# Exercise Solutions

**Question 1:** Using the formulae F = qv x B and B = $\mu_0 i/2\pi r$ show that the S.I. units of the magnetic field B and the permeability constant μ0 may be written as N $m^{-1}A^{-1}$ 2nd $NA^{-1}$ respectively.

**Solution:** Using given formulas,
Magnetic field = B = F/qv = F/itv

[we know, Charge: q = it]

Now,
Units of Force (F) = N
Units of Current (i) = A
Units of time (t) = s
Units of velocity (v) = m/s

Unit of Magnetic field : B = N $m^{-1}A^{-1}$.

Again, using relation, B = $\mu_0 i/2\pi r$

or $\mu_0$ = B2πr/i = N $m^{-1}A^{-1}$ x m/A = $NA^{-2}$

Hence proved

**Question 2:** A current 10A is established in a long wire along the positive z-axis. Find the magnetic filed B at the point (1m, 0, 0).

**Solution:**
Ampere's Law for a current carrying straight wire: B = $\mu_0 i/2\pi d$

$\mu_0 = 4\pi \times 10^{-7}$ T $mA^{-1}$
and d = distance between the current carrying wire and the required point.

=> B = $[4\pi \times 10^{-7} \times 10]/[2\pi \times 1]$

=> B = $2 \times 10^{-6}$ T

**Question 3:** A copper wire of diameter 1.6 mm carries a current of 20 A. Find the maximum magnitude of the magnetic field due to this current.

**Solution:**
Diameter of the copper wire: d = 1.6mm = $1.6\times 10^{-3}$ m
Current through the wire: I = 20 A

Ampere's Law for a current carrying wire of cross section area: $B = \mu_0 i/2\pi r$

$\mu_0 = 4\pi \times 10^{-7}$ T mA$^{-1}$ and Radius of wire : $r = d/2 = 8\times 10^{-3}$ m

$B = [4\pi \times 10^{-7} \times 20]/[2\pi \times 8\times10^{-3}]$

=> $B = 5 \times 10^{-3}$ T

**Question 4:** A transmission wire carries a current of 100 A. What would be the magnetic field B at a point on the road if the wire is 8 m above the road?

**Solution:**
Diameter of the copper wire: d = 8 m
Current through the transmission wire: I = 100 A

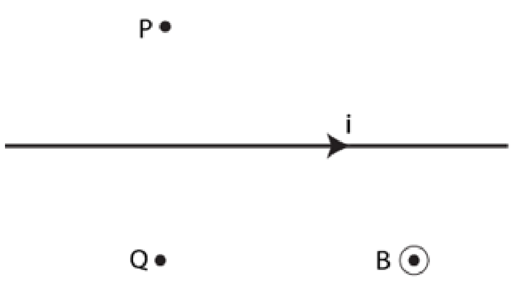
Ampere's Law for a current carrying wire of cross section area: $B = \mu_0 i/2\pi d$
$\mu_0 = 4\pi \times 10^{-7}$ T mA$^{-1}$

Substituting values, we get

$B = [4\pi \times 10^{-7} \times 100]/[2\pi \times 8]$

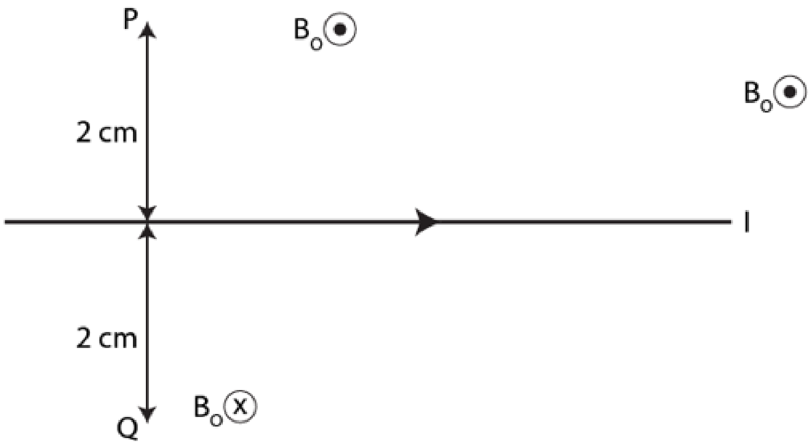
=> $B = 2.5 \times 10^{-6}$ T

**Question 5:** A long, straight wire carrying a current of 1.0 A is placed horizontally in a uniform magnetic field $B = 1.0 \times 10^{-8}$ T pointing vertically upward figure. Find the magnitude of the resultant magnetic field at the points P and Q, both situated at a distance of 2.0 cm from the wire in the same horizontal plane.




**Solution:**
Magnitude of Horizontal magnetic field: $B_o = 1.0 \times 10^{-5}$ T
Current in the wire = i = 1.0 A

Distance between points P, Q and the wire = d = 2.0 cm = 0.02 m




Cross determines the field going into the plane and dot determines the field coming out of the field.

$B_w$ is the magnetic field due to wire:

Ampere's Law for a current carrying wire: $B = \mu_0 i/2\pi d$

$\mu_0 = 4\pi \times 10^{-7}$ T $mA^{-1}$

$B_w = [4\pi \times 10^{-7} \times 1]/[2\pi \times 0.02]$

=> $B_w = 1 \times 10^{-5}$ T

Resultant magnetic field at point P,

$B_P = B_o + B_w$

$B_P = 1\times 10^{-5} + 1\times 10^{-5} = 2 \times 10^{-5}$ T

Magnetic field at point Q due to wire,

$Bw = -[4\pi \times 10^{-7} \times 1]/[2\pi \times 0.02]$

=> $B_w = -1 \times 10^{-5}$ T

The -ve sign is due to the field at Q is opposite to the direction of horizontal field and at P.
Resultant magnetic field at point Q,

$B_Q = B_o + B_w$

$B_Q = 1\times 10^{-5} - 1\times 10^{-5} = 0$

**Question 6:** A long, straight wire of radius r carries a current i and is placed horizontally in a uniform magnetic field B pointing vertically upward. The current is uniformly distributed over its cross section.
(a) At what points will the resultant magnetic field have maximum magnitude?
(b) What will be the minimum magnitude of the resultant magnetic field?

**Solution:** Wire carrying current generate a magnetic field around it. If wire is straight then it will be maximum at the mid point.

Magnetic field generated by wire = $\mu_0 i/2\pi r$

net magnetic field = $B + \mu_0 i/2\pi r$

(b) B= 0 when $r < \mu_0 i/2\pi B$

Also, B = 0 when $r = \mu_0 i/2\pi B$

But when $r > \mu_0 i/2\pi B$
Net magnetic field = $B - \mu_0 i/2\pi r$

**Question 7:** A long, straight wire carrying a current of 30 A is placed in an external, uniform magnetic field of $4.0 \times 10^{-4}$ T parallel to the current. Find the magnitude of the resultant magnetic field at a point 2.0 cm away from the wire.

**Solution:**
Current in the wire = I = 30 A
External Uniform magnetic field = $B_O = 4.0 \times 10^{-4}$ T
Distance between the point and wire = d = 2 cm = 0.02 m
Here B is the magnetic field due to wire. $B_O$ is parallel to the current in the wire.
And, $B_O$ is perpendicular to B.

Magnetic field due to current in wire = $B = \mu_0 i/2\pi d$

$= [2 \times 10^{-7} \times 30]/0.02$

$= 3 \times 10^{-4}$ T

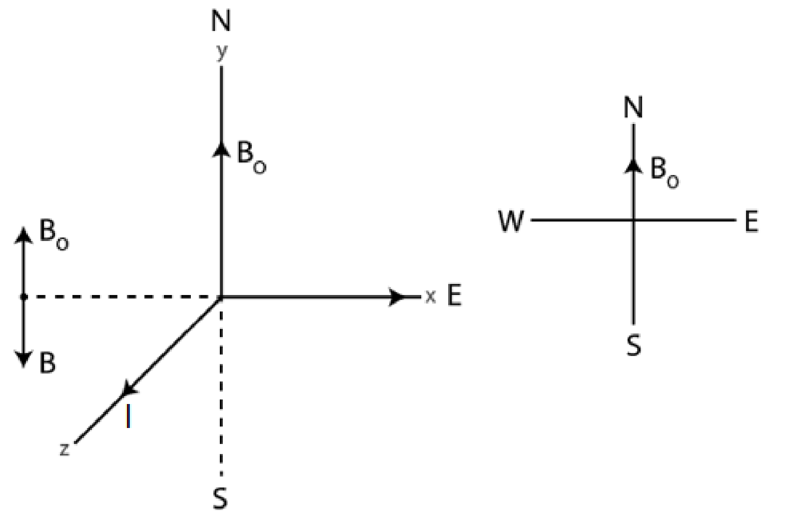
So, resultant magnetic field:
$B_{net}^2 = B^2 + B_0^2$

$B_{net}^2 = (4 \times 10^{-4})^2 + (3 \times 10^{-4})^2$

Or $B_{net} = 5 \times 10^{-4}$ T

**Question 8:** A long, vertical wire carrying a current of 10 A in the upward direction is placed in a region where a horizontal magnetic field of magnitude $2.0 \times 10^{-3}$ T exists from south to north. Find the point where the resultant magnetic field is zero.

**Solution:**
For the resultant magnetic field to be zero, the magnetic field due to the wire should be of the same magnitude as $B_O$ and in the direction north to south.




Magnetic field due to current in wire = B = $\mu_0 i/2\pi d$

We have $B = B_o$ to get zero resultant.

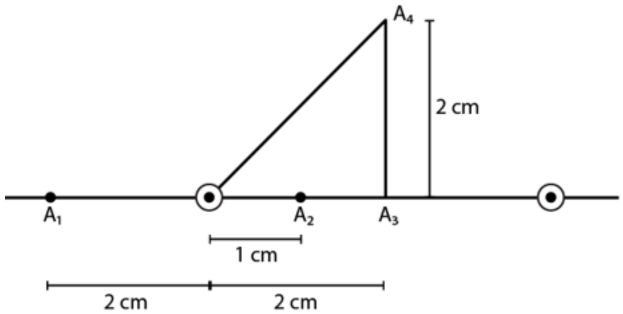
=> $2 \times 10^{-3} = \mu_0 i/2\pi d$

=> $d = [4\pi \times 10^{-7} \times 10]/[2\pi \times 2 \times 10^{-3}]$

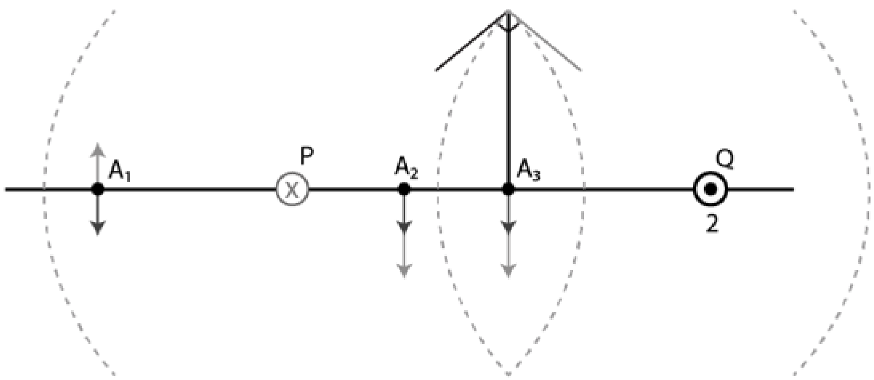
=> d = 0. 001 m = 1 mm

Point should be place at a distance of 0.001 m from the wire in west direction

**Question 9:** Figure shows two parallel wires separated by a distance of 4.0 cm and carrying equal currents of 10 A along opposite directions. Find the magnitude of the magnetic field B at the points $A_1$, $A_2$, $A_3$ and $A_4$.




**Solution:**




P and Q are the two wires having opposite direction of current.
P has current in the plane and Q has current that comes from the plane. According to the right rule, P has a field that rises at $A_1$ and falls at $A_2$ and $A_3$ and is tangent to $A_4$, as shown by the red arrow. Similarly, Q will have a field going down as $A_1$, $A_2$ and $A_3$ and touching $A_4$ as shown by the blue arrow.

Magnetic field due to current in wire = B = $\mu_0 i/2\pi d$ [Formula used]

At $A_1$ :
Net magnetic field due to P and Q

$B_{net} = B_P - B_Q$

$= \mu_0 i/2\pi d_1 - \mu_0 i/2\pi d_2$

Given:
$PA_1 = d_1 = 0.02$ m
$QA_1 = d_2 = 0.06$m

=> $B_{net} = \mu_0 i/2\pi\ [1/0.02 - 1/0.06]$

=> $B_{net} = [(4\pi \times 10^{-7} \times 10)/2\pi] \times 33.33 = 6.66 \times 10^{-5}$ T

At $A_2$ :
Net magnetic field due to P and Q

$B_{net} = B_P + B_Q$

$= \mu_0 i/2\pi d_1 + \mu_0 i/2\pi d_2$

Given:
$PA_2 = d_1 = 0.01$ m
$QA_3 = d_2 = 0.03$ m

$\Rightarrow B_{net} = \mu_0 i/2\pi\ [1/0.01 + 1/0.03]$

$\Rightarrow B_{net} = [(4\ \pi x\ 10^{-7} \times 10)/2\ \pi] \times 133.33 = 2.66 \times 10^{-4}$ T

At $A_3$ :
Net magnetic field due to P and Q

$B_{net} = B_P + B_Q$

$= \mu_0 i/2\pi d_1 + \mu_0 i/2\pi d_2$

Given:
$PA_3 = d_1 = 0.02$ m
$QA_3 = d_2 = 0.02$ m

$\Rightarrow B_{net} = \mu_0 i/2\pi\ [1/0.02 + 1/0.02]$

$\Rightarrow B_{net} = [(4\ \pi x\ 10^{-7} \times 10)/2\ \pi] \times 100 = 2.0 \times 10^{-4}$ T

At $A_4$ :
Net magnetic field due to P and Q

$B_{net}^2 = B_P^2 + B_Q^2$ ...(1)
[By Pythagoras theorem]

$= \mu_0 i/2\pi d_1 + \mu_0 i/2\pi d_2$

$(PA_4)^2 = (PA_3)^2 + (A_3A_4)^2 = 0.02^2 + 0.02^2$

$\Rightarrow (PA_4) = 0.028$ m $= d_1$

Now, $B_P = \mu_0 i/2\pi d_1 = [(4 \pi x\ 10^{-7} \times 10)/(2 \pi x 0.028] = 7.14 \times 10^{-5}$ T

Similarly, $QA_4 = 0.028$ m $= d_2$

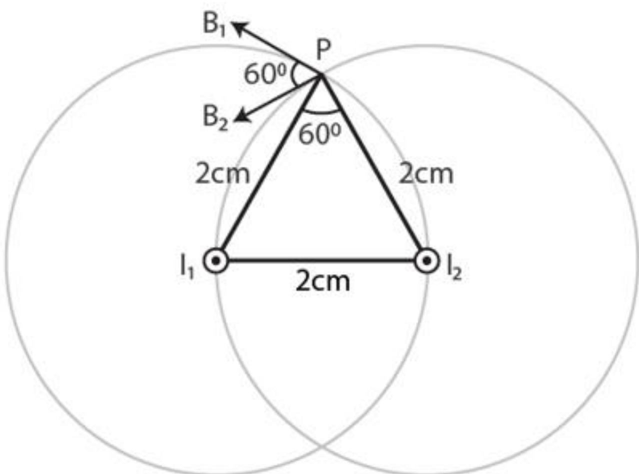
So, $B_Q = \mu_0 i/2\pi d_2 = [(4 \pi x\ 10^{-7} \times 10)/(2 \pi x 0.028] = 7.14 \times 10^{-5}$ T

On substituting the values, (1)=>

$B_{net} = 1 \times 10^{-4}$ T, which is the magnetic field at $A_4$.

**Question 10:** Two parallel wires carry equal currents of 10 A along the same direction and are separated by a distance of 2.0 cm. Find the magnetic field at a point which is 2.0 cm away from each of these wires.

**Solution:**




Above circles show the magnetic field in a plane generated due to wires carrying current.

Here triangle $PI_1I_2$ is an equilateral triangle.

=> Each angle of the triangle = $60^0$.

The magnetic field at point P:

$B_{net}^2 = B_1^2 + B_2^2 + 2 B_1 B_2 \cos 60^o$ ……(1)

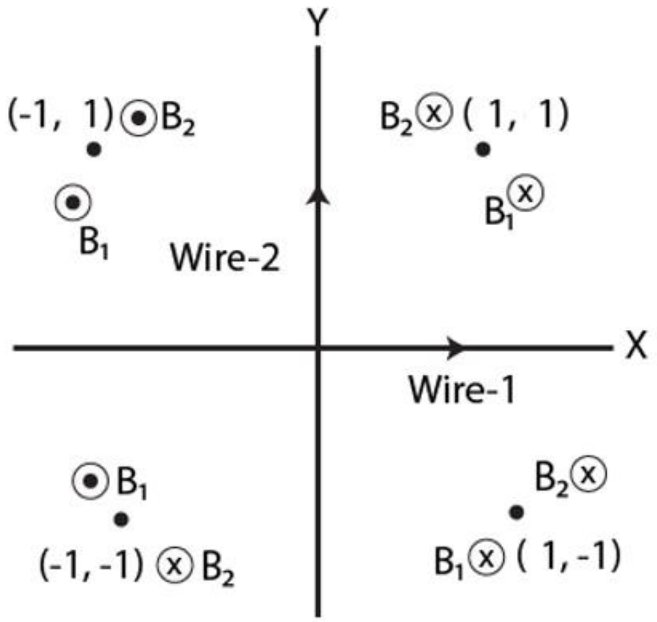
Here $B_1 = B_2 = [2\times10^{-7}\times10]/[2\times10^{-2}]$ and $\cos60^o = ½$

On substituting the values and solving, we have

$B_{net} = \sqrt{3} \times 10^{-4}$ T $= 1.732 \times 10^{-4}$ T

**Question 11:** Two long, straight wires, each carrying a current of 5A, are placed along the x-and y-axis respectively. The currents point along the positive directions of the axes.
Find the magnetic fields at the points (a) (1m, –1m), (b) (–1m, 1m) (c) (–1m, –1m) and (d) (1m, –1m).

**Solution:**
The magnitude of magnetic field due to current in the wire = $B_1 = B_2 = \mu_0 i/2\pi d$




(a) At point (1m, 1m)
The magnetic fields due to wire 1 and 2 are in opposite direction and same magnitude. Hence, net magnetic field is zero.

(b) At (-1m, 1m)
The magnetic field due to wire 1 and 2 add as direction of magnetic field is same.

$B_{net} = B_1 + B_2$

$= [2\text{x}10^{-7}\text{x}5]/1 + [2\text{x}10^{-7}\text{x}5]/1$

$= 2 \times 10^{-6}$ T (along z-axis)

(c) At (-1m, -1m)
The magnetic field due to wire 1 and 2 are same in magnitude and opposite in direction.

$B_{net} = B_1 + B_2 = 0$

(d) At (1m, 1m)

The magnetic field due to wire 1 and 2 are in upward direction.

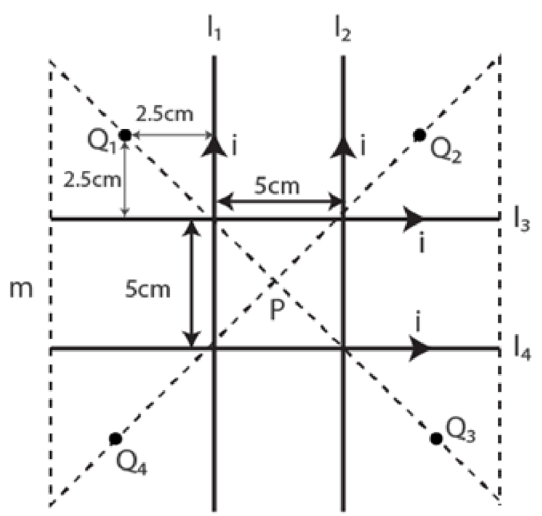
$B_{net} = B_1 + B_2$

$= [2\times10^{-7}\times5]/1 + [2\times10^{-7}\times5]/1$

$= 2 \times 10^{-6}$ T (along negative z-axis)

**Question 12:** Four long, straight wires, each carrying a current of 5.0 A, are placed in a plane as shown in figure. The points of intersection from a square of side 5.0 cm.
(a) Find the magnetic field at the centre P of the square
(b) $Q_1$, $Q_2$, $Q_3$ and $Q_4$ are points situated on the diagonal of the square and at a distance from P that to equal to the length of the diagonal of the square. Find the magnetic fields at these points.

**Solution:**




(a) Consider point P.
Magnetic field due to wires, $l_1$ and $l_2$ are the same in magnitude but opposite in direction. In same way, magnetic field due to wires, $l_3$ and $l_4$ are the same in magnitude but opposite in direction.

Hence, net magnetic field is zero.

(b) Consider point $Q_1$.

Distance between wire $l_1$ and $Q_1 = d_1 = 0.025$ m
Distance between wire $l_2$ and $Q_1 = d_2 = 0.075$ m
Distance between wire $l_3$ and $Q_1 = d_3 = 0.025$ m
Distance between wire $l_4$ and $Q_1 = d_4 = 0.075$ m

Net magnetic field at $Q_1$ due to all 4 wires:
$B_{Q1} = B_{l1} + B_{l2} + B_{l3} + B_{l4}$

$= \mu_0 i/2\pi d_1 + \mu_0 i/2\pi d_2 + \mu_0 i/2\pi d_3 + \mu_0 i/2\pi d_4$

$= [4\pi x10^{-7}x5]/2\pi\ [1/0.025 + 1/0.075 + 1/0.025 + 1/0.075]$

$= 1.06 \times 10^{-4}$ T [upward direction]

Consider point Q2.

Distance between wire $l_1$ and $Q_2 = d_1 = 0.075$ m
Distance between wire $l_2$ and $Q_2 = d_2 = 0.025$ m
Distance between wire $l_3$ and $Q_2 = d_3 = 0.025$ m
Distance between wire $l_4$ and $Q_2 = d_4 = 0.075$ m

Net magnetic field at $Q_2$ due to all 4 wires:
$B_{Q2} = B_{l1} + B_{l2} + B_{l3} + B_{l4}$

$= \mu_0 i/2\pi d_1 + \mu_0 i/2\pi d_2 - \mu_0 i/2\pi d_3 - \mu_0 i/2\pi d_4$

$= [4\pi x10^{-7}x5]/2\pi\ [1/0.075 + 1/0.025 - 1/0.025 - 1/0.075]$

$= 0$

Consider point $Q_3$.

Distance between wire $l_1$ and $Q_3 = d_1 = 0.075$ m
Distance between wire $l_2$ and $Q_3 = d_2 = 0.025$ m
Distance between wire $l_3$ and $Q_3 = d_3 = 0.075$ m
Distance between wire $l_4$ and $Q_3 = d_4 = 0.025$ m

Net magnetic field at $Q_3$ due to all 4 wires:
$B_{Q3} = B_{l1} + B_{l2} + B_{l3} + B_{l4}$

$= \mu_0 i/2\pi d_1 + \mu_0 i/2\pi d_2 + \mu_0 i/2\pi d_3 + \mu_0 i/2\pi d_4$

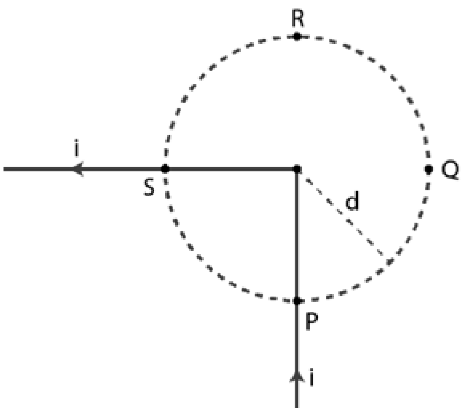
$= [4\pi x10^{-7}x5]/2\pi\ [1/0.075 + 1/0.025 + 1/0.075 + 1/0.025]$

$= 1.06 \times 10^{-4}$ T [downward direction]

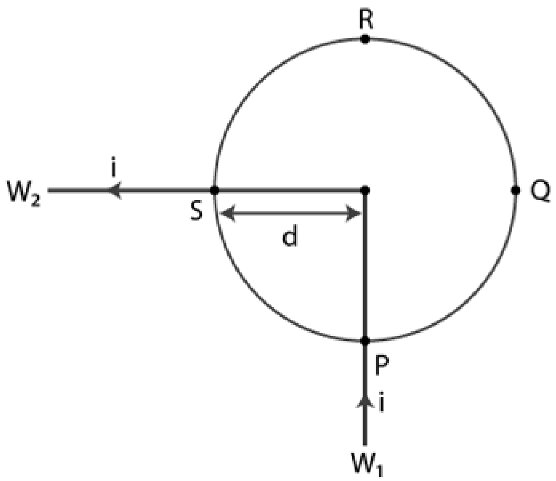
At $Q_4$

Magnetic field at $Q_1$ and $Q_3$ have same magnitude of $1.06 \times 10^{-4}$ T but in opposite direction and magnetic field at $Q_2$ and $Q_4$ is zero.

**Question 13:** Figure shows a long wire bent at the middle to from a right angle. Show that the magnitudes of the magnetic fields at the points, P, Q, R and S are equal and find this magnitude.




**Solution:**




At point P,
Magnetic field at P due to $W_1$ is zero as P is on the axis of $W_1$.
$B_1 = 0$

Magnetic field at P due to $W_2$:

$B_2 = \mu_0 i/4\pi d$
Which is perpendicular to the plane in outward direction.

So $B_{net} = B_1 + B_2 = \mu_0 i/4\pi d$ [perpendicular to the plane in outward direction.]

At point Q:

Magnetic field due to $W_1$:

$B_1 = \mu_0 i/4\pi d$  [perpendicular to the plane in inward direction]

Magnetic field due to $W_2$:

$B_2 = 0$

=> $B_{net} = B_1 + B_2 = \mu 0i/4\pi d$  [perpendicular to the plane in inward direction]

At point R:

=> $B_{net} = \mu_0 i/4\pi d$  [perpendicular to the plane in inward direction]

At point S:

Magnetic field due to $W_1$:

$B_1 = \mu_0 i/4\pi d$  [perpendicular to the plane in outward direction]

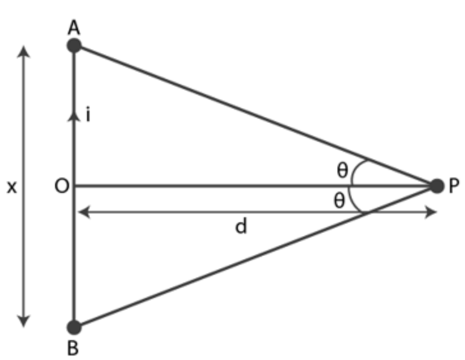
Magnetic field due to $W_2$:

$B_2 = 0$

=> $B_{net} = B_1 + B_2 = \mu_0 i/4\pi d$  [perpendicular to the plane in outward direction]

**Question 14:** Consider a straight piece of length x of a wire carrying a current i. Let P be a point on the perpendicular bisector of the piece, situated at a distance d from its middle point. Show that for d >> x, the magnetic field at P varies at $1/d^2$ whereas for d<<x, it varies as 1/d.

**Solution:**



The magnetic field on perpendicular bisector:

$B = \mu_0 i/4\pi d\ [\sin\theta + \sin\theta]$

$= \mu_0 i/4\pi d\ [2x/\sqrt{(x^2+4d^2)}]$

If d >>> x, then

$B = \mu_0 i/4\pi d\ [2x/2d]$

$B \propto 1/d^2$

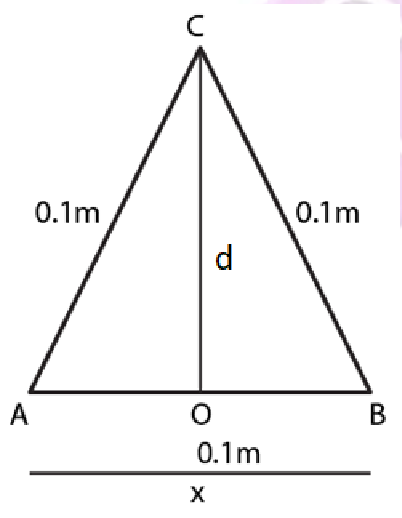
If d << x, then

$B = \mu_0 i/4\pi d\ [2x/x]$

$B \propto 1/d$

**Question 15:** Consider a 10 cm long piece of a wire which carries a current of 10A. Find the magnitude of the magnetic field due to the piece at a point which makes an equilateral triangle with the ends of the piece.

**Solution:**




Magnetic field at a point on the perpendicular bisector:

$B = \mu_0 i/2\pi d\ [x/\sqrt{(x^2+4d^2)}]$ ....(1)

where, x = length of the wire.

Consider, d is the distance between point C and the midpoint O.

Using Pythagoras theorem,

$(BC)^2 = (OB)^2 + (OC)^2$

$(OC)^2 = (0.1)^2 - (0.05)^2$

OC = 0.086 m = d

(1)=>

$$B = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.086} \times \frac{0.01}{\sqrt{0.01^2 + 4 \times (0.086)^2}}$$

$B = 2.32 \times 10^{-5} \times 0.502 = 1.16 \times 10^{-5}$ T

**Question 16:** A long, straight wire carries a current i. Let $B_1$ be the magnetic field at a point P at a distance d from the wire. Consider a section of length ℓ of this wire such that the point P lies on the perpendicular bisector of the section. Let $B_2$ be the magnetic field at this point due to this section only. Find the value of d/ℓ so that $B_2$ differs from $B_1$ by 1%.

**Solution:**
The magnetic field induced in the wire:

$B_1 = \mu_0 i/2\pi d$

The magnetic field due to a section of length l on the perpendicular bisector:

$B_2 = \mu_0 i/4\pi d \times [2l/\sqrt{(l^2+4d^2)}]$

$$B_2 = \frac{\mu_0 i}{4\pi d} \times \frac{2l}{\sqrt{l^2 + 4d^2}}$$

$$= \frac{\mu_0 i}{4\pi d} \times \frac{2l}{d\sqrt{\frac{l^2}{d^2} + 4}}$$

$$= \frac{\mu_0 il}{4\pi d^2} \times \frac{2}{\sqrt{2}}$$

$$= \frac{\sqrt{2}\mu_0 il}{4\pi d^2}$$

Now, $B_1 > B_2$

Also, we are given with $(B_1 – B_2)/B_1 = 1/100$

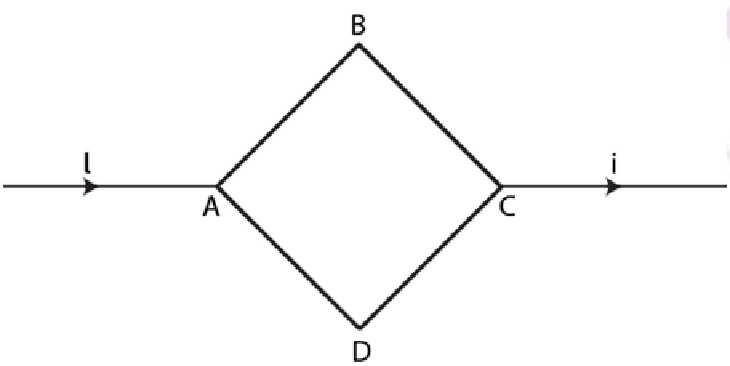
=> $B_2 = 0.99 B_1$

On substituting $B_1$ and $B_2$ in above equation, it turned as

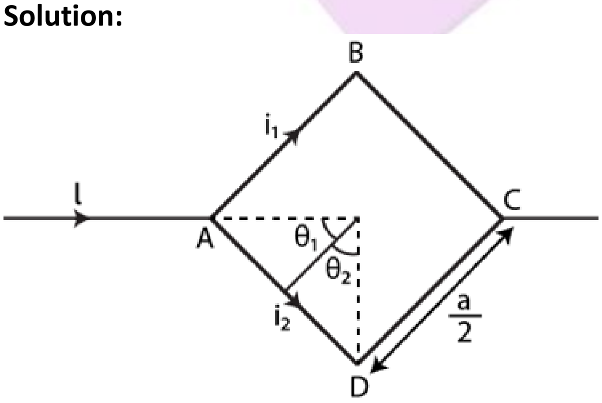
$$\frac{\sqrt{2}\mu_0 il}{4\pi d^2} = 0.99 \times \frac{\mu_0 i}{2\pi d}$$

$$\frac{d}{l} = \frac{1.414}{1.98} = 0.71$$

**Question 17:** Figure shows a square loop ABCD with edge length a. The resistance of the wire ABC is r and that of ADC is 2r. Find the magnetic field B at the centre of the loop assuming uniform wires.




**Solution:**




Let $i_1$ and $i_2$ currents and r and 2r be the resistances in wires ABC and ADC, $i_1/i_2 = 2/1$

$i_1 - 2i_2 = 0$ ...(1)

$i_1 + i_2 = i$ ....(2)

Solving (1) and (2), we have

$i_1 = 2i/3$ and $i_2 = i/3$

And, magnetic field at centre O due to wire AB and BC

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2i_1 \sin \sin 45}{\frac{a}{2}}$$

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}i_1}{a}$$

Magnetic field at centre o due to wire AD and DC

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2i_2 \sin \sin 45}{\frac{a}{2}}$$

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}i_2}{a}$$

Also, $i_1 = 2i_2$
and $B_1 = B_2 > B_3 = B_4$

Net magnetic field at centre:

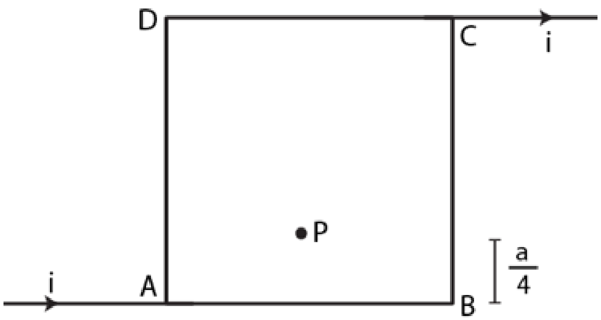
$B_{net} = B_1 + B_2 - (B_3 + B_4)$

$$B_{net} = \left(2 \times \frac{\mu_0}{4\pi} \times \frac{2\sqrt{3} \times \frac{2}{3}(i)}{a}\right) - \left(\frac{\mu_0}{4\pi} \times \frac{2\sqrt{3} \times \frac{1}{3}(2)}{a}\right)$$

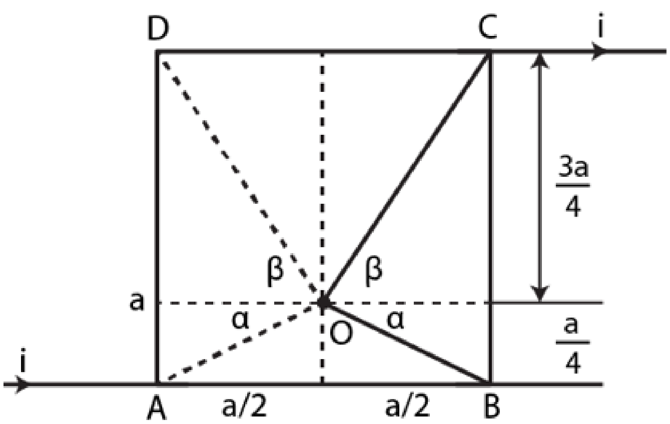
$$B_{net} = \frac{\mu_0}{4\pi} \times \frac{4\sqrt{2}i}{3a}(2-1)$$

$$B_{net} = \frac{\sqrt{2}\mu_0 i}{3\pi a}$$

**Question 18:** Figure shows a square loop of edge a made of a uniform wire. A current i enters the loop at the point A and leaves it at the point C. Find the magnetic field at the point P which is not the perpendicular bisector of AB at a distance a/4 from it.




**Solution:**




Magnetic field due to AB:

$B_{AB} = \mu_0 i/4\pi$  x i/2(a/4) x [2sin(90- α)] = $\mu_0 i/4\pi$  x i/2(a/4) x [2 cos α]  ...(1)

Cos α = (a/2)/AO

From  figure,

$$AO = \sqrt{\frac{a^2}{16} + \frac{a^2}{4}} = \sqrt{\frac{5a^2}{16}} = \frac{a\sqrt{5}}{4}$$

(1)=>

$$B_{AB} = \frac{\mu_0 \times 2i}{4\pi a} \times 2\frac{\frac{a}{2}}{a\left(\frac{\sqrt{5}}{4}\right)} = \frac{2i\mu_0}{\pi\sqrt{5}}$$

Similarly, magnetic field due to DC

$B_{DC}$ = [$\mu_0$ix4x2]/[4πx3a] x [cos β] ...(2)

From figure,
Cos β = (a/2)/DO

$$DO = \sqrt{\left(\frac{3a}{4}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{\frac{9a^2}{16} + \frac{a^2}{4}} = \sqrt{\frac{13a^2}{16}} = \frac{a\sqrt{13}}{4}$$

(2)=>

$$B_{DC} = \frac{\mu_0 \times i}{3\pi a} \times \frac{\frac{a}{2}}{\left(\frac{\sqrt{13}a}{4}\right)} = \frac{2i\mu_0}{a^3\pi\sqrt{13}}$$

The magnetic field due to AD and BC are equal and opposite, so net magnetic field

$B_{net} = B_{AB} - B_{DC}$

$$B_{net} = \frac{2i\mu_0}{\pi a} \times \left[\frac{1}{\sqrt{5}} - \frac{1}{3\sqrt{13}}\right]$$

**Question 19:** Consider the situation described in the previous problem. Suppose the current i enters the loop at the point A and leaves it at the point B. Find the magnetic field at the centre of the loop.

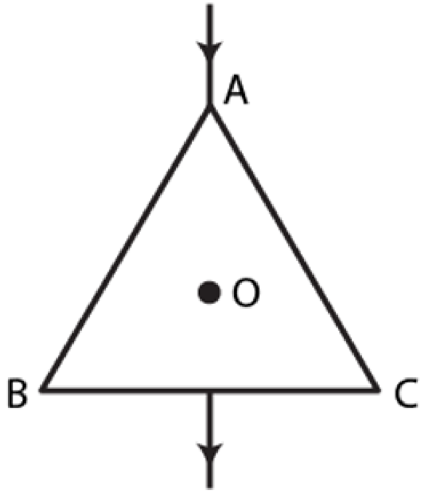
**Solution:**

Magnetic vector B due to BC and due to AD at PT equal in magnitude and opposite in direction. So, net vector B = 0

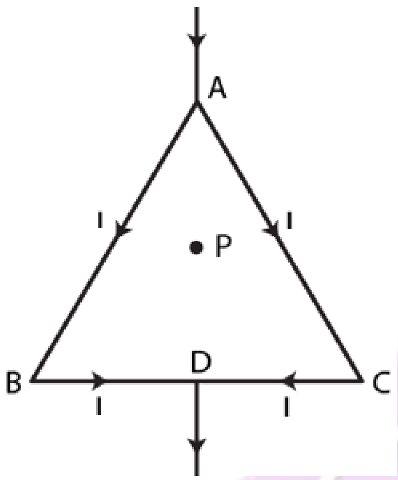
Similarly, due to AB and CD at P = 0

The net vector B at the centre of the square loop = 0

**Question 20:** The wire ABC shown in figure forms an equilateral triangle. Find the magnetic field B at the centre O of the triangle assuming the wire to be uniform.




**Solution:**




Let current 2I flow through the circuit.
Since the wire is uniform, the current is dividing equally between AB and AC arms.

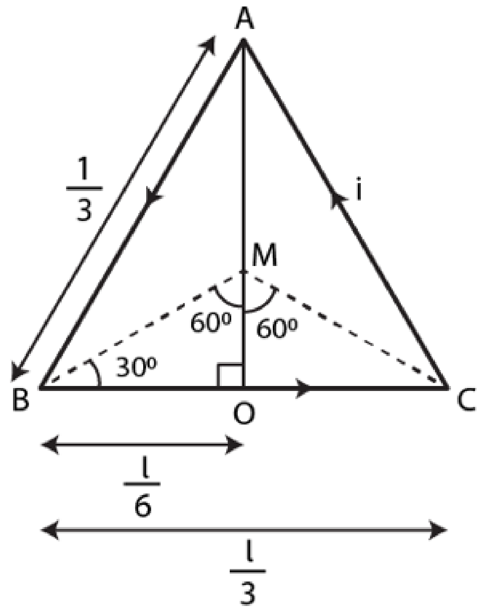
Let "B" be the magnetic field induced at point P due to wire AB and wire AC

and B' be the magnetic field at P due to wire BD and DC.

So, Net magnetic field at P = B+ B' - B - B' = 0

**Question 21:** A wire of length ℓ is bent in the form of an equilateral triangle and carries an electric current i. (a) Find the magnetic field B at the centre.
(b) If the wire is bent in the form of a square, what would be the value of B at the centre?

**Solution:**




Let ABC be the equilateral triangle with each side measure l/3 and angle 60 degrees.

(a) In triangle AOB,

$$AO = \sqrt{\left(\frac{l}{3}\right)^2 - \left(\frac{l}{6}\right)^2} = l\sqrt{\frac{1}{12}}$$

$$MO = \frac{1}{3} \times l\sqrt{\frac{1}{12}} \quad = \frac{l}{6\sqrt{3}}$$

Let "d" be the separation of the point from the wire.

d= MO = l/(6√3)

Magnetic field induced due to current in wire BC:

B = $\mu_0$i/4πd   x 2 sin 60°

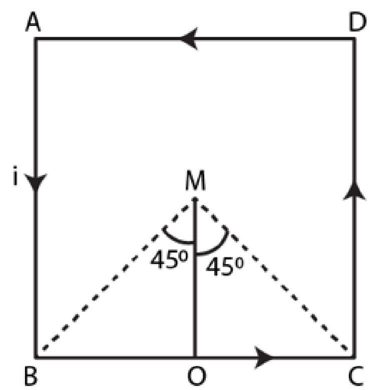
=> B = $\mu_0$i/4πl  x 6√3 x 2√3 = 36$\mu_0$i/4πl

Since all the wires are equal, so the net magnetic field at point M due to AB, BC and CA is

$B_{net} = 3B = 3 \times 36\mu_0 i/4\pi l = 27\mu_0 i/\pi l$

It is perpendicular to the plane outward when the current flows anticlockwise and inward when the current flows clockwise.

(b)




The induced magnetic field due to electric current in wire BC:

$B = \mu_0 i/4\pi d \times 2 \sin 45^\circ$

From figure, $d = l/8$

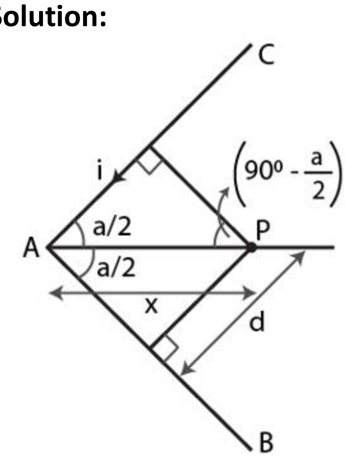
$\Rightarrow B = [2\sqrt{2}\, \mu_0 i]/\pi l$

Given: All the wires are equal, so net magnetic field at point M = 4 × Magnetic field due to wire BC.

$\Rightarrow B_{net} = 4B = [8\sqrt{2}\, \mu_0 i]/\pi l$

**Question 22:** A very long wire carrying a current i is bent to form a plane at an angle α. Find the magnetic field B at a point P on the bisector of this angle situated at a distance x from the vertex A.

**Solution:**



From figure, $\sin(\alpha/2) = d/x$

=> $d = x \sin(\alpha/2)$
Where d = separation of the point from the wire.

Let $\theta_1$ and $\theta_2$ angles made by A and C with P:

$\theta_1 = 90 - \alpha/2$ and $\theta_2 = 90^0$

Now, the magnetic field due to current in wire AC:

$$B = \frac{\mu_0 i}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$

Using values of $\theta_1$ and $\theta_2$

$$= \frac{\mu_0 i}{4\pi x \sin\left(\frac{\alpha}{2}\right)}\left[\cos\frac{\alpha}{2} + 1\right]$$

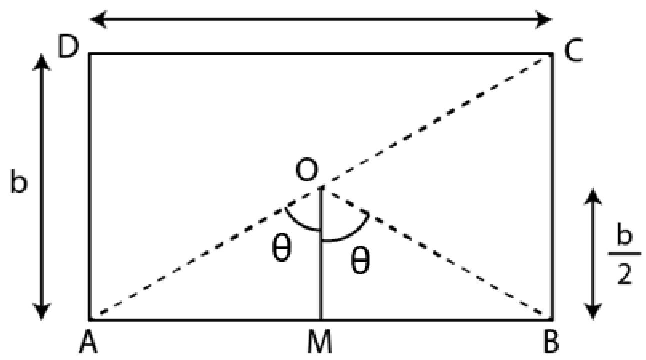
$$= \frac{\mu_0 i 2\cos^2\left(\frac{\alpha}{4}\right)}{4\pi x 2\sin\left(\frac{\alpha}{4}\right)\cos\left(\frac{\alpha}{4}\right)}$$

$$= \frac{\mu_0 i \cot\left(\frac{\alpha}{4}\right)}{4\pi x}$$

The net B due to wires AB and AC: $B_{net} = 2B = \mu_0 i \cot(\alpha/2)/2\pi x$

**Question 23:** Find the magnetic field B at the centre of a rectangular loop of length ℓ and width b, carrying a current i.

**Solution:**




Here OM = d = b/2

In triangle, AOM,

$$AO = \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = \frac{1}{2}\sqrt{b^2 + l^2}$$

$$sin\theta = \frac{\frac{l}{2}}{\frac{1}{2}\sqrt{b^2 + l^2}} = \frac{l}{\sqrt{b^2 + l^2}}$$

Induced magnetic field due to AB:

$B = \mu_0 i/4\pi d \times 2 \sin\theta$

$B = \mu_0 i/4\pi(b/2) \times 2l/\sqrt{[l^2+b^2]}$

$B = \mu_0 i/\pi b \times l/\sqrt{[l^2+b^2]}$

Similarly, the induced magnetic field due to current in BC:

$B = \mu_0 i/\pi l \times b/\sqrt{[l^2+b^2]}$

Now,
Magnetic field due to CD = Magnetic field due to current in wire AB = B
Magnetic field due to current in wire DA = Magnetic field due to current in wire BC = B'

$B_{net} = l(B + B')$

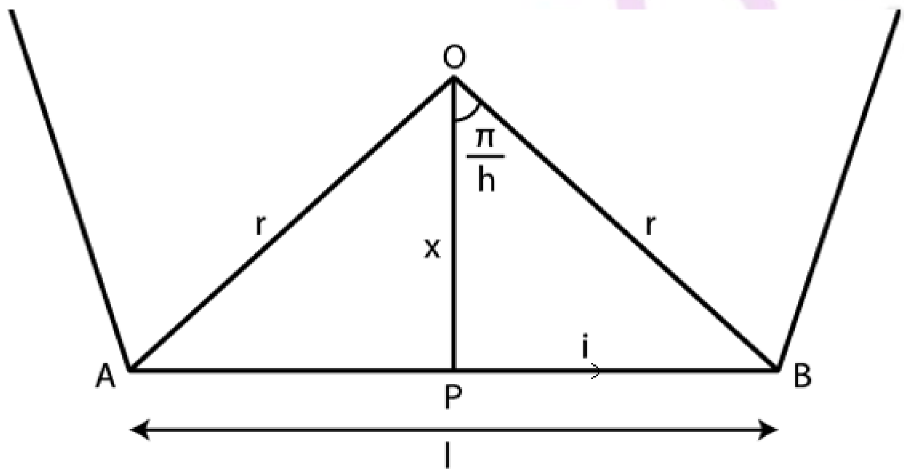
$$= 2\left[\frac{\mu_0 i}{\pi b} \times \frac{l}{\sqrt{l^2+b^2}} + \frac{\mu_0 i}{\pi l} \times \frac{b}{\sqrt{l^2+b^2}}\right]$$

$$= \frac{2\mu_0 i}{\pi\sqrt{l^2+b^2}}\left[\frac{l}{b} + \frac{b}{l}\right]$$

$$= \frac{2\mu_0 i\sqrt{b^2+l^2}}{\pi b l}$$

**Question 24:** A polygon of n equal sides is formed by bending a current carrying wire to total length 2πr which carries a current i.
(a) Find the magnetic field B at the center of the n-sided polygon.
(b) By letting n → ∞, derive the expression for the induced magnetic field at the center of a circular current carrying wire.

**Solution:**




(a) For a polygon of n equal sides:

Central angle = 2π/n

Tan θ = l/2x => x = l/(2 Tan θ)

If angle is small, then l/2 = πr/n

Using Biot–Savart's law for one side of the n-sided polygon,

$$B = \frac{\mu_0}{4\pi}\frac{idlsin\theta}{x^2}$$

$$= \frac{\mu_0}{4\pi}\frac{i(sin\theta + sin\theta)}{x^2}$$

$$= \frac{\mu_0 i\, 2(tan\theta)(2sin\theta)}{4\pi l}$$

$$= \frac{\mu_0 i\, 2n\left[tan\left(\frac{\pi}{n}\right)\right]\left[2sin\left(\frac{\pi}{n}\right)\right]}{4\pi(2\pi r)}$$

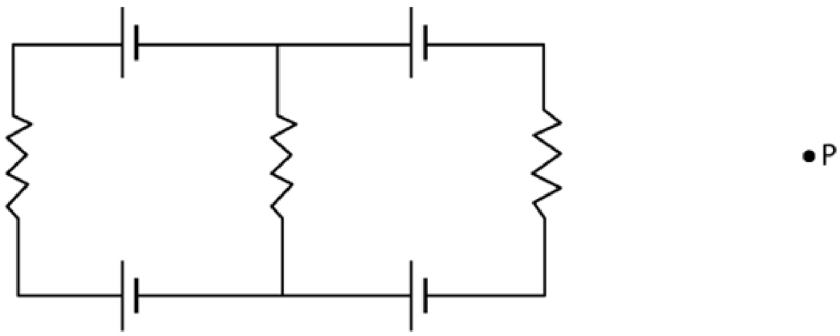
[Putting given values]

For n-sided polygon:
B' = nB
Substitute the value of B and get your answer.

(b) When n → ∞, polygon tends to a circle with radius r and magnetic field will tend toward

$B = \mu_0 i/2r$

**Question 25:** Each of the batteries shown in figure has an emf 5V. Prove that the magnetic field B at the point P is zero for any set of values of the resistances.




**Solution:**

By applying Kirchhoff's Voltage Law, net current in the circuit = 0

So, Net magnetic field "A" at the point P = 0

Induced magnetic field at point P is independent of the values of the various resistances in the electric circuit.