Exercise Solutions

Question 1: The amount of charge passed in time t through a cross-section of a wire is $Q(t) = At^2 + Bt + C$.

(a) Write the dimensional formulae for A, B and C.
(b) If the numerical values of A, B and C are 5, 3 and 1 respectively in S.I. units, find the value of the current at t = 5 s.

Solution:

Amount of the charge: $Q(t) = At^2 + Bt + C$

On comparing the dimensions of each term with the dimension of the charge, we have

$Q = At^2$

$A = Q/t^2 = It/t^2 = I/t$

$\Rightarrow A = [AT^{-1}]$

Again, $Bt = Q$

$\Rightarrow B = Q/t = I = [A]$

Also, $Q = C$

$\Rightarrow C = [AT]$

(b) $I = dQ/dt = d/dt(At^2+Bt+C)$

$= 2At + B$

Here A = 5, B = 3 and t = 5s

$\Rightarrow I = 53$ A

Question 2: An electron gun emits 2.0×10^{16} electrons per second. What electric current does this correspond to?

Solution:
In time, t = 1s, seconds 2.0×10^{16} electrons are emitted.

Total charge flowing from the gun = q = ne

= 2 x 10^{16} x 1.6 x 10^{-19} C

= 3.2 x 10^{-3} C

Corresponding current = i = q/t = [3.2x10^{-3}]/1 = 3.2 x 10^{-3} A

Question 3: The electric current existing in a discharge tube is 2.0 μA. How much charge is transferred across a cross-section of the tube in 5 minutes?

Solution:
Time (t) for which charge transfer = 5 min = 300s
Electric current in the tube = 2.0 μA

Charge transferred across the cross-section in 300s = q = i t = 2 x 300 = 6 x 10^{-4} C

Question 4: The current through a wire depends on time as
$i = i_0 + \alpha t$,
Where i_0 = 10 A and α = 4 As^{-1}. Find the charge crossed through a section of the wire in 10 seconds.

Solution:

The current through a wire depends on time
$i = i_0 + \alpha t$ ...(1)

Where, i_0 = 10A, t = 10s, and a = 4 As^{-1}

t = time for which current passes

For the given expression of current, charge q is,

q = ∫i dt

From 0_s = 0 to t_s and using (1), we get

$q = i_0 t + at^2/2$

= 10 x 10 + (4x10^{-4})/2

= 300 C

Therefore, charge crossed through a section of the wire in 10 seconds is 300C.

Question 5: A current of 1.0 A exists in a copper wire of cross-section 1.0 mm^2. Assuming one free electron per atom calculate the drift speed of the free electrons in the wire. The density of copper is 9000 kg m^{-3}.

Solution:
Cross section of the wire, A= 1mm^2 = 10^{-6} m^2

Density of Copper, ρ = 9000 kg m^{-3}

current in the wire, i = 1 A

We know, i = $nAeV_d$ .....(1)

Which is the current due to 'n' freely bounded electrons per unit volume with a drift speed 'V_d'

and e = charge of the electron = 1.6×10^{-19}C.

First find "n":

we know, 63.5 grams of Copper have 6.022×10^{23} of atoms. So 'm' Kilograms have,

[6.022×10^{23} x mKg]/[63.5×10^{-3} Kg]

Mass in terms of density, m = unit volume x ρ

The number of free electrons = [6.022×10^{23} x 9000Kgm^{-3}]/[63.5×10^{-3} Kg]
[by dividing expression by unit volume]

Now, solving above we have n = 8.535 x 10^{28}

(1)=> V_d = i/nAe

Substituting the known values, we have

V_d = 7.32 x 10^{-5} ms^{-1} = 0.074 mms^{-1}

Question 6: A wire of length 1 m and radius 0.1 mm has a resistance of 100 Ω. Find the resistivity of the material.

Solution:
We know, the resistivity, ρ, of a wire with cross-sectional area A and length l,

$\rho = RA/l$ ...(1)

Where : R is the resistance offered by the wire.

Area of cross-section of the wire = $A = \pi r^2 = \pi (0.1 \times 10^{-3}\ m)^2 = 3.14 \times 10^{-8}\ m^2$

$R = 100\Omega$ and $l = 1m$ (Given)

(1)=> $\rho = 3.14 \times 10^{-6}\ \Omega\ m$

Question 7: A uniform wire of resistance 100 Ω is melted and recast in a wire of length double that of the original. What would be the resistance of the wire?

Solution:
Let R_1 = Initial Resistance of the wire = 100 Ω

l_1= Initial length of wire and l_2 = Final length of wire

Therefore, $l_2 = 2\ l_1$

We know, resistance of a wire = $R = \rho l/A$ ....(1)

We know that the volume remains same after the recast in a wire.

So, $A_1\ l_1 = A_2 \times l_2$

using $l_2 = 2\ l_1$

=> $A_2 = A_1/2$

Initial and final Resistance of the wire: $R_1 = \rho_1 l_1/A_1$ and $R_2 = \rho_2 l_2/A_2$

Since $\rho 1 = \rho_2$.

=> $R_2/R_1 = l_2A_2/l_1A_1 = 2l_1A_1/l_1(A_2/2)$

=> $R_2 = 4R_1 = 4 \times 100 = 400\Omega$

Question 8: Consider a wire of length 4 m and cross-sectional area 1 mm^2 carrying a current of 2 A. If each cubic meter of the material contains 10^{29} free electrons, find the average time taken by an electron to cross the length of the wire.

Solution:
Current through the wire = I = 2A
Area of cross-section = A = 1 mm^2 = 10^{-6} m^2
Number of free electron per unit volume= n = 10^{29}
Length of wire = l= 4m

We know, V_d = i/nAe

V_d = drift velocity

Here, e is the charge of 1 electron= 1.6×10^{-19} C

V_D in terms of length and time, V_D = l/t

=> i/nAe = l/t

t = nAel/i

= $[10^{29} \times 10^{-6} \times 1.6\times10^{-19}\times4]/2$ = 32000 s = 8.89 hours

Therefore, average time taken by an electron to cross the length of the wire is approximately 8.9 hours.

Question 9: What length of a copper wire of cross-section area 0.01 mm^2 will be needed to prepare a resistance of 1 kΩ? Resistivity of copper = 1.7×10^{-8} Ω m.

Solution:
Resistivity of wire = 1000Ω
Resistivity of copper, = 1.7×10^{-8} Ω m
Area of cross-section = A=0.01 mm2 = 0.01×10^{-6} m^2

We know, R = ρl/A

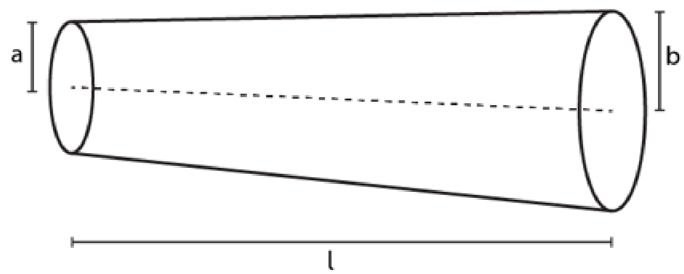
or l = AR/ρ

= $[(0.01\times10^{-6})\times1000]/[1.7\times10^{-8}]$

= 588.23 m

= 0.6 km (approx)

Question 10: Figure shows conductor of length ℓ having a circular cross-section. The radius of cross-section varies linearly from a to b. The resistivity of the material is ρ. Assuming that b – a << ℓ, find the resistance of the conductor.



Solution:

Resistance of a cylinder with length 'l' and area of cross-section A is,

R = ρl/A

From the figure, relation connecting x and y: (b-a)/l = (y-a)/x

or x(b-a) = l(y-a)

Differentiating above expression w.r.t. x and rearranging, we have

dx = ldy/(b-a) ...(1)

Let us assume that, cone is made up of an infinite number of cylinders with length 'dx'.

So for that element, resistance dR:

dR = ρdx/A

Again, resistance for an element with a radius 'y' at x distance from left with length dx:

dR = ρdx/A In this case, A = πy^2

=> dR = $\rho dx/\pi y^2$ ...(2)

To get total resistance (R) integrate above expression form the range a to b and using (1), we have

$$R = \int_a^b \frac{\rho \times \frac{l}{(b-a)}}{\pi y^2}\, dy$$

$$= \rho \times \frac{l}{\pi(b-a)} \int_a^b \frac{dy}{y^2}$$

$$= \rho \times \frac{l}{\pi(b-a)} \times \left(\frac{-1}{y}\right)_a^b$$

$$= \rho \times \frac{l}{\pi ab}$$

Which is the resistance of the truncated cone.

Question 11: A copper wire of radius 0.1 mm and resistance 1 kΩ is connected across a power supply of 20 V.
(a) How many electrons are transferred per second between the supply and the wire at one end?
(b) Write down the current density in the wire.

Solution:
r= 0.1mm = 0.1×10^{-3}m
R= 1000Ω and V = 20V

(a) We know, ne/t = V/R

[using relations, I = V/R, I = q/t and q = ne]

n = Vt/eR

Where, n = number of electrons passed in 1s through the copper wire with resistance 1000Ω

For time t= 1s

n = [20x1]/[1.6×10^{-19}x1000]

= 1.25×10^{17} electrons

(b)
area of cross-section of a wire = A = πr^2

= $\pi (0.1\times10^{-3})^2 = 3.1415 \times 10^{-8}\ m^2$

Therefore, I = 20/1000 = 0.02 A

[using relation, I = V/R]

Now, current density, J = I/A

=> $J = 0.02/[3.1415 \times 10^{-8}]$

$= 6.366 \times 10^{5}\ Am^{-2}$

Question 12: Calculate the electric field in a copper wire of cross-section area 2.0 mm^2 carrying a current of 1 A. The resistivity of copper = $1.7 \times 10^{-8}\ \Omega$ m.

Solution:
Area of cross-section of wire = A=2.0 $mm^2 = 2\times10^{-6}\ m^2$
Current flowing through the wire = I = 1A
Resistivity of copper = $\rho = 1.7 \times 10^{-8}\ \Omega$ m

We know, the Electric field, E = V/l ...(1)
Where l is the distance over which potential difference V

Again from Ohm's law, we have R = ρl/A ...(2)
[relation connecting resistance, R and resistivity, ρ]

Where 'l' is the length of the wire.

And, potential difference, V is V = IR ....(3)

using (2) and (3) relations, we have

E = Iρl/Al = Iρ/A

$= [1 \times 1.7 \times 10^{-8}]/[2 \times 10^{-6}]$

=> $E = 0.0085\ m^{-1}$

Question 13: A wire has a length of 2.0 m and a resistance of 5.0 Ω. Find the electric field existing inside the wire if it carries a current of 10 A.

Solution:
Length of the wire= l = 2m
Resistance of the wire= R = 5 Ω

Current passing through the wire= I = 10A

We know, the Electric field, E = V/l ...(1)
Where l is the distance over which potential difference V

We know that the potential difference, V = IR = 50 V
[Using given values]

=> E = 50/2 = 25 V/m

Question 14: The resistances of an iron wire and a copper wire at 20°C and 3.9 Ω and 4.1 Ω respectively. At what temperature will the resistances be equal? Temperature coefficient of resistivity for iron is $5.0 \times 10^{-3}\ K^{-1}$ and for copper it is $4.0 \times 10^{-3}\ K^{-1}$. Neglect any thermal expansion.

Solution:
The resistances of an iron wire and a copper wire at 20°C and 3.9 Ω and 4.1 Ω respectively.

$R_{(Fe,\ i)}$ at 20°C = 3.9 Ω and $R_{(Cu,\ i)}$ at 20°C = 4.1 Ω

Initial temperature of both the wires = T_i = 20°C
Temp coefficient of resistivity for copper = $\alpha_{Cu} = 4.0 \times 10^{-3}\ K^{-1}$
Temp coefficient of resistivity for iron = $\alpha_{Fe} = 5.0 \times 10^{-3}\ K^{-1}$

Relation connecting the resistance with the change in temperature:

$R_f = R_i(1 + \alpha\Delta T)$ ...(1)

Where α is the Temperature coefficient of resistivity and ΔT is the change in temperature

and $\Delta T = T_f - T_i$

Now,

Final resistance of iron wire: $R_{Fe,f} = R_{Fe,i}(1 + \alpha_{Fe}\ \Delta T)$

$= 3.9\ x(1 + (5x10^{-3})\ \Delta T)$

Final resistance of copper wire: $R_{Cu,f} = R_{Cu,i}(1 + \alpha_{Cu}\ \Delta T)$

$= 4.1\ x(1 + (4x10^{-3})\ \Delta T)$

At the final temperature, it is given that the resistance of both the wires are same.

=> $R_{Fe,f} = R_{Cu,f}$

$3.9 \times (1 + (5\times10^{-3})\ \Delta T) = 4.1 \times (1 + (4\times10^{-3})\ \Delta T)$
On solving above, we have

$\Delta T = 64.5^oC$

Now, $\Delta T = T_f - T_i$

=> $T_f = 64.5^oC + 20^oC = 84.5^oC$, at this temperature resistances are equal.

Question 15: The current in a conductor and the potential difference across its ends are measured by an ammeter and a voltmeter. The meters draw negligible currents. The ammeter is accurate but the voltmeter has a zero error (that is, it does not read zero when no potential difference is applied). Calculate the zero error if the readings for two different conditions are 1.75 A, 14.4 V and 2.75 A, 22.4 V.

Solution:
Ammeter reading of current in the 1st case = i_1 =1.75A
Voltmeter reading for current i_1, V_1=14.4V

Ammeter reading of current in the 2nd case= i_2 = 2.75A
Voltmeter reading for current i_2, V_2 = 22.4V

Let V be the magnitude of zero error in the voltmeter reading.

Under given conditions, the resistance of the wire will remain same. So

$R_1 = R_2$

=> $I_1/I_2 = (V_1-V)/(V_2-V)$

[using relation, V = IR]

=> 1.75/2.75 = (14.4-V)/(22.4-V)

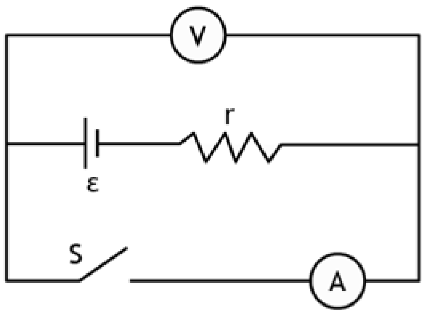
=> 7/11 = (14.4-V)/(22.4-V)

Solving above for V, we get

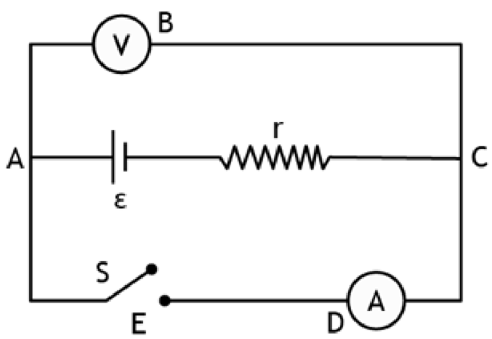
V = 0.4 V

Therefore, the zero error associated with the measurement is 0.4V. This value can be -ve or +ve, these values just indicate that the needle of the voltmeter is the left or right of the zero if zero voltage applied across the voltmeter.

Question 16: Figure shows an arrangement to measure the emf ϵ and internal resistance r of a battery. The voltmeter has a very high resistance and the ammeter also has some resistance. The voltmeter reads 1.52 V when the switch S is open. When the switch is closed the voltmeter reading drops to 1.45 V and the ammeter reads 1.0 A. Find the emf and the internal resistance of the battery.



Solution:



Case 1: When switch is open, the current would circulate through loop ABCA only. Since the internal resistance is very small compared to the resistance of the voltmeter, the voltage drop occurs entirely at the voltmeter. This voltage drop is measured in the measuring device and almost corresponds to the emf of the cell.

=> e.m.f of cell= volt meter reading.

So, emf = 1.52 V

Case 2: When the switch is closed, current 'i' will flow through the loop ACDEA. The voltmeter shows the potential drop across the cell and the internal resistance together. Let us find the internal resistance of the cell, V= ε - ir

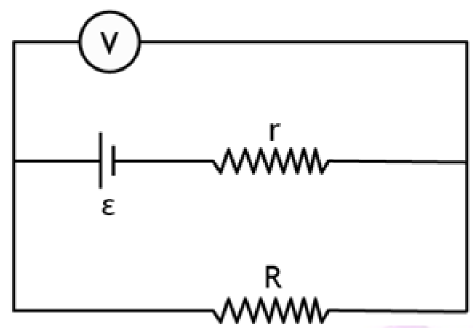
Where V = Volt meter reading = 1.45V, ϵ = 1.52V and i = Ammeter reading = 1A

or r = (ε-V)/i

or r = [1.52-1.45]/1 = 0.07 Ω

Question 17: The potential difference between the terminals of a battery of emf 6.0 V and internal resistance 1 Ω drops to 5.8 V when connected across an external resistor. Find the resistance of the external resistor.

Solution:



The current flowing through the internal and external resistance are the same.
We know, The voltage across the cell will be equal to that across the external resistance.
So, E - ir = iR = V

In this case, E - ir = V

6V - i x 1Ω = 5.8 V

or i = 0.2 A

Also, we know V = iR

0.2 A x R = 5.8 V

=> R = 29 Ω, is the resistance of the external resistor.

Question 18: The potential difference between the terminals of a 6.0 V battery is 7.2 V when it is being charged by a current of 2.0 A. What is the internal resistance of the battery?

Solution:
The current flowing through the internal and external resistance are the same.
We know, The voltage across the cell will be equal to that across the external resistance.

So, E - ir = iR = V

In this case, r = (V-E)/i

Given: V = 7.2V ; E = 6V and i = 2A

=> r = 0.6Ω, is the internal resistance of the cell.

Question 19: The internal resistance of an accumulator battery of emf 6 V is 10 Ω when it is fully discharged. As the battery gets charged up, its internal resistance decreases to 1Ω. The battery in its completely discharged state is connected to a charger which maintains a constant potential difference of 9V. Find the current through the battery
(a) just after the connections are made and
(b) after a long time when it is completely charged.

Solution:
(a) When the battery is being charged, the net emf, E_{net} across the resistance, r_1 is

$E_{net} = E_c - E$

E = emf of the battery = 6V
E_c = Potential difference provided by the charger = 9V

=> E_{net} = 3 V

Also, we know $V = ir_1$

or $i = V/r_1 = 3/10 = 0.3$ A

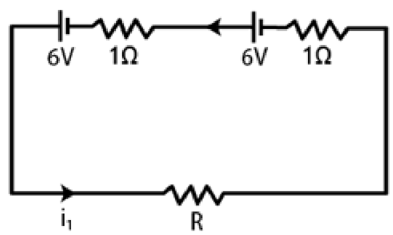
Given: The internal resistance of the battery, r_1 when discharged = 10Ω

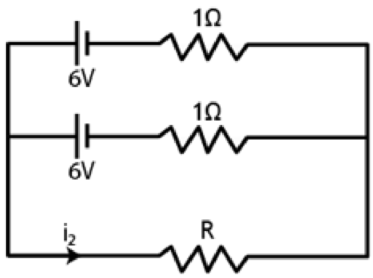
(b) When the battery is completely charged, the internal resistance r_2 will be 1Ω.

Now, i = 3/1 = 3 A

Therefore, current through the internal resistance while charging and after completely charged are 0.3A and 3A respectively.

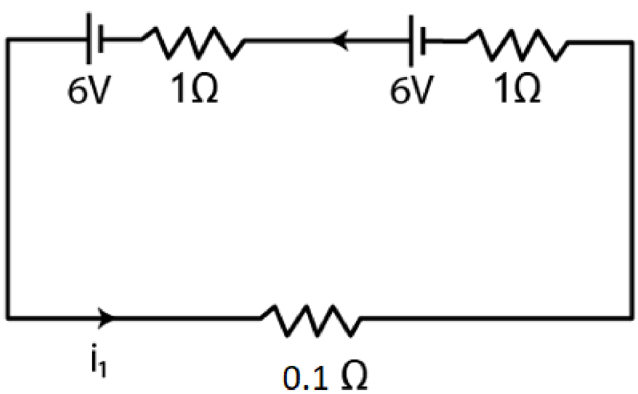
Question 20: Find the value of i_1/i_2 in figure if
(a) R = 0.1Ω, (b) R = 1 Ω (c) R = 10 Ω.
Note from answers that in order to get more current from a combination of two batteries they should be joined in parallel if the external resistance is small and in series if the external resistance is large as compared to the internal resistances.





Solution:

(a)



Applying Loop rule, we have

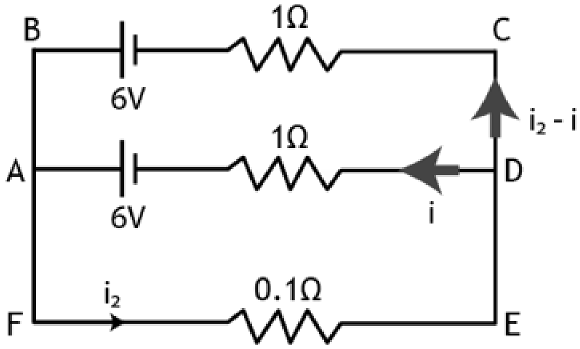
$6\text{ V} + 6\text{V} = i_1 \times 0.1\Omega + i_1 \times 1\Omega + i_1 \times 1\Omega$

[Using relation, V = iR]

or $12\text{ V} = i_1\, 2.1\ \Omega$

or $i_1 = 5.71$ A

Now, from figure



Current i passes through the middle branch and i_2 - i will pass through the upper branch, apply loop rule in AFEDA

$6\ V = i_2 \times 0.1\Omega + i \times 1\Omega$

or $i = 6 - 0.1\ i_2$

Again, apply loop rule in ADCBA

$-6V - i \times 1\Omega + (i_2 - i) \times 1\Omega + 6V = 0$

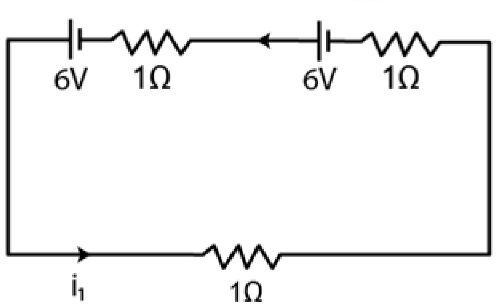
or $i_2 = 2i$

Using results of both the loops,

$i_2 = 10$ A and $i_1 = 5.71$ A

Their ratio: $i_1/ i_2 = 5.71\ A/10\ A = 0.57$

(b)

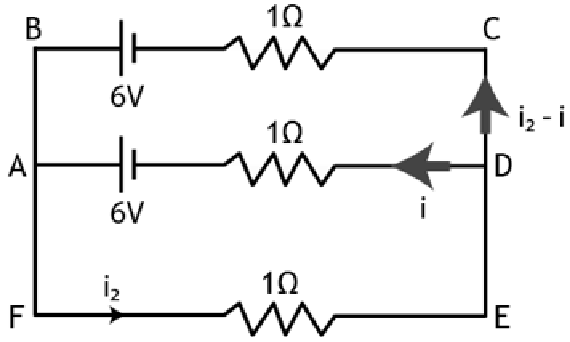


Applying Loop rule,
$6\ V + 6V = i_1 \times 1\Omega + i_1 \times 1\Omega + i_1 \times 1\Omega$

$12V/3\ \Omega = i_1$

Or $i_1 = 4$ A

From second figure,



By applying loop rule in AFEDA,

$6\text{ V} = i_2 \times 1\Omega + i \times 1\Omega$

or $i = 6 - i_2$

Again, apply loop rule in ADCBA

$-6V - i \times 1\Omega + (i_2 - i) \times 1\Omega + 6V = 0$

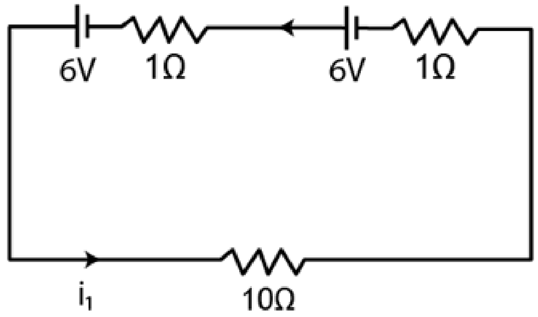
or $i_2 = 2i$

Using results of both the loops,

$i_2 = 4$ A and $i_1 = 4$ A

Their ratio: $i_1/ i_2 = 4\text{ A}/4\text{ A} = 1$

(c)

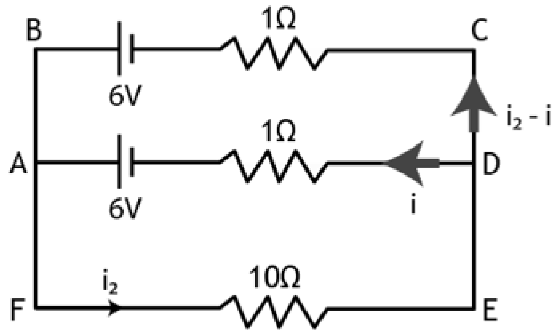


Applying Loop rule,
$6\ V + 6V = i_1 \times 10\Omega + i_1 \times 1\Omega + i_1 \times 1\Omega$

$12V/12\ \Omega = i_1$

Or $i_1 = 1\ A$

From second figure,



By applying loop rule in AFEDA,

$6\ V = i_2 \times 10\Omega + i \times 1\Omega$

or $i = 6 - 10\ i_2$

Again, apply loop rule in ADCBA

$-6V - i \times 1\Omega + (i_2 - i) \times 1\Omega + 6V = 0$

or $i_2 = 2i$

Using results of both the loops,

$i_2 = 0.57\ A$ and $i_1 = 1\ A$

Their ratio: $i_1/ i_2 = 4\ A/0.57\ A = 1.75$

Question 21: Consider $N = n_1 n_2$ identical cells, each of emf ϵ and internal resistance r. Suppose n_1 cell are joined in series to form a line and n_2 such lines are connected in parallel. The combination drives a current in an external resistance R.
(a) Find the current in the external resistance.
(b) Assuming that n_1 and n_2 can be continuously varied, find the relation between n_1, n_2, R and r for which the current in R is maximum.

Solution:

(a) From given, Total emf in every branches are the same.

Let us calculate for one branch:
When n_1 cells each with emf 'E' are connected in series.

Total emf, E_{net} is $E_{net} = n_1 E$

The resistance of n_1 cells each with resistance 'r' in series is $R_o = n_1 r$

Total resistance for such n_2 number of branches, connected in parallel:

$R_{eff} = n_1 r/n_2$

Total net resistance when whole setup is connected to an external resistance R. will be,

$R_{net} = n_1 r/n_2 + R$

Let i be the current through the external resistor i.e. $i = E_{net}/R_{net}$

or $i = n_1 n_2 E/[n_1 r+n_2 R]$ ...(1)

(b)
To get the minimum current through the resistor R, the denominator of equation (1) must be minimum.

Now, $n_1 r+n_2 R = (\sqrt{n_1 r} - \sqrt{n_2 R})^2 + 2\sqrt{(n_1 n_2 r)}$

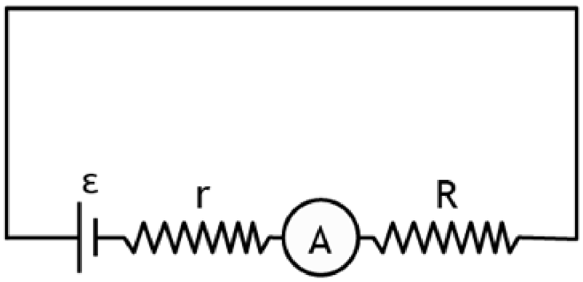
$n_1 r + n_2 R$ is only minimum when $\sqrt{n_1 r} = \sqrt{n_2 R}$

or $n_1 r = n_2 R$

Therefore, i is maximum when $n_1 r = n_2 R$

Question 22: A battery of emf 100 V and a resistor of resistance 10 kΩ are joined in series. This system is used as a source to supply current to an external resistance R. If R is not greater than 100 Ω, the current through it is constant up to two significant digits. Find its value. This is the basic principle of a constant-current source.

Solution:



We know that, current i for a series resistor connection: i_o = E/r

Given: E = 100 V and r= 10kΩ = 10000kΩ

i = 0.01 A

Consider R as 2 Ω

i = 100V/R_{tot}

Where R_{tot} is the effective resistance across the battery.

R_{tot} = r + R,

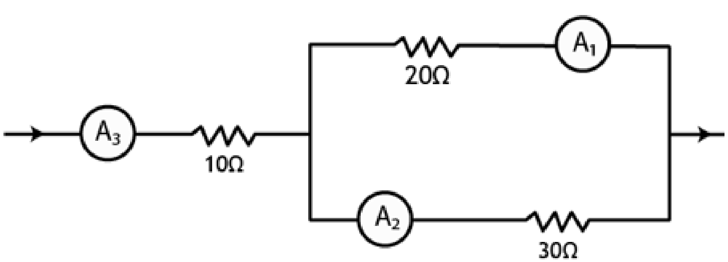
So i = 100V/(r+R) = 100/(10000+2) = 0.01 A (approx.)

Similarly, on putting R=100Ω , the highest possible, we have

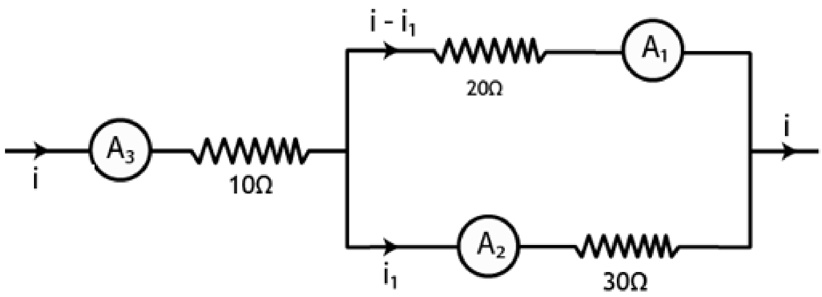
i = 100V/(r+R) = 100/(10000+100) = 0.01 A (approx.)

We can see that up to R = 100 Ω, the current does not change up to two significant digits

Question 23: If the reading of ammeter A_1 in figure below, what will the ammeters A_2 and A_3 read? Neglect the resistances of the ammeters.



Solution:
Current through A_1 ammeter, i – i_1 = 2.4A



The current passing through Ammeter A_3 will divide into two branches. To find the current in both the branches by equating the voltage in each branches.

$(i - i_1) \times 20\Omega = i_1 \times 30\Omega$

Put $i - i_1 = 2.4A$, here i_1 goes through ammeter A_2

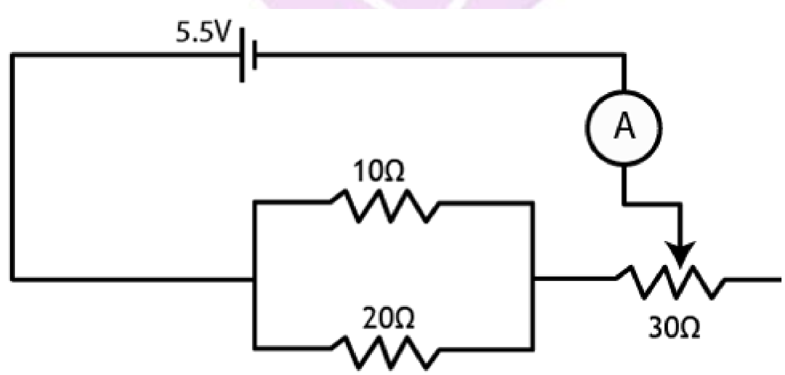
$i_1 = [2.4x20]/30 = 1.6$ A

Total current i, passing through A_3

$i = (i-i_1) + i_1 = 2.4A + 1.6A = 4A$

The current through A_2 and A_3 are 1.6A and 4A.

Question 24: The resistance of the rheostat shown in figure is 30 Ω. Neglecting the meter resistance, find the minimum and maximum currents through the ammeter as the rheostat is varied.



Solution:

The 10 Ω and 20 Ω connected parallel to each other. The effective resistance, R_{eff} between 10 Ω and 20 Ω will be,

R_{eff} = [10x20]/[10+20] = 6.667Ω

The minimum current will be marked when the total resistance is maximum, which happens when rheostat resistance, R=30Ω.

So, current = i = E/R_tot and $R_{tot} = R_{eff}$ + 30Ω = 36.667Ω

The minimum current, i_{min} = 5.5V/36.667Ω = 0.15A

Similarly, maximum current, i_{max} can be find when Rheostat resistance R is minimum, R = 0Ω.

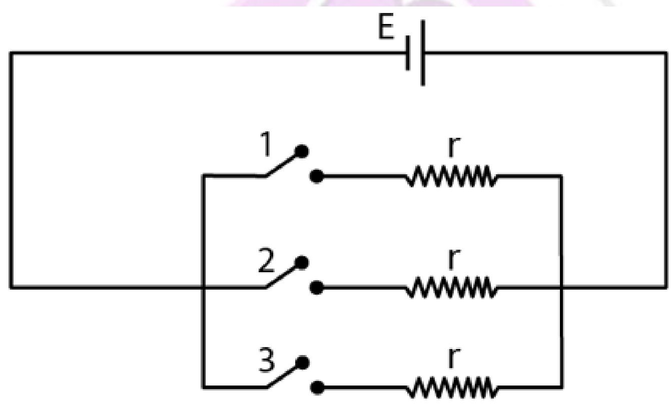
$R_{tot} = R_{eff}$ + 0Ω = 6.667Ω

Therefore, $i_{max} = E/R_{tot}$ = 5.5V/6.667Ω = 0.83A

The current in the ammeter vary from 0.15A to 0.83A

Question 25: Three bulbs, each having a resistance of 180 Ω, are connected in parallel to an ideal battery of emf 60 V. Find the current delivered by the battery when
(a) all the bulbs are switched on,
(b) two of the bulbs are switched on and
(c) only one bulb is switched on.

Solution:



(a) When all the switches are closed. Current will flow equally across each of the resistors. And total current will be the ratio between the potential difference and the effective resistance.

The effective resistance = R_{eff} = r/3

= 180Ω/3

= 60Ω

and current = i = E/R_{eff} = 60V/60Ω = 1 A

(b) Two resistors are connected in parallel.

Effective resistance = R_{eff} = 180Ω/2 = 90Ω

and current = i = E/R_{eff} = 60V/90Ω = 0.67 A

(c) Only one resistor is connected to the battery.

Effective resistance = R_{eff} = r = 180Ω

and current = i = E/R_{eff} = 60V/180Ω = 0.33 A