Exercise Solutions

Question 1: A discharge tube contains helium at a low pressure. A large potential difference is applied across the tube. Consider a helium atom that has just been ionized due to the detachment of an atomic electron. Find the ratio of the distance travelled by the free electron to that by the positive ion in a short time dt after the ionization.

Solution:

Let the mass of the electron and positive helium ion be m_e and m_h.

$m_e = 9.1\times10^{-31}$kg and m_h = 4×mass of proton = $4\times1.6\times10^{-27}$kg

Magnitude of force experienced by electron: F = qE

=> m_e a = qE

=> a = qE/m_e

and Magnitude of force experienced by positive helium ion: F = qE

=> m_h a = qE

=> a = qE/m_h

We know, $s = ut + at^2/2$

Since initial velocity of electron and helium ion is zero.

=> $s = at^2/2$ ...(1)

Distance travelled by free electron and positive helium ion for dt duration:

(using (1))

$s_e = qE/2m_e\ (dt)^2$ and

$s_h = qE/2m_h\ (dt)$

The required ratio is:

$$\frac{S_e}{S_h} = \frac{\frac{qE}{2m_e} \times dt^2}{\frac{qE}{2m_h} \times dt^2}$$

$$= \frac{m_h}{m_e}$$

$$= \frac{4 \times 1.6 \times 10^{-27}}{9.1 \times 10^{-31}}$$

$$\frac{S_e}{S_h} = \frac{m_h}{m_e} = 7340.6$$

Question 2: A molecule of a gas, filled in a discharge tube, gets ionized when an electron is detached from it. An electric field of 5.0 kV m^{-1} exists in the vicinity of the event.
(a) Find the distance travelled by the free electron in 1 µs assuming no collision.
(b) If the mean free path of the electron is 1.0 mm, estimate the time of transit of the free electron between successive collisions

Solution:
(a) Electric field =E = 5 kV m^{-1} = 5×10^3 V m^{-1}
Time = t = 1 µs = 1×10^{-6} s.

Let m be the mass of electron and q be the charge of electron.

where m = 9.1×10^{-31} kg and q = 1.6×10^{-19} C

We know, F = qE and F= ma

on comparing, we get

=> ma = qE

=> a = qE/m

Also, distance travelled by an electron, at u = 0 m/s

=> s = (1/2) at^2

=> s = (1/2) (qE/m) t^2
=> s = $[1.6\text{x}10^{-19}\text{x}5\text{x}10^3\text{x}10^{-12}]/[2\text{x}9.1\text{x}10^{-31}]$

=> s = 439.5 m

(b) Mean free path S travelled by electron, s = 1mm = 10^{-3} m

a = qE/m = $[1.6\times10^{-19}\times5\times10^{3}]/[9.1\times10^{-31}] = 0.87\times10^{15}$

Also, s = (1/2) at^2

Or $t^2 = 2s/a = [2 \times 10^{-3}]/[0.87\times10^{15}] = 2.29\times10^{-18}$

Or t = 1.51×10^{-9}s = 1.5 ns

Question 3: The mean free path of electrons in the gas in a discharge tube is inversely proportional to the pressure inside it. The Crookes dark space occupies half the length of the discharge tube when the pressure is 0.02 mm of mercury. Estimate the pressure at which the dark space will fill the whole tube.

Solution:
Let the mean free path be L and pressure be P.

When P becomes half, L doubles, i.e. the whole tube is filled with Crook's dark space.

Thus, the required pressure = 0.02/2 = 0.01 m of Hg.

Question 4: Two discharge tubes have identical material structure and the same gas is filled in them. The length of one tube is 10 cm and that of the other tube is 20 cm. Sparking starts in both the tubes when the potential difference between the cathode and the anode is 100 V. If the pressure in the shorter tube is 1.0 mm of mercury, what is the pressure in the longer tube?

Solution:
Let d_1 and p_1 be the length and pressure of short tube.
And, d_2 and p_2 be the length and pressure of long tube.

So, p_1 = 1.0 mm , d_1 = 10 cm and d_2 = 20 cm

According to Paschen's law, V = f pd

Sparking potential of a gas in a discharge tube is the function of the product of pressure of the gas and separation between the electrodes.

$V = p_1d_1 = p_2d_2$

=> $p_2 = p_1d_1/d_2 = [1\times10]/20 = 0.5$ mm

Question 5: Calculate n(T)/n (1000 K) for tungsten emitter at T = 300 K, 2000 K and 3000 K where n(T) represents the number of thermions emitted per second by the surface at temperature T. Work function of tungsten is 4.52 eV.

Solution:

According to Richardson-Dushman Equation thermionic current:

$i = ne = AST^2\, e^{-\phi/kT}$

where, A is the constant depend on nature of metal, S is the surface area, n is the thermions emitted, k is the Boltzmann constant, T is the absolute temperature and ϕ is the work function.

For T = 300 K

$$\frac{n(T)}{n(1000)} = \frac{AS300^2\, e^{\frac{-4.52\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}}}{AS1000^2\, e^{\frac{-4.52\times1.6\times10^{-19}}{1.38\times10^{-23}\times1000}}} = 7.057\times10^{-55}$$

For T = 2000 K

$$\frac{n(T)}{n(1000)} = \frac{AS2000^2\, e^{\frac{-4.52\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000}}}{AS1000^2\, e^{\frac{-4.52\times1.6\times10^{-19}}{1.38\times10^{-23}\times1000}}} = 9.59\times10^{11}$$

For T = 3000 K

$$\frac{n(T)}{n(1000)} = \frac{AS3000^2\, e^{\frac{-4.52\times1.6\times10^{-19}}{1.38\times10^{-23}\times3000}}}{AS1000^2\, e^{\frac{-4.52\times1.6\times10^{-19}}{1.38\times10^{-23}\times1000}}} = 1.34\times10^{16}$$

Question 6: The saturation current from a thoriated-tungsten cathode at 2000 K is 100 mA. What will be the saturation current for a pure-tungsten cathode of the same surface area operating at the same temperature? The constant A in the Richardson-Dushman equation is 60×10^4 A $m^{-2}K^{-2}$ pure tungsten and 3.0×10^4 A $m^{-2}K^{-2}$ for thoriated tungsten. The work function of pure tungsten is 4.5 eV and that of thoriated tungsten is 2.6 eV.

Solution:

According to Richardson-Dushman Equation thermionic current:

i = ne = AST^2 e$^{-\phi/kT}$

where, A is the constant depend on nature of metal, S is the surface area, n is the thermions emitted, k is the Boltzmann constant, T is the absolute temperature and ϕ is the work function.

$$i = 60 \times 10^4 \times S \times 2000^2 \times e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000}}$$

$$100 \times 10^{-3} = 3 \times 10^4 \times S \times 2000^2 \times e^{\frac{-2.6\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000}}$$

Dividing above equations, we get

$$\frac{i}{100 \times 10^{-3}} = 20 \times e^{\left(\frac{-4.5\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000} - \frac{-2.6\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000}\right)}$$

$$= 20 \times e^{\frac{(2.6-4.5)\times1.6\times10}{2\times1.38}}$$

$$= 20 \times e^{\frac{8\times(-1.9)}{1.38}}$$

$$\Rightarrow i = 32.9 \times 10^{-6} \text{ A} = 33\,\mu\text{A}$$

Question 7: A tungsten cathode and a thoriated-tungsten cathode have the same geometrical dimensions and are operated at the same temperature. The thoriated-tungsten cathode gives 5000 times more current than the other one. Find the operating temperature. Take relevant data from the previous problem.

Solution:

According to Richardson-Dushman Equation thermionic current:

i = ne = AST^2 e$^{-\phi/kT}$

where, A is the constant depend on nature of metal, S is the surface area, n is the thermions emitted, k is the Boltzmann constant, T is the absolute temperature and ϕ is the work function.

=> $I_{ttungsten} = 5000\ I_{ptungsten}$

$$=> S \times 3 \times 10^4 \times T^2 \times e^{\frac{-2.6\times1.6\times10^{-19}}{1.38\times10^{-23}\times T}} = 5000 \times 60 \times 10^4 \times S \times T^2 \times e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}}$$

$$=> e^{\frac{-2.5\times1.6\times10^{-19}}{1.38\times10^{-23}\times T}} = 10^5 \times e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times10^{-23}\times T}}$$

Applying "*ln*" both sides to eliminate exponential function.

$[-2.89\times10^4]/T = 11.51 + [-5.22\times10^4]/T$

=> T = 2024 K

Question 8: If the temperature of a tungsten filament is raised from 2000 K to 2010 K, by what factor does the emission current change? Work function of tungsten is 4.5 eV.

Solution:
Let initial and final temperature of tungsten filament be I_1 and I_2, then

$$\frac{I_1}{I_2} = \frac{A \times S \times 2000^2\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000}}}{A \times S \times 2010^2\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times10^{-23}\times2010}}}$$

$$= \frac{2000^2\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times10^{-23}}\left(\frac{1}{2000}-\frac{1}{2010}\right)}}{2010^2}$$

$$= \frac{2000^2 \times 0.87828}{2010^2} = .869564$$

The required factor is $I_2/I_1 = 1/0.8696 = 1.14$ (approx)

Question 9: The constant A in the Richardson-Dushman equation of tungsten is 60×10^4 A m^{-2} K^{-2}. The work function surface area 2.0×10^{-5} m^2 is heated by a 24 W electric equals the energy input by the heater and the temperature becomes constant. Assuming that the cathode radiates like a blackbody, calculate the saturation current due to thermions. Take Stefan constant $=6 \times 10^{-8}$ W m^{-2} K^{-4}. Assume that the thermions take only a small fraction of the heat supplied.

Solution:
The power radiated: $P = S\sigma T^4$

[Using Stefen's law]

=> $24 = 6 \times 10^{-8} \times 2 \times 10^{-5} \times T^4$

=> $T = 2.1147 \times 10^3$ K

Again, According to Richardson-Dushman Equation thermionic current i

$i = ne = AST^2 e^{-\phi/kT}$

=>$i = 60 \times 10^4 \times 2\times10^{-5} \times 2114.7^2 \times e^{\{[-4.5\times1.6\times10^{-19}]/[1.38 \times10^{-23}\times2114.7]\}}$

=> $i = 1.034 \times 10^{-3}$ A = 1 mA (approx.)

Question 10: A plate current of 10 mA is obtained when 60 volts are applied across a diode tube. Assuming the Langmuir-Child relation $i_p \propto V_P^{(3/2)}$ to hold, find the dynamic resistance r_p in this operating condition.

Solution:

Given: $i_p \propto V_P^{(3/2)}$

If k = proportionality constant , then

$i_p = k V_P^{(3/2)}$ ....(1)

Derivate above equation with respect to i_p, we have

$1 = (3/2) k(V_P)^{(1/2)} \times dV_P/dI_P$ ...(2)

Dividing (2) by (1)

$1/i_P = (3/2V) dV_P/dI_P$

At V_g = constant

=> $(dV_P/dI_P) = 2V/3i_P = [2\times60]/[3\times10\times10^{-3}]$ = 4 k ohms

Question 11: The plate current in a diode is 20 mA when the plate voltage is 50 V or 60 V. What will be the current if the plate voltage is 70V?

Solution:
For a given temperature current remains same for all voltages i.e. for 70 V current is 20 mA.

Question 12: The power delivered in the plate circuit of a diode is 1.0 W when the plate voltage is 36 V. Find the power delivered if the plate voltage is increased to 49 V. Assume Langmuir – Child equation to hold.

Solution:
Let i_{p1} is the plate current and V_{p1} is the plate voltage.

Here $P_1 = 1.0$ W, $V_{p1} = 36$ V, $V_{p2} = 49$ V

We know, P = VI

=> $P_1 = V_1\, i_{p1}$

=> $i_{p1} = P_1/V_1 = 1/36$

Using Langmuir-Child equation, we have

$$\frac{i_{p1}}{i_{p2}} = \frac{V_{p1}^{\frac{3}{2}}}{V_{p2}^{\frac{3}{2}}} = \frac{36^{\frac{3}{2}}}{49^{\frac{3}{2}}} = \frac{216}{343}$$

$$=> i_{p2} = \frac{343}{36 \times 216} = 0.04411$$

$P_2 = i_{P2} \times V_{P2} = 0.04411 \times 49 = 2.16$

Or $P_2 = 2.2$ W

Question 13: A triode valve operates at $V_p = 225$ V and $V_g = –0.5$ V. The plate current remains unchanged if the plate voltage is increased to 250 V and the grid voltage is decreased to –2.5 V. Calculate the amplification factor.

Solution:
We know, amplification factor for triode = - $(\Delta V_p/\Delta V_g)$

Here $\Delta V_p = \Delta V_{p1} - \Delta V_{p2} = 225 - 250 = -25$

$\Delta V_g = \Delta V_{g1} - \Delta V_{g2} = -0.5 + 2.5 = 2$

=> amplification factor for triode = 12.5

Question 14: Calculate the amplification factor of a triode valve which has plate resistance of 2 kΩ and transconductance of 2 millimho.

Solution:
r_p = 2 KΩ = 2×10^3 (plate resistance)

g_m =2 millimho = 2×10^{-3} mho(transconductance)

Amplification factor for triode = $\mu = r_p \times g_m = 2 \times 10^3 \times 2 \times 10^{-3} = 4$

Question 15: The dynamic plate resistance of a triode valve is 10 kΩ. Find the change in the plate current if the plate voltage is changed from 200 V to 220 V.

Solution:

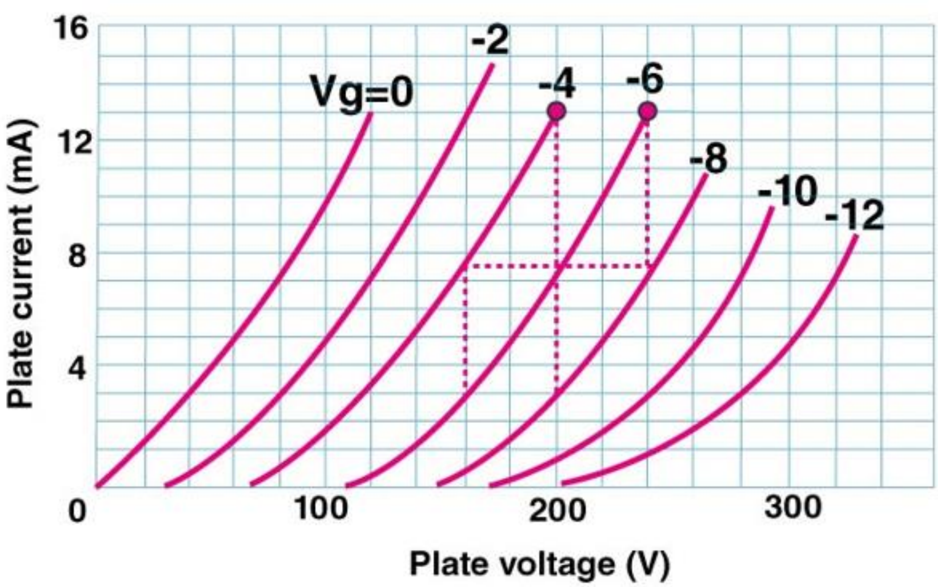
The dynamic resistance of triode

$r_p = (\Delta V_p/\Delta I_p)$ [at V_g = constant]

$\Rightarrow 10^4 = [220-200]/ \Delta I_p$

$\Rightarrow \Delta I_p = 20/10^4 = 2$ mA

Question 16: Find the values of r_p, μ and g_m of a triode operating at plate voltage 200 V and grid voltage –6 V. The plate characteristics are shown in figure.



Solution:

The dynamic resistance of triode

$r_p = (\Delta V_p/\Delta I_p)$ [at V_g = constant]

From figure, considering the line $V_g = -6$ V, we get two value of V_P i.e.

$V_{P1} = 160$ V and $V_{P2} = 240$ V

$i_{p1} = 3$ mA and $i_{p2} = 13$ mA

=> $r_p = [240-160]/[(13-3)\times10^{-3}] = 8$ K ohm

Mutual inductance g_m of a triode valve:

$g_m = (\Delta I_p/\Delta V_g)$ [at V_p = constant]

Taking any two points on the 200 V line, in the graph,
$V_p = 200$ V, $i_{p1} = 13$ mA, $i_{p2} = 3$mA and $V_{g1} = -4$ V, $V_{g2} = -8$ V

=>$g_m = [(13-3)\times10^{-3}]/(-8+4) = 2.5$ milli mho

Amplification factor for triode:

$\mu = (\Delta V_p/\Delta V_G)$ [at I_p = constant]

$\mu = (100-180)/(-6-(-10) = 20$

Question 17: The plate resistance of a triode is 8 kΩ and the transconductance is 2.5 milli mho.
(a) If the plate voltage in increased by 48 V, and the grid voltage is kept constant, what will be the increase in the plate current?
(b) With plate voltage kept constant at this increased value, how much should the grid voltage be decreased in order to bring the plate current back to its initial value?

Solution:

(a) The dynamic resistance of triode:

$r_p = (\Delta V_p/\Delta I_p)$ [at V_g = constant]

When $\Delta V_P = 48$ V, V_g = Constant

$=> 8000 = 48/\Delta I_p$

$= \Delta I_p = 6$ mA

(b) When $g_m = 2.5$ millimho = 0.0025 mho and
$\Delta I_p = 6$ mA = 0.006 A

$g_m = (\Delta I_p/\Delta V_g)$ [at V_p = constant]

$=> 0.0025 = 0.006/\Delta V_g$

$=> \Delta V_g = 2.4$ V

Question 18: The plate resistance and the amplification factor of a triode are 10 kΩ and 20. The tube is operated at plate voltage 250 V and grid voltage –7.5 V. The plate current is 10 mA
(a) To what value should the grid voltage be changed so as to increase the plate current to 15 mA?
(b) To what value should the plate voltage be changed to take the plate current back to 10 mA?

Solution:

(a) Here $i_{p2} = 15$ mA ; V_P = Constant, and $i_{p1} = 10$ mA

Find: V_{g2} = ?

We know, $g_m = \mu/r_p$

$=> g_m = 20/10000 = 2 \times 10^{-3}$ mho

Also,
$g_m = (\Delta I_p/\Delta V_g)$ [at V_p = constant]

$0.002 = [(15-10) \times 10^{-3}]/\Delta V_g$

$=> \Delta V_g = 0.005/0.002 = 2.5$ V

Also we have $\Delta V_g = V_{g2} - V_{g1}$

$=> V_{g2} = 2.5 – 7.5 = -5$ V

(b) The dynamic resistance of triode:

$r_p = (\Delta V_p/\Delta I_p)$ [at V_g = constant]

Here i_{p3} = 10 mA, i_{p2} = 15mA and i_{p3} = 10mA

Find : V_{P3}

=>10000 = $\Delta V_p/[(10-15)x10^{-3}]$

= ΔV_p = -50

Also, $\Delta V_p = V_{p3} - V_{p2}$
(Given : V_{P2} = 250 V)

=> V_{p3} = 200 V

Question 19: The plate current, plate voltage and grid voltage of a 6F6 triode tube are related as $i_p = 41(V_p + 7 V_g)^{1.41}$, Where V_p and V_g are in volts and i_p in microamperes. The tube is operated at V_p = 250 V, V_g = –20 V. Calculate
(a) the tube current,
(b) the plate resistance,
(c) the mutual conductance and
(d) the amplification factor.

Solution:
(a) Here, V_P = 250 V, V_g = –20 V
And, $i_p = 41(V_p + 7 V_g)^{1.41}$ .....(1)

=> $i_p = 41(250 - 140)^{1.41}$ = 30984.71 μA = 30 mA

b) Differentiating equation (1) w.r.t. i_p

$1 = 41 \times 1.41(V_p + 7 V_g)^{1.41} \times dV_p/di_p$ ....(2)

[Here, V_g = constant]

The dynamic resistance of triode, $r_p = dV_p/di_p$
When , V_g = constant

From equation (2),

$dV_p/di_p = [1x10^6]/[41x1.41x110^{0.41}]$ = 2.5 K Ω

(c) Differentiating equation (1) w.r.t. V_g

$di_p/dV_g = 41 \times 1.41(V_p + 7 V_g)^{1.41} \times 7$

where, V_p constant

We know, $g_m = di_p/dV_g$ at V_p constant

=> $41 \times 1.41 \times 110^{0.41} \times 7 = 2780.18$ μ mho = 2.78 millimho

(d) Amplification factor for triode = $\mu = r_p \times g_m$

$= 2.5 \times 10^3 \times 2.78 \times 10^{-3}$

= 6.95

μ = 7 (approx.)

Question 20: The plate current in a triode can be written as $i_p = k(V_g + V_p/\mu)^{3/2}$. Show that the mutual conductance is proportional to the cube root of the plate current.

Solution:
Given: $i_p = k(V_g + V_p/\mu)^{3/2}$

We know, $g_m = di_p/dV_g$ at V_p constant

Where, g_m is the mutual conductance, i_p is the plate current, V_p is the plate current and V_g is the grid voltage.

$g_m = di_p/dV_g = (3/2)k[V_g + V_p/\mu]^{(1/2)}$

Cubing both the sides, we get

$g_m^3 = (27/8)k^3[V_g + V_p/\mu]^{(3/2)} = (27/8)\, k^2 i_p$

=>$g_m \propto i_p^{1/3}$

Hence proved.

Question 21: A triode has mutual conductance = 2.0 millimho and plate resistance = 20 kΩ. It is desired to amplify a signal by a factor of 30. What load resistance should be added in the circuit?

Solution:
A triode has mutual conductance = 2.0 millimho and plate resistance = 20 kΩ.

g_m = 2.0 millimho = 2×10^{-3} mho and $r_p = 20\Omega$ and A = 30

We know, $\mu = r_p \times g_m = 20 \times 10^3 \times 2 \times 10^{-3} = 40$

and $r_P/R_L = 20000/R_L$

Where R_L = load resistance

Also, we know

A (Voltage gain) = $\mu/[1+ r_P/R_L]$

Substituting the values, we have

$30 = 40/[1+20000/R_L]$

$\Rightarrow R_L = 60\ k\Omega$

Question 22: The gain factor of an amplifier is increased from 10 to 12 as the load resistance is changed from 4 kΩ to 8 KΩ. Calculate (a) the amplification factor and (b) the plate resistance.

Solution:
We know,

A (Voltage gain) = $\mu/[1+ r_P/R_L]$ ….(1)

When:
$A_1 = 10$, $R_{l1} = 4k\Omega$

(1)=> $10 = \mu/[1+r_p/4000]$

$\Rightarrow 10(4000 + r_p) = 4000\ \mu$

Or $400\ \mu - r_p = 4000$ …(2)

When: $A_2 = 1_2$, $R_{l2} = 8k\Omega$

(1)=> $1_2 = \mu/[1+r_p/8000]$

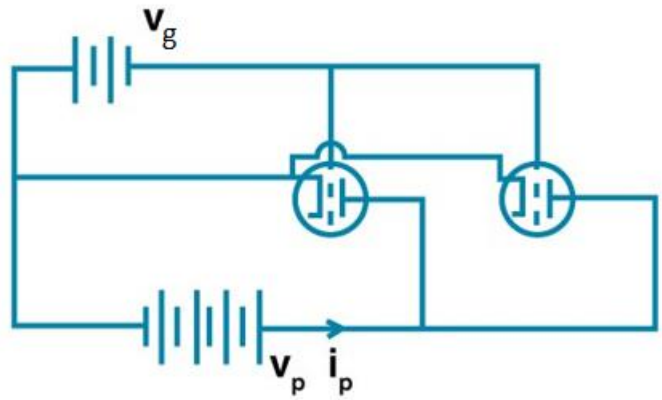
$\Rightarrow 12(8000 + r_p) = 8000\ \mu$

Or $8000\ \mu - 12r_p = 96000$ …(3)

Solving, (2) and (3), we have

$\mu = 15$ and r_p is 2000Ω or 2 kΩ.

Question 23: Figure shows two identical triode tubes connected in parallel. The anodes are connected together, the grids are connected together and the cathodes are connected together. Show that the equivalent plate resistance is half to the individual plate resistance. The equivalent mutual conductance is double the individual mutual conductance and the equivalent amplification factor is the same as the individual amplification factor



Solution:

Given the anodes are connected together, and two triodes have same voltage and same current, this implies, $r_{P1} = r_{P2} = r$

The equivalent resistance,

$$R = \frac{(r_{P1} \times r_{P2})}{(r_{p1} + r_{p2})} = \frac{r^2}{2r} = \frac{r}{2}$$

Let g_{m1} and g_{m2} be the individual conductance.

From figure, $g_{m1} = g_{m2} = g$.

Since, two triodes are parallel, we have equivalent conductance.

=> $G = g_{m1} + g_{m2} = g + g = 2g$

Now, $\mu_1 = g_{m1} r_{p1} = gr$ and $\mu_2 = g_{m2} r_{p2} = gr$

So, equivalence amplification factor = $\mu = RG = (r/2) \times 2g = rg$

Therefore, equivalent amplification factor is the same as the individual amplification factor.