Exercise Solutions

Question 1: The steam point and the ice point of a mercury thermometer are marked at 80° and 20°. What will be the temperature in centigrade mercury scale when this thermometer reads 32°?

Solution:
Ice point = L_o = 20^o and L_1 = 32^o
Steam point = L_{100} = 80^o
Let T be the temperature in centigrade scale corresponding to 32^o

Using formula,
$T = [L_1 - L_0]/[L_{100} - L_0] \times 100$

= [32-20]/[80-20]

= 20^{o}C

Question 2: A constant volume thermometer registers pressure of 1.500×10^4 Pa at the triple point of water and a pressure of 2.050×10^4 Pa at the normal boiling point. What is the temperature at the normal boiling point?

Solution:

The temperature at triple point pressure "P_{tp}" of water is standardized as, T_{tp} = 273.16 K.

We know, for constant volume gas themometer,

$T = P/P_{tr} \times 273.16$ K

$= [2.050 \times 10^4]/[1.5 \times 10^4] \times 273.16$

= 373.31

Temperature at normal Boiling point is 373.31 K

Question 3: A gas thermometer measures the temperature from the variation of pressure of a sample of gas. If the pressure measured at the melting point of lead is 2.20 times the pressure measured at the triple point of water, find the melting point of lead.

Solution:
Pressure at Melting point= 2.20×Pressure at triple point of water (Given)

$T = P/P_{tr} \times 273.16$ k

$= [2.2 \times p_{tr}]/P_{tr} \times 273.16$ k

= 600.952 K

Question 4: The pressure measured by a constant volume gas thermometer is 40 kPa at the triple point of water. What will be the pressure measured at the boiling point of water (100°C)?

Solution:
Pressure at triple point of water = p_{tr} = 40kPa = 40×10^3 Pa
Boiling point of water = T = 100°C = 373 K
Find the value of P:

We know, $T = P/P_{tr} \times 273.16$ k

=> $P = [T \times P_{tr}]/273.16$ k

$= [373 \times 49 \times 10^3]/273.16$

= 54620 Pa

= 54.64 KPa

Question 5: The pressure of the gas in a constant volume gas thermometer is 70 kPa at the ice point. Find the pressure at the steam point.

Solution:
We know, $T = P/P_{tr} \times 273.16$ k

The pressure at ice point = P_1 = 70kPa
Now,
$T_1 = P_1/P_{tr} \times 273.16$ k

=> $273 = [70\times10^3]/P_{tr} \times 273.16$

=> $P_{tr} = [70\times10^3\times273.16]/273$

Again, $T_2 = P_2/P_{tr} \times 273.16$ k

=> $373 = [P_2\times273]/[70\times10^3\times273.16]$

=> $P_2 = [373\times70\times10^3]/273$

=> P_2 = 95.6 K Pa

Question 6: The pressures of the gas in a constant volume gas thermometer are 80 cm, 90 cm and 100 cm of mercury at the ice point, the steam point and in a heated wax bath respectively. Find the temperature of the wax bath.

Solution:
P_{ice} = Pressure at ice point = 80cm of Hg
T_{ice} = Ice point temperature = 0°C
P_{steam} = Pressure at steam point = 90cm of Hg
T_{steam} = Steam point temperature = 100°C
Pressure of wax bath = P = 100cm of Hg
Let "T" be the temperature of the wax bath.

Find T.

Now,

$$\frac{(P - P_{ice})}{(P_{steam} - P_{ice})} = \frac{(T - T_{ice})}{(T_{steam} - T_{ice})}$$

=>

$$T = \frac{100cm - 80cm}{90cm - 80cm} \times 100°C = 200°C$$

Question 7: In a Callender's compensated constant pressure air thermometer, the volume of the bulb is 1800 cc. When the bulb is kept immersed in a vessel, 200 cc of mercury has to be poured out. Calculate the temperature of the vessel.

Solution:
$v/[v-v'] = T/T_o$

Where, T = temperature of vessel
V = Initial volume = 1800CC
V'= The poured out amount for compensation=200CC
T0 = Ice point temperature= 273K

=> T = 1800/(1800-200) x 273

= 307.125 K

Question 8: A platinum resistance thermometer reads 0° when its resistance is 80Ω and 100° is when resistance is 90 Ω. Find the temperature at the platinum scale at which the resistance is 86 Ω.

Solution:
We know, $T = [R_T - R_o]/[R_{100}-R_o] \times 100$

Where, R_T = Resistance at which temperature is to be measured.
R_0 = Resistance at 0°C
R_{100} = Resistance at 100°C

T = (86-80)/(90-80) x 100

= 60° C

Temperature at 86Ω is 60° C.

Question 9: A resistance thermometer reads R = 20.0Ω, 27.5Ω, and 50.0Ω at the ice point (0°C), the steam point (100°C) and the zinc point (420°C) respectively. Assuming that the resistance varies with temperature as $R_\theta = R_0 (I + \alpha\theta + \beta\theta^2)$, find the values of R_0, α and β. Here θ represents the temperature on the Celsius scale.

Solution:

$R_\theta = R_0(1 + \alpha\,\theta + \beta\,\theta^2)$

The values for 3 resistance (R_θ) are given at 3 temperatures (θ).

At temperature 0° C:

$R_{\theta 1} = R_o(1 + \alpha \times 0 + \beta \times 0^2)$ ….(1)

At temperature 100° C:

$R_{\theta 2} = R_o(1 + \alpha \times 100 + \beta \times 100^2)$ ….(2)

At temperature 420° C:

$R_{\theta 3} = R_o(1 + \alpha \times 420 + \beta \times 420^2)$ ….(3)

Solving these three equations simultaneously for three unknowns,

From (1), $R_{\theta 1} = R_o = 20\ \Omega$

Put value of R_o in (2) and (3)

(2)=> $27.5\ \Omega = 20\ \Omega(1 + \alpha \times 100 + \beta \times 100^2)$ and

(3)=> $50\ \Omega = 20\ \Omega\ (1 + \alpha \times 420 + \beta \times 420^2)$

from eqn.2 after re arranging, we get,

$\alpha = [0.375 - 100^2 \times \beta]/100$ …………(4) and

putting value of α in (3), we get

$50\ \Omega = 20\ \Omega\ (1 + [[0.375-100^2 x\ \beta]/100\ ]\ x420 + \beta\ x\ 420^2)$

Or
$1.5 = 4.2 \times 0.375 - 100^2 \times \beta + 420^2 \times \beta$

Or
$\beta = -5.58 \times 10^{-7}$

Putting the value of β in (4) , we get

$\alpha = 3.806 \times 10^{-3}$

Hence, the required values are
$R_0 = 20\ \Omega$
$\alpha = 3.806 \times 10^{-3}$
$\beta = -5.58 \times 10^{-7}$

Question 10: A concrete slab has a length of 10 m on a winter night when the temperature is 0°C. Find the length of the slab on a summer day when the temperature is 35°C. The coefficient of linear expansion of concrete is 1.0×10^{-5}°C^{-1}

Solution:
The length of the lab at 0° C = L_0 = 10m
Change in temperature of slab from 0°C = θ = 35

Coefficient of linear expansion:
Using relation, $L_t = L_o(1 + \alpha\theta)$ ...(1)
Where,
L_0 = Length at a reference temperature = 10m
α = coefficient of linear expansion = 1.0×10^{-5} °C^{-1}

(1)=> $L_t = 10(1 + 1 \times 10^{-5} \times 35) = 10.0035$ m

Therefore, the length of the slab at 35°C is 10.0035m

Question 11: A metre scale made of steel is calibrated at 20°C to give a correct reading. Find the distance between 50 cm mark and 51 cm mark if the scale is used at 10°C. Coefficient of linear expansion of steel is $1.1 \times 10^{-5}\ ^{o}C^{-1}$.

Solution:

Using relation, $L_t = L_o(1 + \alpha\theta)$ ...(1)
Where,
L_o = Length at a reference temperature
α = coefficient of linear expansion and
θ = Change in temperature.

Here, 20°C can be taken as reference temperature and L_o =length between adjacent centimeter markings = 1 cm or 0.01 m

Also given, θ= -10° C and $\alpha = 1.1 \times 10^{-5}\ °C^{-1}$,

(1) => $L_t = 0.01(1 + 1.1 \times 10^{-5} \times (-10)) = 0.99989$cm

Thus, final length between 50cm and 51cm marks will be 0.99989cm.

Question 12: A railway track (made of iron) is laid in winter when the average temperature is 18°C. The track consists of sections of 12.0 m placed one after the other. How much gap should be left between two such sections so that there is no compression during summer when the maximum temperature goes to 48°C? Coefficient of linear expansion of iron = $11 \times 10^{-6}\ °C^{-1}$

Solution:

Using relation, $L_t = L_o (1 + \alpha\theta)$ ...(1)
Where,
L_o = Length at a reference temperature
α = coefficient of linear expansion and
θ = Change in temperature.

Here, L_o = 12 m and $\alpha = 11 \times 10^{-6}\ °C^{-1}$

At temp 18^o C

(1) => $L_{18} = 12(1 + 11 \times 10^{-6} \times 18) = 12.00237$ m

At temp 48^o C

(1) => $L_{48} = 12(1 + 11 \times 10^{-6} \times 48) = 12.006336$ m

Now, $\Delta L = L_{48} - L_{18} = 12.006336 - 12.00237 = 0.4$ cm

So, then there must be a gap of 0.4 cm between the rails.

Question 13: A circular hole of diameter 2.00 cm is made in an aluminium plate at 0°C. What will be the diameter at 100°C? α for Aluminium = 2.3×10^{-5} $^oC^{-1}$.

Solution:
$d_2 = d_1(1 + \alpha \Delta t)$ ...(1)
Here,
$d_1 = 2m = 2 \times 10^{-2}$ cm
$t_1 = 0$ ^oC and $t_2 = 100$ ^oC
$\alpha = 2.3 \times 10^{-5}$ ^0C

(1) => $d_2 = (2 \times 10^{-2}) (1 + (2.3 \times 10^{-5}) 10^2)$

= 0.020046 m = 2.0046 cm

Question 14: Two-metre scales, one of steel and the other of aluminium, agree at 20^{o}C . Calculate the ratio aluminium -centimetre/steel-centimetre at (a) 0°C, (b) 40°C and (c) 100°C.
α for steel = 1.1×10^{-5} $^oC^{-1}$ and for aluminium 2.3×10^{-5} $^oC^{-1}$

Solution:

Using relation, $L_t = L_o(1 + \alpha\theta)$ ...(1)
Where,
L_0 = Length at a reference temperature

α = Coefficient of linear expansion and
θ = Change in temperature.

Here, L_0 = 12 m and α = 11 × 10^{-6} °C^{-1}

Let 'Al' and 'S' will be used as subscripts to denote Aluminium and Steel respectively.

(a) At temp 0°C

$L_{Al}/L_S = [L_o(1 + \alpha_{Al} (-20))] /[L_o(1 + \alpha_S(-20))]$

$= [1 - 2.3x10^{-5}x20)] /[1 - 1.1x10^{-5} x20]$

= 0.99977

(b) At temp 40°C

$L_{Al}/L_S = [L_o(1 + \alpha_{Al} (20))] /[L_o(1 + \alpha_S(20))]$

$= [1 + 2.3x10^{-5}x20)] /[1 + 1.1x10^{-5} x20]$

= 1.000249

(c) At temp 100°C

$L_{Al}/L_S = [L_o(1 + \alpha_{Al} (80))] /[L_o(1 + \alpha_S(80))]$

$= [1 + 2.3x10^{-5}x80)] /[1 + 1.1x10^{-5} x80]$

= 1.000954

Question 15: A meter scale is made up of steel and measures correct length at 16°C. What will be the percentage error if this scale is used (a) on a summer day when the temperature is 46°C and (b) on a winter say when the temperature is 6°C? Coefficient of linear expansion of steel = 11 × 10^{-6} °C^{-1}.

Solution:
T_0 = 16° = 289 K and α = 11× 10^{-6} C^{-1}

(a) The temperature at which the scale measures during a summer day = T_s = 46°= 319 K.

Therefore, ΔT = 319 – 289 = 30K
The change in length due to linear expansion, ΔL = lα ΔT

Where l is length of the meter scale.

=> ΔL/l = α ΔT

Now, percentage error = ((ΔL/l) x 100)%

=> α ΔT x 100 = 11 x 10^{-6} x 30 x 100 = 0.033%

(b) T_w = 6° = 279 K

and Δ T = 289 – 279 = 10K

Now, percentage error = ((ΔL/l) x 100)%

> α ΔT x 100 = 11 x 10^{-6} x 10 x 100 = 0.011%

Question 16: A metre scale made of steel reads accurately at 20°C. In a sensitive experiment, distances up to 0.055 mm in 1 m are required. Find the range of temperature in which the experiment can be performed with this metre scale. Coefficient of linear expansion of steel = 11 × 10^{-6} °C^{-1}.

Solution:
We know, ΔL = L_o α ΔT
Where Δ L is the change in length and Δ T is the change in temperature.

0.055 x 10^{-3} = 11 x 10^{-6} x 1 x (20 ± T_2)

=> 5 = 20 ± T_2

=> $T_2 = 15^oC$
Either $T_2 = 15^oC$ or $T_2 = 25^oC$

The experiment can be performed with the given metre scale is 15° C to 25° C.
Question 17: The density of water at 0°C is 0.998 g cm^{-3} and at 4°C is 1.000g cm^{-3}. Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C.

Solution:
Density of water at 0°C = ρ_0 = 0.998 g cm^{-3}
Density of water at 4°C: ρ_4 = 1.000 g cm^{-3}
Temperature Range= θ = 4°C

We know, $f_4 = f_0(1 + \gamma\Delta t)$

=> $f_o = f_4/(1 + \gamma\Delta t)$

=> $0.998 = 1/(1 + \gamma \times 4)$

=> $\gamma = 0.0005 = 5 \times 10^{-4}\ {}^oC^{-1}$

As the density decreases,

$\gamma = -5 \times 10^{-4}\ {}^oC^{-1}$

Hence the average Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C is $-5 \times 10^{-4}\ {}^oC^{-1}$.

Question 18: Find the ratio of the lengths of an iron rod and an aluminium rod for which the difference in the lengths is independent of temperature. Coefficients of linear expansion of iron and aluminium are $12 \times 10^{-6}\ °C^{-1}$ and $23 \times 10^{-6}\ °C^{-1}$ respectively.

Solution:
Let the original length of iron rod be L_{Fe} and
original length of aluminium rods be L_{Al}
Let, L'_{Fe} and L'_{Al} be the changed lengths (ΔL) when temperature is changed by ΔT.

Given:
Coefficient of Linear Expansion of iron rod = α_{Fe} = 12 x 10^{-6} °C^{-1}
Coefficient of Linear Expansion of Aluminium Rod = α_{Al} = 20 x 10^{-6} °C^{-1}

Now,
$L'_{Fe} = L_{Fe}(1 + \alpha_{Fe}\Delta T)$ and $L'_{Al} = L_{Al}(1 + \alpha_{Al}\Delta T)$

Difference in lengths is independent of temperature it shows that their difference is constant.
=> $L'_{Fe} - L'_{Al} = L_{Fe} - L_{Al}$ ....(A)

Therefore, $L'_{Fe} - L'_{Al} = L_{Fe}(1 + \alpha_{Fe}\Delta T) - L_{Al}(1 + \alpha_{Al}\Delta T)$

$= L_{Fe} - L_{Al} + L_{Fe}\alpha_{Fe}\Delta T - L_{Al}\alpha_{Al}\Delta T$ ......(1)

Using equation (A)

(1)=> $L_{Fe}\alpha_{Fe}\Delta T = L_{Al}\alpha_{Al}\Delta T$

=> $L_{Fe} / L_{Al} = 23/12$

Ratio of the lengths of an iron rod and an aluminium rod is 23:12.

Question 19: A pendulum clock gives correct time at 20°C at a place where g = 9.800 m s^{-2}. The pendulum consists of a light steel rod connected to a heavy ball. It is taken to a different place where g = 9.788 m s^{-2}. At what temperature will it give correct time? Coefficient of linear expansion of steel = 12 × 10^{-6} °C^{-1}.

Solution:
From question, we are given
Temperature at which the pendulum shows correct time = T_1 = 20°C
Value of gravitational acceleration at a place where T_1 is 20°C = g_1 = 9.8 m s^{-2}
Value of g at different places = g_2 = 9.788 m s^{-2}
Coefficient of linear expansion of steel = α = 12 x 10-6 °C^{-1}

Let, T_2 be the temperature at a place where value of g = 9.788 m s^{-2}.

We know, $t = 2\pi \sqrt{(l/g)}$
Where, l is the length of the rod.

Now, $t_1 = 2\pi \sqrt{(l_1/g_1)}$ and $t_2 = 2\pi \sqrt{(l_2/g_2)}$

For obtaining correct time both the time periods should be same. This implies: $t_1 = t_2$.

$\Rightarrow 2\pi \sqrt{(l_1/g_1)} = 2\pi \sqrt{(l_2/g_2)}$

$\Rightarrow \sqrt{(l_1/g_1)} = \sqrt{(l_2/g_2)}$ ...(1)

Also, we know, changes in length due to linear expansion, $l_2 = l_1(1 + \alpha\Delta T)$

Substituting this result in equation (1), we get

$$\sqrt{\frac{l_1}{g_1}} = \sqrt{\frac{l_1(1+\alpha\Delta T)}{g_2}}$$

$$\frac{l_1}{g_1} = \frac{l_1(1+\alpha\Delta T)}{g_2}$$

$(g_2/g_1) = 1 + 12 \times 10^{-6}\ {}^{o}C^{-1} \times \Delta T$

$\Rightarrow [9.788/9.8] = 1 + 12 \times 10^{-6}\ {}^{o}C^{-1} \times \Delta T$

$\Rightarrow \Delta T = -102.04\ {}^{o}C$
Because: $\Delta T = T_2 - T_1$

$\Rightarrow T_2 - T_1 = -102.04$

$\Rightarrow T_2 = -102.04 + 20 = -82.04\ {}^{o}C$

Question 20: An aluminium plate fixed in a horizontal position has a hole of diameter 2.000 cm. A steel sphere of diameter 2.005 cm rests on this hole. All the lengths refer to a temperature of 10°C. The temperature of the entire system is slowly increased. At what temperature will the ball fall down? Coefficient of linear expansion of aluminium is $23 \times 10^{-6}\ °C^{-1}$ and that of steel is $11 \times 10^{-6}\ °C^{-1}$.

Solution:

From question, we are given

Let d_{Al} be diameter of the hole in the aluminium plate and d_{st} be diameter of the Steel sphere resting on the hole.

Initial temperature = $T_1 = 10\ °\ C$

α_{Al} and α_{st} be the Coefficient of linear expansion of aluminium and Coefficient of linear expansion of Steel respectively.

Here,

$d_{Al} = 2.000$ cm, $d_{st} = 2.005$ cm, $\alpha_{Al} = 23 \times 10^{-6}\ °C^{-1}$ and $\alpha_{st} = 11 \times 10^{-6}\ °C^{-1}$

Now, change in diameter of Steel sphere and aluminium plate:

$d'_{st} = d_{st}(1 + \alpha_{st}\ \Delta T)$ and $d'_{Al} = d_{Al}\ (1 + \alpha_{Al}\ \Delta T)$

Where d'_{st} and d'_{Al} changed diameters.

For the steel ball to fall through the hole the changed diameter of the aluminum plate and steel ball should be equal.

=> $d'_{st} = d'_{Al}$

=> $d_{st}(1 + \alpha_{st}\ \Delta T) = d_{Al}\ (1 + \alpha_{Al}\ \Delta T)$

=> $2.005(1 + 11\text{x}10^{-6} \text{ x } \Delta T) = 2\ (1 + 23\text{x}10^{-6} \text{ x } \Delta T)$

=> $-5 \text{ x } 10^{-3} = -23.945 \text{ x } 10^{-6} \text{ x } \Delta T$

=> $\Delta T = 208.811$

Because: $\Delta T = T_2 - T_1$

=> $T_2 - T_1 = 208.811$

=> $T_2 = 208.811 + 10 = 218.811\ ^oC$

Question 21: A glass window is to be fit in an aluminium frame. The temperature on the working day is 40°C and the glass window measures exactly 20 cm × 30 cm. What should be the size of the aluminium frame so that there is no stress on the glass in winter even if the temperature drops to 0°C? Coefficients of linear expansion for glass and aluminium are $9.0 \times 10^{-6}\ °C^{-1}$ and $24\times10^{-6}\ °C^{-1}$ respectively.

Solution:

From question, we are given
Let l_{Gl} and b_{Gl} be length and breadth of the Glass window.

α_{Gl} and α_{Al} be the Coefficient of linear expansion for glass and aluminium respectively.

Here,
l_{Gl} = 20 cm, b_{Gl} = 30 cm, $\alpha_{Gl} = 9.0 \times 10^{-6}\ °C^{-1}$ and $\alpha_{Al} = 24 \times 10^{-6}\ °C^{-1}$

Now, $l' = l(1 - \alpha\ \Delta T)$
[l' is the changed length and the value of α is negative]

Now,Let the changed length of aluminium frame = Changed length of glass window.

=> $l'_{AL} = l'_{Gl}$

=> $l_{Al}(1 - \alpha_{Al}\ \Delta T) = l_{Gl}\ (1 - \alpha_{Gl}\ \Delta T)$

$$l_{Al} = \frac{20\ cm \times (1 - 9.0 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40\ )}$$

$$l_{Al} = \frac{20 \times 0.99964}{0.99904}\ cm$$

=> l_{Al} = 20.01 cm

Similarly, changed breadth of aluminium frame = Changed breadth of Glass window

=> $b'_{AL} = b'_{Gl}$

=> $b_{Al}(1 - \alpha_{Al}\,\Delta T) = b_{Gl}\,(1 - \alpha_{Gl}\,\Delta T)$

$$b_{Al} = \frac{30\ cm \times (1 - 9.0 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40)}$$

$$b_{Al} = \frac{30 \times 0.99964}{0.99904}\ cm$$

=> b_{Al} = 30.018 cm

Therefore, dimensions are: 20.01 cm x 30.018 cm.

Question 22: The volume of a glass vessel is 1000 cc at 20°C. What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature? Coefficients of cubical expansion of mercury and glass are $1.8 \times 10^{-4}\ °C^{-1}$ and $9.0 \times 10^{-6}\ °C^{-1}$ respectively.

Solution:
Volume of remaining space = $V'_g - V'_{Hg}$

Here V_g = 1000 CC, $T_1 = 20^oC$, $\gamma_{Hg} = 1.8x10^{-4}\ ^oC$ and $\gamma_g = 9.0 \times 10^{-6}\ °C^{-1}$.
V_{Hg} = ?

The remaining space should not change with temperature which means: Initial = Final

$V'_g - V'_{Hg} = V_g - V_{Hg}$ ...(1)

Now,

$V'_g = v_g(1 + \gamma_g\,\Delta T)$

$V'_{Hg} = v_{Hg}(1 + \gamma_{Hg}\,\Delta T)$

=> $V'_g - V'_{Hg} = v_g(1 + \gamma_g \Delta T) - v_{Hg}(1 + \gamma_{Hg} \Delta T)$

Using (1),

=> $v_g/v_{Hg} = \gamma_{Hg}/\gamma_g$

=> $(\gamma_g v_g)/\gamma_{Hg} = v_{Hg}$

=> $v_{Hg} = [1000\text{x}9\text{x}10^{-6}]/[1.8\text{x}10^{-4}] = 50$ CC

Question 23: An aluminium can of cylindrical shape contains 500 cm^3 of water. The area of the inner cross section of the can is 125 cm^2. All measurements refer to 10°C. Find the rise in the water level if the temperature increases to 80°C. The coefficient of linear expansion of aluminium = 23×10^{-6} °C^{-1} and the average coefficient of volume expansion of water = 3.2×10^{-4} °C^{-1} respectively.

Solution:
Volume of water in Aluminium Can : $V_w = 500$ cm^3
Area of the inner cross section of the can : A = 125 cm^2.
Initial Temperature while measuring: $T_1 = 10$ ° C.
Increased temperature : $T_2 = 80$ °C
Thus, Change in Temperature : $\Delta T = T_2 - T_1 = 80-10 = 70$ ° C.
The coefficient of linear expansion of aluminium : $\alpha = 23\times10^{-6}$ °C^{-1}.

The Average coefficient of volume expansion of water:
$\Gamma = 3.2\times10^{-4}$ °C^{-1}.

Now,
increased volume of water= $v'_w = v_w(1 + \gamma\Delta T)$

$= 500(1 + 3.2 \text{ x } 10^{-4} \text{ x } 70)$

$= 511.2$ cm^3
The aluminium can will be expanded too due to temperature increase.

Rise in volume of water = increased volume of water - Volume of water in Aluminium Can

= 511.2 - 500

= 11.2 cm^3

Now,

Rise in water level = [Rise in volume]/[Ares of cross section]

= 11.2 cm^3/125cm^2

= 0.0896 cm

Question 24: A glass vessel measures exactly 10 cm × 10 cm × 10 cm at 0°C. It is filled completely with mercury at this temperature. When the temperature is raised to 10°C, 1.6 cm^3 of mercury overflows. Calculate the coefficient of volume expansion of mercury. Coefficient of linear expansion of glass = 6.5×10^{-6} $°C^{-1}$.

Solution:
V_g = 10cm x 10cm x 10cm = 1000 cm^3.
V_{Hg} = 1000 cm^3.
T_1 = 0 °C and T_2 = 10 ° C.

v_g = volume of a glass vessel and v_{Hg} = Volume of mercury in the vessel

T_1 = Temperature at which glass is filled completely with mercury:

T_2 = Increased temperature

Change in temperature: $\Delta T = T_2 - T_1$ = 10 ° C.

Coefficient of Volume Expansion of Glass: γ_g: $3\times \alpha_g = 3\times 6.5\times 10^{-6}$ $°C^{-1}$

Now,
Volume of mercury overflown is equal to the difference in changed volumes of mercury and glass respectively.

$v'_{Hg} = v'_g = 1.6$ cm^3

$v_{Hg}(1 + \gamma_{Hg}\,\Delta T) - v_g(1 + \gamma_g\,\Delta T) = 1.6$ cm^3

$1000(1 + \gamma_{Hg} \times 10) - 1000(1 + 3 \times 6.5 \times 10^{-6} \times 10) = 1.6$ cm^3

$10000 \times \gamma_{Hg} - 0.196 = 1.6$

$\Rightarrow \gamma_{Hg} = [1.6+0.195]/10000 = 1.795 \times 10^{-4}\ {}^{o}C^{-1}$

Question 25: The densities of wood and benzene at 0°C are 880 kg m^3 and 900 kg m^{-3} respectively. The coefficients of volume expansion are 1.2×10^{-3} °C^{-1} for wood and 1.5×10^{-3} °C^{-1} for benzene. At what temperature will a piece of wood just sink in benzene?

Solution:
Let V' is the changed volume due to change in temperature ΔT.

For wood: $v'_w = v_w(1 + \gamma_w\,\Delta T)$
For Benzene: $v'_b = v_b(1 + \gamma_b\,\Delta T)$

Using Density-Volume Relation,
Changed Density of wood: $\rho'_w = M_w / v'_w$
Changed Density of benzene: $\rho'_b = M_b / v'_b$

Let's equate the final densities of wood and benzene to obtain the required condition, i.e. $\rho'_w = \rho'_b$

$\Rightarrow M_w / v'_w = M_b / v'_b$

$$\frac{M_w}{V_w(1+\gamma_w \Delta T)} = \frac{M_b}{V_b(1+\gamma_b \Delta T)}$$

$$\frac{\rho_w}{(1+\gamma_w \Delta T)} = \frac{\rho_b}{(1+\gamma_b \Delta T)}$$

$$\frac{880}{1+1.2 \times 10^{-3} \times \Delta T} = \frac{900}{(1+1.5 \times 10^{-3} \times \Delta T)}$$

Solving above equations for ΔT, we get

$\Delta T = 20/0.24$

or $\Delta T = T_2 - T_1 = 20/0.24$

=> $T_2 = 83.33 + 0^oC = 83.33$

Therefore, At 83.33 °C the piece of wood will just sink in benzene.

Question 26: A steel rod of length 1 m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed?

Solution:

When the steel rod is heated up to 100 °C , due to thermal expansion it’s length will increase with temperature.
A longitudinal strain develops when there exists an opposing force to the expansion of the length. Hence, there will be zero longitudinal strain as there is no opposite in this case.

Question 27: A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is unstrained at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel = 1.2×10^{-5} °C^{-1}.

Solution:
for linear thermal expansion of a body, $\Delta L = L\alpha\Delta T$

Where ΔL is the Change in Length and L is the initial length at T_1.

Given: $\alpha = 1.2 \times 10^{-5}$ °C^{-1} and $\Delta T = T_1 - T_2 = 30$ °C

=> $\Delta L/L = \alpha\ \Delta T$

$= 1.2 \times 10^{-5} \times 30$

$= 3.6 \times 10^{-4}$

But, Strain is Change in length divided by original length:
$S = \Delta L/L$

So, $S = 3.6 \times 10^{-4}$

Hence, Longitudinal strain developed in the rod is 3.6×10^{-4}

Question 28: A steel wire of cross-sectional area 0.5 mm^2 is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is 1.2×10^{-5} °C^{-1} and its Young's modulus is 2.0×10^{11} N m^{-2}.

Solution:
$A = 0.5\ mm^2 = 0.5 \times 10^{-6}\ m^2$.
$T_1 = 20$ ° C.
$T_2 = 0$ °C
$\Delta T = 20$ ° C.
$\alpha_s = 1.2 \times 10^{-5}$ °C^{-1}
and $Y = 2.0 \times 10^{11}$ N m^{-2}.

We know, Young's Modulus: Y = Stress/Strain = $(F/A)/(\Delta L/L)$

=> $Y = [FL]/[A\Delta L]$ ...(1)

Where, F is the force or tension between the wire and the fixed supports and
A is the cross -section area.

For Linear Expansion: $\Delta L = L\alpha\ \Delta T$ ....(2)

ΔL is the Change in Length due to decrease in temperature and L is the original length at T1.

Using (2) in (1), we have

$Y = [FL]/[A\ \alpha\ L\ \Delta T]$

or F = YA L α_s ΔT

F = YAα_s ΔT
= 2 x 10^{11} x 0.5 x 10^{-6} x 1.2 x 10^{-5} x 20
= 24

=> F = 24 N, the tension in the wire when the temperature falls to 0°C is 24 N.

Question 29: A steel rod is rigidly clamped at its two ends. The rod is under zero tension at 20°C. If the temperature rises to 100°C, what force will be rod exert on one of the clamp? Area of cross section of the rad = 2.00 mm^2. Coefficient of linear expansion of steel = 12.0 × 10^{-6} $°C^{-1}$ and Young's modulus of steel = 2.00 × 10^{11} N m^{-2}.

Solution:
T_1 = 20 ° C, and T_2 = 100 ° C.
So, Δ T= $T_2 - T_1$ = 100 - 20 = 80 ° C.

A = 2.00 mm^2 = 2.00 × 10^{-6} m^2.
α = 12.0 × 10^{-6} $°C^{-1}$
Y = 2.00 × 10^{11} N m^{-2}.

We know, Young's Modulus: Y = Stress/Strain = (F/A)/(ΔL/L)

=> Y = [FL]/[AΔL] ...(1)

Where, F is the force or tension between the wire and the fixed supports and
A is the cross -section area.

For Linear Expansion: ΔL = Lα ΔT ....(2)

Using (2) in (1), we have

Y = F/[A α ΔT]

or F = YA L α ΔT

= 2 x 10^{11} x 2 x 10^{-6} x 12 x 10^{-6} x 80

= 384

=> F = 384 N

when the temperature is increased to 100° C , the rod will exert a force of 384N on one of the champ.

Question 30: Two steel rods and an aluminium rod of equal length l_0, and equal cross-section are joined rigidly at their ends as shown in the figure bellow. All the rods are in a state of zero tension at 0°C. Find the length of the system when the temperature is raised to θ. Coefficient of linear expansion of aluminium and steel are α_a and α_s respectively. Young's modulus of aluminium is Y_a and steel is Y_s.

Steel
Aluminium
Steel

Solution:
At $T_1 = 0°$ C
Length of two steel rods and one aluminium rod joined rigidly at T_1= l
Coefficient of linear expansion of aluminium : α_a
Coefficient of linear expansion of steel are : α_s
Young's modulus of aluminium : Y_a
Young's modulus of steel : Y_s
Increased Temperature : $T_2 = \theta$
Change in temperature : $\Delta T = \theta - 0 = \theta$

We know, Young's Modulus: Y = Stress/Strain = (F/A)/(Δl/l)

=> Y = [Fl]/[AΔl] ...(1)

For Linear Expansion: Δl = lα ΔT ....(2)

Using (2) in (1), we have

Y = F/[A α ΔT]

or F/A = stress = Y α ΔT ...(1)

The Total Young's Modulus of whole system:

Total Y = [Total Stress]/[Total Strain]

Now, Total stress = stress due to two steel rods + stress due to one aluminium rod

= (2 Y_s x α_s x θ) + Y_a x α_a x θ) ....(2)
[using (1)]

Total Young's Modulus = (2 x Y_s) + Y_a .....(3)

Total Strain = [Total Stress]/Total Y]

$$\frac{\Delta l}{l} = \frac{2Y_s\alpha_s\theta + Y_a\alpha_a\theta}{2Y_s + Y_a}$$

$$l' - l = l \times \left(\frac{2Y_s\alpha_s\theta + Y_a\alpha_a\theta}{2Y_s + Y_a}\right)$$

$$l' = l\left(1 + \frac{2Y_s\alpha_s\theta + Y_a\alpha_a\theta}{2Y_s + Y_a}\right)$$

Which is the length of the system when temperature is increase to θ.

Question 31: A steel ball initially at a pressure of 1.0×10^5 Pa is heated from 20°C to 120°C keeping its volume constant. Find the pressure inside the ball. Coefficient of linear expansion of steel = 12×10^{-6} °C^{-1} and bulk modulus of steel = 1.6×10^{11} N m^{-2}.

Solution:
Pressure = P = BγΔθ
Where P is the pressure inside the ball.
B = Bulk modulus of steel = 1.6 x 10^{11} Nm^{-2}
Δθ = Change in temp = 100°C

(Here, $\gamma = 3\alpha$; given $\alpha = 12 \times 10^{-6}\ {}^{o}C$)

=> $P = 1.6 \times 10^{11} \times 3 \times 12 \times 10^{-6} \times 100$
$= 5.8 \times 10^{8}$ Pa

Question 32: Show that moment of inertia of a solid body of any shape changes with temperature as $I = I_0(1 + 2\alpha\theta)$, where I_0 is the moment of inertia at 0°C and α is the coefficient of linear expansion of the solid.

Solution:
Moment of inertia at 0° C = I_0
Change in temperature : $\Delta T = \theta - 0 = \theta$

We know, $I_0 = MR_o^2$ ...(1)

Where M is the mass of the body, I_0 is the initial moment of inertia at 0° and R_0 is the radius of gyration at 0° C.

When temperature of the solid body increases, the radius of gyration also changes due to thermal expansion.

Let I be the changed moment of inertia when temperature is θ.

$I = MR'^2$ ....(2)

Where $R' = R_o(1 + \alpha\ \Delta T)$; R' is the changed radius of gyration due to expansion.

(2)=> $I = M[R_o(1 + \alpha\ \Delta T)]^2$

$= MR_o^2\ (1 + 2\alpha\theta + \alpha^2\ \theta^2)$

[Given: $\Delta T = \theta$]

$\alpha^2\ \theta^2$ is neglected, we get

$I = I_0 (1 + 2\alpha\theta)$
[Using (1)]

Hence Proved.

Question 33: A torsional pendulum consists of a solid disc connected to a thin wire ($\alpha = 2.4 \times 10^{-5}$ °C^{-1}) at its centre. Find the percentage change in the time period between peak winter (5°C) and peak summer (45°C).

Solution:
Temperature change in winter: $\Delta T_1 = 5°$ C
Temperature change in summer : $\Delta T_2 = 45°$ C
and $\alpha = 2.4 \times 10^{-5}$ °C^{-1}

Due to change in temperature the moment of inertia will also change.
So,
$I' = I_o(1 + 2\alpha \Delta T)$

Where, I' is the changed moment of inertia and I_0 is the initial moment of inertia at 0 ° C

We know, time period for torsional pendulum: $T = 2\pi\sqrt{[I/K]}$ ...(1)
Here, k is torque constant of the wire.

During winter:

$I' = I_o(1 + 2\alpha \Delta T_1)$

(1)=> $t_1 = 2\pi\sqrt{[I'/K]}$

$= 2\pi\sqrt{[(I_0 (1 + 2\alpha \Delta T_1)/K]}$

$$t_1 = 2\pi\sqrt{\frac{I_0(1 + 2 \times 2.4 \times 10^{-6} \times 5)}{\mathrm{k}}}$$

$$t_1 = 2\pi\sqrt{\frac{I_0(1.000024)}{k}}$$

During Summer:

I' = I_0 (1 + 2α ΔT_2)

(1)=> $t_1 = 2\pi\sqrt{[I'/K]}$

$$t_2 = 2\pi\sqrt{\frac{I_0(1+2\alpha\Delta T_2)}{k}}$$

$$t_2 = 2\pi\sqrt{\frac{I_0(1+2\times 2.4\times 10^{-5}\times 45)}{k}}$$

$$t_2 = 2\pi\sqrt{\frac{I_o(1.000216)}{k}}$$

Now, Let us find percentage change in the time period between peak winter (5°C) and peak summer (45°C) :

% change = [change in quantity]/[original quantity] = $[t_2 - t_1]/t_1$

or % change = $(t_2/t_1) - 1)$ x 100
Using values of t_1 and t_2, we get

$$\text{\% Change} = \left(\frac{2\pi\sqrt{\frac{I_o(1.00216)}{k}}}{2\pi\sqrt{\frac{I_0(1.000024)}{k}}} - 1\right)\times 100$$

$$= \left(\frac{\sqrt{1.00216}}{\sqrt{1.00024}} - 1\right)\times 100$$

$$= (1.00095931 - 1)\times 100$$

=> % change = $9.59 \times 10^{-4} \times 100 = 0.0959$ %

Question 34: A circular disc made of iron is rotated about its axis at a constant velocity ω. Calculate the percentage change in the linear speed of a particle of the rim as the disc is slowly heated from 20°C to 50°C keeping the angular velocity constant. Coefficient of linear expansion of iron = 1.2×10^{-5} °C^{-1}.

Solution:
We know, Angular Velocity: $\omega = v/r$
Where v and r be the velocity and radius of the particle.

When temperature increased, the disc undergoes thermal expansion.
Let v be the velocity of particle at T_1 and v' be the velocity of particle at T_2.
and r be the radius of particle at T_1 and r' be the changed radius of particle at T_2.

From question, we have
$T_1 = 20$ ° C
$T_2 = 50$ ° C
$\Delta T = 50-20 = 30$° C
$\alpha = 1.2 \times 10^{-5}$ °C^{-1}

Now,
Angular Velocity at T_1: $\omega_1 = v/r$
Angular Velocity at T_2 : $\omega_2 = v'/r'$

Also we know, Thermal linear expansion of radius is $r' = r(1 + \alpha\ \Delta T)$

ω is constant even after heating the disc, so

$\omega = v/r = v'/r'$

=> $v/r = v'/[r(1 + \alpha\ \Delta T)]$

=> $v' = v(1 + \alpha\ \Delta T)$

$= v(1 + 1.2\times10^{-5}\times30) = 1.00036\ v$

=> v' = 1.00036 v

Let us find the percentage change in the linear speed of a particle of the rim when the disc is slowly heated from 20°C to 50°C :

% change = [change in quantity]/[original quantity] = [(v' - v)/v] x 100

= [(1.00036v - v)/v] x 100

= (1.00036 -1) x 100

= 0.036