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1. Classify the following numbers as rational or irrational:

(i) 2 –√5

Solution:

We know that, √5 = 2.2360679…

Here, 2.2360679…is non-terminating and non-recurring.

Now, substituting the value of √5 in 2 –√5, we get,

2-√5 = 2-2.2360679… = -0.2360679

Since the number, – 0.2360679…, is non-terminating non-recurring, 2 –√5 is an irrational number.

(ii) (3 +√23)- √23

Solution:

(3 +√23) –√23 = 3+√23–√23
= 3
= 3/1

Since the number 3/1 is in p/q form, **(3 +√23)- √23** is rational.

(iii) 2√7/7√7

Solution:

2√7/7√7 = (2/7)× (√7/√7)

We know that (√7/√7) = 1

Hence, (2/7)× (√7/√7) = (2/7)×1 = 2/7

Since the number, 2/7 is in p/q form, 2√7/7√7 is rational.

(iv) 1/√2

Solution:

Multiplying and dividing numerator and denominator by √2 we get,

(1/√2) ×(√2/√2)= √2/2 (since √2×√2 = 2)

We know that, √2 = 1.4142…

Then, √2/2 = 1.4142/2 = 0.7071..

Since the number , 0.7071..is non-terminating non-recurring, 1/√2 is an irrational number.

(v) 2π

Solution:

We know that, the value of π = 3.1415

Hence, 2π = 2×3.1415.. = 6.2830…

Since the number, 6.2830…, is non-terminating non-recurring, 2π is an irrational number.

2. Simplify each of the following expressions:

(i) (3+√3)(2+√2)

Solution:

$(3+\sqrt{3})(2+\sqrt{2})$

Opening the brackets, we get, $(3\times2)+(3\times\sqrt{2})+(\sqrt{3}\times2)+(\sqrt{3}\times\sqrt{2})$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(2+\sqrt{2})$

Solution:

$(3+\sqrt{3})(2+\sqrt{2}) = 3^2-(\sqrt{3})^2 = 9-3$

$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Solution:

$(\sqrt{5}+\sqrt{2})^2 = \sqrt{5}^2+(2\times\sqrt{5}\times\sqrt{2})+\sqrt{2}^2$

$= 5+2\times\sqrt{10}+2 = 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Solution:

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5}^2-\sqrt{2}^2) = 5-2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter, (say d). That is, π =c/d. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Solution:

There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realize whether c or d is irrational. The value of π is almost equal to 22/7 or 3.142857…

4. Represent (√9.3) on the number line.

Solution:

Step 1: Draw a 9.3 units long line segment, AB. Extend AB to C such that BC=1 unit.
Step 2: Now, AC = 10.3 units. Let the centre of AC be O.
Step 3: Draw a semi-circle of radius OC with centre O.
Step 4: Draw a BD perpendicular to AC at point B intersecting the semicircle at D. Join OD.
Step 5: OBD, obtained, is a right angled triangle.

Here, OD 10.3/2 (radius of semi-circle), OC = 10.3/2 , BC = 1

OB = OC – BC

$\Longrightarrow (10.3/2)-1 = 8.3/2$

Using Pythagoras theorem,

We get,

$OD^2=BD^2+OB^2$

$\Longrightarrow (10.3/2)^2 = BD^2+(8.3/2)^2$

$\Longrightarrow BD^2 = (10.3/2)^2-(8.3/2)^2$

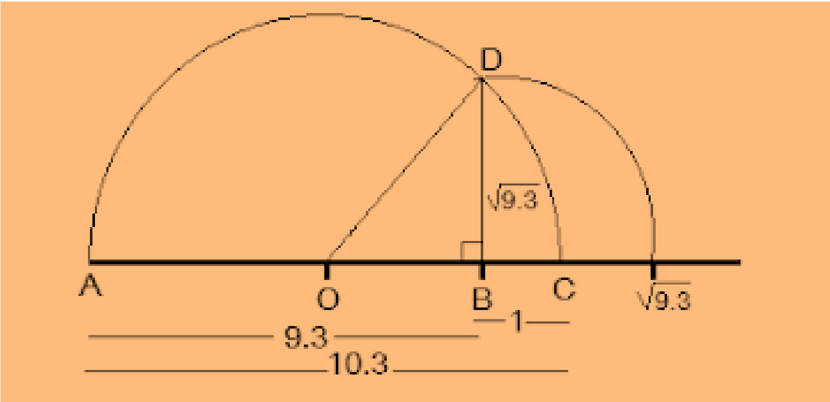
$\Longrightarrow (BD)^2 = (10.3/2)-(8.3/2)(10.3/2)+(8.3/2)$

$\Longrightarrow BD^2 = 9.3$

$\Longrightarrow BD = \sqrt{9.3}$

Thus, the length of BD is √9.3.

Step 6: Taking
BD as radius and B as centre draw an arc which touches the line segment. The point where it touches the line segment is at a distance of √9.3 from O as shown in the figure.



5. Rationalize the denominators of the following:

(i) 1/√7

Solution:

Multiply and divide 1/√7 by √7

(1×√7)/(√7×√7) = √7/7

(ii) 1/(√7-√6)

Solution:

Multiply and divide 1/(√7-√6) by (√7+√6)

[1/(√7-√6)]×(√7+√6)/(√7+√6) = (√7+√6)/(√7-√6)(√7+√6)

= (√7+√6)/$\sqrt{7}^2$-$\sqrt{6}^2$ [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√7+√6)/(7-6)

= (√7+√6)/1

= √7+√6

(iii) 1/(√5+√2)

Solution:

Multiply and divide 1/(√5+√2) by (√5-√2)

[1/(√5+√2)]×(√5-√2)/(√5-√2) = (√5-√2)/(√5+√2)(√5-√2)

= (√5-√2)/($\sqrt{5}^2$-$\sqrt{2}^2$) [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√5-√2)/(5-2)

= (√5-√2)/3

(iv) 1/(√7-2)

Solution:

Multiply and divide 1/(√7-2) by (√7+2)

1/(√7-2)×(√7+2)/(√7+2) = (√7+2)/(√7-2)(√7+2)

= (√7+2)/($\sqrt{7}^2$-2^2) [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√7+2)/(7-4)

= (√7+2)/3