CHAPTER TEST

1. Find the expansions of the following :
(i) $(2x + 3y + 5)(2x + 3y - 5)$
(ii) $(6 - 4a - 7b)^2$
(iii) $(7 - 3xy)^3$
(iv) $(x + y + 2)^3$
Solution:
(i) $(2x + 3y + 5)(2x + 3y - 5)$
Let us simplify the expression, we get
$(2x + 3y + 5)(2x + 3y - 5) = [(2x + 3y) + 5][(2x - 3y) - 5]$
By using the formula, $(a)^2 - (b)^2 = [(a + b)(a - b)]$
$= (2x + 3y)^2 - (5)^2$
$= (2x)^2 + (3y)^2 + 2 \times 2x \times 3y - 5 \times 5$
$= 4x^2 + 9y^2 + 12xy - 25$

(ii) $(6 - 4a - 7b)^2$
Let us simplify the expression, we get
$(6 - 4a - 7b)^2 = [6 + (-4a) + (-7b)]^2$
$= (6)^2 + (-4a)^2 + (-7b)^2 + 2(6)(-4a) + 2(-4a)(-7b) + 2(-7b)(6)$
$= 36 + 16a^2 + 49b^2 - 48a + 56ab - 84b$

(iii) $(7 - 3xy)^3$
Let us simplify the expression
By using the formula, we get
$(7 - 3xy)^3 = (7)^3 - (3xy)^3 - 3(7)(3xy)(7 - 3xy)$
$= 343 - 27x^3y^3 - 63xy(7 - 3xy)$
$= 343 - 27x^3y^3 - 441xy + 189x^2y^2$

(iv) $(x + y + 2)^3$
Let us simplify the expression
By using the formula, we get
$(x + y + 2)^3 = [(x + y) + 2]^3$
$= (x + y)^3 + (2)^3 + 3(x + y)(2)(x + y + 2)$
$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6(x + y)[(x + y) + 2]$
$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6(x + y)^2 + 12(x + y)$
$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6(x^2 + y^2 + 2xy) + 12x + 12y = x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6x^2 + 6y^2 + 12xy + 12x + 12y$
$= x^3 + y^3 + 3x^2y + 3xy^2 + 8 + 6x^2 + 6y^2 + 12x + 12y + 12xy$

2. Simplify: $(x - 2)(x + 2)(x^2 + 4)(x^4 + 16)$

Solution:

Let us simplify the expression, we get

$(x - 2)(x + 2)(x^4 + 4)(x^4 + 16) = (x^2 - 4)(x^4 + 4)(x^4 + 16)$
$= [(x^2)^2 - (4)^2](x^4 + 16)$
$= (x^4 - 16)(x^4 + 16)$
$= (x^4)^2 - (16)^2$
$= x^8 - 256$

3. Evaluate 1002 × 998 by using a special product.

Solution:

Let us simplify the expression, we get

$1002 \times 998 = (1000 + 2)(1000 - 2)$
$= (1000)^2 - (2)^2$
$= 1000000 - 4$
$= 999996$

4. If $a + 2b + 3c = 0$, Prove that $a^3 + 8b^3 + 27c^3 = 18abc$

Solution:

Given:

$a + 2b + 3c = 0$, $a + 2b = -3c$

Let us cube on both the sides, we get

$(a + 2b)^3 = (-3c)^3$
$a^3 + (2b)^3 + 3(a)(2b)(a + 2b) = -27c^3$
$a^3 + 8b^3 + 6ab(-3c) = -27c^3$
$a^3 + 8b^3 - 18abc = -27c^3$
$a^3 + 8b^3 + 27c^3 = 18abc$

Hence proved.

5. If $2x = 3y - 5$, then find the value of $8x^3 - 27y^3 + 90xy + 125$.

Solution:

Given:

$2x = 3y - 5$
$2x - 3y = -5$

Now, let us cube on both sides, we get

$(2x - 3y)^3 = (-5)^3$
$(2x)^3 - (3y)^3 - 3 \times 2x \times 3y(2x - 3y) = -125$
$8x^3 - 27y^3 - 18xy(2x - 3y) = -125$

Now, substitute the value of $2x - 3y = -5$
$8x^3 – 27y^3 – 18xy (-5) = -125$
$8x^3 – 27y^3 + 90xy = -125$
$8x^3 – 27y^3 + 90xy + 125 = 0$

6. If $a^2 – 1/a^2 = 5$, evaluate $a^4 + 1/a^4$
Solution:
It is given that,
$a^2 – 1/a^2 = 5$
So,
By using the formula, $(a + b)^2$
$[a^2 – 1/a^2]^2 = a^4 + 1/a^4 – 2$
$[a^2 – 1/a^2]^2 + 2 = a^4 + 1/a^4$
Substitute the value of $a^2 – 1/a^2 = 5$, we get
$5^2 + 2 = a^4 + 1/a^4$
$a^4 + 1/a^4 = 25 + 2$
$= 27$

7. If $a + 1/a = p$ and $a – 1/a = q$, Find the relation between p and q.
Solution:
It is given that,
$a + 1/a = p$ and $a – 1/a = q$
so,
$(a + 1/a)^2 – (a – 1/a)^2 = 4(a) (1/a)$
$= 4$
By substituting the values, we get
$p^2 – q^2 = 4$
Hence the relation between p and q is that $p^2 – q^2 = 4$.

8. If $(a^2 + 1)/a = 4$, find the value of $2a^3 + 2/a^3$
Solution:
It is given that,
$(a^2 + 1)/a = 4$
$a^2/a + 1/a = 4$
$a + 1/a = 4$
So by multiplying the expression by 2a, we get
$2a^3 + 2/a^3 = 2[a^3 + 1/a^3]$
$= 2 [(a + 1/a)^3 – 3 (a) (1/a) (a + 1/a)]$
$= 2 [(4)^3 – 3(4)]$

= 2 [64 - 12]
= 2 (52)
= 104

9. If x = 1/(4 - x), find the value of
(i) x + 1/x
(ii) $x^3 + 1/x^3$
(iii) $x^6 + 1/x^6$
Solution:
It is given that,
x = 1/(4 - x)
So,
(i) x(4 - x) = 1
$4x – x^2 = 1$
Now let us divide both sides by x, we get
4 – x = 1/x
4 = 1/x + x
1/x + x = 4
1/x + x = 4

(ii) $x^3 + 1/x^3 = (x + 1/x)^2 – 3(x + 1/x)$
By substituting the values, we get
$= (4)^3 – 3(4)$
= 64 – 12
= 52

(iii) $x^6 + 1/x^6 = (x^3 + 1/x^3)^2 – 2$
$= (52)^2 – 2$
= 2704 – 2
= 2702

10. If x – 1/x = 3 + 2√2, find the value of ¼ $(x^3 – 1/x^3)$
Solution:
It is given that,
x – 1/x = 3 + 2√2
So,
$x^3 – 1/x^3 = (x – 1/x)^3 + 3(x – 1/x)$
$= (3 + 2\sqrt{2})^3 + 3(3 + 2\sqrt{2})$
By using the formula, $(a+b)^3 = a^3 + b^3 + 3ab (a + b)$

$= (3)^3 + (2\sqrt{2})^3 + 3\,(3)\,(2\sqrt{2})\,(3 + 2\sqrt{2}) + 3(3 + 2\sqrt{2})$
$= 27 + 16\sqrt{2} + 54\sqrt{2} + 72 + 9 + 6\sqrt{2}$
$= 108 + 76\sqrt{2}$

Hence,
$\frac{1}{4}(x^3 – 1/x^3) = \frac{1}{4}(108 + 76\sqrt{2})$
$= 27 + 19\sqrt{2}$

11. If $x + 1/x = 3\ 1/3$, find the value of $x^3 – 1/x^3$

Solution:

It is given that,
$x + 1/x = 3\ 1/3$
we know that,
$(x – 1/x)^2 = x^2 + 1/x^2 – 2$
$= x^2 + 1/x^2 + 2 – 4$
$= (x + 1/x)^2 – 4$
But $x + 1/x = 3\ 1/3 = 10/3$
So,
$(x – 1/x)^2 = (10/3)^2 – 4$
$= 100/9 – 4$
$= (100 - 36)/9$
$= 64/9$
$x – 1/x = \sqrt{(64/9)}$
$= 8/3$
Now,
$x^3 – 1/x^3 = (x – 1/x)^3 + 3\,(x)\,(1/x)\,(x – 1/x)$
$= (8/3)^3 + 3\,(8/3)$
$= ((512/27) + 8)$
$= 728/27$
$= 26\ 26/27$

12. If $x = 2 - \sqrt{3}$, then find the value of $x^3 – 1/x^3$

Solution:

It is given that,
$x = 2 - \sqrt{3}$
so,
$1/x = 1/(2 - \sqrt{3})$
By rationalizing the denominator, we get
$= [1(2 + \sqrt{3})] / [(2 - \sqrt{3})\,(2 + \sqrt{3})]$
$= [(2 + \sqrt{3})] / [(2^2) – (\sqrt{3})^2]$

$= [(2 + \sqrt{3})] / [4 - 3]$
$= 2 + \sqrt{3}$
Now,
$x – 1/x = 2 - \sqrt{3} – 2 - \sqrt{3}$
$= - 2\sqrt{3}$
Let us cube on both sides, we get
$(x – 1/x)^3 = (-2\sqrt{3})^3$
$x^3 – 1/x^3 – 3 (x) (1/x) (x – 1/x) = 24\sqrt{3}$
$x^3 – 1/x^3 – 3(-2\sqrt{3}) = -24\sqrt{3}$
$x^3 – 1/x^3 + 6\sqrt{3} = -24\sqrt{3}$
$x^3 – 1/x^3 = -24\sqrt{3} - 6\sqrt{3}$
$= -30\sqrt{3}$
Hence,
$x^3 – 1/x^3 = -30\sqrt{3}$

13. If the sum of two numbers is 11 and sum of their cubes is 737, find the sum of their squares.
Solution:
Let us consider x and y be two numbers
Then,
$x + y = 11$
$x^3 + y^3 = 735$ and $x^2 + y^2 =?$
Now,
$x + y = 11$
Let us cube on both the sides,
$(x + y)^3 = (11)^3$
$x^3 + y^3 + 3xy (x + y) = 1331$
$737 + 3x \times 11 = 1331$
$33xy = 1331 – 737$
$= 594$
$xy = 594/33$
$xy = 8$
We know that, $x + y = 11$
By squaring on both sides, we get
$(x + y)^2 = (11)^2$
$x^2 + y^2 + 2xy = 121\ ^2\ x^2 + y^2 + 2 \times 18 = 121$
$x^2 + y^2 + 36 = 121$
$x^2 + y^2 = 121 – 36$
$= 85$

Hence sum of the squares = 85

14. If $a - b = 7$ and $a^3 - b^3 = 133$, find:
(i) ab
(ii) $a^2 + b^2$
Solution:
It is given that,
$a - b = 7$
let us cube on both sides, we get
(i) $(a - b)^3 = (7)^3$
$a^3 + b^3 - 3ab (a - b) = 343$
$133 - 3ab \times 7 = 343$
$133 - 21ab = 343$
$- 21ab = 343 - 133$ $21ab$
$= 210$
$ab = -210/21$
$ab = -10$

(ii) $a^2 + b^2$
Again $a - b = 7$
Let us square on both sides, we get
$(a - b)^2 = (7)^2$
$a^2 + b^2 - 2ab = 49$
$a^2 + b^2 - 2 \times (- 10) = 49$
$a^2 + b^2 + 20 = 49$
$a^2 + b^2 = 49 - 20$
$= 29$
Hence, $a^2 + b^2 = 29$

15. Find the coefficient of x^2 expansion of $(x^2 + x + 1)^2 + (x^2 - x + 1)^2$
Solution:
Given:
The expression, $(x^2 + x + 1)^2 + (x^2 - x + 1)^2$
$(x^2 + x + 1)^2 + (x^2 - x + 1)^2 = [((x^2 + 1) + x)^2 + [(x^2 + 1) - x)^2]$
$= (x^2 + 1)^2 + x^2 + 2 (x^2 + 1) (x) + (x^2 + 1)^2 + x^2 - 2 (x^2 + 1) (x)$
$= (x^2)^2 + (1)^2 + 2 \times x^2 \times 1 + x^2 + (x^2)^2 + 1 + 2 \times x^2 + 1 + x^2$
$= x^4 + 1 + 2x^2 + x^2 + x^4 + 1 + 2x^2 + x^2$
$= 2x^4 + 6x^2 + 2$
$\therefore$ Co-efficient of x^2 is 6.