EXERCISE 9.1

1. Convert the following to logarithmic form:
(i) $5^2 = 25$
(ii) $a^5 = 64$
(iii) $7^x = 100$
(iv) $9^0 = 1$
(v) $6^1 = 6$
(vi) $3^{-2} = 1/9$
(vii) $10^{-2} = 0.01$
(viii) $(81)^{3/4} = 27$
Solution:
(i) $5^2 = 25$
Let us apply log, we get
$\text{Log}_5 25 = 2$

(ii) $a^5 = 64$
Let us apply log, we get
$\text{Log}_a 64 = 5$

(iii) $7^x = 100$
Let us apply log, we get
$\text{Log}_7 100 = x$

(iv) $9^0 = 1$
Let us apply log, we get
$\text{Log}_9 1 = 0$

(v) $6^1 = 6$
Let us apply log, we get
$\text{Log}_6 6 = 1$

(vi) $3^{-2} = 1/9$
Let us apply log, we get
$\text{Log}_3 1/9 = -2$

(vii) $10^{-2} = 0.01$
Let us apply log, we get
$\text{Log}_{10} 0.01 = -2$

(viii) $(81)^{3/4} = 27$
Let us apply log, we get
$\text{Log}_{81} 27 = ¾$

2. Convert the following into exponential form:
(i) $\log_2 32 = 5$
(ii) $\log_3 81=4$
(iii) $\log_3 1/3 = -1$
(iv) $\log_3 4 = 2/3$
(v) $\log_8 32 = 5/3$
(vi) $\log_{10} (0.001) = -3$
(vii) $\log_2 0.25 = -2$
(viii) $\log_a (1/a) = -1$
Solution:
(i) $\log_2 32 = 5$
The exponential form of the expression is
$2^5 = 32$

(ii) $\log_3 81=4$
The exponential form of the expression is
$3^4 = 81$

(iii) $\log_3 1/3 = -1$
The exponential form of the expression is
$3^{-1} = 1/3$

(iv) $\log_3 4 = 2/3$
The exponential form of the expression is
$(8)^{2/3} = 4$

(v) $\log_8 32 = 5/3$
The exponential form of the expression is
$(8)^{5/3} = 32$

(vi) $\log_{10} (0.001) = -3$
The exponential form of the expression is
$10^{-3} = 0.001$

(vii) $\log_2 0.25 = -2$
The exponential form of the expression is
$2^{-2} = 0.25$

(viii) $\log_a (1/a) = -1$
The exponential form of the expression is
$a^{-1} = 1/a$

3. By converting to exponential form, find the values of:
(i) $\log_2 16$
(ii) $\log_5 125$
(iii) $\log_4 8$
(iv) $\log_9 27$
(v) $\log_{10} (.01)$
(vi) $\log_7 1/7$
(vii) $\log_{.5} 256$
(viii) $\log_2 0.25$
Solution:
(i) $\log_2 16$
Let us consider $\log_2 16 = x$
So,
$(2)^x = 16$
$= 2 \times 2 \times 2 \times 2$
$2^x = 2^4$
By comparing the powers,
$x = 4$

(ii) $\log_5 125$
Let us consider $\log_5 125 = x$
So,
$(5)^x = 125$
$= 5 \times 5 \times 5$
$5^x = 5^3$
By comparing the powers,
$x = 3$

(iii) $\log_4 8$
Let us consider $\log_4 8 = x$
So,

$(4)^x = 8$
$(2 \times 2)^x = 2 \times 2 \times 2$
$2^{2x} = 2^3$
By comparing the powers,
$2x = 3$
$x = 3/2$

(iv) $\log_9 27$
Let us consider $\log_9 27 = x$
So,
$(9)^x = 27$
$(3 \times 3)^x = 3 \times 3 \times 3$
$3^{2x} = 3^3$
By comparing the powers,
$2x = 3$
$x = 3/2$

(v) $\log_{10} (.01)$
Let us consider $\log_{10} (.01) = x$
So,
$(10)^x = 1/100$
$= 1/10 \times 1/10$
$10^x = 1/(10)^2$
$10^x = 10^{-2}$
By comparing the powers,
$x = -2$

(vi) $\log_7 1/7$
Let us consider $\log_7 1/7 = x$
So,
$(7)^x = 1/7$
$7^x = 7^{-1}$
By comparing the powers,
$x = -1$

(vii) $\log_{.5} 256$
Let us consider $\log_{.5} 256 = x$
So,
$(.5)^x = 256$

$(5/10)^x = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$(1/2)^x = 2^8$
$(2)^{-x} = 2^8$
By comparing the powers,
$-x = 8$
$x = -8$

(viii) $\log_2 0.25$
Let us consider $\log_2 0.25 = x$
So,
$(2)^x = 0.25$
$= 25/100$
$2^x = 1/4$
$2^x = (2)^{-2}$
By comparing the powers,
$x = -2$

4. Solve the following equations for x:
(i) $\log_3 x = 2$
(ii) $\log_x 25 = 2$
(iii) $\log_{10} x = -2$
(iv) $\log_4 x = ½$
(v) $\log_x 11 = 2.5$
(vi) $\log_x ¼ = -1$
(vii) $\log_{81} x = 3/2$
(viii) $\log_9 x = 2.5$
(ix) $\log_4 x = -1.5$
Solution:
(i) $\log_3 x = 2$
Let us simplify the expression,
$(3)^2 = x$
$x = 9$

(ii) $\log_x 25 = 2$
Let us simplify the expression,
$(x)^2 = 25$
$= 5 \times 5$
$x^2 = 5^2$
Since the powers are same,

So,
$x = 5$

(iii) $\log_{10} x = -2$
Let us simplify the expression,
$(10)^{-2} = x$
$x = 1/(10)^2$
$= 1/100$
$x = 0.01$

(iv) $\log_4 x = ½$
Let us simplify the expression,
$(4)^{1/2} = x$
$x = (2 \times 2)^{1/2}$
$= (2)^{2 \times 1/2}$
$x = 2$

(v) $\log_x 11 = 2.5$
Let us simplify the expression,
$(x)^1 = 11$
$x = 11$

(vi) $\log_x ¼ = -1$
Let us simplify the expression,
$(x)^{-1} = ¼$
$x^{-1} = 4^{-1}$
Since the powers are same,
So,
$x = 4$

(vii) $\log_{81} x = 3/2$
Let us simplify the expression,
$(81)^{3/2} = x$
$x = 81^{3/2}$
$= (3^4)^{3/2}$
$= 3^{4 \times 3/2}$
$= 3^6$
$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 729$

$x = 729$

(viii) $\log_9 x = 2.5$
$\log_9 x = 5/2$
Let us simplify the expression,
$(9)^{5/2} = x$
$x = (3^2)^{5/2}$
$= 3^{2 \times 5/2}$
$= 3^5$
$= 3 \times 3 \times 3 \times 3 \times 3$
$= 234$
$x = 234$

(ix) $\log_4 x = -1.5$
$\log_4 x = -3/2$
Let us simplify the expression,
$(4)^{-3/2} = x$
$x = (2^2)^{-3/2}$
$= 2^{2 \times -3/2}$
$= 2^{-3}$
$= 1/2^3$
$= 1/(2 \times 2 \times 2)$
$= 1/8$
$x = 1/8$

5. Given $\log_{10} a = b$, express 10^{2b-3} in terms of a.
Solution:
Given:
$\log_{10} a = b$
$(10)^b = a$
Now,
$10^{2b-3} = (10)^{2b} / (10)^3$
$= (10^b)^2 / (10 \times 10 \times 10)$
Substitute the value of $(10)^b = a$, we get
$= a^2/1000$

6. Given $\log_{10} x = a$, $\log_{10} y = b$ and $\log_{10} z = c$,
(i) write down 10^{2a-3} in terms of x.
(ii) write down 10^{3b-1} in terms of y.

(iii) if $\log_{10} P = 2a + b/2 – 3c$, express P in terms of x, y and z.

Solution:

Given:

$\log_{10} x = a$

⇨ $(10)^a = x$

$\log_{10} y = b$

⇨ $(10)^b = y$

$\log_{10} z = c$

⇨ $(10)^c = z$

(i) Write down 10^{2a-3} in terms of x.

$10^{2a-3} = (10)^{2a} / (10)^3$

$= (10^a)^2 / (10 \times 10 \times 10)$

Substitute the value of $(10)^a = x$, we get

$= x^2/1000$

(ii) Write down 10^{3b-1} in terms of y.

$10^{3b-1} = (10)^{3b} / (10)^1$

$= (10^b)^3 / (10)$

Substitute the value of $(10)^b = y$, we get

$= y^3/10$

(iii) If $\log_{10} P = 2a + b/2 – 3c$, express P in terms of x, y and z.

we know that,

$(10)^a = x$

$(10)^b = y$

$(10)^c = z$

By substituting the values

$\log_{10} P = 2a + b/2 – 3c$

$= 2 \log_{10} x + ½ \log_{10} y – 3 \log_{10} z$

$= \log_{10} x^2 + \log_{10} y^{1/2} - \log_{10} z^3$

$= \log_{10} (x^2 + y^{1/2}) - \log_{10} z^3$

$= \log_{10} [(x^2\sqrt{y})/z^3]$

$P = (x^2\sqrt{y})/z^3$

7. If $\log_{10}x = a$ and $\log_{10}y = b$, find the value of xy.

Solution:

Given:

$\log_{10}x = a$

$(10)^a = x$

$\log_{10}y = b$
$(10)^b = y$
Then,
$xy = (10)^a \times (10)^b$
$= (10)^{a+b}$

8. Given $\log_{10} a = m$ and $\log_{10} b = n$, express a^3/b^2 in terms of m and n.
Solution:
Given:
$\log_{10} a = m$
$(10)^m = a$

$\log_{10} b = n$
$(10)^n = b$
So,
$a^3/b^2 = (10^m)^3 / (10^n)^2$
$= 10^{3m} / 10^{2n}$
$= 10^{3m-2n}$

9. Given $\log_{10}a= 2a$ and $\log_{10}y = -b/2$
(i) write 10^a in terms of x.
(ii) write 10^{2b+1} in terms of y.
(iii) if $\log_{10}P= 3a - 2b$, express P in terms of x and y.
Solution:
Given:
$\log_{10}a= 2a$
$(10)^{2a} = a$

$\log_{10}y = -b/2$
$(10)^{-b/2} = y$
(i) Write 10^a in terms of x.
$10^a = (10^{2a})^{1/2}$
$= (x)^{1/2}$
$= \sqrt{x}$

(ii) Write 10^{2b+1} in terms of y.
$10^{2b+1} = 10^{2b} \times 10^1$

$= 10^{4(b/2)} \times 10^1$
$= (10^{b/2})^4 \times 10^1$
$= y^4 \times 10^1$
$= 10y^4$

(iii) If $\log_{10}P = 3a - 2b$, express P in terms of x and y.
$\log_{10}P = 3a - 2b$
Substitute the values,
$\log_{10}P = 3/2\ (2a) – 4(b/2)$
$= 3/2\ (\log_{10} x) – 4\ (\log_{10} y)$
$= (\log_{10} x)^{3/2} - (\log_{10} y)^4$
$= \log_{10} [(x^{3/2}) / y^4]$
$P = (x^{3/2}) / y^4$

10. If $\log_2 y = x$ and $\log_3 z = x$, find 72^x in terms of y and z.
Solution:
Given:
$\log_2 y = x$
$2^x = y$

$\log_3 z = x$
$3^x = z$
So,
$72^x = (2 \times 2 \times 2 \times 3 \times 3)^x$
$= (2^3 \times 3^2)^x$
$= 2^{3x} \times 3^{2x}$
$= (2^x)^3 \times (3^x)^2$
$= y^3 \times z^2$
$= y^3z^2$

11. If $\log_2 x = a$ and $\log_5 y = a$, write 100^{2a-1} in terms of x and y.
Solution:
Given:
$\log_2 x = a$
$2^a = x$

$\log_5 y = a$
$5^a = y$
So,

$100^{2a-1} = (2 \times 2 \times 5 \times 5)^{2a-1}$
$= (2^2 \times 5^2)^{2a-1}$
$= 2^{4a-2} \times 5^{a-2}$
$= (2^{4a})/2^2 \times (5^{4a})/5^2$
$= [(2^a)^4 \times (5^a)^4] / (4\times25)$
$= (x^4y^4)/100$

EXERCISE 9.2

1. Simplify the following:
(i) log a^3 – log a^2
(ii) log a^3 ÷ log a^2
(iii) log 4/log 2
(iv) (log 8 log 9)/log 27
(v) log 27/log √3
(vi) (log 9 – log 3)/log 27
Solution:
(i) log a^3 – log a^2
By using Quotient law,
log a^3 – log a^2 = log (a^3/a^2)
= log a

(ii) log a^3 ÷ log a^2
By using power law,
log a^3 ÷ log a^2 = 3log a ÷ 2 log a
= 3log a / 2log a
= 3/2

(iii) log 4/log 2
Let us simplify the expression,
log 4/log 2 = log(2×2)/log 2
By using power law,
= 2 log 2/log 2
= 2

(iv) (log 8 log 9)/log 27
Let us simplify the expression,
(log 8 log 9)/log 27 = (log 2^3. log 3^2)/log 3^3
By using power law,
= [(3 log 2).(2 log 3)]/(3 log 3)
= [(log 2).2] / 1
= 2 log 2
= log 2^2
= log 4

(v) log 27/log √3

Let us simplify the expression,

$\log 27/\log \sqrt{3} = \log(3\times3\times3)/\log(3)^{1/2}$

$= \log 3^3/\log 3^{1/2}$

By using power law

$= 3\log 3/((1/2)\log 3)$

$= (3\times2)/1 (\log 3/\log 3)$

$= (6) (1)$

$= 6$

(vi) (log 9 – log 3)/log 27

Let us simplify the expression,

$(\log 9 - \log 3)/\log 27 = [\log(3\times3) - \log 3] / \log(3\times3\times3)$

$= [\log 3^2 - \log 3] / \log 3^3$

By using power law

$= [2 \log 3 - \log 3]/3\log 3$

$= \log 3/3\log 3$

$= 1/3$

2. Evaluate the following:

(i) $\log (10 \div \sqrt[3]{10})$

(ii) $2 + ½ \log(10^{-3})$

(iii) 2 log 5 + log 8 – ½ log 4

(iv) $2 \log 10^3 + 3 \log 10^{-2} - 1/3 \log 5^{-3} + ½ \log 4$

(v) 2 log 2 + log 5 – ½ log 36 – log 1/30

(vi) 2 log 5 + log 3 + 3 log 2 – ½ log 36 – 2 log 10

(vii) log 2 + 16 log 16/15 + 12 log 25/24 + 7 log 81/80

(viii) $2 \log_{10} 5 + \log_{10} 8 - ½ \log_{10} 4$

Solution:

(i) $\log (10 \div \sqrt[3]{10})$

Let us simplify the expression,

$\log (10 \div \sqrt[3]{10}) = \log (10 \div 10^{1/3})$

$= \log (10^{1 - 1/3})$

$= \log (10^{2/3})$

$= 2/3 \log 10$

$= 2/3 (1)$

$= 2/3$

(ii) $2 + ½ \log(10^{-3})$

Let us simplify the expression,

$2 + ½ \log(10^{-3}) = 2 + ½ \times (-3) \log 10$
$= 2 – 3/2 \log 10$
$= 2 – 3/2 (1)$
$= 2 – 3/2$
$= (4-3)/2$
$= ½$

(iii) $2 \log 5 + \log 8 – ½ \log 4$
Let us simplify the expression,
$2 \log 5 + \log 8 – ½ \log 4 = \log 5^2 + \log 8 – ½ \log 2^2$
$= \log 25 + \log 8 – ½ 2\log 2$
$= \log 25 + \log 8 – \log 2$
$= \log (25\times8)/2$
$= \log (25\times4)$
$= \log 100$
$= \log 10^2$
$= 2 \log 10$
$= 2 (1)$
$= 2$

(iv) $2 \log 10^3 + 3 \log 10^{-2} - 1/3 \log 5^{-3} + ½ \log 4$
Let us simplify the expression,
$2 \log 10^3 + 3 \log 10^{-2} - 1/3 \log 5^{-3} + ½ \log 4 = 2\times3 \log 10 + 3(-2)\log 10 – 1/3 (-3) \log 5 + ½ \log 2^2$
$= 6 \log 10 – 6 \log 10 + \log 5 + ½ 2 \log 2$
$= 6 \log 10 – 6 \log 10 + \log 5 + \log 2$
$= 0 + \log 5 + \log 2$
$= \log (5\times2)$
$= \log 10$
$= 1$

(v) $2 \log 2 + \log 5 – ½ \log 36 – \log 1/30$
Let us simplify the expression,
$2 \log 2 + \log 5 – ½ \log 36 – \log 1/30 = \log 2^2 + \log 5 – ½ \log 6^2 – \log (1/30)$
$= \log 4 + \log 5 – \log 6 – \log 1/30$
$= \log 4 + \log 5 – \log 6 – (\log 1 – \log 30)$
$= \log 4 + \log 5 – \log 6 – \log 1 + \log 30$
$= \log 4 + \log 5 + \log 30 – (\log 6 + \log 1)$
$= \log (4\times5\times30) – \log (6\times1)$

= log (4×5×30)/(6×1)
= log (4×5×5)
= log 100
= log 10^2
= 2 log 10
= 2 (1)
= 2

(vi) 2 log 5 + log 3 + 3 log 2 – ½ log 36 – 2 log 10
Let us simplify the expression,
2log 5 + log 3 + 3log 2 – ½ log 36 – 2log 10 = log 5^2 + log 3 + log 2^3 – ½ log 6^2 – log 10^2
= log 25 + log 3 + log 8 – log 6 – log 100
= log (25×3×8) – log(6×100)
= log (25×3×8)/ (6×100)
= log (1×3×8) / (6×4)
= log 24/24
= log 1
= 0

(vii) log 2 + 16 log 16/15 + 12 log 25/24 + 7 log 81/80
Let us simplify the expression,
log 2 + 16 log 16/15 + 12 log 25/24 + 7 log 81/80 = log 2 + 16(log 16 – log 15) + 12(log 25 – log 24) + 7(log 81 – log 80)
= log 2 + 16 (log 2^4 – log (3×5)) + 12 (log 5^2 – log (3×2×2×2)) + 7 (log (3×3×3×3) – log (2^4 × 5))
= log 2 + 16(4log 2 – (log 3 + log 5)) + 12 (2log 5 – log (3×2^3)) + 7 (log 3^4 – (log 2^4 + log 5))
= log 2 + 16 (4log 2 – log 3 – log 5) + 12 (2log 5 – (log 3 + 3log 2)) + 7 (4log 3 – 4log 2 – log 5)
= log 2 + 64log 2 – 16log 3 – 16log 5 + 24log 5 – 12log 3 – 36log 2 + 28log 3 – 28log 2 – 7log 5
= (log 2 + 64log 2 – 36log 2 – 28log 2) + (-16log 3 – 12log 3 + 28log 3) + (-16log 5 + 24log 5 – 7log 5)
= (65log 2 – 64log 2) + (-28log 3 + 28log 3) + (-23log 5 + 24log 5)
= log 2 + 0 + log 5
= log (2×5)
= log 10
= 1

(viii) $2 \log_{10} 5 + \log_{10} 8 – ½ \log_{10} 4$
Let us simplify the expression,
$2 \log_{10} 5 + \log_{10} 8 – ½ \log_{10} 4 = \log_{10} 5^2 + \log_{10} 8 – \log_{10} 4^{1/2}$
$= \log_{10} 25 + \log_{10} 8 – \log_{10} (2)^{2 \times 1/2}$
$= \log_{10} 25 + \log_{10} 8 – \log_{10} 2$
$= \log_{10} [(25\times8)/2]$
$= \log_{10} (25\times4)$
$= \log_{10} 100$
$= \log_{10} 10^2$
$= 2 \log_{10} 10$
$= 2 (1)$
$= 2$

3. Express each of the following as a single logarithm:
(i) 2 log 3 – ½ log 16 + log 12
(ii) $2 \log_{10} 5 – \log_{10} 2 + 3 \log_{10} 4 + 1$
(iii) ½ log 36 + 2 log 8 – log 1.5
(iv) ½ log 25 – 2 log 3 + 1
(v) ½ log 9 + 2 log 3 – log 6 + log 2 - 2
Solution:
(i) 2 log 3 – ½ log 16 + log 12
Let us simplify the expression into single logarithm,
$2 \log 3 – ½ \log 16 + \log 12 = 2 \log 3 – ½ \log 4^2 + \log 12$
$= 2 \log 3 – \log 4 + \log 12$
$= \log 3^2 – \log 4 + \log 12$
$= \log 9 - \log 4 + \log 12$
$= \log (9\times12)/4$
$= \log (9\times3)$
$= \log 27$

(ii) $2 \log_{10} 5 – \log_{10} 2 + 3 \log_{10} 4 + 1$
Let us simplify the expression into single logarithm,
$2 \log_{10} 5 – \log_{10} 2 + 3 \log_{10} 4 + 1 = \log_{10} 5^2 – \log_{10} 2 + \log_{10} 4^3 + \log_{10} 10$
$= \log_{10} 25 – \log_{10} 2 + \log_{10} 64 + \log_{10} 10$
$= \log_{10} (25\times64\times10) – \log_{10} 2$
$= \log_{10} (16000) – \log_{10} 2$
$= \log_{10} (16000/2)$
$= \log_{10} 8000$

(iii) ½ log 36 + 2 log 8 – log 1.5

Let us simplify the expression into single logarithm,

½ log 36 + 2 log 8 – log 1.5 = log $36^{1/2}$ + log 8^2 – log 1.5
= log $(6)^{2 \times 1/2}$ + log 64 – log 1.5
= log 6 + log 64 – log (15/10)
= log 6 + log 64 – (log 15 – log 10)
= log (6×64) – log 15 + log 10
= log (6×64×10) – log 15
= log [(6×64×10)/15]
= log (4×64)
= log 256

(iv) ½ log 25 – 2 log 3 + 1

Let us simplify the expression into single logarithm,

½ log 25 – 2 log 3 + 1 = log $25^{1/2}$ – log 3^2 + log 10
= log $(5)^{2 \times 1/2}$ – log 9 + log 10
= log 5 – log 9 + log 10
= log (5×10) – log 9
= log ((5×10)/9)
= log 50/9

(v) ½ log 9 + 2 log 3 – log 6 + log 2 - 2

Let us simplify the expression into single logarithm,

½ log 9 + 2 log 3 – log 6 + log 2 – 2 = log $9^{1/2}$ + log 3^2 – log 6 + log 2 – log 100
= log $3^{2 \times 1/2}$ + log 9 – log 6 + log 2 – log 100
= log 3 + log 9 – log 6 + log 2 – log 100
= log [(3×9×2)/(6×100)]
= log 9/100

4. Prove the following:
(i) $\log_{10} 4 \div \log_{10} 2 = \log_3 9$
(ii) $\log_{10} 25 + \log_{10} 4 = \log_5 25$
Solution:

(i) $\log_{10} 4 \div \log_{10} 2 = \log_3 9$

Let us consider LHS, $\log_{10} 4 \div \log_{10} 2$

$\log_{10} 4 \div \log_{10} 2 = \log_{10} 2^2 \div \log_{10} 2$
$= 2 \log_{10} 2 \div \log_{10} 2$
$= 2 \log_{10} 2 / \log_{10} 2$

$= 2\,(1)$
$= 2$

Now let us consider RHS,

$\log_3 9 = \log_3 3^2$
$= 2 \log_3 3$
$= 2(1)$
$= 2$

∴ LHS = RHS

Hence proved.

(ii) $\log_{10} 25 + \log_{10} 4 = \log_5 25$

Let us consider LHS, $\log_{10} 25 + \log_{10} 4$

$\log_{10} 25 + \log_{10} 4 = \log_{10} (25\times4)$
$= \log_{10} 100$
$= \log_{10} 10^2$
$= 2 \log_{10} 10$
$= 2(1)$
$= 2$

Now, let us consider RHS,

$\log_5 25 = \log_5 5^2$
$= 2 \log_5 5$
$= 2\,(1)$
$= 2$

∴ LHS = RHS

Hence proved.

5. If $x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$, express $\log [(10\sqrt{y})/x^2z^3]$ in terms of a, b, c.

Solution:

Given:

$x = (100)^a = (10^2)^a = 10^{2a}$
$y = (10000)^b = (10^4)^b = 10^{4b}$
$z = (10)^c$

It is given that, $\log [(10\sqrt{y})/x^2z^3]$

$\log [(10\sqrt{y})/x^2z^3] = (\log 10 + \log \sqrt{y}) - (\log x^2 + \log z^3)$
$= (1 + \log(y)^{1/2}) - (\log x^2 + \log z^3)$ [we know that, $\log 10 = 1$]
$= (1 + ½ \log y) - (2 \log x + 3 \log z)$

Now substitute the values of x, y, z, we get

$= (1 + ½ \log 10^{4b}) - (2 \log 10^{2a} + 3 \log 10^c)$
$= (1 + ½\, 4b \log 10) - (2\times2a \log 10 + 3\times c \log 10)$

$= (1 + \frac{1}{2} 4b) – (2\times2a + 3c)$ [Since, log 10 = 1]
$= (1 + 2b) – (4a + 3c)$
$= 1 + 2b – 4a – 3c$

6. If $a = \log_{10}x$, find the following in terms of a :
(i) x
(ii) $\log_{10} \sqrt[5]{x^2}$
(iii) $\log_{10} 5x$
Solution:
Given:
$a = \log_{10}x$
(i) x
$10^a = x$
$\therefore x = 10^a$

(ii) $\log_{10} \sqrt[5]{x^2}$
$\log_{10} \sqrt[5]{x^2} = \log_{10} (x^2)^{1/5}$
$= \log_{10} (x)^{2/5}$
$= 2/5 \log_{10} x$
$= 2/5 (a)$
$= 2a/5$

(iii) $\log_{10} 5x$
$x = (10)^a$
$= \log_{10} 5x$
$= \log_{10} 5(10)^a$
$= \log_{10} 5 + \log_{10} 10$
$= \log_{10} 5 + a(1)$
$= a + \log_{10} 5$

7. If a =log 2/3, b = log 3/5 and c = 2 log √(5/2). Find the value of
(i) a + b + c
(ii) 5^{a+b+c}
Solution:
Given:
a =log 2/3
b = log 3/5
c = 2 log √(5/2)
(i) a + b + c

Let us substitute the given values, we get

$a + b + c = \log 2/3 + \log 3/5 + 2 \log \sqrt{(5/2)}$

$= (\log 2 - \log 3) + (\log 3 - \log 5) + 2 \log (5/2)^{1/2}$

$= \log 2 - \log 3 + \log 3 - \log 5 + 2 \times ½ (\log 5 - \log 2)$

$= \log 2 - \log 3 + \log 3 - \log 5 + \log 5 - \log 2$

$= 0$

(ii) 5^{a+b+c}

$5^{a+b+c} = 5^0$

$= 1$

8. If x = log 3/5, y = log 5/4 and z = 2 log √3/2, find the value of

(i) x + y – z

(ii) 3^{x+y-z}

Solution:

Given:

$x = \log 3/5 = \log 3 - \log 5$

$y = \log 5/4 = \log 5 - \log 4$

$z = 2 \log \sqrt{3}/2 = \log (\sqrt{3}/2)^2 = \log ¾ = \log 3 - \log 4$

(i) x + y – z

Let us substitute the given values, we get

$x + y - z = \log 3 - \log 5 + \log 5 - \log 4 - (\log 3 - \log 4)$

$= \log 3 - \log 5 + \log 5 - \log 4 - \log 3 + \log 4$

$= 0$

(ii) 3^{x+y-z}

$3^{x+y-z} = 3^0$

$= 1$

9. If x = $\log_{10} 12$, y = $\log_4 2 \times \log_{10} 9$ and z = $\log_{10} 0.4$, find the values of

(i) x – y – z

(ii) 7^{x-y-z}

Solution:

Given:

$x = \log_{10} 12$

$y = \log_4 2 \times \log_{10} 9$

$z = \log_{10} 0.4$

(i) x – y – z

Let us substitute the given values, we get

$x – y – z = \log_{10} 12 - \log_4 2 \times \log_{10} 9 - \log_{10} 0.4$
$= \log_{10} (3\times4) – \log_4 4^{1/2} \times \log_{10} 3^2 – \log_{10} 4/10$
$= \log_{10} 3 + \log_{10} 4 – ½ \log_4 4 \times 2 \log_{10} 3 – (\log_{10} 4 – \log_{10} 10)$
$= \log_{10} 3 + \log_{10} 4 – ½ \times 1 \times 2 \log_{10} 3 – \log_{10} 4 + 1$
$= \log_{10} 3 + \log_{10} 4 - \log_{10} 3 – \log_{10} 4 + 1$
$= 1$

(ii) 7^{x-y-z}
$7^{x-y-z} = 7^1$
$= 7$

10. If log V + log 3 = log π + log 4 + 3 log r, find V in terms of other quantities.
Solution:
Given:
log V + log 3 = log π + log 4 + 3 log r
Let us simplify the given expression to find V,
$\log (V \times 3) = \log (\pi \times 4 \times r^3)$
$\log 3V = \log 4\pi r^3$
$3V = 4\pi r^3$
$V = 4\pi r^3/3$

11. Given 3 (log 5 – log 3) – (log 5 - 2 log 6) = 2 – log n, find n.
Solution:
Given:
3 (log 5 – log 3) – (log 5 - 2 log 6) = 2 – log n
Let us simplify the given expression to find n,
3 log 5 – 3 log 3 – log 5 + 2 log 6 = 2 – log n
2 log 5 – 3 log 3 + 2 log 6 = 2 (1) – log n
$\log 5^2 – \log 3^3 + \log 6^2 = 2 \log 10 – \log n$ [Since, 1 = log 10]
$\log 25 – \log 27 + \log 36 – \log 10^2 = - \log n$
log n = - log 25 + log 27 – log 36 + log 100
= (log 100 + log 27) – (log 25 + log 36)
= log (100×27) – log (25×36)
= log (100×27)/(25×36)
log n = log 3
n = 3

12. Given that $\log_{10} y + 2 \log_{10} x = 2$, express y in terms of x.
Solution:

Given:
$\log_{10} y + 2 \log_{10} x = 2$
Let us simplify the given expression,
$\log_{10} y + \log_{10} x^2 = 2(1)$
$\log_{10} y + \log_{10} x^2 = 2 \log_{10} 10$
$\log_{10} (y\times x^2) = \log_{10} 10^2$
$yx^2 = 100$
$y = 100/x^2$

13. Express $\log_{10} 2 + 1$ in the form $\log_{10}x$.
Solution:
Given:
$\log_{10} 2 + 1$
Let us simplify the given expression,
$\log_{10} 2 + 1 = \log_{10} 2 + \log_{10} 10$ [Since, $1 = \log_{10} 10$]
$= \log_{10} (2\times10)$
$= \log_{10} 20$

14. If $a^2 = \log_{10} x$, $b^2 = \log_{10} y$ and $a^2/2 – b^2/3 = \log_{10} z$. Express z in terms of x and y.
Solution:
Given:
$a^2 = \log_{10} x$
$b^2 = \log_{10} y$
$a^2/2 – b^2/3 = \log_{10} z$
Let us substitute the given values in the expression, we get
$\log_{10} x/2 - \log_{10} y/3 = \log_{10} z$
$\log_{10} x^{1/2} – \log_{10} y^{1/3} = \log_{10} z$
$\log_{10} \sqrt{x} – \log_{10} \sqrt[3]{y} = \log_{10} z$
$\log_{10} \sqrt{x}/\sqrt[3]{y} = \log_{10} z$
$\sqrt{x}/\sqrt[3]{y} = z$
$z = \sqrt{x}/\sqrt[3]{y}$

15. Given that log m = x + y and log n = x - y, express the value of log m^2n in terms of x and y.
Solution:
Given:
$\log m = x + y$
$\log n = x – y$
$\log m^2n$

Let us simplify the given expression,
$\log m^2n = \log m^2 + \log n$
$= 2 \log m + \log n$
By substituting the given values, we get
$= 2 (x + y) + (x - y)$
$= 2x + 2y + x – y$
$= 3x + y$

16. Given that $\log x = m + n$ and $\log y = m – n$, express the value of $\log (10x/y^2)$ in terms of m and n.
Solution:
Given:
$\log x = m + n$
$\log y = m – n$
$\log (10x/y^2)$
Let us simplify the given expression,
$\log (10x/y^2) = \log 10x – \log y^2$
$= \log 10 + \log x – 2 \log y$
$= 1 + \log x – 2 \log y$
$= 1 + (m + n) – 2(m - n)$
$= 1 + m + n – 2m + 2n$
$= 1 – m + 3n$

17. If $\log x/2 = \log y/3$, find the value of y^4/x^6.
Solution:
Given:
$\log x/2 = \log y/3$
Let us simplify the given expression,
By cross multiplying, we get
$3 \log x = 2 \log y$
$\log x^3 = \log y^2$
so, $x^3 = y^2$
now square on both sides, we get
$(x^3)^2 = (y^2)^2$
$x^6 = y^4$
$y^4/x^6 = 1$

18. Solve for x:
(i) $\log x + \log 5 = 2 \log 3$

(ii) $\log_3 x - \log_3 2 = 1$
(iii) x = log 125/log 25
(iv) (log 8/log 2) × (log 3/log√3) = 2 log x
Solution:
(i) log x + log 5 = 2 log 3
Let us solve for x,
Log x = 2 log 3 – log 5
= $\log 3^2$ – log 5
= log 9 – log 5
= log (9/5)
∴ x = 9/5

(ii) $\log_3 x - \log_3 2 = 1$
Let us solve for x,
$\log_3 x = 1 + \log_3 2$
= $\log_3 3 + \log_3 2$ [Since, 1 can be written as $\log_3 3 = 1$]
= $\log_3 (3\times2)$
= $\log_3 6$
∴ x = 6

(iii) x = log 125/log 25
x = $\log 5^3/\log 5^2$
= 3 log 5/ 2 log 5
= 3/2 [Since, log 5/log 5 = 1]
∴ x = 3/2

(iv) (log 8/log 2) × (log 3/log√3) = 2 log x
($\log 2^3$/log 2) × ($\log 3/\log 3^{1/2}$) = 2 log x
(3log 2/log 2) × (log 3/½ log 3) = 2 log x
3 × 1/(½) = 2 log x
3 × 2 = 2 log x
6 = 2 log x
log x = 6/2
log x = 3
x = $(10)^3$
= 1000
∴ x = 1000

19. Given $2 \log_{10} x + 1 = \log_{10} 250$, find

(i) x
(ii) $\log_{10}2x$
Solution:
Given:
$2 \log_{10} x + 1 = \log_{10} 250$
(i) let us simplify the above expression,
$\log_{10} x^2 + \log_{10} 10 = \log_{10} 250$ [Since, 1 can be written as $\log_{10} 10$]
$\log_{10} (x^2 \times 10) = \log_{10} 250$
$(x^2 \times 10) = 250$
$x^2 = 250/10$
$x^2 = 25$
$x = \sqrt{25}$
$= 5$
$\therefore x = 5$

(ii) $\log_{10} 2x$
We know that, $x = 5$
So, $\log_{10} 2x = \log_{10} 2\times5$
$= \log_{10} 10$
$= 1$

20. If $\log x/\log 5 = \log y^2/\log 2 = \log 9/\log (1/3)$, find x and y.
Solution:
Given:
$\log x/\log 5 = \log y^2/\log 2 = \log 9/\log (1/3)$
let us consider,
$\log x/\log 5 = \log 9/\log (1/3)$
$\log x = (\log 9\times\log 5)/\log (1/3)$
$= (\log 3^2 \times \log 5) / (\log 1 - \log 3)$
$= (2 \log 3 \times \log 5) / (-\log 3)$ [$\log 1 = 0$]
$= -2 \times \log 5$
$= \log 5^{-2}$
$x = 5^{-2}$
$= 1/5^2$
$= 1/25$
Now,
$\log y^2/\log 2 = \log 9/\log (1/3)$
$\log y^2 = (\log 9\times\log2)/\log (1/3)$
$= (\log 3^2 \times \log 2) / (\log 1 - \log 3)$

= (2 log 3 × log 2) / (-log 3) [log 1 = 0]
= -2 × log 2
= log 2^{-2}
$y^2 = 2^{-2}$
= $1/2^2$
= ¼
= √(1/4)
= ½

21. Prove the following:
(i) $3^{\log 4} = 4^{\log 3}$
(ii) $27^{\log 2} = 8^{\log 3}$
Solution:
(i) $3^{\log 4} = 4^{\log 3}$
Let us take log on both sides,
If log $3^{\log 4}$ = log $4^{\log 3}$
log 4 . log 3 = log 3 . log 4
log 2^2 . log 3 = log 3 . log 2^2
2 log 2 . log 3 = log 3 . 2 log 2
Which is true.
Hence proved.

(ii) $27^{\log 2} = 8^{\log 3}$
Let us take log on both sides,
If log $27^{\log 2}$ = log $8^{\log 3}$
log 2 . log 27 = log 3 . log 8
log 2 . log 3^3 = log 3 . log 2^3
log 2 . 3 log 3 = log 3 . 3 log 2
3 log2 . log 3 = 3 log2 . log 3
Which is true.
Hence proved.

22. Solve the following equations:
(i) log (2x + 3) = log 7
(ii) log (x +1) + log (x – 1) = log 24
(iii) log (10x + 5) – log (x – 4) = 2
(iv) $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$
(v) log (4y – 3) = log (2y + 1) – log 3
(vi) $\log_{10} (x + 2) + \log_{10} (x – 2) = \log_{10} 3 + 3 \log_{10} 4$

(vii) log (3x + 2) + log (3x – 2) = 5 log 2

Solution:

(i) log (2x + 3) = log 7

Let us simplify the expression,

2x + 3 = 7

2x = 7 – 3

2x = 4

x = 4/2

= 2

(ii) log (x +1) + log (x – 1) = log 24

Let us simplify the expression,

log [(x +1) (x – 1)] = log 24

log $(x^2 - 1)$ = log 24

$(x^2 - 1) = 24$

$x^2 = 24 + 1$

= 25

x = √25

= 5

(iii) log (10x + 5) – log (x – 4) = 2

Let us simplify the expression,

log (10x + 5) / (x – 4) = 2 log 10

log (10x + 5) / (x – 4) = log 10^2

(10x + 5) / (x – 4) = 100

10x + 5 = 100 (x - 4)

10x + 5 = 100x – 400

5 + 400 = 100x – 10x

90x = 405

x = 405/90

= 81/18

= 9/2

= 4.5

(iv) $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$

Let us simplify the expression,

$\log_{10} [5\times (5x + 1)] = \log_{10} (x + 5) + \log_{10} 10$

$\log_{10} [5\times (5x + 1)] = \log_{10} [(x + 5) \times 10]$

$[5\times (5x + 1)] = [(x + 5) \times 10]$

$25x + 5 = 10x + 50$
$25x – 10x = 50 – 5$
$15x = 45$
$x = 45/15$
$= 3$

(v) $\log (4y – 3) = \log (2y + 1) – \log 3$
Let us simplify the expression,
$\log (4y – 3) = \log (2y + 1) / 3$
$(4y – 3) = (2y + 1) / 3$
By cross multiplying, we get
$3(4y - 3) = 2y + 1$
$12y – 9 = 2y + 1$
$12y – 2y = 9 + 1$
$10y = 10$
$y = 10/10$
$= 1$

(vi) $\log_{10} (x + 2) + \log_{10} (x – 2) = \log_{10} 3 + 3 \log_{10} 4$
Let us simplify the expression,
$\log_{10} [(x + 2) \times (x - 2)] = \log_{10} 3 + \log_{10} 4^3$
$\log_{10} [(x + 2) \times (x - 2)] = \log_{10} (3\times4^3)$
$[(x + 2) \times (x - 2)] = (3\times4^3)$
$(x^2 - 4) = (3\times4\times4\times4)$
$(x^2 - 4) = 192$
$x^2 = 192 + 4$
$= 196$
$x = \sqrt{196}$
$= 14$

(vii) $\log (3x + 2) + \log (3x – 2) = 5 \log 2$
Let us simplify the expression,
$\log (3x + 2) + \log (3x – 2) = \log 2^5$
$\log [(3x + 2) \times (3x - 2)] = \log 32$
$\log (9x^2 - 4) = \log 32$
$(9x^2 - 4) = 32$
$9x^2 = 32 + 4$
$9x^2 = 36$
$x^2 = 36/9$

$x^2 = 4$

$x = \sqrt{4}$

$= 2$

23. Solve for x:

$\log_3 (x + 1) – 1 = 3 + \log_3 (x – 1)$

Solution:

Given:

$\log_3 (x + 1) – 1 = 3 + \log_3 (x – 1)$

Let us simplify the expression,

$\log_3 (x + 1) – \log_3 (x – 1) = 3 + 1$

$\log_3 (x + 1) / (x – 1) = 4 \log_3 3$ [Since, $\log_3 3 = 1$]

$\log_3 (x + 1) / (x – 1) = \log_3 3^4$

$(x + 1) / (x – 1) = 3^4$

By cross multiplying, we get

$(x + 1) = 81 (x - 1)$

$x + 1 = 81x – 81$

$81x – x = 1 + 81$

$80x = 82$

$x = 82/80$

$= 41/40$

$= 1\ 1/40$

24. Solve for x:

$5^{\log x} + 3^{\log x} = 3^{\log x+1} – 5^{\log x – 1}$

Solution:

Given:

$5^{\log x} + 3^{\log x} = 3^{\log x+1} – 5^{\log x – 1}$

Let us simplify the expression,

$5^{\log x} + 3^{\log x} = 3^{\log x} . 3^1 – 5^{\log x} . 5^{-1}$

$5^{\log x} + 3^{\log x} = 3.3^{\log x} – 1/5 . 5^{\log x}$

$5^{\log x} + 1/5 . 5^{\log x} = 3.3^{\log x} – 3^{\log x}$

$(1 + 1/5) 5^{\log x} = (3 - 1) 3^{\log x}$

$(6/5) 5^{\log x} = 2(3^{\log x})$

$5^{\log x} / 3^{\log x} = (2×5)/6$

$(5/3)^{\log x} = 10/6$

$(5/3)^{\log x} = 5/3$

$(5/3)^{\log x} = (5/3)^1$

So, by comparing the powers

$\log x = 1$
$\log x = \log 10$
$x = 10$

25. If log (x-y)/2 = ½ (log x + log y), prove that $x^2 + y^2 = 6xy$
Solution:
Given:
$\log (x-y)/2 = ½ (\log x + \log y)$
Let us simplify,
$\log (x-y)/2 = ½ (\log x×y)$
$\log (x-y)/2 = ½ \log xy$
$\log (x-y)/2 = \log (xy)^{1/2}$
$(x-y)/2 = (xy)^{1/2}$
By squaring on both sides, we get
$[(x-y)/2]^2 = [(xy)^{1/2}]^2$
$(x - y)^2/4 = xy$
By cross multiplying, we get
$(x - y)^2 = 4xy$
$x^2 + y^2 – 2xy = 4xy$
$x^2 + y^2 = 4xy + 2xy$
$x^2 + y^2 = 6xy$
Hence proved.

26. If $x^2 + y^2 = 23xy$, Prove that log (x + y)/5 = ½ (log x + log y)
Solution:
Given:
$x^2 + y^2 = 23xy$
So, the above equation can be written as
$x^2 + y^2 = 25xy – 2xy$
$x^2 + y^2 + 2xy = 25xy$
$(x + y)^2 = 25xy$
$(x + y)^2 / 25 = xy$
Now by taking log on both sides, we get
$\log [(x + y)^2 / 25] = \log xy$
$\log [(x + y)/5]^2 = \log xy$
$2 \log (x+y)/5 = \log x + \log y$
$\log (x+y)/5 = ½ \log x + \log y$
Hence proved.

27. If p = $\log_{10}$ 20 and q = $\log_{10}$ 25, find the value of x if 2 $\log_{10}$ (x + 1) = 2p – q

Solution:

Given:

$p = \log_{10} 20$

$q = \log_{10} 25$

Then,

$2 \log_{10} (x + 1) = 2p – q$

Now substitute the values of p and q, we get

$2 \log_{10} (x + 1) = 2 \log_{10} 20 - \log_{10} 25$

$= 2 \log_{10} 20 - \log_{10} 5^2$

$= 2 \log_{10} 20 – 2 \log_{10} 5$

$2 \log_{10} (x + 1) = 2 (\log_{10} 20 – \log_{10} 5)$

$\log_{10} (x + 1) = (\log_{10} 20 – \log_{10} 5)$

$= \log_{10} (20/5)$

$\log_{10} (x + 1) = \log_{10} 4$

$(x + 1) = 4$

$x = 4 - 1$

$= 3$

28. Show that:

(i) $1/\log_2 42 + 1/\log_3 42 + 1/\log_7 42 = 1$

(ii) $1/\log_8 36 + 1/\log_9 36 + 1/\log_{18} 36 = 2$

Solution:

(i) $1/\log_2 42 + 1/\log_3 42 + 1/\log_7 42 = 1$

Let us consider LHS:

$1/\log_2 42 + 1/\log_3 42 + 1/\log_7 42$

By using the formula, $\log_n m = \log_m / \log_n$

$1/\log_2 42 + 1/\log_3 42 + 1/\log_7 42 = 1/(\log 42/\log_2) + 1/(\log 42/\log_3) + 1/(\log 42/\log_7)$

$= \log_2/\log 42 + \log_3/\log 42 + \log_7/\log 42$

$= (\log_2 + \log_3 + \log_7)/\log 42$

$= (\log 2×3×7)/\log 42$

$= \log 42 / \log 42$

$= \log 42/\log 42$

$= 1$

= RHS

(ii) $1/\log_8 36 + 1/\log_9 36 + 1/\log_{18} 36 = 2$

Let us consider LHS:

$1/\log_8 36 + 1/\log_9 36 + 1/\log_{18} 36$

By using the formula, $\log_n m = \log_m / \log_n$

$1/\log_8 36 + 1/\log_9 36 + 1/\log_{18} 36 = 1/(\log 36/\log_8) + 1/(\log 36/\log_9) + 1/(\log 36/\log_{18})$

$= \log_8/\log 36 + \log_9/\log 36 + \log_{18}/\log 36$

$= (\log_8 + \log_9 + \log_{18})/\log 36$

$= (\log 8\times9\times18)/\log 36$

$= \log 36^2/\log 36$

$= 2 \log 36/\log 36$

$= 2$

= RHS

29. Prove the following identities:

(i) $1/\log_a abc + 1/\log_b abc + 1/\log_c abc = 1$

(ii) $\log_b a. \log_c b. \log_d c = \log_d a$

Solution:

(i) $1/\log_a abc + 1/\log_b abc + 1/\log_c abc = 1$

Let us consider LHS:

$1/\log_a abc + 1/\log_b abc + 1/\log_c abc$

By using the formula, $\log_n m = \log_m / \log_n$

$1/\log_a abc + 1/\log_b abc + 1/\log_c abc = 1/(\log abc/\log_a) + 1/(\log abc/\log_b) + 1/(\log abc/\log_c)$

$= \log_a/ \log abc + \log_b/ \log abc + \log_c/ \log abc$

$= (\log_a + \log_b + \log_c)/ \log abc$

$= (\log a\times b\times c)/ \log abc$

$= \log abc/\log abc$

$= 1$

= RHS

(ii) $\log_b a. \log_c b. \log_d c = \log_d a$

Let us consider LHS:

$\log_b a. \log_c b. \log_d c = (\log a/ \log b) \times (\log b/ \log c) \times (\log c/ \log d)$

$= \log a/\log d$

$= \log_d a$

= RHS

30. Given that $\log_a x = 1/ \alpha$, $\log_b x = 1/\beta$, $\log_c x = 1/\gamma$, find $\log_{abc} x$.

Solution:

It is given that:

$\log_a x = 1/ \alpha$, $\log_b x = 1/\beta$, $\log_c x = 1/\gamma$

So,

$\log_a x = 1/ \alpha \Rightarrow \log x/\log_a = 1/ \alpha \Rightarrow \log_a = \alpha \log x$

$\log_b x = 1/\beta \Rightarrow \log x/\log_b = 1/\beta \Rightarrow \log_b = \beta \log x$

$\log_c x = 1/\gamma \Rightarrow \log x/\log_c = 1/\gamma \Rightarrow \log_c = \gamma \log x$

Now,

$\log_{abc} x = \log x/\log abc$

$= \log x/(\log a + \log b + \log c)$

$= \log x/(\alpha \log x + \beta \log x + \gamma \log x)$

$= \log x/\log x(\alpha + \beta + \gamma)$

$= 1/(\alpha + \beta + \gamma)$

31. Solve for x:

(i) $\log_3 x + \log_9 x + \log_{81} x = 7/4$

(ii) $\log_2 x + \log_8 x + \log_{32} x = 23/15$

Solution:

(i) $\log_3 x + \log_9 x + \log_{81} x = 7/4$

let us simplify the expression,

$1/\log_x 3 + 1/\log_x 9 + 1/\log_x 81 = 7/4$

$1/\log_x 3^1 + 1/\log_x 3^2 + 1/\log_x 3^4 = 7/4$

$1/\log_x 3 + 1/2\log_x 3 + 1/4\log_x 3 = 7/4$

$1/\log_x 3\ [1 + ½ + ¼] = 7/4$

$1/\log_x 3\ [(4+2+1)/4] = 7/4$

$\log_3 x\ [7/4] = 7/4$

$\log_3 x = (7/4) \times (4/7)$

$\log_3 x = 1$

$\log_3 x = \log_3 3$ [Since, $1= \log_a a$]

On comparing, we get

$x = 3$

(ii) $\log_2 x + \log_8 x + \log_{32} x = 23/15$

let us simplify the expression,

$1/\log_x 2 + 1/\log_x 8 + 1/\log_x 32 = 23/15$

$1/\log_x 2^1 + 1/\log_x 2^3 + 1/\log_x 2^5 = 23/15$

$1/\log_x 2 + 1/3\log_x 2 + 1/5\log_x 2 = 23/15$

$1/\log_x 2\ [1 + 1/3 + 1/5] = 23/15$

$\log_2 x\ [(15 + 5 + 3)/15] = 23/15$

$\log_2 x\ [23/15] = 23/15$

$\log_2 x = (23/15) \times (15/23)$

$\log_2 x = 1$

$\log_2 x = \log_2 2$ [Since, $1= \log_a a$]

On comparing, we get
$x = 2$