Exercise Solutions

Question 1: Find the time required for a 50 Hz alternating current to change its value from zero to the rms value.

Solution:
Current at any time = $i = i_0 \sin(2\pi ft)$

Where,
i = Current at any time t.
i_0 = Maximum value of the current in the circuit, and
f = Frequency of the alternating current

Here, $i_{rms} = i_0/\sqrt{2}$

$\Rightarrow i_0 \sin(2\pi ft) = i_0/\sqrt{2}$

or $\sin(2\pi ft) = 1/\sqrt{2} = \sin(\pi/4)$

$\Rightarrow 2\pi ft = \pi/4$

$\Rightarrow 2ft = 1/4$

$\Rightarrow 2 \times 50 \times t = 1/4$

$\Rightarrow t = 1/400 = 2.5 \times 10^{-3}$ s

Question 2: The household supply of electricity is at 220 V (rms value) and 50 Hz. Find the peak voltage and the least possible time in which the voltage can change from the rms value to zero.

Solution:
Peak voltage = $E_0 = \sqrt{2} \times E_{rms}$

$\Rightarrow E_0 = \sqrt{2} \times 220$

$\Rightarrow E_0 = 1.414 \times 220$

$\Rightarrow E_0 = 311$ V (approx)

Let us find the, time taken to reach the zero value from rms:

$i_{rms} = i_0/\sqrt{2} = i_0 \sin \omega t$

or $\omega t = \pi/4$

=> $t = \pi/[4 \times 2\pi f]$

[use, angular velocity = $\omega = 2\pi f$]

=> $t = \pi/[8\pi \times 50] = 1/400 = 2.5 \times 10^{-3}$ s

Question 3: A bulb rated 60W at 220V is connected across a household supply of alternating voltage of 220V. Calculate the maximum instantaneous current through the filament

Solution:
We know, $P = V^2/R$

or $R = V^2/P = [220]^2/60 = 806.67$

The maximum instantaneous current through the filament:

$i_o = \varepsilon_0/R$ ....(1)

ε_0 = Instantaneous voltage = $\sqrt{2} \times 220 = 311.08$

(1)=> $i_0 = 311.08/806.67 = 0.39$ A

Question 4: An electric bulb is designed to operate at 12 volts DC. If this bulb is connected to an AC source and given normal brightness, what would be the peak voltage of the source?

Solution:
Given: Voltage = 12V.

The peak voltage under given condition will be

Peak voltage = $E_0 = \sqrt{2} \times E = \sqrt{2} \times 12 = 16.97 = 17$ V (approx)

[Use $\sqrt{2} = 1.414$]

Question 5: The peak power consumed by a resistive coil when connected to an AC source is 80W. Find the energy consumed by the coil in 100 seconds which is many times larger than the time period of the source.

Solution:
Peak Power = P_0 = 80 W

Then, instantaneous power = P_{rms} = $P_0/2$ = 40 W

Energy consumed by the coil in t = 100 s = P x t = 40 x 100 = 4000 J

Question 6: The dielectric strength of air is 3.0×10^6 V/m. A parallel-plate air-capacitor has area 20 cm^2 and plate separation 0.10 mm. Find the maximum rms voltage of an AC source which can be safely connected to this capacitor.

Solution:

Dielectric strength of air = E = 3.0×10^6 V/m
Separation width = d = 0.10 mm and
Area = A = 20 cm^2

Now,

Potential difference across the capacitor = V = E x d

= $3 \times 10^6 \times 1 \times 10^{-4}$

= 300 V

The maximum rms voltage of an AC source which can be safely connected to this capacitor = V_{rms} = $V/\sqrt{2}$

= $300/\sqrt{2}$

=> V_{rms} = 212 V

Question 7: The current in a discharging LR circuit is given by $i = i_0 e^{(-t/\tau)}$ where τ is the time constant of the circuit. Calculate the rms current for the period t = 0 to t = τ.

Solution:

The current in a discharging LR circuit $i = i_0 e^{(-t/\tau)}$

So, the rms current for the period t = 0 to t = T :

$$I_{rms}^2 = \frac{1}{T}\int_0^T I_0^2 e^{-2t/T}\,dt$$

$$= \frac{I_0^2}{T} \times \left[\frac{T}{2} e^{-2t/T}\right]_0^T$$

$$= \frac{I_0^2}{T} \times \frac{T}{2} \times [e^{-2T/T} - 1]$$

$$= \frac{I_0^2}{2}\left(1 - \frac{1}{e^2}\right)$$

or,

$$I_{rms} = \frac{I_0}{e}\left(\sqrt{\frac{e^2 - 1}{2}}\right)$$

Question 8: A capacitor of capacitance 10 μF is connected to an oscillator giving an output voltage ϵ = (10V) sin ωt. Find the peak currents in the circuit for ω = 10 s^{-1}, 100 s^{-1}, 500 s^{-1}, 1000 s^{-1}.

Solution:

Capacitance of the capacitor = C = 10 μF

Output voltage of the oscillator = ϵ = (10V) sin ωt

On comparing the output voltage of the oscillator with, $\epsilon = \epsilon_0$

Peak voltage = ϵ_0 = 10V

For capacitive circuit, peak current is $i_0 = \epsilon_0/x_c$

Where $x_c = 1/\omega c$

[ω = angular frequency and c = capacitance of capacitor]

At ω = 10 s^{-1}

$x_c = 1/\omega c = 1/[10 \times 10^{-5}]$

$\epsilon_0 = 10$

=> $i_0 = \epsilon_0/x_c = 10^{-3}$ A

At ω = 100 s^{-1}

$x_c = 1/\omega c = 1/[100 \times 10^{-5}]$

$\epsilon_0 = 10$

=> $i_0 = \epsilon_0/x_c = 10^{-2}$ A

At ω = 500 s^{-1}

$x_c = 1/\omega c = 1/[500 \times 10^{-5}]$

$\epsilon_0 = 10$

=> $i_0 = \epsilon_0/x_c = 5 \times 10^{-2}$ A

At ω = 1000 s^{-1}

$x_c = 1/\omega c = 1/[1000 \times 10^{-5}]$

$\epsilon_0 = 10$

=> $i_0 = \epsilon_0/x_c = 10^{-1}$ A

Question 9: A coil of inductance 5.0 mH and negligible resistance is connected to the oscillator of the previous problem. Find the peak currents in the circuit for ω= 100 s^{-1}, 500 s^{-1}, 1000s^{-1}.

Solution:
Inductance of a coil = 5.0 mH and Peak voltage = ϵ_0 =10 V

Let X_L be the reactance of a coil and i_o be the peak current.

At ω = 100 s^{-1}

$X_L = \omega c = 0.005 \times 100 = 0.5$ ohm

=> $i_0 = \epsilon_0/x_L = 20$ A

At ω = 500 s^{-1}

$X_L = \omega c = 0.005 \times 500 = 2.5$ ohm

=> $i_0 = \epsilon_0/x_L = 4$ A

At ω = 1000 s^{-1}

$X_L = \omega c = 0.005 \times 1000 = 5$ ohm

=> $i_0 = \epsilon_0/x_L = 2$ A

Question 10: A coil has a resistance of 10 Ω and an inductance of 0.4 Henry. It is connected to an AC source of 6.5 V, 30/π Hz. Find the average power consumed in the circuit.

Solution:
Resistance= R = 10 Ω
Inductance = L = 0.4 Henry
Frequency = f = 30/π Hz
It is connected to an AC source (E) of 6.5 V

Impedance of a coil = $Z = \sqrt{[R^2+X_L^2]}$

$= \sqrt{[R^2+(2\pi fL)^2]}$

rms current = $i_{rms} = 6.5/Z$ and $\cos\phi = R/Z$

Average power consumed in the circuit:

Power = $V_{rms}\, i_{rms} \cos\phi$

$$Power = \frac{(6.5)^2 \times R}{\left(\sqrt{\{R^2 + (2\pi f L)^2\}}\right)^2}$$

$$= \frac{6.5 \times 6.5 \times 10}{(10 \times 10) + \left(2\pi \times 0.4 \times \frac{30}{\pi}\right)^2}$$

$$= 0.625 \text{ W}$$

Question 11: A resistor of resistance 100 Ω is connected to an AC source ϵ = (12 V) sin (250 π s^{-1}) t. Find the energy dissipated as heat during t = 0 to t = 1.0 ms.

Solution:
Energy dissipated (E) as heat during t = 0 to t = 1.0 ms = 10^{-3} s:

$$E = \int_0^t \frac{\epsilon^2}{R}$$

$$= \int_0^t \frac{[(12\,V) sin\,(250\,\pi\,t)]^2\,dt}{R}$$

$$= \frac{144}{100}\int_0^t \frac{1 - cos\,2 \times 250\pi t}{2}\,dt$$

$$= \frac{144}{100}\int_0^t (1 - cos\,500\pi t\,)dt$$

$$E = \frac{144}{100}\left[t - \frac{sin\,500\pi t}{500\pi}\right]$$

$$At\ t = 10^{-3}s$$

$$E = \frac{144}{100}\left[\frac{1}{1000} - \frac{1}{500 \times 3.14}\right]$$

$$E = 2.61 \times 10^{-4} J$$

[Using $sin^2A = (1-cos^2A)/2$]

Question 12: In a series RC circuit with an AC source R = 300Ω, C = 25 μF, ϵ_0 = 50 V and v = 50/π Hz. Find the peak current and the average power dissipated in the circuit.

Solution:
Resistance = R = 300Ω,
Capacitance = C = 25 μF = 25 x 10^{-6} F
Frequency = v = 50/π Hz
Rms voltage = ϵ_0 = 50/π V

We know, Reactance = X_C = 1/ωC

= 1/2πvc

= 1/[2π x 25 x 10^{-6} x 50/π]

= 10^4/25

Now,

Impedance, Z
We know, $Z^2 = R^2 + X_C^2$

$Z^2 = (300)^2 + (10^4/25)^2$

$Z^2 = 250000$

Or Z = 500

Peak Current, $i_o = \varepsilon_0/Z = 50/500 = 0.1$ A

Average power dissipated:

Power = $V_{rms}\, i_{rms} \cos \phi$

$$\text{Power} = \frac{E_0}{\sqrt{2}} \times \frac{E_0}{\sqrt{2} \times Z} \times \frac{R}{Z}$$

$$= \frac{50 \times 50 \times 300}{2 \times 500 \times 500}$$

$$= \frac{3}{2} = 1.5\text{W}$$

Question 13: An electric bulb is designed to consume 55 W when operated at 110 volts. It is connected to a 220 V, 50 Hz line through a choke coil in series. What should be the inductance of the coil for which the bulb gets correct voltage?

Solution:
Power = P = 55W,
Bulb operated at voltage =V = 110V,
Voltage supplied =E = 220V
Frequency = f = 50Hz
Let "L" is the inductance of the coil for which the bulb gets correct voltage.

We know Resistance = $R = V^2/P = (110)^2/55 = 220$ ohm

Angular velocity = $\omega = 2\pi f = 2\pi(50) = 100\pi$

Current in the circuit = $i = E/Z = E/\sqrt{[R^2+(\omega L)^2]}$

Voltage drop across the resistor = $V = iR = ER/\sqrt{[R^2+(\omega L)^2]}$

=> $110 = [220 \times 220]/\sqrt{[(220)^2 + (100\pi L)^2]}$

=> $(220)^2 + (100\pi L)^2 = (440)^2$

=> $L^2 = 1.4726$

or L = 1.213 Hz

Question 14: In a series LCR circuit with an AC source, R = 300Ω, C = 20 µF, L = 1.0 Henry, ϵrms = 50 V and v = 50/π Hz. Find
(a) The rms current in the circuit and
(b) The rms potential differences across the capacitor, the resistor and the inductor. Note that the sum of the rms potential differences across the three elements is greater than the rms voltage of the source.

Solution:
Given:
R = 300Ω, C = 20 µF, L = 1.0 Henry, ϵrms = 50 V and v = 50/π Hz.

(a) rms current in the circuit:

$I_{rms} = \varepsilon/Z$

Where Z is impedance in the circuit

We know, $Z^2 = R^2 + (X_C - X_L)^2$ ...(1)

Given R = 300Ω

$X_C = 1/(2\pi fC) = 1/(2\pi \times 50/\pi \times 20 \times 10^{-6}) = 10^4/20$

$X_L = 2\pi fL = 2\pi \times 50/\pi \times 1 = 100$

(1)=> $Z^2 = (300)^2 + (10^4/20 - 100)^2$

Or $Z^2 = 250000$

Or Z = 500

Thus, $I_o = 50/500 = 0.1$ A

(b) Potential difference across the capacitor

We know $V_C = I_o X_C = 0.1 \times 500 = 50$ V

Potential difference across the resistor

$V_R = I_o \times R = 0.1 \times 300 = 30$ V

Potential difference across the inductor

$V_L = I_o X_L = 0.1 \times 100 = 10$ V

Now, Net potential drops = $V_C + V_R + V_L = 50+30+10 = 90$ V

Rms Voltage = $\varepsilon_{rms} = 50$ V

=> Sum of all potential drops more than rms potential applied.

Question 15: Consider the situation of the previous problem. Find the average electric field energy stored in the capacitor and the average magnetic field energy stored in the coil.

Solution:
Resistance = R = 300Ω
Capacitance of capacitor =C = 20 μF
Frequency v = 50/π Hz
Inductance of inductor =L = 1.0 Henry
Voltage across the circuit ϵrms = 50 V

Current = $I_0 = 0.1$ A [From previous problem]
Electric energy stored in capacitor: E_C = (1/2) CV^2

$= (1/2) \times 20 \times 10^{-6} \times 50 \times 50$

$= 25 \times 10^{-3}$ J

= 25 mJ

Magnetic field energy stored in the coil: E_M = (1/2) LI_0^2

$= (1/2) \times 1 \times (0.1)^2 \times 5 \times 10^{-3}$

$= 5 \times 10^{-3}$ J
= 5 mJ

Question 16: An inductance of 2.0 H, a capacitance of 18 μF and a resistance of 10 kΩ are connected to an AC source of 20 V with adjustable frequency.
(a) What frequency should be chosen to maximize the current in the circuit?
(b) What is the value of this maximum current?

Solution:
(a) For current to be maximum in a circuit: $X_C = X_L$

Where, X_C and X_L resonance across capacitor and inductor respectively.

$1/\omega C = \omega L$

$\Rightarrow \omega^2 = 1/LC$

$\Rightarrow \omega^2 = 1/[2 \times 18 \times 10^{-6}]$

$\Rightarrow \omega^2 = 10^6/36$

$\Rightarrow \omega = 10^3/6$

Also $\omega = 2\pi f$

So, $f = 1000/[6 \times 2\pi] = 27$ Hz

(b) Max. Current = $I = E/R$

$\Rightarrow I = 20/(10 \times 10^3) = 2$ mA

Question 17: An inductor-coil, a capacitor and an AC source of rms voltage 24 V are connected in series. When the frequency of the source is varied, a maximum rms current of 6.0 A is observed. If this inductor coil is connected to a battery of emf 12 V and internal resistance 4.0Ω, what will be the current?

Solution:
rms voltage = E_{rms} = 24 V
Internal resistance = r = 4 ohm
rms current = I_{rms} = 6A

rms resistance = $R = E_{rms}/I_{rms} = 24/6 = 4$ ohm

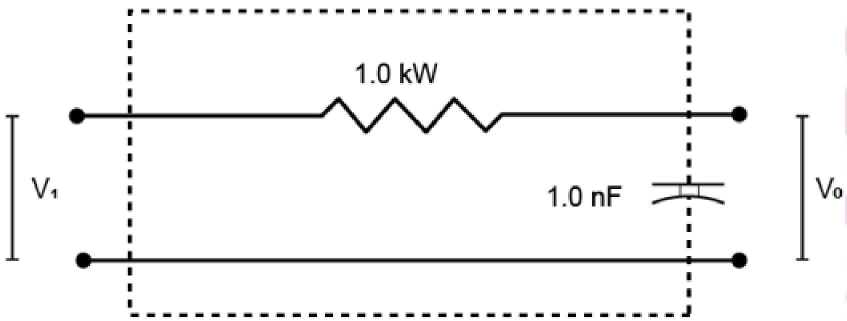
If this inductor coil is connected to a battery of emf 12 V and internal resistance 4.0Ω, say R' = 4.0Ω, the steady current

I = E/R'

Net resistance = R' = R + r' = 4 + 4 = 8 ohm

The steady current = I = 12/8 = 1.5 A

Question 18: Figure shows a typical circuit for low-pass filter. An AC input Vi = 10 mV is applied at the left end and the output V0 is received at the right end. Find the output voltages for v = 10 kHz, 100 k Hz, 1.0 MHz and 10.0 MHz Note that as the frequency is increased the output decreases and hence the name low-pass filter.



Solution:
Voltage = V = 10×10^{-3} V
Resistance = R = 1×10^{3} ohm
Capacitance = C = 10×10^{-9} F

At frequency, f = 10 kHz

Reactance = $X_C = 1/\omega C$
Where ω = angular velocity = 2πf

Here, $\omega C = 2\pi fC = 2\pi \times 10 \times 10^3 \times 10 \times 10^{-9} = 10^4/2\pi$

=> $X_C = 5000/\pi$

Again, Impedance = $Z = \sqrt{[R^2+X_C^2]}$

Here, $R^2 + X_C^2 = (1\times10^3)^2 + (5000/\pi)^2$

$R^2 + X_C^2 = 10^6 + (5000/\pi)^2$

=> $Z = \sqrt{[10^6 + (5000/\pi)^2]}$

Current = I_o = V/Z

$I_o = [10 \times 10^{-3}]/ \sqrt{[10^6 + (5000/\pi)^2]}$

Now, Output voltage = $V_0 = I_0 X_C$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5000}{\pi}\right)^2}} \times \frac{5000}{\pi}$$

V_o = 16.12 V = 16.1 mV

At frequency, f = 100 kHz

Reactance = $X_C = 1/\omega C$
Where ω = angular velocity = 2πf

$X_C = 1/2\pi fC = 1/[2\pi \times 100 \times 10^3 \times 10 \times 10^{-9}] = 10^3/2\pi$

=> $X_C = 500/\pi$

Again, Impedance = $Z = \sqrt{[R^2+X_C^2]}$

Here, $R^2 + X_C^2 = (1 \times 10^3)^2 + (500/\pi)^2$

$R^2 + X_C^2 = 10^6 + (500/\pi)^2$

=> $Z = \sqrt{[10^6 + (500/\pi)^2]}$

Current = I_o = V/Z

$I_o = [10 \times 10^{-3}]/ \sqrt{[10^6 + (500/\pi)^2]}$

Now, Output voltage = $V_0 = I_0 X_C$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{500}{\pi}\right)^2}} \times \frac{500}{\pi}$$

V_o = 1.612 V = 1.6 mV

At frequency, f = 1 MHz

Reactance = $X_C = 1/\omega C$
Where ω = angular velocity = 2πf

$X_C = 1/2\pi fC = 1/[2\pi \times 10^6 \times 10 \times 10^{-9}] = 10^2/2\pi$

=> $X_C = 50/\pi$

Again, Impedance = $Z = \sqrt{[R^2+X_C^2]}$

Here, $R^2 + X_C^2 = (1\times10^3)^2 + (50/\pi)^2$

$R^2 + X_C^2 = 10^6 + (50/\pi)^2$

=> $Z = \sqrt{[10^6 + (50/\pi)^2]}$

Current = $I_o = V/Z$

$I_o = [10 \times 10^{-3}]/\sqrt{[10^6 + (50/\pi)^2]}$

Now, Output voltage = $V_0 = I_0 X_C$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{50}{\pi}\right)^2}} \times \frac{50}{\pi}$$

V_o = 0.1612 V = 0.16 mV

At frequency, f = 10 MHz

Reactance = $X_C = 1/\omega C$
Where ω = angular velocity = 2πf

$X_C = 1/2\pi fC = 1/[2\pi \times 10^7 \times 10 \times 10^{-9}] = 10/2\pi$

=> $X_C = 5/\pi$

Again, Impedance = $Z = \sqrt{[R^2+X_C^2]}$

Here, $R^2 + X_C^2 = (1\times10^3)^2 + (5/\pi)^2$

$R^2 + X_C^2 = 10^6 + (5/\pi)^2$

=> $Z = \sqrt{[10^6 + (5/\pi)^2]}$

Current = $I_o = V/Z$

$I_o = [10 \times 10^{-3}]/ \sqrt{[10^6 + (5/\pi)^2]}$

Now, Output voltage = $V_0 = I_0 X_C$

$$V_0 = \frac{10 \times 10^{-3}}{\sqrt{10^6 + \left(\frac{5}{\pi}\right)^2}} \times \frac{5}{\pi}$$

$V_o = 0.01612$ V = 16 μV

Question 19: A transformer has 50 turns in the primary and 100 in the secondary. If the primary is connected to a 220 V DC supply, what will be the voltage across the secondary?

Solution:
If the primary is connected to a 220 V DC supply, the voltage across the secondary is zero.
Reason: A transformer works on the principle of electromagnetic induction, which is only possible on AC.

The induced current in the secondary coil is zero. Thus, output voltage will be zero.