So the reduced weight = (91 × 5x)/ 7x = 65 kg

(ii) Amount collected for charity = Rs 2100
Here the ratio between orphanage and a blind school = 3: 4
Sum of ratios = 3 + 4 = 7

We know that
Orphanage schools share = 2100 × 3/7 = Rs 900
Blind schools share = 2100 × 4/7 = Rs 1200

16. (i) The sides of a triangle are in the ratio 7: 5: 3 and its perimeter is 30 cm. Find the lengths of sides.
(ii) If the angles of a triangle are in the ratio 2: 3: 4, find the angles.
Solution:

(i) It is given that
Perimeter of triangle = 30 cm
Ratio among sides = 7: 5: 3
Here the sum of ratios = 7 + 5 + 3 = 15

We know that
Length of first side = 30 × 7/15 = 14 cm
Length of second side = 30 × 5/15 = 10 cm
Length of third side = 30 × 3/15 = 6 cm

Therefore, the sides are 14 cm, 10 cm and 6 cm.

(ii) We know that
Sum of all the angles of a triangle = 180^0
Here the ratio among angles = 2: 3: 4
Sum of ratios = 2 + 3 + 4 = 9

So we get
First angle = 180 × 2/9 = 40^0
Second angle = 180 × 3/9 = 60^0
Third angle = 180 × 4/9 = 80^0

Hence, the angles are 40^0, 60^0 and 80^0.

17. Three numbers are in the ratio 1/2: 1/3: ¼. If the sum of their squares is 244, find the numbers.
Solution:

It is given that
Ratio of three numbers = 1/2: 1/3: 1/4
= (6: 4: 3)/ 12
= 6: 4: 3

Consider first number = 6x
Second number = 4x

Third number = 3x
So based on the condition
$(6x)^2 + (4x)^2 + (3x)^2 = 244$
$36x^2 + 16x^2 + 9x^2 = 244$
So we get
$61x^2 = 244$
$x^2 = 244/61 = 4 = 2^2$
x = 2

Here
First number = 6x = 6 × 2 = 12
Second number = 4x = 4 × 2 = 8
Third number = 3x = 3 × 2 = 6

18. (i) A certain sum was divided among A, B and C in the ratio 7: 5: 4. If B got Rs 500 more than C, find the total sum divided.
(ii) In a business, A invests Rs 50000 for 6 months, B Rs 60000 for 4 months and C Rs 80000 for 5 months. If they together earn Rs 18800 find the share of each.
Solution:

(i) It is given that
Ratio between A, B and C = 7: 5: 4
Consider A share = 7x
B share = 5x
C share = 4x
So the total sum = 7x + 5x + 4x = 16x

Based on the condition
5x – 4x = 500
x = 500
So the total sum = 16x = 16 × 500 = Rs 8000

(ii) 6 months investment of A = Rs 50000
1 month investment of A = 50000 × 6 = Rs 300000

4 months investment of B = Rs 60000
1 month investment of B = 60000 × 4 = Rs 240000

5 months investment of C = Rs 80000
1 month investment of C = 80000 × 5 = Rs 400000

Here the ratio between their investments = 300000: 240000: 400000
= 30: 24: 40
Sum of ratio = 30 = 24 + 40 = 94
Total earnings = Rs 18800

So we get
A share = 30/94 × 18800 = Rs 6000
B share = 24/94 × 18800 = Rs 4800
C share = 40/94 = 18800 = Rs 8000

19. (i) In a mixture of 45 litres, the ratio of milk to water is 13: 2. How much water must be added to this mixture to make the ratio of milk to water as 3: 1?
(ii) The ratio of the number of boys to the numbers of girls in a school of 560 pupils is 5: 3. If 10 new boys are admitted, find how many new girls may be admitted so that the ratio of the number of boys to the number of girls may change to 3: 2.
Solution:

(i) It is given that
Mixture of milk to water = 45 litres
Ratio of milk to water = 13: 2
Sum of ratio = 13 + 2 = 15
Here the quantity of milk = (45 × 13)/ 15 = 39 litres
Quantity of water = 45 × 2/15 = 6 litres

Consider x litre of water to be added, then water = (6 + x) litres
Here the new ratio = 3: 1
39: (6 + x) = 3: 1
We can write it as
39/ (6 + x) = 3/1
By cross multiplication
39 = 18 + 3x
3x = 39 – 18 = 21
x = 21/3 = 7 litres

Hence, 7 litres of water is to be added to the mixture.

(ii) It is given that
Ratio between boys and girls = 5: 3
Number of pupils = 560
So the sum of ratios = 5 + 3 = 8

We know that
Number of boys = 5/8 × 560 = 350
Number of girls = 3/8 × 560 = 210
Number of new boys admitted = 10
So the total number of boys = 350 + 10 = 360

Consider x as the number of girls admitted
Total number of girls = 210 + x
Based on the condition
360: 210 + x = 3: 2
We can write it as
360/ 210 + x = 3/2
By cross multiplication
630 + 3x = 720
3x = 720 – 630 = 90
So we get
x = 90/3 = 30

Hence, 30 new girls are to be admitted.

20. (i) The monthly pocket money of Ravi and Sanjeev are in the ratio 5: 7. Their expenditures are in the ratio 3: 5. If each saves Rs 80 per month, find their monthly pocket money.
(ii) In class X of a school, the ratio of the number of boys to that of the girls is 4: 3. If there were 20 more boys and 12 less girls, then the ratio would have been 2: 1. How many students were there in the class?
Solution:

(i) Consider the monthly pocket money of Ravi and Sanjeev as 5x and 7x
Their expenditure is 3y and 5y respectively.
5x – 3y = 80 …… (1)
7x – 5y = 80 …… (2)
Now multiply equation (1) by 7 and (2) by 5
Subtracting both the equations
35x – 21y = 560
35x – 25y = 400
So we get
4y = 160
y = 40

In equation (1)
5x = 80 + 3 × 40 = 200
x = 40
Here the monthly pocket money of Ravi = 5 × 40 = 200

(ii) Consider x as the number of students in class
Ratio of boys and girls = 4: 3
Number of boys = 4x/7
Number of girls = 3x/7

Based on the problem
(4x/7 + 20): (3x/7 – 12) = 2: 1
We can write it as
(4x + 140)/ 7: (3x – 84)/ 7 = 2: 1
So we get
(4x + 140)/ 7 × 7/ (3x – 84) = 2/1
(4x + 140)/ (3x – 84) = 2/1
6x – 168 = 4x + 140
6x – 4x = 140 + 168
2x = 308
x = 308/2 = 154

Therefore, 154 students were there in the class.

21. In an examination, the ratio of passes to failures was 4: 1. If 30 less had appeared and 20 less passed, the ratio of passes to failures would have been 5: 1. How many students appeared for the examination.
Solution:

Consider number of passes = 4x
Number of failures = x
Total number of students appeared = 4x + x = 5x

In case 2
Number of students appeared = 5x – 30
Number of passes = 4x – 20
So the number of failures = (5x – 30) – (4x – 20)
By further calculation
= 5x – 30 – 4x + 20
= x – 10

Based on the condition
(4x – 20)/ (x – 10) = 5/1
By cross multiplication
5x – 50 = 4x – 20
5x – 4x = - 20 + 50
x = 30

No. of students appeared = 5x = 5 × 30 = 150

EXERCISE 7.2

1. Find the value of x in the following proportions:
(i) 10: 35 = x: 42
(ii) 3: x = 24: 2
(iii) 2.5: 1.5 = x: 3
(iv) x: 50 :: 3: 2
Solution:

(i) 10: 35 = x: 42
We can write it as
35 × x = 10 × 42
So we get
x = (10 × 42)/ 35
x = 2 × 6
x = 12

(ii) 3: x = 24: 2
We can write it as
x × 24 = 3 × 2
So we get
x = (3 × 2)/ 24
x = ¼

(iii) 2.5: 1.5 = x: 3
We can write it as
1.5 × x = 2.5 × 3
So we get
x = (2.5 × 3)/ 1.5
x = 5.0

(iv) x: 50 :: 3: 2
We can write it as
x × 2 = 50 × 3
So we get
x = (50 × 3)/ 2
x = 75

2. Find the fourth proportional to
(i) 3, 12, 15
(ii) 1/3, 1/4, 1/5
(iii) 1.5, 2.5, 4.5
(iv) 9.6 kg, 7.2 kg, 28.8 kg
Solution:

(i) 3, 12, 15
Consider x as the fourth proportional to 3, 12 and 15
3: 12 :: 15: x
We can write it as
3 × x = 12 × 15

So we get
x = (12 × 15)/ 3
x = 60

(ii) 1/3, 1/4, 1/5
Consider x as the fourth proportional to 1/3, 1/4 and 1/5
1/3: 1/4:: 1/5: x
We can write it as
1/3 × x = 1/4 × 1/5
So we get
x = 1/4 × 1/5 × 3/1
x = 3/20

(iii) 1.5, 2.5, 4.5
Consider x as the fourth proportional to 1,5, 2.5 and 4.5
1.5: 2.5 :: 4.5: x
We can write it as
1.5 × x = 2.5 × 4.5
So we get
x = (2.5 × 4.5)/ 1.5
x = 7.5

(iv) 9.6 kg, 7.2 kg, 28.8 kg
Consider x as the fourth proportional to 9.6, 7.2 and 28.8
9.6: 7.2 :: 28.8: x
We can write it as
9.6 × x = 7.2 × 28.8
So we get
x = (7.2 × 28.8)/ 9.6
x = 21.6

3. Find the third proportional to
(i) 5, 10
(ii) 0.24, 0.6
(iii) Rs. 3, Rs. 12
(iv) 5 ¼ and 7.
Solution:

(i) Consider x as the third proportional to 5, 10
5: 10 :: 10: x
It can be written as
5 × x = 10 × 10
x = (10 × 10)/ 5 = 20

Hence, the third proportional to 5, 10 is 20.

(ii) Consider x as the third proportional to 0.24, 0.6
0.24: 0.6 :: 0.6: x
It can be written as
0.24 × x = 0.6 × 0.6

x = (0.6 × 0.6)/ 0.24 = 1.5

Hence, the third proportional to 0.24, 0.6 is 1.5.

(iii) Consider x as the third proportional to Rs. 3 and Rs. 12
3: 12 :: 12: x
It can be written as
3 × x = 12 × 12
x = (12 × 12)/ 3 = 48

Hence, the third proportional to Rs. 3 and Rs. 12 is Rs. 48

(iv) Consider x as the third proportional to 5 ¼ and 7
5 ¼: 7 :: 7: x
It can be written as
21/4 × x = 7 × 7
x = (7 × 7 × 4)/ 21 = 28/3 = 9 1/3

Hence, the third proportional to 5 ¼ and 7 is 9 1/3.

4. Find the mean proportion of:
(i) 5 and 80
(ii) 1/12 and 1/75
(iii) 8.1 and 2.5
(iv) (a – b) and (a^3 – a^2b), a > b
Solution:

(i) Consider x as the mean proportion of 5 and 80
5: x :: x: 80
It can be written as
x^2 = 5 × 80 = 400
x = √400 = 20

Therefore, mean proportion of 5 and 80 is 20.

(ii) Consider x as the mean proportion of 1/12 and 1/75
1/12: x :: x: 1/75
It can be written as
x^2 = 1/12 × 1/75 = 1/900
x = √1/900 = 1/30

Therefore, mean proportion of 1/12 and 1/75 is 1/30.

(iii) Consider x as the mean proportion of 8.1 and 2.5
8.1: x :: x: 2.5
It can be written as
x^2 = 8.1 × 2.5 = 20.25
x = √20.25 = 4.5

Therefore, mean proportion of 8.1 and 2.5 is 4.5.

(iv) Consider x as the mean proportion of $(a - b)$ and $(a^3 - a^2b)$, $a > b$
$(a - b): x :: (a^3 - a^2b)$
It can be written as
$x^2 = (a - b)(a^3 - a^2b)$
So we get
$x^2 = (a - b) a^2 (a - b)$
$x^2 = a^2 (a - b)^2$
Here
$x = a(a - b)$

Therefore, mean proportion of $(a - b)$ and $(a^3 - a^2b)$, $a > b$ is $a(a - b)$.

5. If a, 12, 16 and b are in continued proportion find a and b.
Solution:

It is given that
a, 12, 16 and b are in continued proportion
a/12 = 12/16 = 16/b

We know that
a/12 = 12/16
By cross multiplication
16a = 144
a = 144/16 = 9

Similarly
12/16 = 16/b
By cross multiplication
12b = 16 × 16 = 256
b = 256/12 = 64/3 = 21 1/3

Therefore, a = 9 and b = 64/3 or 21 1/3.

6. What number must be added to each of the numbers 5, 11, 19 and 37 so that they are in proportion?
Solution:

Consider x to be added to 5, 11, 19 and 37 to make them in proportion
5 + x: 11 + x :: 19 + x: 37 + x
It can be written as
(5 + x) (37 + x) = (11 + x) (19 + x)
By further calculation
$185 + 5x + 37x + x^2 = 209 + 11x + 19x + x^2$
$185 + 42x + x^2 = 209 + 30x + x^2$
So we get
$42x - 30x + x^2 - x^2 = 209 - 185$
12x = 24
x = 2

Hence, the least number to be added is 2.

7. What number should be subtracted from each of the numbers 23, 30, 57 and 78 so that the remainders are in proportion?
Solution:

Consider x be subtracted from each term
23 – x, 30 – x, 57 – x and 78 – x are proportional
It can be written as
23 – x: 30 – x :: 57 – x: 78 – x
(23 – x)/ (30 – x) = (57 – x)/ (78 – x)
By cross multiplication
(23 – x) (78 – x) = (30 – x) (57 – x)
By further calculation
$1794 – 23x – 78x + x^2 = 1710 – 30x – 57x + x^2$
$x^2 – 101x + 1794 – x^2 + 87x – 1710 = 0$
So we get
- 14x + 84 = 0
14x = 84
x = 84/14 = 6

Therefore, 6 is the number to be subtracted from each of the numbers.

8. If 2x – 1, 5x – 6, 6x + 2 and 15x – 9 are in proportion, find the value of x.
Solution:

It is given that
2x – 1, 5x – 6, 6x + 2 and 15x – 9 are in proportion
We can write it as
(2x – 1) (15x – 9) = (5x – 6) (6x + 2)
By further calculation
$30x^2 – 18x – 15x + 9 = 30x^2 + 10x – 36x – 12$
$30x^2 – 33x + 9 = 30x^2 – 26x – 12$
$30x^2 – 33x – 30x^2 + 26x = - 12 – 9$
So we get
-7x = - 21
x = -21/-7 = 3

Therefore, the value of x is 3.

9. If x + 5 is the mean proportion between x + 2 and x + 9, find the value of x.
Solution:

It is given that
x + 5 is the mean proportion between x + 2 and x + 9
We can write it as
$(x + 5)^2 = (x + 2) (x + 9)$
By further calculation
$x^2 + 10x + 25 = x^2 + 11x + 18$
$x^2 + 10x – x^2 – 11x = 18 – 25$
So we get
- x = - 7

x = 7

Hence, the value of x is 7.

10. What number must be added to each of the numbers 16, 26 and 40 so that the resulting numbers may be in continued proportion?
Solution:

Consider x be added to each number
16 + x , 26 + x and 40 + x are in continued proportion
It can be written as
(16 + x)/ (26 + x) = (26 + x)/ (40 + x)
By cross multiplication
(16 + x) (40 + x) = (26 + x) (26 + x)
On further calculation
$640 + 16x + 40x + x^2 = 676 + 26x + 26x + x^2$
$640 + 56x + x^2 = 676 + 52x + x^2$
$56x + x^2 – 52x – x^2 = 676 – 640$
So we get
4x = 36
x = 36/4 = 9

Hence, 9 is the number to be added to each of the numbers.

11. Find two numbers such that the mean proportional between them is 28 and the third proportional to them is 224.
Solution:

Consider a and b as the two numbers
It is given that 28 is the mean proportional
a: 28 :: 28: b
We get
$ab = 28^2 = 784$
Here a = 784/b …… (1)

We know that 224 is the third proportional
a: b :: b: 224
So we get
$b^2 = 224a$ ….. (2)

Now by substituting the value of a in equation (2)
$b^2 = 224 × 784/b$
So we get
$b^3 = 224 × 784$
$b^3 = 175616 = 56^3$
b = 56

By substituting the value of b in equation (1)
a = 784/56 = 14

Therefore, 14 and 56 are the two numbers.

12. If b is the mean proportional between a and c, prove that a, c, $a^2 + b^2$ and $b^2 + c^2$ are proportional.
Solution:

It is given that
b is the mean proportional between a and c
We can write it as
$b^2 = a \times c$
$b^2 = ac$ ….. (1)

We know that
a, c, $a^2 + b^2$ and $b^2 + c^2$ are in proportion
It can be written as
$a/c = (a^2 + b^2)/ (b^2 + c^2)$
By cross multiplication
$a (b^2 + c^2) = c (a^2 + b^2)$
Using equation (1)
$a (ac + c^2) = c (a^2 + ac)$
So we get
$ac (a + c) = a^2c + ac^2$
Here ac (a + c) = ac (a + c) which is true.

Therefore, it is proved.

13. If b is the mean proportional between a and c, prove that (ab + bc) is the mean proportional between $(a^2 + b^2)$ and $(b^2 + c^2)$.
Solution:

It is given that
b is the mean proportional between a and c
$b^2 = ac$ …. (1)

Here (ab + bc) is the mean proportional between $(a^2 + b^2)$ and $(b^2 + c^2)$
$(ab + bc)^2 = (a^2 + b^2) (b^2 + c^2)$

Consider LHS = $(ab + bc)^2$
Expanding using formula
$= a^2b^2 + b^2c^2 + 2ab^2c$
Using equation (1)
$= a^2 (ac) + ac (c)^2 + 2a. ac. c$
$= a^3c + ac^3 + 2a^2c^2$
Taking ac as common
$= ac (a^2 + c^2 + 2ac)$
$= ac (a + c)^2$

RHS = $(a^2 + b^2) (b^2 + c^2)$
Using equation (1)
$= (a^2 + ac) (ac + c^2)$
Taking common terms out

= a (a + c) c (a + c)
= ac $(a + c)^2$
Hence, LHS = RHS.

14. If y is mean proportional between x and z, prove that xyz $(x + y + z)^3 = (xy + yz + zx)^3$.
Solution:

It is given that
y is mean proportional between x and z
We can write it as
$y^2 = xz$ …… (1)

Consider
LHS = xyz $(x + y + z)^3$
It can be written as
= xz. y $(x + y + z)^3$
Using equation (1)
= y^2 y $(x + y + z)^3$
= $y^3 (x + y + z)^3$
So we get
= $[y (x + y + z)]^3$
By further calculation
= $(xy + y^2 + yz)^3$
Using equation (1)
= $(xy + yz + zx)^3$
= RHS

Hence, it is proved.

15. If a + c = mb and 1/b + 1/d = m/c, prove that a, b, c and d are in proportion.
Solution:

It is given that
a + c = mb and 1/b + 1/d = m/c

a + c = mb
Dividing the equation by b
a/b + c/d = m ……. (1)

1/b + 1/d = m/c
Multiplying the equation by c
c/b + c/d = m …… (2)

Using equation (1) and (2)
a/b + c/b = c/b + c/d
So we get
a/b = c/d

Therefore, it is proved that a, b, c and d are in proportion.

16. If x/a = y/b = z/c, prove that

$$(i) \frac{x^3}{a^2} + \frac{y^3}{b^2} + \frac{z^3}{c^2} = \frac{(x+y+z)^3}{(a+b+c)^2}$$

$$(ii) [\frac{a^2x^2 + b^2y^2 + c^2z^2}{a^3x + b^3y + c^3z}]^3 = \frac{xyz}{abc}$$

$$(iii) \frac{ax - by}{(a+b)(x-y)} + \frac{by - cz}{(b+c)(y-z)} + \frac{cz - ax}{(c+a)(z-x)} = 3$$

Solution:

It is given that

x/a = y/b = z/c

We can write it as

x = ak, y = bk and z = ck

$$(i) LHS = \frac{x^3}{a^2} + \frac{y^3}{b^2} + \frac{z^3}{c^2}$$

It can be written as

$$= \frac{a^3k^3}{a^2} + \frac{b^3k^3}{b^2} + \frac{c^3k^3}{c^2}$$

So we get

$$= ak^3 + bk^3 + ck^3$$

Taking common terms

$$= k^3(a+b+c)$$

$$RHS = \frac{(x+y+z)^3}{(a+b+c)^2}$$

It can be written as

$$= \frac{(ak+bk+ck)^3}{(a+b+c)^2}$$

So we get

$$= \frac{k^3(a+b+c)^3}{(a+b+c)^2}$$

$$= k^3(a+b+c)$$

Therefore, LHS = RHS.

$$(ii) LHS = [\frac{a^2x^2 + b^2y^2 + c^2z^2}{a^3x + b^3y + c^3z}]^3$$

$It\ can\ be\ written\ as$

$$= [\frac{a^2a^2k^2 + b^2b^2k^2 + c^2c^2k^2}{a^3.ak + b^3.bk + c^3.ck}]^3$$

$By\ further\ calculation$

$$= [\frac{a^4k^2 + b^4k^2 + c^4k^2}{a^4k + b^4k + c^4k}]^3$$

$So\ we\ get$

$$= [\frac{k^2(a^4 + b^4 + c^4)}{k(a^4 + b^4 + c^4)}]^3$$

$$= k^3$$

$$RHS = \frac{xyz}{abc}$$

$We\ can\ write\ it\ as$

$$= \frac{ak.bk.ck}{abc}$$

$$= k^3$$

Therefore, LHS = RHS.

$$(iii) LHS = \frac{ax - by}{(a + b)(x - y)} + \frac{by - cz}{(b + c)(y - z)} + \frac{cz - ax}{(c + a)(z - x)}$$

$It\ can\ be\ written\ as$

$$= \frac{ax - by}{(a + b)(ak - bk)} + \frac{by - cz}{(b + c)(bk - ck)} + \frac{cz - ax}{(c + a)(ck - ak)}$$

$By\ further\ calculation$

$$= \frac{a^2k - b^2k}{(a+b)k(a-b)} + \frac{b^2k - c^2k}{(b+c)k(b-c)} + \frac{c^2k - a^2k}{(c+a)k(c-a)}$$

$Taking\ common\ terms$

$$= \frac{k(a^2 - b^2)}{k(a^2 - b^2)} + \frac{k(b^2 - c^2)}{k(b^2 - c^2)} + \frac{k(c^2 - a^2)}{k(c^2 - a^2)}$$

$So\ we\ get$

$$= 1 + 1 + 1$$
$$= 3$$
$$= RHS$$

Therefore, LHS = RHS.

17. If a/b = c/d = e/f prove that:
(i) $(b^2 + d^2 + f^2)(a^2 + c^2 + e^2) = (ab + cd + ef)^2$

$$(ii) \frac{(a^3 + c^3)^2}{(b^3 + d^3)^2} = \frac{e^6}{f^6}$$

$$(iii) \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2} = \frac{ac}{bd} + \frac{ce}{df} + \frac{ae}{df}$$

$$(iv) bdf\left[\frac{a+b}{b} + \frac{c+d}{d} + \frac{c+f}{f}\right]^3 = 27(a+b)(c+d)(e+f)$$

Solution:

Consider
a/b = c/d = e/f = k
So we get
a = bk, c = dk, e = fk

(i) LHS = $(b^2 + d^2 + f^2)(a^2 + c^2 + e^2)$
We can write it as
$= (b^2 + d^2 + f^2)(b^2k^2 + d^2k^2 + f^2k^2)$
Taking out the common terms
$= (b^2 + d^2 + f^2) k^2 (b^2 + d^2 + f^2)$
So we get
$= k^2 (b^2 + d^2 + f^2)$

RHS = $(ab + cd + ef)^2$
We can write it as
$= (b.\ kb + dk.\ d + fk.\ f)^2$
So we get
$= (kb^2 + kd^2 + kf^2)$
Taking out common terms
$= k^2 (b^2 + d^2 + f^2)^2$

Therefore, LHS = RHS.

$$(ii) LHS = \frac{(a^3+c^3)^2}{(b^3+d^3)^2}$$

$$It\ can\ be\ written\ as$$

$$= \frac{(b^3k^3+d^3k^3)^2}{(b^3+d^3)^2}$$

$$Taking\ out\ the\ common\ terms$$

$$= \frac{[k^3(b^3+d^3)]^2}{(b^3+d^3)^2}$$

$$So\ we\ get$$

$$= \frac{k^6(b^3+d^3)^2}{(b^3+d^3)^2}$$

$$= k^6$$

$$RHS = \frac{e^6}{f^6}$$

$$We\ get$$

$$= \frac{f^6k^6}{f^6}$$

$$= k^6$$

Therefore, LHS = RHS.

$$(iii) LHS = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$

$$It\ can\ be\ written\ as$$

$$= \frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$$

$$So\ we\ get$$

$$= k^2 + k^2 + k^2$$

$= 3k^2$

$RHS = \frac{ac}{bd} + \frac{ce}{df} + \frac{ae}{bf}$

$It\ can\ be\ written\ as$

$= [\frac{bk.dk}{bd} + \frac{dk.fk}{df} + \frac{bk.fk}{bf}]$

$So\ we\ get$

$= k^2 + k^2 + k^2$

$= 3k^2$

Therefore, LHS = RHS.

$(iv) LHS = bdf[\frac{a+b}{b} + \frac{c+d}{d} + \frac{e+f}{f}]^3$

$It\ can\ be\ written\ as$

$= bdf[\frac{bk+b}{b} + \frac{dk+d}{d} + \frac{fk+f}{f}]^3$

$Taking\ out\ the\ common\ terms$

$= bdf[\frac{b(k+1)}{b} + \frac{d(k+1)}{d} + \frac{f(k+1)}{f}]^3$

So we get
= bdf $(k + 1 + k + 1 + k + 1)^3$
By further calculation
= bdf $(3k + 3)^3$
= 27 bdf $(k + 1)^3$

RHS = 27 (a + b) (c + d) (e + f)
It can be written as
= 27 (bk + b) (dk + d) (fk + f)
Taking out the common terms
= 27 b (k + 1) d (k + 1) f (k + 1)
So we get
= 27 bdf $(k + 1)^3$

Therefore, LHS = RHS.

18. If ax = by = cz; prove that

$$\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=\frac{bc}{a^2}+\frac{ca}{b^2}+\frac{ab}{c^2}$$

Solution:

Consider ax = by = cz = k
It can be written as
x = k/a, y = k/b, z = k/c

$$LHS=\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$$

Substituting the values

$$=\frac{\frac{k^2}{a^2}}{\frac{k}{b}\cdot\frac{k}{c}}+\frac{\frac{k^2}{b^2}}{\frac{k}{c}\cdot\frac{k}{a}}+\frac{\frac{k^2}{c^2}}{\frac{k}{a}\cdot\frac{k}{b}}$$

By further calculation

$$=\frac{\frac{k^2}{a^2}}{\frac{k^2}{bc}}+\frac{\frac{k^2}{b^2}}{\frac{k^2}{ca}}+\frac{\frac{k^2}{c^2}}{\frac{k^2}{ab}}$$

It can be written as

$$=\frac{k^2}{a^2}\times\frac{bc}{k^2}+\frac{k^2}{b^2}\times\frac{ca}{k^2}+\frac{k^2}{c^2}\times\frac{ab}{k^2}$$

So we get

$$=\frac{bc}{a^2}+\frac{ca}{b^2}+\frac{ab}{c^2}$$

$$=RHS$$

19. If a, b, c and d are in proportion, prove that:
(i) (5a + 7v) (2c – 3d) = (5c + 7d) (2a – 3b)
(ii) (ma + nb): b = (mc + nd): d
(iii)$(a^4 + c^4):(b^4 + d^4) = a^2c^2:b^2d^2$

$$(iv)\frac{a^2+ab}{c^2+cd}=\frac{b^2-2ab}{d^2-2cd}$$

$$(v)\frac{(a+c)^3}{(b+d)^3}=\frac{a(a-c)^2}{b(b-d)^2}$$

$$(vi)\frac{a^2+ab+b^2}{a^2-ab+b^2}=\frac{c^2+cd+d^2}{c^2-cd+d^2}$$

$$(vii)\frac{a^2+b^2}{c^2+d^2}=\frac{ab+ad-bc}{bc+cd-ad}$$

$$(viii)abcd[\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{d^2}]=a^2+b^2+c^2+d^2$$

Solution:

It is given that
a, b, c, d are in proportion
Consider a/b = c/d = k
a = b, c = dk

(i) LHS = (5a + 7b) (2c – 3d)
Substituting the values
= (5bk + 7b) (2dk – 3d)
Taking out the common terms
= k (5b + 7b) k (2d – 3d)
So we get
= k^2 (12b) (-d)
= - 12 bd k^2

RHS = (5c + 7d) (2a – 3b)
Substituting the values
= (5dk + 7d) (2kb – 3b)
Taking out the common terms
= k (5d + 7d) k (2b – 3b)
So we get
= k^2 (12d) (-b)
= - 12 bd k^2

Therefore, LHS = RHS.

(ii) (ma + nb): b = (mc + nd): d
We can write it as

$$\frac{ma+nb}{b}=\frac{mc+nd}{d}$$

$Consider$

$$LHS=\frac{mbk+nb}{b}$$

$Taking\ out\ the\ common\ terms$

$$=\frac{b(mk+n)}{b}$$

$$=mk+n$$

$$RHS = \frac{mc + nd}{d}$$

$$It\ can\ be\ written\ as$$

$$= \frac{mdk + nd}{d}$$

$$Taking\ out\ the\ common\ terms$$

$$= \frac{d(mk + n)}{d}$$

$$= mk + n$$

Therefore, LHS = RHS.

(iii)$(a^4 + c^4): (b^4 + d^4) = a^2c^2: b^2d^2$
We can write it as

$$\frac{a^4 + c^4}{b^4 + d^4} = \frac{a^2c^2}{b^2d^2}$$

$$Consider$$

$$LHS = \frac{a^4 + c^4}{b^4 + d^4}$$

$$Substituting\ the\ values$$

$$= \frac{b^4k^4 + d^4k^4}{b^4 + d^4}$$

$$Taking\ out\ the\ common\ terms$$

$$= \frac{k^4(b^4 + d^4)}{(b^4 + d^4)}$$

$$= k^4$$

$$RHS = \frac{a^2c^2}{b^2d^2}$$

$$We\ can\ write\ it\ as$$

$$= \frac{k^2b^2.k^2d^2}{b^2d^2}$$

$$= k^4$$

Therefore, LHS = RHS.

$$(iv) LHS = \frac{a^2 + ab}{c^2 + cd}$$

$$It\ can\ be\ written\ as$$

$$= \frac{k^2b^2 + bk.b}{k^2d^2 + dk.d}$$

$$Taking\ out\ the\ common\ terms$$

$$= \frac{kb^2(k+1)}{d^2k(k+1)}$$

$$= \frac{b^2}{d^2}$$

$$RHS = \frac{b^2 - 2ab}{d^2 - 2cd}$$

$$It\ can\ be\ written\ as$$

$$= \frac{b^2 - 2bkb}{d^2 - 2dkd}$$

$$Taking\ out\ the\ common\ terms$$

$$= \frac{b^2(1-2k)}{d^2(1-2k)}$$

$$= \frac{b^2}{d^2}$$

Therefore, LHS = RHS.

$$(v) LHS = \frac{(a+c)^3}{(b+d)^3}$$

$$We\ can\ write\ it\ as$$

$$= \frac{(bk+dk)^3}{(b+d)^3}$$

$$= k^3$$

$$RHS = \frac{a(a-c)^2}{b(b-d)^2}$$

$We\ can\ write\ it\ as$

$$= \frac{bk(bk-dk)^2}{b(b-d)^2}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{bk.k^2(b-d)^2}{b(b-d)^2}$$

$$= k^3$$

Therefore, LHS = RHS.

$$(vi) LHS = \frac{a^2+ab+b^2}{a^2-ab+b^2}$$

$We\ can\ write\ it\ as$

$$= \frac{b^2k^2+bk.b+b^2}{b^2k^2-bk.b+b^2}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{b^2(k^2+k+1)}{b^2(k^2-k+1)}$$

$So\ we\ get$

$$= \frac{(k^2+k+1)}{(k^2-k+1)}$$

$$RHS = \frac{c^2+cd+d^2}{c^2-cd+d^2}$$

$We\ can\ write\ it\ as$

$$= \frac{d^2k^2+dkd+d^2}{d^2k^2-dkd+d^2}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{d^2(k^2+k+1)}{d^2(k^2-k+1)}$$

$So\ we\ get$

$$= \frac{(k^2+k+1)}{(k^2-k+1)}$$

Therefore, LHS = RHS.

$$(vii) LHS = \frac{a^2+b^2}{c^2+d^2}$$

$We\ can\ write\ it\ as$

$$= \frac{b^2k^2+b^2}{d^2k^2+d^2}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{b^2(k^2+1)}{d^2(k^2+1)}$$

$$= \frac{b^2}{d^2}$$

$$RHS = \frac{ab+ad-bc}{bc+cd-ad}$$

$We\ can\ write\ it\ as$

$$= \frac{bk.b+bk.d-b.dk}{bk.d+dk.d-bk.d}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{k(b^2+bd-bd)}{k(bd+d^2-bd)}$$

$$= \frac{b^2}{d^2}$$

Therefore, LHS = RHS.

$$(viii) LHS = abcd[\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}]$$

$We\ can\ write\ it\ as$

$$= bk.b.dk.d[\frac{1}{b^2k^2} + \frac{1}{b^2} + \frac{1}{d^2k^2} + \frac{1}{d^2}]$$

$By\ further\ calculation$

$$= k^2b^2d^2[\frac{d^2 + d^2k^2 + b^2 + b^2k^2}{b^2d^2k^2}]$$

So we get
= $d^2 (1 + k^2) + b^2 (1 + k^2)$
= $(1 + k^2) (b^2 + d^2)$

RHS = $a^2 + b^2 + c^2 + d^2$
We can write it as
= $b^2k^2 + b^2 + d^2k^2 + d^2$
Taking out the common terms
= $b^2 (k^2 + 1) + d^2 (k^2 + 1)$
= $(k^2 + 1) (b^2 + d^2)$

Therefore, LHS = RHS.

20. If x, y, z are in continued proportion, prove that:
$(x + y)^2/ (y + z)^2 = x/z$.
Solution:

It is given that
x, y, z are in continued proportion
Consider x/y = y/z = k
So we get
y = kz
x = yk = kz × k = k^2z

$$LHS = \frac{(x + y)^2}{(y + z)^2}$$

$We\ can\ write\ it\ as$

$$= \frac{(k^2z + kz)^2}{(kz + z)^2}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{[kz(k + 1)]^2}{[z(k + 1)]^2}$$

$So\ we\ get$

$$= \frac{k^2 z^2 (k+1)^2}{z^2 (k+1)^2}$$

$$= k^2$$

$$RHS = \frac{x}{z}$$

$We\ can\ write\ it\ as$

$$= \frac{k^2 z}{z}$$

$$= k^2$$

Therefore, LHS = RHS.

21. If a, b, c are in continued proportion, prove that:

$$\mathbf{\frac{pa^2 + qab + rb^2}{pb^2 + qbc + rc^2} = \frac{a}{c}}$$

Solution:

It is given that
a, b, c are in continued proportion

$$\frac{pa^2 + qab + rb^2}{pb^2 + qbc + rc^2} = \frac{a}{c}$$

Consider a/b = b/c = k
So we get
a = bk and b = ck ….. (1)
From equation (1)
a = (ck) k = ck^2 and b = ck

We know that

$$LHS = \frac{pa^2 + qab + rb^2}{pb^2 + qbc + rc^2}$$

$We\ can\ write\ it\ as$

$$= \frac{p(ck^2)^2 + q(ck^2)ck + r(ck)^2}{p(ck)^2 + q(ck)c + rc^2}$$

$By\ further\ calculation$

$$= \frac{pc^2k^4 + qc^2k^3 + rc^2k^2}{pc^2k^2 + qc^2k + rc^2}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{c^2k^2}{c^2}[\frac{pk^2 + qk + r}{pk^2 + qk + r}]$$

$$= k^2$$

$$RHS = \frac{a}{c}$$

$We\ can\ write\ it\ as$

$$= \frac{ck^2}{c}$$

$$= k^2$$

Therefore, LHS = RHS.

22. If a, b, c are in continued proportion, prove that:

(i) $\mathbf{\frac{a+b}{b+c} = \frac{a^2(b-c)}{b^2(a-b)}}$

(ii) $\mathbf{\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} = \frac{a}{b^2c^2} + \frac{b}{c^2a^2} + \frac{c}{a^2b^2}}$

(iii) a: c = $(a^2 + b^2)$: $(b^2 + c^2)$

(iv) $a^2b^2c^2 (a^{-4} + b^{-4} + c^{-4}) = b^{-2} (a^4 + b^4 + c^4)$

(v) abc $(a + b + c)^3 = (ab + bc + ca)^3$

(vi) (a + b + c) (a – b + c) = $a^2 + b^2 + c^2$

Solution:

It is given that
a, b, c are in continued proportion
So we get
a/b = b/c = k

$$(i) LHS = \frac{a+b}{b+c}$$

$We\ can\ write\ it\ as$

$$= \frac{ck^2 + ck}{ck + c}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{ck(k+1)}{c(k+1)}$$

$$= k$$

$$RHS = \frac{a^2(b-c)}{b^2(a-b)}$$

$We\ can\ write\ it\ as$

$$= \frac{(ck^2)^2(ck-c)}{(ck)^2(ck^2-ck)}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{c^2k^4c(k-1)}{c^2k^2ck(k-1)}$$

$So\ we\ get$

$$= \frac{c^3k^4(k-1)}{c^3k^3(k-1)}$$

$$= k$$

Therefore, LHS = RHS.

$$(ii) LHS = \frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}$$

$We\ can\ write\ it\ as$

$$= \frac{1}{(ck^2)^3} + \frac{1}{(ck)^3} + \frac{1}{c^3}$$

$$= \frac{1}{c^3k^6} + \frac{1}{c^3k^3} + \frac{1}{c^3}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{1}{c^3}[\frac{1}{k^6} + \frac{1}{k^3} + \frac{1}{1}]$$

$$RHS = \frac{a}{b^2c^2} + \frac{b}{c^2a^2} + \frac{c}{a^2b^2}$$

$We\ can\ write\ it\ as$

$$= \frac{ck^2}{(ck)^2c^2} + \frac{ck}{c^2(ck^2)^2} + \frac{c}{(ck^2)^2(ck)^2}$$
$$= \frac{ck^2}{c^4k^2} + \frac{ck}{c^4k^4} + \frac{c}{c^4k^6}$$

$So\ we\ get$

$$= \frac{1}{c^3} + \frac{1}{c^3k^3} + \frac{1}{c^3k^6}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{1}{c^3}[1 + \frac{1}{k^3} + \frac{1}{k^6}]$$
$$= \frac{1}{c^3}[\frac{1}{k^6} + \frac{1}{k^3} + 1]$$

Therefore, LHS = RHS.

(iii) a: c = $(a^2 + b^2)$: $(b^2 + c^2)$
We can write it as

$$\frac{a}{c} = \frac{a^2 + b^2}{b^2 + c^2}$$

$We\ know\ that$

$$LHS = \frac{a}{c} = \frac{ck^2}{c} = k^2$$
$$RHS = \frac{a^2 + b^2}{b^2 + c^2}$$

$We\ can\ write\ it\ as$

$$= \frac{(ck^2)^2 + (ck)^2}{(ck)^2 + c^2}$$

$So\ we\ get$

$$= \frac{c^2k^4 + c^2k^2}{c^2k^2 + c^2}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{c^2k^2(k^2 + 1)}{c^2(k^2 + 1)}$$

$$= k^2$$

Therefore, LHS = RHS.

(iv) $a^2b^2c^2 (a^{-4} + b^{-4} + c^{-4}) = b^{-2} (a^4 + b^4 + c^4)$

$$LHS = a^2b^2c^2(a^{-4}b^{-4}c^{-4})$$

$We\ can\ write\ it\ as$

$$= a^2b^2c^2[\frac{1}{a^4} + \frac{1}{b^4} + \frac{1}{c^4}]$$

$So\ we\ get$

$$= \frac{a^2b^2c^2}{a^4} + \frac{a^2b^2c^2}{b^4} + \frac{a^2b^2c^2}{c^4}$$

$By\ further\ calculation$

$$= \frac{b^2c^2}{a^2} + \frac{c^2a^2}{b^2} + \frac{a^2b^2}{c^2}$$

$It\ can\ be\ written\ as$

$$= \frac{(ck)^2c^2}{(ck^2)^2} + \frac{c^2(ck^2)^2}{(ck)^2} + \frac{(ck^2)^2(ck)^2}{c^2}$$

$$= \frac{c^2k^2c^2}{c^2k^4} + \frac{c^2c^2k^4}{c^2k^2} + \frac{c^2k^4c^2k^2}{c^2}$$

$On\ further\ simplification$

$$= \frac{c^2}{k^2} + \frac{c^2k^2}{1} + \frac{c^2k^6}{1}$$

$Taking\ out\ the\ common\ terms$

$$= c^2[\frac{1}{k^2} + k^2 + k^6]$$

$$= \frac{c^2}{k^2}[1 + k^4 + k^8]$$

$$RHS = b^{-2}[a^4 + b^4 + c^4]$$

$We\ can\ write\ it\ as$

$$= \frac{1}{b^2}[a^4 + b^4 + c^4]$$

$So\ we\ get$

$$= \frac{1}{(ck)^2}[(ck^2)^4 + (ck)^4 + c^4]$$

$By\ further\ calculation$

$$= \frac{1}{c^2k^2}[c^4k^8 + c^4k^4 + c^4]$$

$Taking\ out\ the\ common\ terms$

$$= \frac{c^4}{c^2k^2}[k^8 + k^4 + 1]$$

$$= \frac{c^2}{k^2}[1 + k^4 + k^8]$$

Therefore, LHS = RHS.

(v) LHS = $abc\ (a + b + c)^3$
We can write it as
$= ck^2.\ ck.\ c\ [ck^2 + ck + c]^3$
Taking out the common terms
$= c^3\ k^3\ [c\ (k^2 + k + 1)]^3$
So we get
$= c^3\ k^3.\ c^3\ (k^2 + k + 1)^3$
$= c^6\ k^3\ (k^2 + k + 1)^3$

RHS = $(ab + bc + ca)^3$
We can write it as
$= (ck^2.\ ck + ck.\ c + c.\ ck^2)^3$
So we get
$= (c^2k^3 + c^2k + c^2k^2)^3$
$= (c^2k^3 + c^2k^2 + c^2k)^3$
Taking out the common terms
$= [c^2k\ (k^2 + k + 1)]^3$

$= c^6k^3 (k^2 + k + 1)^3$

Therefore, LHS = RHS.

(vi) LHS = (a + b + c) (a – b + c)
We can write it as
$= (ck^2 + ck + c) (ck^2 – ck + c)$
Taking out the common terms
$= c (k^2 + k + 1) c (k^2 – k + 1)$
$= c^2 (k^2 + k + 1) (k^2 – k + 1)$
So we get
$= c^2 (k^4 + k^2 + 1)$

$RHS = a^2 + b^2 + c^2$
We can write it as
$= (ck^2)^2 + (ck)^2 + (c)^2$
So we get
$= c^2k^4 + c^2k^2 + c^2$
Taking out the common terms
$= c^2 (k^4 + k^2 + 1)$

Therefore, LHS = RHS.

23. If a, b, c, d are in continued proportion, prove that:

$$(i)\frac{a^3 + b^3 + c^3}{b^3 + c^3 + d^3} = \frac{a}{d}$$

(ii) $(a^2 – b^2) (c^2 – d^2) = (b^2 – c^2)^2$
(iii) (a + d) (b + c) – (a + c) (b + d) = $(b – c)^2$
(iv) a: d = triplicate ratio of (a – b): (b – c)

$$(v)\left(\frac{a-b}{c} + \frac{a-c}{b}\right)^2 - \left(\frac{d-b}{c} + \frac{d-c}{b}\right)^2 = (a-d)^2\left(\frac{1}{c^2} - \frac{1}{b^2}\right)$$

Solution:

It is given that
a, b, c, d are in continued proportion
Here we get
a/b = b/c = c/d = k
$c = dk, b = ck = dk . k = dk^2$
$a = bk = dk^2 . k = dk^3$

$$(i) LHS = \frac{a^3 + b^3 + c^3}{b^3 + c^3 + d^3}$$

We can write it as

$$= \frac{(dk^3)^3 + (dk^2)^3 + (dk)^3}{(dk^2)^3 + (dk)^3 + d^3}$$

So we get

$$= \frac{d^3k^9 + d^3k^6 + d^3k^3}{d^3k^6 + d^3k^3 + d^3}$$

$$Taking\ out\ the\ common\ terms$$

$$= \frac{d^3k^3(k^6 + k^3 + 1)}{d^3(k^6 + k^3 + 1)}$$

$$= k^3$$

$$RHS = \frac{a}{d} = \frac{dk^3}{d} = k^3$$

Therefore, LHS = RHS.

(ii) LHS = $(a^2 – b^2) (c^2 – d^2)$
We can write it as
= $[(dk^3)^2 – (dk^2)^2] [(dk)^2 – d^2]$
By further calculation
= $(d^2k^6 – d^2k^4) (d^2k^2 – d^2)$
Taking out the common terms
= $d^2k^4 (k^2 – 1) d^2 (k^2 – 1)$
= $d^4k^4 (k^2 – 1)^2$

RHS = $(b^2 – c^2)^2$
We can write it as
= $[(dk^2)^2 – (dk)^2]^2$
By further calculation
= $[d^2k^4 – d^2k^2]^2$
Taking out the common terms
= $[d^2k^2 (k^2 – 1)]^2$
= $d^4 k^4 (k^2 – 1)^2$

Therefore, LHS = RHS.

(iii) LHS = (a + d) (b + c) – (a + c) (b + d)
We can write it as
= $(dk^3 + d) (dk^2 + dk) – (dk^3 + dk) (dk^2 + d)$
Taking out the common terms
= $d (k^3 + 1) dk (k + 1) – dk (k^2 + 1) d (k^2 + 1)$
By further simplification
= $d^2k (k + 1) (k^3 + 1) – d^2k (k^2 + 1) (k^2 + 1)$
So we get
= $d^2k (k^4 + k^3 + k + 1 – k^4 – 2k^2 – 1)$
= $d^2k (k^3 – 2k^2 + k)$
Taking k as common
= $d^2k^2 (k^2 – 2k + 1)$
= $d^2k^2 (k – 1)^2$
RHS = $(b – c)^2$

We can write it as
$= (dk^2 – dk)^2$
Taking out the common terms
$= d^2k^2 (k – 1)^2$

Therefore, LHS = RHS.

(iv) a: d = triplicate ratio of (a – b): (b – c) = $(a – b)^3: (b – c)^3$
We know that

$$LHS = a : d = \frac{a}{d}$$

$$It\ can\ be\ written\ as$$

$$= \frac{dk^3}{d}$$

$$= k^3$$

$$RHS = \frac{(a-b)^3}{(b-c)^3}$$

$$We\ can\ write\ it\ as$$

$$= \frac{(dk^3 - dk^2)^3}{(dk^2 - dk)^3}$$

$$Taking\ out\ the\ common\ terms$$

$$= \frac{d^3k^6(k-1)^3}{d^3k^3(k-1)^3}$$

$$= k^3$$

Therefore, LHS = RHS.

(v)

$$LHS = (\frac{a-b}{c} + \frac{a-c}{b})^2 - (\frac{d-b}{c} + \frac{d-c}{b})^2$$

$$We\ can\ write\ it\ as$$

$$= (\frac{dk^3 - dk^2}{dk} + \frac{dk^3 - dk}{dk^2})^2 - (\frac{d-dk^2}{dk} + \frac{d-dk}{dk^2})^2$$

$$Taking\ out\ the\ common\ terms$$

$$= (\frac{dk^2(k-1)}{dk} + \frac{dk(k^2-1)}{dk^2})^2 - (\frac{d(1-k^2)}{dk} + \frac{d(1-k)}{dk^2})^2$$

$By\ further\ calculation$

$$= (k(k-1) + \frac{(k^2-1)}{k})^2 - (\frac{1-k^2}{k} + \frac{(1-k)}{k^2})^2$$

$Taking\ LCM\ we\ get$

$$= (\frac{k^2(k-1)+(k^2-1)}{k})^2 - (\frac{k(1-k^2)+1-k}{k^2})^2$$

$So\ we\ get$

$$= (\frac{k^3-k^2+k^2-1}{k})^2 - (\frac{k-k^3+1-k}{k^2})^2$$

$$= \frac{(k^3-1)^2}{k^2} - \frac{(-k^3+1)^2}{k^4}$$

$$= \frac{(k^3-1)^2}{k^2} - \frac{(1-k^3)^2}{k^4}$$

$On\ further\ simplification$

$$= (\frac{k^3-1}{k^2})^2(1-\frac{1}{k^2})$$

$We\ get$

$$= \frac{(k^3-1)^2(k^2-1)}{k^4}$$

$$RHS = (a-d)^2(\frac{1}{c^2} - \frac{1}{b^2})$$

$We\ can\ write\ it\ as$

$$= (dk^3-d)^2(\frac{1}{d^2k^2} - \frac{1}{d^2k^4})$$

$So\ we\ get$

$$= d^2(k^3-1)^2(\frac{k^2-1}{d^2k^4})$$

$$= \frac{(k^3-1)^2(k^2-1)}{k^4}$$

Therefore, LHS = RHS.

EXERCISE 7.3

1. If a: b :: c: d, prove that

(i) $\frac{2a+5b}{2a-5b} = \frac{2c+5d}{2c-5d}$

(ii) $\frac{5a+11b}{5c+11d} = \frac{5a-11b}{5c-11d}$

(iii) (2a + 3b) (2c – 3d) = (2a – 3b) (2c + 3d)

(iv) (la + mb): (lc + mb) :: (la – mb): (lc – mb)

Solution:

(i) We know that
If a: b :: c: d we get a/b = c/d
By multiplying 2/5
2a/5b = 2c/5d
By applying componendo and dividendo
(2a + 5b)/ (2a – 5b) = (2c + 5d)/ (2c – 5d)

(ii) We know that
If a: b :: c: d we get a/b = c/d
By multiplying 5/11
5a/11b = 5c/11d
By applying componendo and dividendo
(5a + 11b)/ (5a – 11b) = (5c + 11d)/ (5c – 11d)
Now by applying alternendo
(5a + 11b)/ (5c + 11d) = (5a – 11b)/ (5c – 11d)

(iii) We know that
If a: b :: c: d we get a/b = c/d
By multiplying 2/3
2a/3b = 2c/3d
By applying componendo and dividendo
(2a + 3b)/ (2a – 3b) = (2c + 3d)/ (2c – 3d)
By cross multiplication
(2a + 3b) (2c – 3d) = (2a – 3b) (2c + 3d)

(iv) We know that
If a: b :: c: d we get a/b = c/d
By multiplying l/m
la/mb = lc/md
By applying componendo and dividendo
(la + mb)/ (la – mb) = (lc + md)/ (lc – md)
Now by applying alternendo
(la + mb)/ (lc + md) = (la – mb)/ (lc – md)
So we get
(la + mb): (lc + md) :: (la – mb): (lc – md)

2.

(i) If $\frac{5x+7y}{5u+7v} = \frac{5x-7y}{5u-7v}$, show that $\frac{x}{y} = \frac{u}{v}$.

(ii) $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$, prove that $\frac{a}{b} = \frac{c}{d}$.

Solution:

$(i) \frac{5x+7y}{5u+7v} = \frac{5x-7y}{5u-7v}$

By applying alternendo

$$\frac{5x+7y}{5x-7y} = \frac{5u+v}{5u-7v}$$

Now by applying componendo and dividendo

$$\frac{5x+7y+5x-7y}{5x+7y-5x+7y} = \frac{5u+7v+5u-7v}{5u+7v-5u+7v}$$

By further calculation

$$\frac{10x}{14y} = \frac{10u}{14v}$$

Dividing by $\frac{10}{14}$

$$\frac{x}{y} = \frac{u}{v}$$

Therefore, it is proved.

$(ii) \frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$

By applying alternendo

$$\frac{8a+5b}{8a-5b} = \frac{8c+5d}{8c-5d}$$

Now by applying componendo and dividendo

$$\frac{8a+5b+8a-5b}{8a+5b-8a+5b} = \frac{8c+5d+8c-5d}{8c+5d-8c+8d}$$

$$By\ further\ calculation$$

$$\frac{16a}{10b} = \frac{16c}{10d}$$

$$Dividing\ by\ \frac{16}{10}$$

$$\frac{a}{b} = \frac{c}{d}$$

Therefore, it is proved.

3. If (4a + 5b) (4c – 5d) = (4a – 5d) (4c + 5d), prove that a, b, c, d are in proportion.
Solution:

It is given that
(4a + 5b) (4c – 5d) = (4a – 5d) (4c + 5d)
We can write it as

$$\frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$$

$$By\ applying\ componendo\ and\ dividendo$$

$$\frac{4a+5b+4a-5b}{4a+5b-4a+5b} = \frac{4c+5d+4c-5d}{4c+5d-4c+5d}$$

$$So\ we\ get$$

$$\frac{8a}{10b} = \frac{8a}{10b}$$

$$Dividing\ by\ \frac{8}{10}$$

$$\frac{a}{b} = \frac{c}{d}$$

Therefore, it is proved that a, b, c, d are in proportion.

4. If (pa + qb): (pc + qd) :: (pa – qb): (pc – qd) prove that a: b :: c: d.
Solution:

It is given that
(pa + qb): (pc + qd) :: (pa – qb): (pc – qd)
We can write it as

$$\frac{pa+qb}{pc+qd} = \frac{pa-qb}{pc-qd}$$
$$\frac{pa+qb}{pa-qb} = \frac{pc+qd}{pc-qd}$$
$$By\ applying\ componendo\ and\ dividendo$$
$$\frac{pa+qb+pa-qb}{pa+qb-pa+qb} = \frac{pc+qd+pc-qd}{pc+qd-pc+qd}$$
$$So\ we\ get$$
$$\frac{2pa}{2qb} = \frac{2pc}{2qd}$$
$$Dividing\ by\ \frac{2p}{2q}$$
$$\frac{a}{b} = \frac{c}{d}$$

Therefore, it is proved that a: b :: c: d.

5. If (ma + nb): b :: (mc + nd): d, prove that a, b, c, d are in proportion.
Solution:

It is given that
(ma + nb): b :: (mc + nd): d
We can write it as
(ma + nb)/ b = (mc + nd)/ d
By cross multiplication
mad + nbd = mbc + nbd
Here mad = mbc
ad = bc
By further calculation
a/b = c/d

Therefore, it is proved that a, b, c, d are in proportion.

6. If $(11a^2 + 13b^2)(11c^2 – 13d^2) = (11a^2 – 13b^2)(11c^2 + 13d^2)$, prove that a: b :: c: d.
Solution:

It is given that
$(11a^2 + 13b^2)(11c^2 – 13d^2) = (11a^2 – 13b^2)(11c^2 + 13d^2)$
We can write it as

$$\frac{11a^2+13b^2}{11a^2-13b^2}=\frac{11c^2+13d^2}{11c^2-13d^2}$$

By applying componendo and dividendo

$$\frac{11a^2+13b^2+11a^2-13b^2}{11a^2+13b^2-11a^2+13b^2}=\frac{11c^2+13d^2+11c^2-13d^2}{11c^2+13d^2-11c^2+13d^2}$$

So we get

$$\frac{22a^2}{26b^2}=\frac{22c^2}{26d^2}$$

Dividing by $\frac{22}{26}$

$$\frac{a^2}{b^2}=\frac{c^2}{d^2}$$

$$\frac{a}{b}=\frac{c}{d}$$

Therefore, it is proved that a: b :: c: d.

7. If (a + 3b + 2c + 6d) (a – 3b – 2c + 6d) = (a + 3b – 2c – 6d) (a – 3b + 2c – 6d), prove that a: b:: c: d.
Solution:

It is given that
(a + 3b + 2c + 6d) (a – 3b – 2c + 6d) = (a + 3b – 2c – 6d) (a – 3b + 2c – 6d)
We can write it as

$$\frac{a+3b+2c+6d}{a-3b+2c-6d}=\frac{a+3b-2c-6d}{a-3b-2c+6d}$$

Applying alternendo

$$\frac{a+3b+2c+6d}{a+3b-2c-6d}=\frac{a-3b+2c-6d}{a-3b-2c+6d}$$

By applying componendo and dividendo

$$\frac{a+3b+2c+6d+a+3b-2c-6d}{a+3b+2c+6d-a-3b+2c+6d}=\frac{a-3b+2c-6d+a-3b-2c+6d}{a-3b+2c-6d-a+3b+2c-6d}$$

By further calculation

$$\frac{2(a+3b)}{2(2c+6d)}=\frac{2(a-3b)}{2(2c-6d)}$$

Dividing by 2

Dividing by 2

$$\frac{a+3b}{2c+6d}=\frac{a-3b}{2c-6d}$$

By applying alternendo

$$\frac{a+3b}{a-3b}=\frac{2c+6d}{2c-6d}$$

Again applying alternendo and dividendo

$$\frac{a+3b+a-3b}{a+3b-a+3b}=\frac{2c+6d+2c-6d}{2c+6d-2c+6d}$$

So we get

$$\frac{2a}{6b}=\frac{4c}{12d}=\frac{2c}{6d}$$

Dividing by $\frac{2}{6}$

$$\frac{a}{b}=\frac{c}{d}$$

Therefore, it is proved that a: b :: c: d.

8.

If $x=\frac{2ab}{a+b}$ find the value of $\frac{x+a}{x-a}+\frac{x+b}{x-b}$.

Solution:

It is given that

$$x=\frac{2ab}{a+b}$$

$$\frac{x}{a}=\frac{2b}{a+b}$$

By applying componendo and dividendo

$$\frac{x+a}{x-a}=\frac{2b+a+b}{2b-a-b}=\frac{3b+a}{b-a}......(1)$$

Similarly

$$\frac{x}{b}=\frac{2a}{a+b}$$

By applying componendo and dividendo

$$\frac{x+b}{x-b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b} \ldots(2)$$

Now adding both the equations

$$\frac{x+a}{x-a} + \frac{x+b}{x-b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

By further calculation

$$= -\frac{a+3b}{a-b} + \frac{3a+b}{a-b}$$

So we get

$$= \frac{-a-3b+3a+b}{a-b}$$

$$= \frac{2a-2b}{a-b}$$

Taking 2 as common

= 2(a – b)/ (a – b)

= 2

9.

If $x = \frac{8ab}{a+b}$ find the value of $\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$

Solution:

It is given that

$$x = \frac{8ab}{a+b}$$

$$\frac{x}{4a} = \frac{2b}{a+b}$$

By applying componendo and dividendo

$$\frac{x+4a}{x-4a} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a} \ldots\ldots(1)$$

Similarly

$$\frac{x}{4b} = \frac{2a}{a+b}$$

By applying componendo and dividendo

$$\frac{x+b}{x-b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b} \ldots(2)$$

$Now\ adding\ both\ the\ equations$

$$\frac{x+4a}{x-4a}+\frac{x+4b}{x-4b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$$

$By\ further\ calculation$

$$=-\frac{a+3b}{a-b}+\frac{3a+b}{a-b}$$

$So\ we\ get$

$$=\frac{-a-3b+3a+b}{a-b}$$

$$=\frac{2a-2b}{a-b}$$

$Taking\ 2\ as\ common$

= 2(a – b)/ (a – b)

= 2

10.

$$If\ x=\frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}\ find\ the\ value\ of\ \frac{x+2\sqrt{2}}{x-2\sqrt{2}}+\frac{x+2\sqrt{3}}{x-2\sqrt{3}}$$

Solution:

It is given that

$$x=\frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$$

We can write it as

$$x=\frac{4\sqrt{2}.\sqrt{3}}{\sqrt{2}+\sqrt{3}}$$

Here

$$\frac{x}{2\sqrt{2}}=\frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}}$$

By applying componendo and dividendo

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}}=\frac{2\sqrt{3}+\sqrt{2}+\sqrt{3}}{2\sqrt{3}-\sqrt{2}-\sqrt{3}}$$

By further calculation

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}}=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\ \ldots\ldots\ (1)$$

Similarly

$$\frac{x}{2\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{3}}$$

By applying componendo and dividendo

$$\frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{2}-\sqrt{3}}$$

By further calculation

$$\frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \ldots\ldots (2)$$

By adding both the equations

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} + \frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

We can write it as

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

So we get

$$= \frac{3\sqrt{3}+\sqrt{2}-3\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

Taking out 2 as common

$$= \frac{2(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2}}$$

$= 2$

11.

Solve for x: $\frac{\sqrt{36x+1}+6\sqrt{x}}{\sqrt{36x+1}-6\sqrt{x}} = 9$

Solution:

It is given that

$$\frac{\sqrt{36x+1}+6\sqrt{x}}{\sqrt{36x+1}-6\sqrt{x}} = 9$$

By applying componendo and dividendo

$$\frac{\sqrt{36x+1}+6\sqrt{x}+\sqrt{36x+1}-6\sqrt{x}}{\sqrt{36x+1}+6\sqrt{x}-\sqrt{36x-1}+6\sqrt{x}}=\frac{9+1}{9-1}$$

By further calculation

$$\frac{2\sqrt{36x+1}}{12\sqrt{x}}=\frac{10}{8}$$

So we get

$$\frac{\sqrt{36x+1}}{6\sqrt{x}}=\frac{5}{4}$$

By squaring both sides

$$\frac{36x+1}{36x}=\frac{25}{16}$$

By cross multiplication
36x × 25 = 16 (36x + 1)
900x = 576x + 16
900x – 576x = 16
So we get
324x = 16
x = 16/324
x = 4/81

12. Using properties of properties, find x from the following equations:

(i) $\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}}=3$

(ii) $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\frac{5}{2}$

(iii) $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=\frac{a}{b}$

(iv) $\frac{\sqrt{12x+1}+\sqrt{2x-3}}{\sqrt{12x+1}-\sqrt{2x-3}}=\frac{3}{2}$

(v) $\frac{3x+\sqrt{9x^2+5}}{3x-\sqrt{9x^2+5}}=5$

(vi) $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}=\frac{c}{d}$

Solution:

$$(i)\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}}=3$$

$By\ applying\ componendo\ and\ dividendo$

$$\frac{\sqrt{2-x}+\sqrt{2+x}+\sqrt{2-x}-\sqrt{2+x}}{\sqrt{2-x}+\sqrt{2+x}-\sqrt{2-x}+\sqrt{2+x}}=\frac{3+1}{3-1}$$

$On\ further\ calculation$

$$\frac{2\sqrt{2-x}}{2\sqrt{2+x}}=\frac{4}{2}$$

$$\frac{\sqrt{2-x}}{\sqrt{2+x}}=\frac{2}{1}$$

$By\ squaring\ on\ both\ sides$

$$\frac{2-x}{2+x}=\frac{4}{1}$$

By cross multiplication
8 + 4x = 2 – x
So we get
4x + x = 2 – 8
5x = - 6
x = -6/5

$$(ii)\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\frac{5}{2}$$

$By\ applying\ componendo\ and\ dividendo$

$$\frac{\sqrt{x+4}+\sqrt{x-10}+\sqrt{x+4}-\sqrt{x-10}}{\sqrt{x+4}+\sqrt{x-10}-\sqrt{x+4}+\sqrt{x-10}}=\frac{5+2}{5-2}$$

$On\ further\ calculation$

$$\frac{2\sqrt{x+4}}{2\sqrt{x-10}}=\frac{7}{3}$$

$$\frac{\sqrt{x+4}}{\sqrt{x-10}}=\frac{7}{3}$$

$By\ squaring\ on\ both\ sides$

$$\frac{x+4}{x-10}=\frac{49}{9}$$

By cross multiplication
49x – 490 = 9x + 36
49x – 9x = 36 + 490
So we get
40x = 526
x = 526/40
x = 263/20

$$(iii)\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}}-\sqrt{1-x}=\frac{a}{b}$$

By applying componendo and dividendo

$$\frac{\sqrt{1+x}+\sqrt{1-x}+\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}-\sqrt{1+x}+\sqrt{1-x}}=\frac{a+b}{a-b}$$

On further calculation

$$\frac{2\sqrt{1+x}}{2\sqrt{1-x}}=\frac{a+b}{a-b}$$

$$\frac{\sqrt{1+x}}{\sqrt{1-x}}=\frac{a+b}{a-b}$$

By squaring on both sides

$$\frac{1+x}{1-x}=\frac{(a+b)^2}{(a-b)^2}$$

By applying componendo and dividendo

$$\frac{1+x+1-x}{1+x-1+x}=\frac{(a+b)^2+(a-b)^2}{(a+b)^2-(a-b)^2}$$

By further calculation

$$\frac{2}{2x}=\frac{2(a^2+b^2)}{4ab}$$

$$\frac{1}{x}=\frac{a^2+b^2}{2ab}$$

So we get

$$x = \frac{2ab}{a^2 + b^2}$$

$$(iv)\frac{\sqrt{12x+1}+\sqrt{2x-3}}{\sqrt{12x+1}-\sqrt{2x-3}} = \frac{3}{2}$$

By applying componendo and dividendo

$$\frac{\sqrt{12x+1}+\sqrt{2x-3}+\sqrt{12x+1}-\sqrt{2x-3}}{\sqrt{12x+1}+\sqrt{2x-3}-\sqrt{12x+1}+\sqrt{2x-3}} = \frac{3+2}{3-2}$$

On further calculation

$$\frac{2\sqrt{12x+1}}{2\sqrt{2x-3}} = \frac{5}{1}$$

$$\frac{\sqrt{12x+1}}{\sqrt{2x-3}} = \frac{5}{1}$$

By squaring on both sides

$$\frac{12x+1}{2x-3} = \frac{25}{1}$$

By cross multiplication

50x – 75 = 12x + 1

50x – 12x = 1 + 75

So we get

38x = 76

x = 76/38 = 2

$$(v)\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}} = \frac{5}{1}$$

By applying componendo and dividendo

$$\frac{3x+\sqrt{9x^2-5}+3x-\sqrt{9x^2-5}}{3x+\sqrt{9x^2-5}-3x+\sqrt{9x^2-5}} = \frac{5+1}{5-1}$$

On further calculation

$$\frac{6x}{2\sqrt{9x^2-5}} = \frac{6}{4}$$

$$\frac{3x}{\sqrt{9x^2-5}} = \frac{3}{2}$$

$By\ squaring\ on\ both\ sides$

$$\frac{9x^2}{9x^2-5}=\frac{9}{4}$$

By cross multiplication
$81x^2 - 45 = 36x^2$
$81x^2 - 36x^2 = 45$
So we get
$45x^2 = 45$
$x^2 = 1$
$x = \pm 1$
$x = 1, -1$

Verification:
(i) If x = 1

$$\frac{3\times 1+\sqrt{9\times 1-5}}{3\times 1-\sqrt{9\times 1-5}}=\frac{3+\sqrt{4}}{3-\sqrt{4}}$$

$So\ we\ get$

$$=\frac{3+2}{3-2}$$

$$=\frac{5}{1}$$

Hence, x = 1.

(ii) If x = -1

$$\frac{3\times(-1)+\sqrt{9\times(-1)^2-5}}{3\times(-1)-\sqrt{9\times(-1)^2-5}}=\frac{-3+\sqrt{9-5}}{-3-\sqrt{9-5}}$$

$By\ further\ calculation$

$$=\frac{-3+\sqrt{4}}{-3-\sqrt{4}}$$

$So\ we\ get$

$$=\frac{-3+2}{-3-2}$$

$$=\frac{-1}{-5}$$

$$=\frac{1}{5}$$

Here 1/5 ≠ 5/1
x = - 1 is not the solution

Therefore, x = 1.

$$(vi)\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}=\frac{c}{d}$$

By applying componendo and dividendo

$$\frac{\sqrt{a+x}+\sqrt{a-x}+\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}-\sqrt{a+x}+\sqrt{a-x}}=\frac{c+d}{c-d}$$

On further calculation

$$\frac{2\sqrt{a+x}}{2\sqrt{a-x}}=\frac{c+d}{c-d}$$

$$\frac{\sqrt{a+x}}{\sqrt{a-x}}=\frac{c+d}{c-d}$$

By squaring on both sides

$$\frac{a+x}{a-x}=\frac{(c+d)^2}{(c-d)^2}$$

By applying componendo and dividendo

$$\frac{a+x+a-x}{a+x-a+x}=\frac{(c+d)^2+(c-d)^2}{(c+d)^2-(c-d)^2}$$

By further calculation

$$\frac{2a}{2x}=\frac{2(c^2+d^2)}{4cd}$$

$$\frac{a}{x}=\frac{c^2+d^2}{2cd}$$

By cross multiplication

$$x(c^2+d^2)=2acd$$

$$x=\frac{2acd}{c^2+d^2}$$

13. Using properties of proportion solve for x. Give that x is positive.

$$\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}}=5$$

Solution:

$$\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}}=\frac{5}{1}$$

By applying componendo and dividendo

$$\frac{3x+\sqrt{9x^2-5}+3x-\sqrt{9x^2-5}}{3x+\sqrt{9x^2-5}-3x+\sqrt{9x^2-5}}=\frac{5+1}{5-1}$$

On further calculation

$$\frac{6x}{2\sqrt{9x^2-5}}=\frac{6}{4}$$

$$\frac{3x}{\sqrt{9x^2-5}}=\frac{3}{2}$$

By squaring on both sides

$$\frac{9x^2}{9x^2-5}=\frac{9}{4}$$

By cross multiplication

$81x^2 - 45 = 36x^2$

$81x^2 - 36x^2 = 45$

So we get

$45x^2 = 45$

$x^2 = 1$

$x = \pm 1$

$x = 1, - 1$

Verification:

(i) If x = 1

$$\frac{3\times1+\sqrt{9\times1-5}}{3\times1-\sqrt{9\times1-5}}=\frac{3+\sqrt{4}}{3-\sqrt{4}}$$

So we get

$$=\frac{3+2}{3-2}$$

$$=\frac{5}{1}$$

Hence, x = 1.

14. Solve

$$\mathbf{\frac{1+x+x^2}{1-x+x^2}=\frac{62(1+x)}{63(1-x)}}$$

Solution:

$$\frac{1+x+x^2}{1-x+x^2}=\frac{62(1+x)}{63(1-x)}$$

We can write it as

$$\frac{(1-x)(1+x+x^2)}{(1+x)(1-x+x^2)}=\frac{62}{63}$$
$$\frac{(1+x)(1-x+x^2)}{(1-x)(1+x+x^2)}=\frac{63}{62}$$
$$\frac{1+x^3}{1-x^3}=\frac{63}{62}$$

By applying componendo and dividendo

$$\frac{1+x^3+1-x^3}{1+x^3-1+x^3}=\frac{63+62}{63-62}$$

On further calculation

$$\frac{2}{2x^3}=\frac{125}{1}$$
$$\frac{1}{x^3}=\frac{125}{1}$$

So we get

$$x^3=(\frac{1}{5})^3$$

x = 1/5

15. Solve for x:

$$\mathbf{16(\frac{a-x}{a+x})^3=\frac{a+x}{a-x}}$$

Solution:

$$16(\frac{a-x}{a+x})^3=\frac{a+x}{a-x}$$

We can write it as

$$(\frac{a+x}{a-x}) \times (\frac{a+x}{a-x})^3 = 16$$

$$(\frac{a+x}{a-x})^4 = 16 = (\pm 2)^4$$

$Here$

$$\frac{a+x}{a-x} = \pm 2$$

$$If\, \frac{a+x}{a-x} = \frac{2}{1}$$

$By\, applying\, componendo\, and\, dividendo$

$$\frac{a+x+a-x}{a+x-a+x} = \frac{2+1}{2-1}$$

$On\, further\, calculation$

$$\frac{2a}{2x} = \frac{3}{1}$$

$$\frac{a}{x} = \frac{3}{1}$$

So we get

3x = a

x = a/3

$$If\, \frac{a+x}{a-x} = \frac{-2}{1}$$

$By\, applying\, componendo\, and\, dividendo$

$$\frac{a+x+a-x}{a+x-a+x} = \frac{-2+1}{-2-1}$$

$On\, further\, calculation$

$$\frac{2a}{2x} = \frac{-1}{-3}$$

$$\frac{a}{x} = \frac{1}{3}$$

So we get

x = 3a

Therefore, x = a/3, 3a.

16.

$$\text{If } x=\frac{\sqrt{a+x}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}},\ \text{using properties of proportion, show that } x^2-2ax+1=0$$

Solution:

It is given that

$$x=\frac{\sqrt{a+x}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$$

We can write it as

$$\frac{x+1}{x-1}=\frac{2\sqrt{a+1}}{2\sqrt{a-1}}$$

By applying componendo and dividendo

$$\frac{(x+1)^2}{(x-1)^2}=\frac{a+1}{a-1}$$

On further calculation

$$\frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2}=\frac{2a}{2}$$

By applying componendo and dividendo

$$\frac{x^2+1+2x+x^2+1-2x}{x^2+1+2x-x^2-1+2x}=a$$

So we get

$$\frac{2x^2+2}{4x}=a$$

Taking out common terms

$$\frac{2(x^2+1)}{4x}=a$$

$$\frac{x^2+1}{2x}=a$$

We get

$2ax = x^2 + 1$

$x^2 - 2ax + 1 = 0$

Therefore, it is proved.

17.

$$\text{Give } x = \frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}} \text{ use componendo and dividendo to prove that } b^2 = \frac{2a^2x}{x^2+1}$$

Solution:

$$\frac{x}{1} = \frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$$

By applying componendo and dividendo

$$\frac{x+1}{x-1} = \frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}+\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}-\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}$$

On further calculation

$$\frac{(x+1)}{(x-1)} = \frac{2\sqrt{a^2+b^2}}{2\sqrt{a^2+b^2}}$$

$$\frac{(x+1)}{(x-1)} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

By squaring both sides

$$\frac{(x+1)^2}{(x-1)^2} = \frac{a^2+b^2}{a^2-b^2}$$

Expanding the equations

$$\frac{x^2+1+2x}{x^2+1-2x} = \frac{a^2+b^2}{a^2-b^2}$$

By applying componendo and dividendo

$$\frac{x^2+1+2x+x^2+1-2x}{x^2+1+2x-x^2-1+2x} = \frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}$$

So we get

$$\frac{2x^2+2}{4x} = \frac{2a^2}{2b^2}$$

Taking out common terms

$$\frac{2(x^2+1)}{4x} = \frac{2a^2}{2b^2}$$

$$\frac{x^2+1}{2x}=\frac{a^2}{b^2}$$

So we get

$$b^2=\frac{2a^2x}{x^2+1}$$

18.

Given that $\frac{a^3+3ab^2}{b^3+3a^2b}=\frac{63}{62}$. Using componendo and dividendo find a : b.

Solution:

It is given that

$$\frac{a^3+3ab^2}{b^3+3a^2b}=\frac{63}{62}$$

By applying componendo and dividendo

$$\frac{a^3+3ab^2+b^3+3a^2b}{a^3+3ab^2-b^3-3a^2b}=\frac{63+62}{63-62}=\frac{125}{1}$$

On further calculation

$$\frac{(a+b)^3}{(a-b)^3}=(\frac{5}{1})^3$$

So we get

$$\frac{(a+b)}{(a-b)}=5$$

By cross multiplication
a + b = 5a – 5b
We can write it as
5a – a – 5b – b = 0
4a – 6b = 0
4a = 6b
We get
a/b = 6/4
a/b = 3/2
a: b = 3: 2

19.

Give $\frac{x^3+12x}{6x^2+8}=\frac{y^3+27y}{9y^2+27}$. Using componendo and dividendo find x : y.

Solution:

It is given that

$$\frac{x^3+12x}{6x^2+8}=\frac{y^3+27y}{9y^2+27}$$

By applying componendo and dividendo

$$\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8}=\frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

On further calculation

$$\frac{(x+2)^3}{(x-2)^3}=\frac{(y+3)^3}{(y-3)^3}$$

We can write it as

$$(\frac{x+2}{x-2})^3=(\frac{y+3}{y-3})^3$$

So we get

$$\frac{x+2}{x-2}=\frac{y+3}{y-3}$$

By applying componendo and dividendo

$$\frac{x+2+x-2}{x+2-x+2}=\frac{y+3+y-3}{y+3-y+3}$$

By further calculation

2x/4 = 2y/3

x/2 = y/3

By cross multiplication

x/y = 2/3

Hence, the required ratio x: y is 2: 3.

20. Using the properties of proportion, solve the following equation for x; given

$$\frac{x^3+3x}{3x^2+1}=\frac{341}{91}$$

Solution:

It is given that

$$\frac{x^3+3x}{3x^2+1}=\frac{341}{91}$$

By applying componendo and dividendo

$$\frac{x^3+3x+3x^2+1}{x^3+3x-3x^2-1}=\frac{341+91}{341-91}$$

On further calculation

$$\frac{x^3+3x^2+3x+1}{x^3-3x^2+3x-1}=\frac{432}{250}=\frac{216}{125}$$

We can write it as

$$\frac{(x+1)^3}{(x-1)^3}=\frac{216}{125}=(\frac{6}{5})^3$$

So we get

$$\frac{x+1}{x-1}=\frac{6}{5}$$

By cross multiplication
6x – 6 = 5x + 5
6x – 5x = 5 + 6
x = 11

21.
If $\frac{x+y}{ax+by}=\frac{y+z}{ay+bz}=\frac{z+x}{az+bx}$, prove that each of these ratio is equal to $\frac{2}{a+b}$ unless $x+y+z=0$.
Solution:

It is given that

$$\frac{x+y}{ax+by}=\frac{y+z}{ay+bz}=\frac{z+x}{az+bx}$$

By addition

$$=\frac{x+y+y+z+z+x}{ax+by+ay+bz+az+bx}$$

By further calculation

$$= \frac{2(x+y+z)}{x(a+b)+y(a+b)+z(a+b)}$$

So we get

$$= \frac{2(x+y+z)}{(a+b)(x+y+z)}$$

$$= \frac{2}{a+b}$$

If $x + y + z \neq 0$

Therefore, it is proved.

CHAPTER TEST

1. Find the compound ratio of:
$(a + b)^2 : (a - b)^2$
$(a^2 - b^2) : (a^2 + b^2)$
$(a^4 - b^4) : (a + b)^4$
Solution:

$(a + b)^2 : (a - b)^2$
$(a^2 - b^2) : (a^2 + b^2)$
$(a^4 - b^4) : (a + b)^4$
We can write it as

$$= \frac{(a+b)^2}{(a-b)^2} \times \frac{a^2-b^2}{a^2+b^2} \times \frac{a^4-b^4}{(a+b)^4}$$

By further calculation

$$= \frac{(a+b)^2}{(a-b)^2} \times \frac{(a+b)(a-b)}{a^2+b^2} \times \frac{(a^2+b^2)(a+b)(a-b)}{(a+b)^4}$$

So we get

$$= \frac{1}{1}$$

$$= 1 : 1$$

2. If (7p + 3q): (3p – 2q) = 43: 2, find p: q.
Solution:

It is given that
(7p + 3q): (3p – 2q) = 43: 2
We can write it as
(7p + 3q)/ (3p – 2q) = 43/2
By cross multiplication
129p – 86q = 14p + 6q
129p – 14p = 6q + 86q
So we get
115p = 92q
By division
p/q = 92/115 = 4/5

Hence, p: q = 4: 5.

3. If a: b = 3: 5, find (3a + 5b): (7a – 2b).
Solution:

It is given that
a: b = 3: 5

We can write it as
a/b = 3/5

Here
(3a + 5b): (7a – 2b)
Now dividing the terms by b

$$3 \times \frac{a}{b} + 5 : 3 \times \frac{a}{b} - 2$$

$$Substituting\ the\ values\ of\ \frac{a}{b}$$

$$3 \times \frac{3}{5} + 5 : 3 \times \frac{3}{5} - 2$$

$$By\ further\ calculation$$

$$(\frac{9}{5} + 5) : (\frac{21}{5} - 2)$$

$$Taking\ LCM$$

$$\frac{9 + 25}{5} : \frac{21 - 10}{5}$$

$$So\ we\ get$$

$$\frac{34}{5} : \frac{11}{5}$$

Here
(3a + 5b): (7a – 2b) = 34: 11

4. The ratio of the shorter sides of a right angled triangle is 5: 12. If the perimeter of the triangle is 360 cm, find the length of the longest side.
Solution:

Consider the two shorter sides of a right-angled triangle as 5x and 12x
So the third longest side

$$= \sqrt{(5x)^2 + (12x)^2}$$

$$= \sqrt{25x^2 + 144x^2}$$

$$= \sqrt{169x^2}$$

= 13x

It is given that
5x + 12x + 13x = 360 cm

By further calculation
30x = 360
We get
x = 360/30 = 12

Here the length of the longest side = 13x
Substituting the value of x
= 13 × 12
= 156 cm

5. The ratio of the pocket money saved by Lokesh and his sister is 5: 6. If the sister saves Rs 30 more, how much more the brother should save in order to keep the ratio of their savings unchanged?
Solution:

Consider 5x and 6x as the savings of Lokesh and his sister.
Lokesh should save Rs y more
Based on the problem
(5x + y)/ (6x + 30) = 5/6
By cross multiplication
30x + 6y = 30x + 150
By further calculation
30x + 6y – 30x = 150
So we get
6y = 150
y = 150/6 = 25

Therefore, Lokesh should save Rs 25 more than his sister.

6. In an examination, the number of those who passed and the number of those who failed were in the ratio of 3: 1. Had 8 more appeared, and 6 less passed, the ratio of passed to failures would have been 2: 1. Find the number of candidates who appeared.
Solution:

Consider the number of passed = 3x
Number of failed = x
So the total candidates appeared = 3x + x = 4x

In the second case
Number of candidates appeared = 4x + 8
Number of passed = 3x – 6
Number of failed = 4x + 8 – 3x + 6 = x + 14
Ratio = 2: 1

Based on the condition
(3x – 6)/ (x + 14) = 2/1
By cross multiplication
3x – 6 = 2x + 28
3x – 2x = 28 + 6
x = 34

Here the number of candidates appeared = 4x = 4 × 34 = 136

7. What number must be added to each of the numbers 15, 17, 34 and 38 to make them proportional?
Solution:

Consider x be added to each number
So the numbers will be
15 + x, 17 + x, 34 + x and 38 + x
Based on the condition
(15 + x)/ (17 + x) = (34 + x)/ (38 + x)
By cross multiplication
(15 + x) (38 + x) = (34 + x) (17 + x)
By further calculation
$570 + 53x + x^2 = 578 + 51x + x^2$
So we get
$x^2 + 53x – x^2 – 51x = 578 – 570$
2x = 8
x = 4

Hence, 4 must be added to each of the numbers.

8. If (a + 2b + c), (a – c) and (a – 2b + c) are in continued proportion, prove that b is the mean proportional between a and c.
Solution:

It is given that
(a + 2b + c), (a – c) and (a – 2b + c) are in continued proportion
We can write it as
(a + 2b + c)/ (a – c) = (a – c)/ (a – 2b + c)
By cross multiplication
$(a + 2b + c) (a – 2b + c) = (a – c)^2$
On further calculation
$a^2 – 2ab + ac + 2av – 4b^2 + 2bc + ac – 2bc + c^2 = a^2 – 2ac + c^2$
So we get
$a^2 – 2ab + ac + 2ab – 4b^2 + 2bc + ac – 2bc + c^2 – a^2 + 2ac – c^2 = 0$
$4ac – 4b^2 = 0$
Dividing by 4
$ac – b^2 = 0$
$b^2 = ac$

Therefore, it is proved that b is the mean proportional between a and c.

9. If 2, 6, p, 54 and q are in continued proportion, find the values of p and q.
Solution:

It is given that
2, 6, p, 54 and q are in continued proportion
We can write it as
2/6 = 6/p = p/54 = 54/q

(i) We know that
2/6 = 6/p
By cross multiplication
2p = 36
p = 18

(ii) We know that
p/54 = 54/q
By cross multiplication
pq = 54 × 54
Substituting the value of p
q = (54 × 54)/ 18 = 162

Therefore, the values of p and q are 18 and 162.

10. If a, b, c, d, e are in continued proportion, prove that: a: e = a^4: b^4.
Solution:

It is given that
a, b, c, d, e are in continued proportion
We can write it as
a/b = b/c = c/d = d/e = k
d = ek, c = ek^2, b = ek^3 and a = ek^4

Here
LHS = a/e
Substituting the values
= ek^4/ e
= k^4

RHS = a^4/b^4
Substituting the values
= $(ek^4)^4/ (ek^3)^4$
So we get
= e^4k^{16}/ e^4k^{12}
= k^{16-12}
= k^4

Hence, it is proved that a: e = a^4: b^4.

11. Find two numbers whose mean proportional is 16 and the third proportional is 128.
Solution:

Consider x and y as the two numbers
Mean proportion = 16
Third proportion = 128
√xy = 16
xy = 256
Here
x = 256/y ….. (1)

$y^2/x = 128$
Here
$x = y^2/128$ …. (2)

Using both the equations
$256/y = y^3/ 128$
By cross multiplication
$y^3 = 256 \times 128 = 32768$
$y^3 = 32^3$
$y = 32$

Substituting the value of y in equation (1)
$x = 256/y$
So we get
$x = 256/32 = 8$

Hence, the two numbers are 8 and 32.

12. If q is the mean proportional between p and r, prove that:

$$\mathbf{p^2 - 3q^2 + r^2 = q^4(\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2})}$$

Solution:

It is given that
q is the mean proportional between p and r
$q^2 = pr$

Here
LHS = $p^2 - 3q^2 + r^2$
We can write it as
$= p^2 – 3pr + r^2$

$$RHS = q^4(\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2})$$

$We\ can\ write\ it\ as$

$$= (q^2)^2(\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2})$$

$Substituting\ the\ value\ of\ q$

$$= (pr)^2(\frac{1}{p^2} - \frac{3}{pr} + \frac{1}{r^2})$$

$Taking\ LCM$

$$= p^2r^2(\frac{r^2 - 3pr + p^2}{p^2r^2})$$

So we get
$= r^2 - 3pr + p^2$
Here
LHS = RHS

Therefore, it is proved.

13. If a/b = c/d = e/f, prove that each ratio is
$$(i)\sqrt{\frac{3a^2-5c^2+7e^2}{3b^2-5d^2+7f^2}}$$
$$(ii)[\frac{2a^3+5c^3+7e^3}{2b^3+5d^3+7f^3}]^{\frac{1}{3}}$$
Solution:

It is given that
a/b = c/d = e/f = k
So we get
a = k, c = dk, e = fk

$$(i)\sqrt{\frac{3a^2-5c^2+7e^2}{3b^2-5d^2+7f^2}}$$
$Substituting\ the\ values$

$$=\sqrt{\frac{3b^2k^2-5d^2k^2+7f^2k^2}{3b^2-5d^2-7f^2}}$$
$Taking\ k\ as\ common$

$$=k\sqrt{\frac{3b^2-5d^2+7f^2}{3b^2-5d^2+7f^2}}$$
$$=k$$

Therefore, it is proved.

$$(ii)[\frac{2a^3+5c^3+7e^3}{2b^3+5d^3+7f^3}]^{\frac{1}{3}}$$
$Substituting\ the\ values$

$$=[\frac{2b^3k^3+5d^3k^3+7f^3k^3}{2b^3+5d^3+7f^3}]^{\frac{1}{3}}$$

$Taking\ k\ as\ common$

$$= k[\frac{2b^3 + 5d^3 + 7f^3}{2b^3 + 5d^3 + 7f^3}]^{\frac{1}{3}}$$

= k

Therefore, it is proved.

14. If x/a = y/b = z/c, prove that

$$\mathbf{\frac{3x^3 - 5y^3 + 4z^3}{3a^3 - 5b^3 + 4c^3} = (\frac{3x - 5y + 4z}{3a - 5b + 4c})^3}$$

Solution:

It is given that

x/a = y/b = z/c = k

So we get

x = ak, y = bk, z = ck

Here

$$LHS = \frac{3x^3 - 5y^3 + 4z^3}{3a^3 - 5b^3 + 4c^3}$$

$Substituting\ the\ values$

$$= \frac{3a^3k^3 - 5b^3k^3 + 4c^3k^3}{3a^3 - 5b^3 + 4c^3}$$

$Taking\ out\ the\ common\ terms$

$$= \frac{k^3(3a^3 - 5b^3 + 4c^3)}{3a^3 - 5b^3 + 4c^3}$$

= k^3

$$RHS = (\frac{3x - 5y + 4z}{3a - 5b + 4c})^3$$

$Substituting\ the\ values$

$$= (\frac{3ak - 5bk + 4ck}{3a - 5b + 4c})^3$$

$Taking\ out\ the\ common\ terms$

$$= (\frac{k(3a - 5b + 4c)}{3a - 5b + 4})^3$$

$= k^3$

Hence, LHS = RHS.

15. If x: a = y: b, prove that

$$\frac{x^4+a^4}{x^3+a^3}+\frac{y^4+b^4}{y^3+b^3}=\frac{(x+y)^4+(a+b)^4}{(x+y)^3+(a+b)^3}$$

Solution:

We know that
x/a = y/b = k
So we get
x = ak, y = bk

Here

$$LHS=\frac{x^4+a^4}{x^3+a^3}+\frac{y^4+b^4}{y^3+b^3}$$

$$Substituting\ the\ values$$

$$=\frac{a^4k^4+a^4}{a^3k^3+a^3}+\frac{b^4k^4+b^4}{b^3k^3+b^3}$$

$$Taking\ out\ the\ common\ terms$$

$$=\frac{a^4(k^4+1)}{a^3(k^3+1)}+\frac{b^4(k^4+1)}{b^3(k^3+1)}$$

$$We\ get$$

$$=\frac{a(k^4+1)}{k^3+1}+\frac{b(k^4+1)}{k^3+1}$$

$$We\ can\ write\ it\ as$$

$$=\frac{a(k^4+1)+b(k^4+1)}{k^3+1}$$

$$=\frac{(k^4+1)(a+b)}{k^3+1}$$

$$RHS=\frac{(x+y)^4+(a+b)^4}{(x+y)^3+(a+b)^3}$$

$$Substituting\ the\ values$$

$$= \frac{(ak + bk)^4 + (a + b)^4}{(ak + bk)^3 + (a + b)^3}$$

$$Taking\ out\ the\ common\ terms$$

$$= \frac{k^4(a + b)^4 + (a + b)^4}{k^3(a + b)^3(a + b)^3}$$

$$We\ get$$

$$= \frac{(a + b)^4(k^4 + 1)}{(a + b)^3(k^3 + 1)}$$

$$We\ can\ write\ it\ as$$

$$= \frac{(a + b)(k^4 + 1)}{k^3 + 1}$$

Here LHS = RHS

Therefore, it is proved.

16.

$$\text{If } \frac{x}{b + c - a} = \frac{y}{c + a - b} = \frac{z}{a + b - c} \text{ prove that each ratio's equal to : } \frac{x + y + z}{a + b + c}$$

Solution:

Consider

$$\frac{x}{b + c - a} = \frac{y}{c + a - b} = \frac{z}{a + b - c} = k$$

So we get

x = k (b + c – a)

y = k (c + a – b)

z = k (a + b – a)

Here

$$\frac{x + y + z}{a + b + c} = \frac{k(b + c - a) + k(c + a - b) + k(a + b - c)}{a + b + c}$$

$$By\ further\ calculation$$

$$= \frac{k(b + c - a + c + a - b + a + b - c)}{a + b + c}$$

$$So\ we\ get$$

$$= \frac{k(a+b+c)}{a+b+c}$$
= k

Therefore, it is proved.

17. If a: b = 9: 10, find the value of

$$\text{(i)} \frac{5a+3b}{5a-3b}$$

$$\text{(ii)} \frac{2a^2-3b^2}{2a^2+3b^2}$$

Solution:

It is given that
a: b = 9: 10
So we get
a/b = 9/10

$$(i) \frac{5a+3b}{5a-3b} = \frac{\frac{5a}{b}+\frac{3b}{b}}{\frac{5a}{b}-\frac{3b}{b}}$$

$$By\ further\ calculation$$

$$= \frac{\frac{5a}{b}+3}{\frac{5a}{b}-3}$$

$$Substituting\ the\ values\ of\ \frac{a}{b}$$

$$= \frac{5 \times \frac{9}{10}+3}{5 \times \frac{9}{10}-3}$$

$$So\ we\ get$$

$$= \frac{\frac{9}{2}+3}{\frac{9}{2}-3}$$

$$= \frac{\frac{15}{2}}{\frac{3}{2}}$$

$$By\ further\ simplification$$

$$= \frac{15}{2} \times \frac{2}{3}$$
= 5

$$(ii)\frac{2a^2-3b^2}{2a^2+3b^2}$$

$Dividing\ by\ b^2$

$$=\frac{\frac{2a^2}{b^2}-\frac{3b^2}{b^2}}{\frac{2a^2}{b^2}+\frac{3b^2}{b^2}}$$

$By\ further\ calculation$

$$=\frac{2(\frac{a}{b})^2-3}{2(\frac{a}{b})^2+3}$$

$Substituting\ the\ values\ of\ \frac{a}{b}$

$$=\frac{2(\frac{9}{10})^2-3}{2(\frac{9}{10})^2+3}$$

$So\ we\ get$

$$=\frac{2\times\frac{81}{100}-3}{2\times\frac{81}{100}+3}$$

$$=\frac{\frac{81}{50}-3}{\frac{81}{50}+3}$$

$By\ further\ simplification$

$$=\frac{\frac{81-150}{50}}{\frac{81+150}{50}}$$

$We\ get$

$$=\frac{-69}{50}\times\frac{50}{231}$$

$$=\frac{-69}{231}$$

$$=\frac{-23}{77}$$

18. If $(3x^2 + 2y^2): (3x^2 - 2y^2) = 11: 9$, find the value of

$$\mathbf{\frac{3x^4+25y^4}{3x^4-25y^4}}$$

Solution:

It is given that

$(3x^2 + 2y^2): (3x^2 - 2y^2) = 11: 9$

We can write it as

$$\frac{3x^2+2y^2}{3x^2-2y^2}=\frac{11}{9}$$

$By\ applying\ componendo\ and\ dividendo$

$$\frac{3x^2+2y^2+3x^2-2y^2}{3x^2+2y^2-3x^2+2y^2}=\frac{11+9}{11-9}$$

$By\ further\ calculation$

$$\frac{6x^2}{4y^2}=\frac{20}{2}$$
$$\frac{3x^2}{2y^2}=10$$

$We\ can\ write\ it\ as$

$$\frac{x^2}{y^2}=10\times\frac{2}{3}=\frac{20}{3}$$

Here

$$\frac{3x^4+25y^4}{3x^4-25y^4}=\frac{\frac{3x^4}{y^4}+\frac{25y^4}{y^4}}{\frac{3x^4}{y^4}-\frac{25y^4}{y^4}}$$

$We\ can\ write\ it\ as$

$$=\frac{3(\frac{x^2}{y^2})^2+25}{3(\frac{x^2}{y^2})^2-25}$$

$By\ substituing\ the\ values$

$$=\frac{3(\frac{20}{3})^2+25}{3(\frac{20}{3})^2-25}$$

$By\ further\ calculation$

$$=\frac{3\times\frac{400}{9}+25}{3\times\frac{400}{9}-25}$$

$Taking\ LCM$

$$= \frac{\frac{400+75}{3}}{\frac{400-75}{3}}$$

So we get

$$= \frac{475}{3} \times \frac{3}{325}$$

$$= \frac{19}{13}$$

19.

If $x = \frac{2mab}{a+b}$, find the value of $\frac{x+ma}{x-ma} + \frac{x+mb}{x-mb}$.

Solution:

It is given that

$$x = \frac{2mab}{a+b}$$

We can write it as

$$= \frac{x}{ma} + \frac{2b}{a+b}$$

By applying componendo and dividendo

$$\frac{x+ma}{x-ma} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a} \ldots.(1)$$

Similarly

$$\frac{x}{mb} = \frac{2a}{a+b}$$

By applying componendo and dividendo

$$\frac{x+mb}{x-mb} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b} \ldots.(2)$$

Now adding both the equations

$$\frac{x+ma}{x-ma} + \frac{x+mb}{x-mb} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$$

By further calculation

$$= \frac{-3b+a}{a-b} + \frac{3a+b}{a-b}$$

$So\ we\ get$

$$= \frac{-3b - a + 3a + b}{a - b}$$

$$= \frac{2a - 2b}{a - b}$$

$Taking\ out\ 2\ as\ common$

$$= \frac{2(a - b)}{a - b}$$

$$= 2$$

20.

$$\textbf{If } x = \frac{pab}{a+b}, \textbf{ prove that } \frac{x+pa}{x-pa} - \frac{x+pb}{x-pb} = \frac{2(a^2 - b^2)}{ab}.$$

Solution:

It is given that

$$x = \frac{pab}{a+b}$$

$We\ can\ write\ it\ as$

$$= \frac{x}{pa} + \frac{b}{a+b}$$

$By\ applying\ componendo\ and\ dividendo$

$$\frac{x+pa}{x-pa} = \frac{b+a+b}{b-a-b} = \frac{a+2b}{-a} \ldots..(1)$$

$Similarly$

$$\frac{x}{pb} = \frac{a}{a+b}$$

$By\ applying\ componendo\ and\ dividendo$

$$\frac{x+pb}{x-pb} = \frac{a+a+b}{a-a-b} = \frac{2a+b}{-b} \ldots.(2)$$

$We\ know\ that$

$$LHS = \frac{x+pa}{x-pa} - \frac{x+pb}{x-pb}$$

$Using\ both\ the\ equations$

$= \frac{a+2b}{-a} - \frac{2a+b}{-b}$

$= \frac{a+2b}{-a} + \frac{2a+b}{b}$

Taking LCM

$= \frac{ab+2b^2-2a^2-ab}{-ab}$

$= \frac{2b^2-2a^2}{-ab}$

So we get

$= \frac{-2a^2+2b^2}{-ab}$

Taking out 2 as common

$= \frac{-2(a^2-b^2)}{-ab}$

$= \frac{2(a^2-b^2)}{ab}$

= RHS

21.

Find x from the equation $\mathbf{\frac{a+x+\sqrt{a^2-x^2}}{a+x-\sqrt{a^2-x^2}} = \frac{b}{x}}$.

Solution:

It is given that

$\frac{a+x+\sqrt{a^2-x^2}}{a+x-\sqrt{a^2-x^2}} = \frac{b}{x}$

By applying componendo and dividendo

$\frac{a+x+\sqrt{a^2-x^2}+a+x-\sqrt{a^2-x^2}}{a+x+\sqrt{a^2-x^2}-a-x+\sqrt{a^2-x^2}} = \frac{b+x}{b-x}$

By further calculation

$\frac{2(a+x)}{2\sqrt{a^2-x^2}} = \frac{b+x}{b-x}$

$Dividing\ by\ 2$

$$\frac{(a+x)}{\sqrt{a^2-x^2}} = \frac{b+x}{b-x}$$

$By\ squaring\ on\ both\ sides$

$$\frac{(a+x)^2}{a^2-x^2} = \frac{(b+x)^2}{(b-x)^2}$$

$We\ can\ write\ it\ as$

$$\frac{(a+x)^2}{(a+x)(a-x)} = \frac{(b+x)^2}{(b-x)^2}$$

$$\frac{a+x}{a-x} = \frac{(b+x)^2}{(b-x)^2}$$

$By\ applying\ componendo\ and\ dividendo$

$$\frac{a+x+a-x}{a+x-a+x} = \frac{(b+x)^2+(b-x)^2}{(b+x)^2-(b-x)^2}$$

$By\ further\ calculation$

$$\frac{2a}{2x} = \frac{2(b^2+x^2)}{4bx}$$

$Dividing\ by\ 2$

$$\frac{a}{x} = \frac{(b^2+x^2)}{2bx}$$

$By\ cross\ multiplication$

$$2abx = x(b^2+x^2)$$

$$2ab = b^2+x^2$$

$$x^2 = 2ab - b^2$$

$$x = \sqrt{2ab-b^2}$$

22.

If $\mathbf{x = \frac{\sqrt[3]{a+1}+\sqrt[3]{a-1}}{\sqrt[3]{a+1}-\sqrt[3]{a-1}}}$, **prove that :** $\mathbf{x^3 - 3ax^2 + 3x - a = 0}$.

Solution:

It is given that

$$x = \frac{\sqrt[3]{a+1}+\sqrt[3]{a-1}}{\sqrt[3]{a+1}-\sqrt[3]{a-1}}$$

By applying componendo and dividendo

$$\frac{x+1}{x-1} = \frac{\sqrt[3]{a+1}+\sqrt[3]{a-1}+\sqrt[3]{a+1}-\sqrt[3]{a-1}}{\sqrt[3]{a+1}-\sqrt[3]{a-1}-\sqrt[3]{a+1}+\sqrt[3]{a-1}}$$

On further calculation

$$\frac{x+1}{x-1} = \frac{2\sqrt[3]{a+1}}{2\sqrt[3]{a-1}}$$

$$\frac{x+1}{x-1} = \frac{\sqrt[3]{a+1}}{\sqrt[3]{a-1}}$$

By cubing on both sides

$$\frac{(x+1)^3}{(x-1)^3} = \frac{a+1}{a-1}$$

By applying componendo and dividendo

$$\frac{(x+1)^3+(x-1)^3}{(x+1)^3-(x-1)^3} = \frac{a+1+a-1}{a+1-a+1}$$

By further calculation

$$\frac{2(x^3+3x)}{2(3x^2+1)} = \frac{2a}{2}$$

Dividing by 2

$$\frac{(x^3+3x)}{(3x^2+1)} = \frac{a}{1}$$

By cross multiplication

$x^3 + 3x = 3ax^2 + a$

$x^3 - 3ax^2 + 3x - a = 0$

Therefore, it is proved.

23.
If $\frac{by+cz}{b^2+c^2}=\frac{cz+ax}{c^2+a^2}=\frac{ax+by}{a^2+b^2}$, prove that each of these ratio is equal to $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$.

Solution:

$$\frac{by+cz}{b^2+c^2}=\frac{cz+ax}{c^2+a^2}=\frac{ax+by}{a^2+b^2}$$

By addition

$$=\frac{2(ax+by+cz)}{2(a^2+b^2+c^2)}$$
$$=\frac{ax+by+cz}{a^2+b^2+c^2}$$

We know that

$$\frac{by+cz}{b^2+c^2}=\frac{ax+by+cz}{a^2+b^2+c^2}$$

By applying alternendo

$$\frac{by+cz}{ax+by+cz}=\frac{b^2+c^2}{a^2+b^2+c^2}$$

We can write it as

$$\frac{by+cz-ax-by-cz}{ax+by+cz}=\frac{b^2+c^2-a^2-b^2-c^2}{a^2+b^2+c^2}$$

So we get

$$\frac{-ax}{ax+by+cz}=\frac{-a^2}{a^2+b^2+c^2}$$
$$\frac{x}{ax+by+cz}=\frac{a}{a^2+b^2+c^2}$$
$$\frac{x}{a}=\frac{ax+by+cz}{a^2+b^2+c^2}\ldots(1)$$

In the same way

$$\frac{y}{b}=\frac{ax+by+cz}{a^2+b^2+c^2}\ldots.(2)$$
$$\frac{z}{c}=\frac{ax+by+cz}{a^2+b^2+c^2}\ldots(3)$$

$Using\ the\ equations$

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$

Therefore, it is proved.