Exercise 1.1 Page: 5

1. Is zero a rational number? Can you write it in the form p/q where p and q are integers and q ≠ 0?

Solution:

We know that, a number is said to be rational if it can be written in the form p/q , where p and q are integers and q ≠ 0.

Taking the case of '0',

Zero can be written in the form 0/1, 0/2, 0/3 … as well as , 0/1, 0/2, 0/3 ..

Since it satisfies the necessary condition, we can conclude that 0 can be written in the p/q form, where q can either be positive or negative number.

Hence, 0 is a rational number.

2. Find six rational numbers between 3 and 4.

Solution:

There are infinite rational numbers between 3 and 4.

As we have to find 6 rational numbers between 3 and 4, we will multiply both the numbers, 3 and 4, with 6+1 = 7 (or any number greater than 6)

i.e., 3 × (7/7) = 21/7

and, 4 × (7/7) = 27/7. ∴The numbers in between 21/7 and 28/7 will be rational and will fall between 3 and 4.

Hence, 22/7, 23/7, 24/7, 25/7, 26/7, 27/7 are the 6 rational numbers between 3 and 4.

3. Find five rational numbers between 3/5 and 4/5.

Solution:

There are infinite rational numbers between 3/5 and 4/5.

To find out 5 rational numbers between 3/5 and 4/5, we will multiply both the numbers 3/5 and 4/5 with 5+1=6 (or any number greater than 5)

i.e., (3/5) × (6/6) = 18/30

and, (4/5) × (6/6) = 24/30

∴The numbers in between18/30 and 24/30 will be rational and will fall between 3/5 and 4/5.

Hence,19/30, 20/30, 21/30, 22/30, 23/30 are the 5 rational numbers between 3/5 and 4/5

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

Solution:

True

Natural numbers- Numbers starting from 1 to infinity (without fractions or decimals)

i.e., Natural numbers= 1,2,3,4…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3…

Or, we can say that whole numbers have all the elements of natural numbers and zero.

∴ Every natural number is a whole number; however, every whole number is not a natural number.

(ii) Every integer is a whole number.

Solution:

False

Integers- Integers are set of numbers that contain positive, negative and 0; excluding fractional and decimal numbers.

i.e., integers= {…-4,-3,-2,-1,0,1,2,3,4…}

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers include whole numbers as well as negative numbers.

∴ Every whole number is an integer; however, every integer is not a whole number.

(iii) Every rational number is a whole number.

Solution:

False

Rational numbers- All numbers in the form p/q, where p and q are integers and q≠0.

i.e., Rational numbers = 0, 19/30 , 2, 9/-3, -12/7…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers includes whole numbers as well as negative numbers.

∴ Every whole numbers are rational, however, every rational numbers are not whole numbers.

Exercise 1.2 Page: 8

1. State whether the following statements are true or false. Justify your answers.
(i) Every irrational number is a real number.

Solution:
True
Irrational Numbers - A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and $q \neq 0$.
i.e., Irrational numbers = 0, 19/30, 2, 9/-3, -12/7, √2, √5, π, 0.102….

Real numbers - The collection of both rational and irrational numbers are known as real numbers.
i.e., Real numbers = √2, √5, π, 0.102…
∴ Every irrational number is a real number, however, every real numbers are not irrational numbers.

(ii) Every point on the number line is of the form √m where m is a natural number.

Solution:
False
The statement is false since as per the rule, a negative number cannot be expressed as square roots.
E.g., √9 =3 is a natural number.
But √2 = 1.414 is not a natural number.
Similarly, we know that there are negative numbers on the number line but when we take the root of a negative number it becomes a complex number and not a natural number.
E.g., √-7 = 7i, where i = √-1
∴ The statement that every point on the number line is of the form √m, where m is a natural number is false.

(iii) Every real number is an irrational number.

Solution:
False
The statement is false, the real numbers include both irrational and rational numbers. Therefore, every real number cannot be an irrational number.
Real numbers - The collection of both rational and irrational numbers are known as real numbers.
i.e., Real numbers = √2, √5, π, 0.102…
Irrational Numbers - A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and $q \neq 0$.
i.e., Irrational numbers = 0, 19/30, 2, 9/-3, -12/7, √2, √5, π, 0.102….
∴ Every irrational number is a real number, however, every real number is not irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution:
No, the square roots of all positive integers are not irrational.
For example,
√4 = 2 is rational.
√9 = 3 is rational.

Hence, the square roots of positive integers 4 and 9 are not irrational. (2 and 3, respectively).

3. Show how √5 can be represented on the number line.

Solution:

Step 1: Let line AB be of 2 unit on a number line.

Step 2: At B, draw a perpendicular line BC of length 1 unit.

Step 3: Join CA

Step 4: Now, ABC is a right angled triangle. Applying Pythagoras theorem,

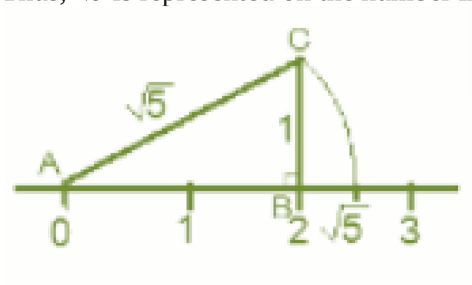
$AB^2+BC^2 = CA^2$

$2^2+1^2 = CA^2 \Rightarrow CA^2 = 5$

$\Rightarrow CA = \sqrt{5}$. Thus, CA is a line of length √5 unit.

Step 4: Taking CA as a radius and A as a center draw an arc touching the number line. The point at which number line get intersected by arc is at √5 distance from 0 because it is a radius of the circle whose center was A.

Thus, √5 is represented on the number line as shown in the figure.



4. Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP1 of unit length. Draw a line segment P1P2 perpendicular to OP_1 of unit length (see Fig. 1.9). Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in Fig. 1.9 :

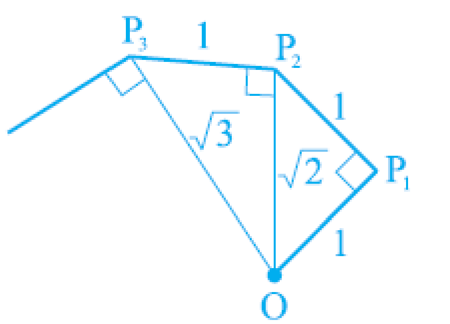
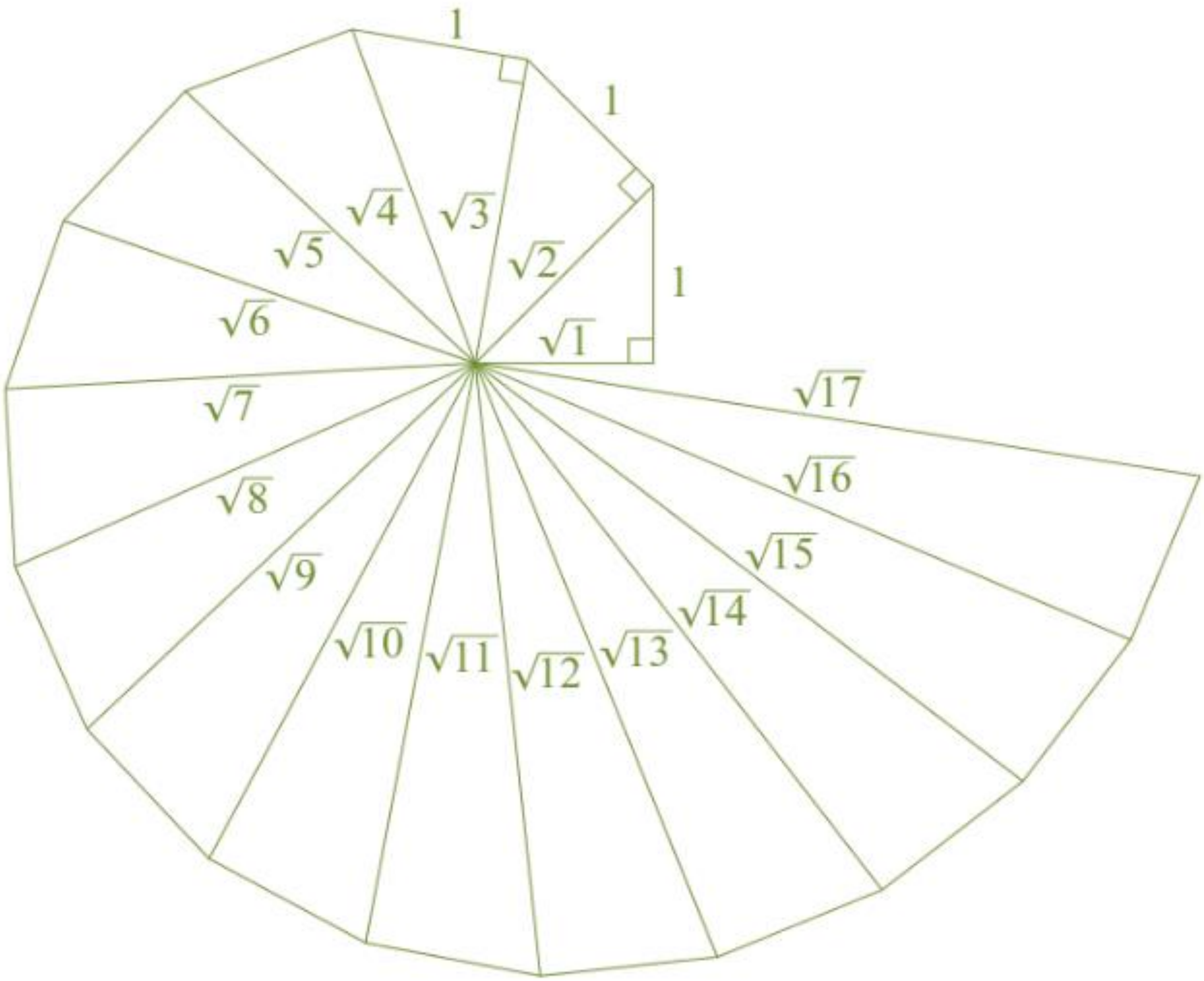


Fig. 1.9 : Constructing square root spiral

Constructing this manner, you can get the line segment P_{n-1}Pn by square root spiral drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points P_2, P_3,....,Pn,... ., and joined them to create a beautiful spiral depicting √2, √3, √4, ...

Solution:



Step 1: Mark a point O on the paper. Here, O will be the center of the square root spiral.
Step 2: From O, draw a straight line, OA, of 1cm horizontally.
Step 3: From A, draw a perpendicular line, AB, of 1 cm.
Step 4: Join OB. Here, OB will be of $\sqrt{2}$
Step 5: Now, from B, draw a perpendicular line of 1 cm and mark the end point C.
Step 6: Join OC. Here, OC will be of $\sqrt{3}$
Step 7: Repeat the steps to draw $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$….

Exercise 1.3 Page: 14

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) 36/100

Solution:

```
       00.36
100 | 360-
    | 300
    |  600-
    |  600
    |    0
```

= 0.36 (Terminating)

(ii)1/11

Solution:

```
     0.0909…
11 | 1
   | 0
   |  10
   |   0
   |  100
   |   99
   |    10
   |     0
   |    100
   |     99
   |      1
```

$= 0.0909\ldots = 0.\overline{09}$ (Non terminating and repeating)

(iii) $4\frac{1}{8}$

Solution:

$$4\frac{1}{8} = \frac{33}{8}$$

```
     4.125
8 | 33
    32
    --
     10
      8
     --
      20
      16
      --
       40
       40
       --
        0
```

= 4.125 (Terminating)

(iv) 3/13

Solution:

```
      0.230769
13 | 30
     26
     --
      40
      39
      --
       10
        0
       --
       100
        91
       ---
         90
         78
         --
         120
         117
         ---
           3
```

$= 0.230769... = 0.\overline{230769}$

(v) 2/11

Solution:

```
     0.18
11 | 2
     0
     --
     20
     11
     --
      90
      88
      --
       2
```

$= 0.181818181818... = 0.\overline{18}$ (Non terminating and repeating)

(vi) 329/400

Solution:

$$
\begin{array}{r|l}
 & 0.8225 \\
400 & 329 \\
 & \underline{0} \\
 & 3290 \\
 & \underline{3200} \\
 & 900 \\
 & \underline{800} \\
 & 1000 \\
 & \underline{800} \\
 & 2000 \\
 & \underline{2000} \\
 & 0
\end{array}
$$

= 0.8225 (Terminating)

2. You know that 1/7 = 0.$\overline{142857}$. Can you predict what the decimal expansions of 2/7, 3/7, 4/7, 5/7, 6/7 are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of 1/7 carefully.]

Solution:

$$1/7 = 0.\overline{142857}$$
$$\therefore 2\times 1/7 = 2\times 0.\overline{142857} = 0.\overline{285714}$$
$$3\times 1/7 = 3\times 0.\overline{142857} = 0.\overline{428571}$$
$$4\times 1/7 = 4\times 0.\overline{142857} = 0.\overline{571428}$$
$$5\times 1/7 = 5\times 0.\overline{142857} = 0.\overline{714285}$$
$$6\times 1/7 = 6\times 0.\overline{142857} = 0.\overline{857142}$$

3. Express the following in the form p/q, where p and q are integers and q ≠ 0.

(i) $\mathbf{0.\overline{6}}$

Solution:

$0.\overline{6} = 0.666\ldots$

Assume that $x = 0.666\ldots$

Then, $10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$x = 2/3$

(ii) $0.\overline{47}$

Solution:

$0.\overline{47} = 0.4777\ldots$

=
(4/10)+(0.777/10)
Assume that $x = 0.777\ldots$
Then, $10x = 7.777\ldots$
$10x = 7 + x$
$x = 7/9$
(4/10)+(0.777../10) = (4/10)+(7/90) ($\therefore$ x = 7/9 and x = 0.777…$\Rightarrow$0.777…/10 = 7/(9×10) = 7/90)
= (36/90)+(7/90) = 43/90

(ii) $0.\overline{001}$

Solution:

$0.\overline{001} = 0.001001\ldots$
Assume that $x = 0.001001\ldots$
Then, $1000x = 1.001001\ldots$
$1000x = 1 + x$
$999x = 1$
$x = 1/999$

4. Express 0.99999.... in the form p/q . Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution:
Assume that $x = 0.9999\ldots$..Eq (a)
Multiplying both sides by 10,
$10x = 9.9999\ldots$. Eq. (b)
Eq.(b) – Eq.(a), we get
$10x = 9.9999\ldots -$
$x = 0.9999\ldots$
$9x = 9$
$x = 1$

The difference between 1 and 0.999999 is 0.000001 which is negligible.
Hence, we can conclude that, 0.999 is too much near 1, therefore, 1 as the answer can be justified.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17 ? Perform the division to check your answer.

Solution:
1/17

Dividing 1 by 17:

```
      0.0588235294117647
17 | 100
     85
     150
     136
      140
      136
        40
        34
         60
         51
          90
          85
           50
           34
           160
           153
             70
             68
              20
              17
               30
               17
               130
               119
                110
                102
                  80
                  68
                  120
                  119
                    100
```

$\frac{1}{17} = 0.\overline{0588235294117647}$

∴ There are 16 digits in the repeating block of the decimal expansion of 1/17.

6. Look at several examples of rational numbers in the form p/q (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solution:

We observe
that when q is 2, 4, 5, 8, 10… Then the decimal expansion is terminating. For example:
1/2 = 0. 5, denominator q = 2^1
7/8 = 0. 875, denominator q =2^3
4/5 = 0. 8, denominator q = 5^1
We can observe that the terminating decimal may be obtained in the situation where prime factorization of the denominator of the given fractions has the power of only 2 or only 5 or both.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.
Solution:
We know that all irrational numbers are non-terminating non-recurring. ∴ three numbers with decimal expansions that are non-terminating non-recurring are:

a) √3 = 1.732050807568

b) √26 =5.099019513592

c) √101 = 10.04987562112

8. Find three different irrational numbers between the rational numbers 5/7 and 9/11.

Solution:

$$\frac{5}{7} = 0.\overline{714285}$$

$$\frac{9}{11} = 0.\overline{81}$$

∴ Three different irrational numbers are:
a) 0.73073007300073000073…

b) 0.75075007300075000075…

c) 0.76076007600076000076…

9. Classify the following numbers as rational or irrational according to their type:

(i)√23
Solution:
√23 = 4.79583152331…

Since the number is non-terminating non-recurring therefore, it is an irrational number.

(ii)√225

Solution:

$\sqrt{225} = 15 = 15/1$

Since the number can be represented in p/q form, it is a rational number.

(iii) 0.3796

Solution:

Since the number,0.3796, is terminating, it is a rational number.

(iv) 7.478478

Solution:

The number,7.478478, is non-terminating but recurring, it is a rational number.

(v) 1.101001000100001…

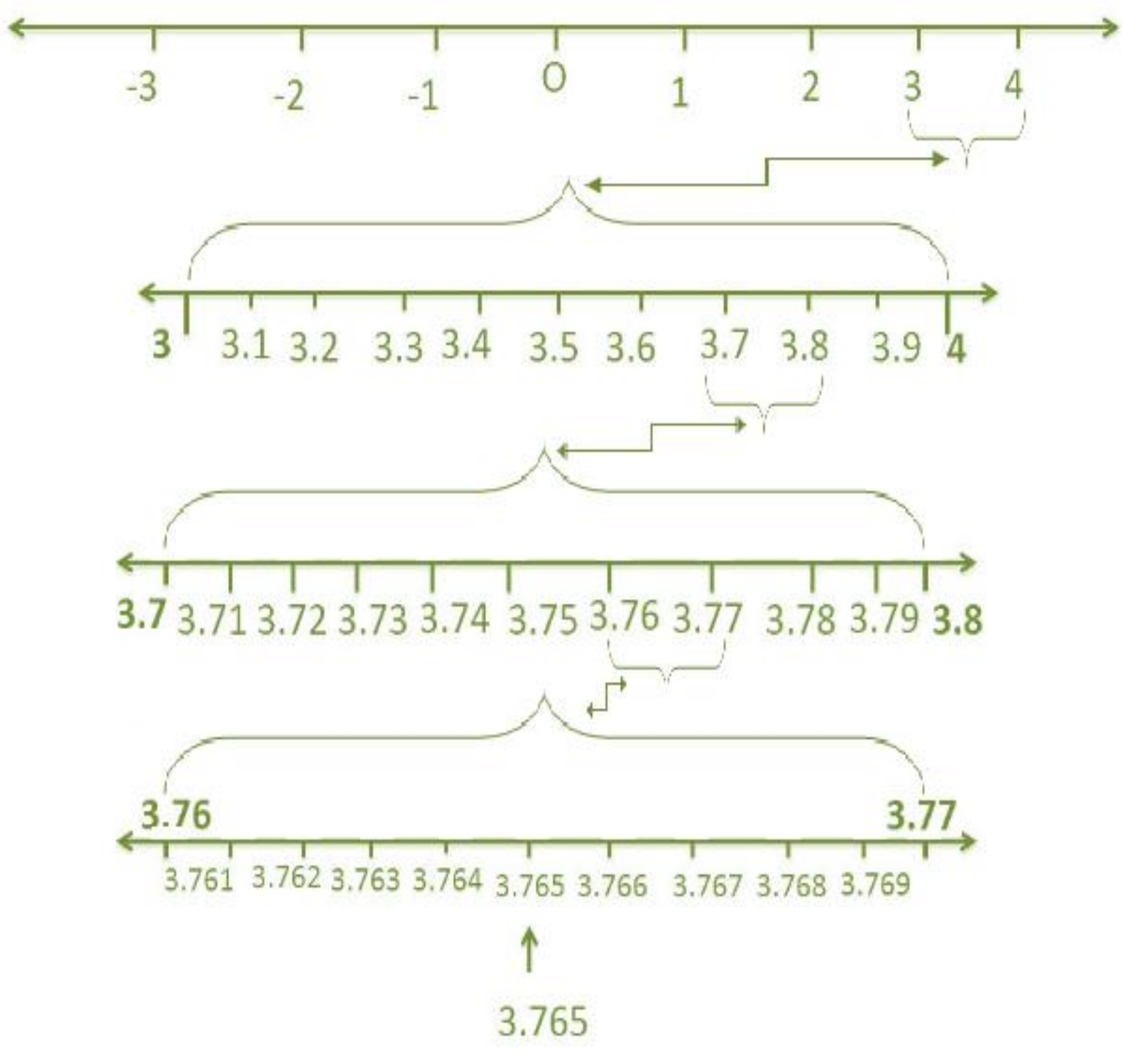
Solution:

Since the number,1.101001000100001…, is non-terminating non-repeating (non-recurring), it is an irrational number.

Exercise 1.4 Page: 18

1. Visualise 3.765 on the number line, using successive magnification.

Solution:

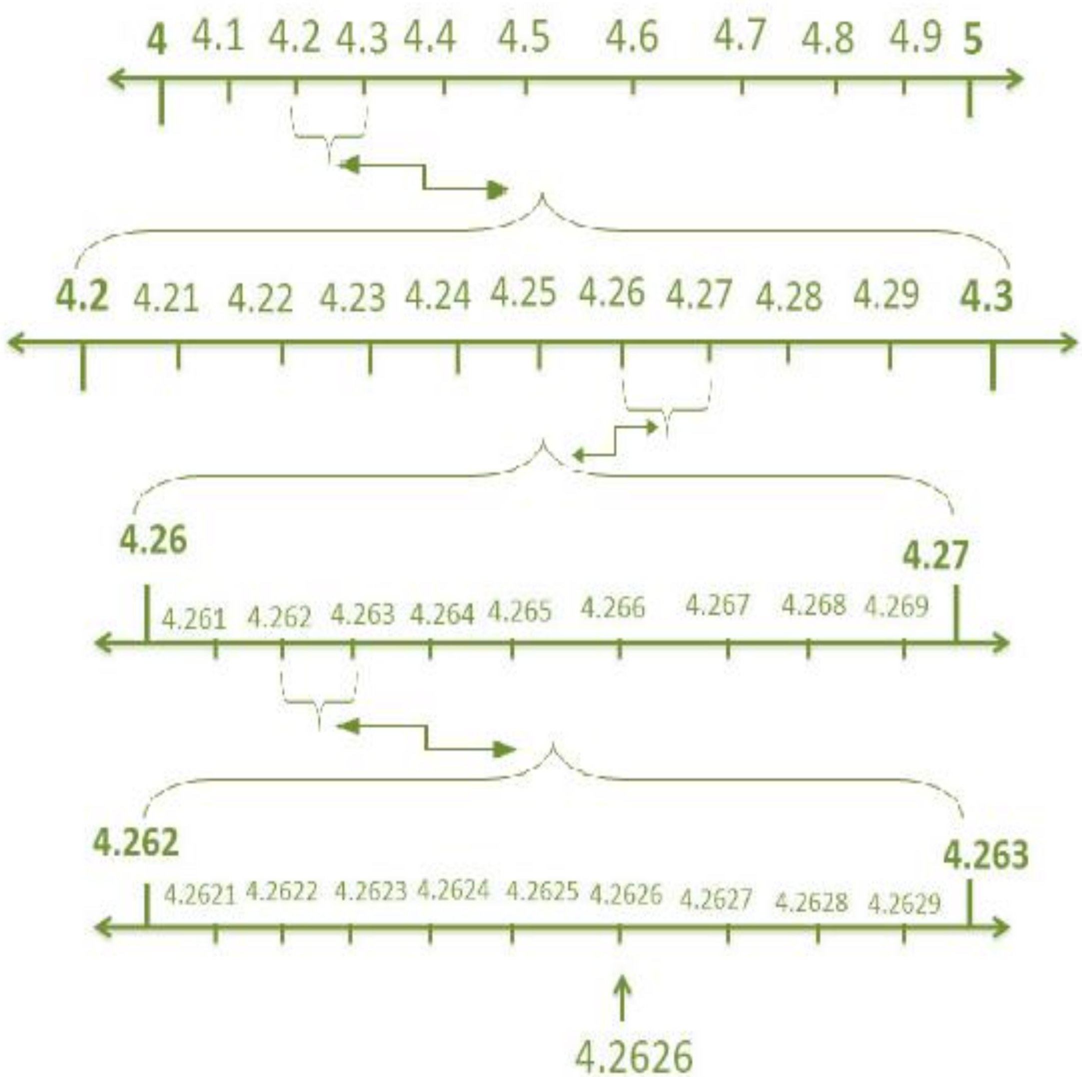


2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Solution:

$4.\overline{26} = 4.26262626.....$

$4.\overline{26}$ up to 4 decimal places= 4.2626



Exercise 1.5 Page: 24

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$

Solution:

We know that, $\sqrt{5} = 2.2360679\ldots$

Here, $2.2360679\ldots$is non-terminating and non-recurring.

Now, substituting the value of $\sqrt{5}$ in $2 - \sqrt{5}$, we get,

$2-\sqrt{5} = 2-2.2360679\ldots = -0.2360679$

Since the number, $-0.2360679\ldots$, is non-terminating non-recurring, $2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

Solution:

$(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23}$

$= 3$

$= 3/1$

Since the number 3/1 is in p/q form, $(3 + \sqrt{23}) - \sqrt{23}$ is rational.

(iii) $2\sqrt{7}/7\sqrt{7}$

Solution:

$2\sqrt{7}/7\sqrt{7} = (2/7) \times (\sqrt{7}/\sqrt{7})$

We know that $(\sqrt{7}/\sqrt{7}) = 1$

Hence, $(2/7) \times (\sqrt{7}/\sqrt{7}) = (2/7) \times 1 = 2/7$

Since the number, 2/7 is in p/q form, $2\sqrt{7}/7\sqrt{7}$ is rational.

(iv) $1/\sqrt{2}$

Solution:

Multiplying and dividing numerator and denominator by $\sqrt{2}$ we get,

$(1/\sqrt{2}) \times (\sqrt{2}/\sqrt{2}) = \sqrt{2}/2$ (since $\sqrt{2} \times \sqrt{2} = 2$)

We know that, $\sqrt{2} = 1.4142\ldots$

Then, $\sqrt{2}/2 = 1.4142/2 = 0.7071..$

Since the number , 0.7071..is non-terminating non-recurring, $1/\sqrt{2}$ is an irrational number.

(v) 2π

Solution:

We know that, the value of $\pi = 3.1415$

Hence, $2\pi = 2 \times 3.1415.. = 6.2830\ldots$

Since the number, 6.2830…, is non-terminating non-recurring, 2π is an irrational number.

2. Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

Solution:

$(3+\sqrt{3})(2+\sqrt{2})$

Opening the brackets, we get, $(3\times 2)+(3\times\sqrt{2})+(\sqrt{3}\times 2)+(\sqrt{3}\times\sqrt{2})$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(2+\sqrt{2})$

Solution:

$(3+\sqrt{3})(2+\sqrt{2}) = 3^2-(\sqrt{3})^2 = 9-3$

$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Solution:

$(\sqrt{5}+\sqrt{2})^2 = \sqrt{5}^2+(2\times\sqrt{5}\times\sqrt{2})+ \sqrt{2}^2$

$= 5+2\times\sqrt{10}+2 = 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Solution:

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5}^2-\sqrt{2}^2) = 5-2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter, (say d). That is, π =c/d. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Solution:

There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realize whether c or d is irrational. The value of π is almost equal to 22/7 or 3.142857…

4. Represent (√9.3) on the number line.

Solution:

Step 1: Draw a 9.3 units long line segment, AB. Extend AB to C such that BC=1 unit.

Step 2: Now, AC = 10.3 units. Let the centre of AC be O.

Step 3: Draw a semi-circle of radius OC with centre O.

Step 4: Draw a BD perpendicular to AC at point B intersecting the semicircle at D. Join OD.

Step 5: OBD, obtained, is a right angled triangle.

Here, OD 10.3/2 (radius of semi-circle), OC = 10.3/2 , BC = 1

OB = OC – BC

$\Rightarrow (10.3/2)-1 = 8.3/2$

Using Pythagoras theorem,

We get,

$OD^2=BD^2+OB^2$

$\Rightarrow (10.3/2)^2 = BD^2+(8.3/2)^2$

$\Rightarrow BD^2 = (10.3/2)^2-(8.3/2)^2$

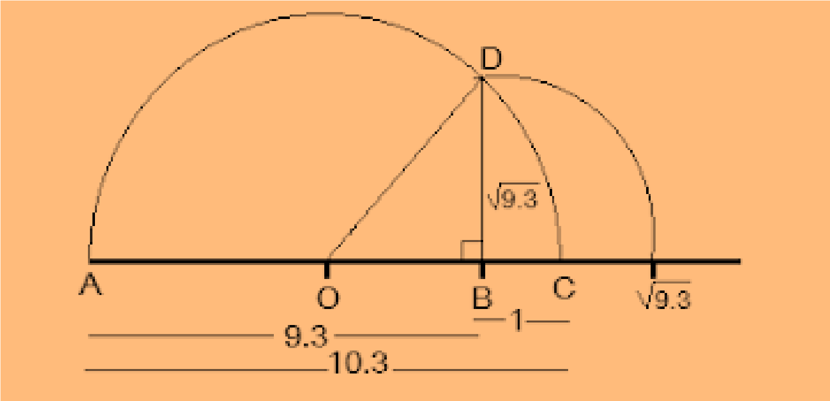
$\Rightarrow (BD)^2 = (10.3/2)-(8.3/2)(10.3/2)+(8.3/2)$

$\Rightarrow BD^2 = 9.3$

$\Rightarrow BD = \sqrt{9.3}$

Thus, the length of BD is √9.3.

Step 6: Taking
BD as radius and B as centre draw an arc which touches the line segment. The point where it touches the line segment is at a distance of √9.3 from O as shown in the figure.



5. Rationalize the denominators of the following:

(i) 1/√7

Solution:

Multiply and divide 1/√7 by √7

(1×√7)/(√7×√7) = √7/7

(ii) 1/(√7-√6)

Solution:

Multiply and divide 1/(√7-√6) by (√7+√6)

[1/(√7-√6)]×(√7+√6)/(√7+√6) = (√7+√6)/(√7-√6)(√7+√6)

= (√7+√6)/$\sqrt{7}^2$-$\sqrt{6}^2$ [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√7+√6)/(7-6)

= (√7+√6)/1

= √7+√6

(iii) 1/(√5+√2)

Solution:

Multiply and divide 1/(√5+√2) by (√5-√2)

[1/(√5+√2)]×(√5-√2)/(√5-√2) = (√5-√2)/(√5+√2)(√5-√2)

= (√5-√2)/($\sqrt{5}^2$-$\sqrt{2}^2$) [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√5-√2)/(5-2)

= (√5-√2)/3

(iv) 1/(√7-2)

Solution:

Multiply and divide 1/(√7-2) by (√7+2)

1/(√7-2)×(√7+2)/(√7+2) = (√7+2)/(√7-2)(√7+2)

= (√7+2)/($\sqrt{7}^2$-2^2) [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√7+2)/(7-4)

= (√7+2)/3

Exercise 1.6

1. Find:
(i)$64^{1/2}$
Solution:
$64^{1/2} = (8\times8)^{1/2}$
$= (8^2)^{½}$
$= 8^1$ [$\because 2\times1/2 = 2/2 = 1$]
$= 8$

(ii)$32^{1/5}$
Solution:
$32^{1/5} = (2^5)^{1/5}$
$= (2^5)^{⅕}$
$= 2^1$ [$\because 5\times1/5 = 1$]
$= 2$

(iii)$125^{1/3}$
Solution:
$(125)^{1/3} = (5\times5\times5)^{1/3}$
$= (5^3)^{⅓}$
$= 5^1$ $(3\times1/3 = 3/3 = 1)$
$= 5$

2. Find:
(i) $9^{3/2}$
Solution:
$9^{3/2} = (3\times3)^{3/2}$
$= (3^2)^{3/2}$
$= 3^3$ [$\because 2\times3/2 = 3$]
$=27$

(ii) $32^{2/5}$
Solution:
$32^{2/5} = (2\times2\times2\times2\times2)^{2/5}$
$= (2^5)^{2/5}$
$= 2^2$ [$\because 5\times2/5 = 2$]
$= 4$

(iii)$16^{3/4}$
Solution:
$16^{3/4} = (2\times2\times2\times2)^{3/4}$
$= (2^4)^{3/4}$
$= 2^3$ [$\because 4\times3/4 = 3$]

$= 8$

(iv) $125^{-1/3}$

$125^{-1/3} = (5\times5\times5)^{-1/3}$

$= (5^3)^{-1/3}$

$= 5^{-1}$ [$\because 3\times-1/3 = -1$]

$= 1/5$

3. Simplify:

(i) $2^{2/3}\times2^{1/5}$

Solution:

$2^{2/3}\times2^{1/5} = 2^{(2/3)+(1/5)}$ [$\because$Since, $a^m\times a^n=a^{m+n}$____ Laws of exponents]

$= 2^{13/15}$ [$\because 2/3 + 1/5 = (2\times5+3\times1)/(3\times5) = 13/15$]

(ii) $(1/3^3)^7$

Solution:

$(1/3^3)^7 = (3^{-3})^7$ [$\because$Since,$(a^m)^n = a^{m \times n}$____ Laws of exponents]

$= 3^{-27}$

(iii) $11^{1/2}/11^{1/4}$

Solution:

$11^{1/2}/11^{1/4} = 11^{(1/2)-(1/4)}$

$= 11^{1/4}$ [$\because(1/2) - (1/4) = (1\times4-2\times1)/(2\times4) = 4-2)/8 = 2/8 = ¼$]

(iv) $7^{1/2}\times8^{1/2}$

Solution:

$7^{1/2}\times8^{1/2} = (7\times8)^{1/2}$ [$\because$Since, $(a^m\times b^m = (a\times b)^m$ ____ Laws of exponents

$= 56^{1/2}$