Exercise Solutions

Question 1: Calculate the volume of 1 mole of an ideal gas at STP.

Solution:
We know ideal gas equation, PV = nRT ...(1)

Pressure = P = 1.01 Pa
Number of moles = n = 1
Temperature = T = 273.15K

We know, R = gas constant = $8.3 JK^{-1}mol^{-1}$

(1)=> v = nRT/P = $[1x8.31x273.15]/[1.01x10^5] = 0.0224\ m^3 = 2.24 \times 10^2\ m^3$

Question 2: Find the number of molecules of an ideal gas in a volume of 1.000 cm^3 at STP.

Solution:
Volume of ideal gas at STP= 22.4L = $22.4 \times 10^3\ cm^3$
Number of molecule in 22.4×10^3 of ideal gas at STP = 6.022×10^{23}

Number of molecules in $1cm^3$ of ideal gas at STP = $[6.022 \times 10^{23}]/[22.4 \times 10^3] = 2.688 \times 10^{19}$.

Question 3: Find the number of molecules in 1 cm^3 of an ideal gas at 0°C and at a pressure of 10^{-5} mm of mercury.

Solution:
Volume of ideal gas = V = $1cm^3 = 10^{-6}\ m^3$

We know ideal gas equation, PV = nRT

Pressure = P = 10^{-5} mm of Hg = 133.32×10^{-5} Pa
Temperature = T = 273.15K

We know, R = gas constant = $8.3 JK^{-1}mol^{-1}$

=> n = PV/RT = $[133.32 \times 10^{-5} \times 10^{-6}]/[8.3 \times 273.15]$

= 3.538×10^{11}

Question 4: Calculate the mass of 1 cm^3 of oxygen kept at STP.

Solution:
Volume of oxygen gas = 1 cm^3 = 10^{-3} m^3 (Given)
Volume of oxygen gas at STP= 22.4L = 22.4×10^3 cm^3

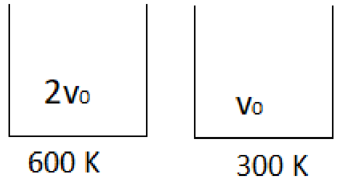
we know, ideal equation for gas: n = PV/RT

= $[1 \times 1 \times 10^{-3}]/[0.082 \times 273]$

= $10^{-3}/22.4$

Now, mass = $[10^{-3}/22.4] \times 32$ grams = 1.428×10^{-3} grams = 1.428 mg

Question 5: Equal masses of air are sealed in two vessels, one of volume V_0 and the other of volume $2V_0$. If the first vessel is maintained at a temperature 300 K and the other at 600 K, find the ratio of the pressures in the two vessels.



Solution:
We know ideal gas equation, PV = nRT

Pressure = P = 1.01 Pa
Number of moles = n = 1
Temperature = T = 273.15K

We know, R = gas constant = $8.3 JK^{-1}mol^{-1}$
Since Masses of both the gas is equal. Therefore, number of moles of both the gas is equal.

=> $n_1 = n_2 = n$

Now,
$P_1 = [nRx300]/V_o$ and

$P_2 = [nRx600]/2V_o$

$P_1/P_2 = [nRx300]/V_o \times [nRx600]/2V_o = 1/1$

So, the ratio of pressure gas in two vessels is 1:1.

Question 6: An electric bulb of volume 250 cc was sealed during manufacturing at a pressure of 10^{-3} mm of mercury at 27°C. Compute the number of air molecules contained in the bulb. Avogadro constant = 6×10^{23} mol^{-1}, density of mercury = 13600 kg m^{-3} and g = 10 m s^{-2}.

Solution:
We know ideal gas equation, PV = nRT
R = gas constant and n = number of moles
Pressure = P = 10^{-3} mm of mercury = 133.32×10^{-3} Pa
Temperature = T = 27^0C = 27 + 273.15 = 300.15 K
and Volume of gas = 250cc = 250×10^{-6} m^3

Now, n = PV/RT = $[133.32 \times 10^{-3} \times 250 \times 10^{-6}]/[8.31 \times 300.15]$

= 13.36×10^{-9}

Number of molecules in electric bulb = Avogadro number x number of moles

= $6 \times 10^{23} \times 13.36 \times 10^{-9}$

= 8.01×10^{15}

Question 7: A gas cylinder has walls that can bear a maximum pressure of 1.0×10^6 Pa. It contains a gas at 8.0×10^5 Pa and 300 K. The cylinder is steadily heated. Neglecting any change in the volume, calculate the temperature at which the cylinder will break.

Solution:
We know ideal gas equation, PV = nRT
R = gas constant and n = number of moles
Pressure = P
Temperature = T

Given, Pressure of gas = $P_1 = 8.0 \times 10^5$ Pa
$P_{max} = P_2 = 10^6$ Pa
Temperature of gas = T_1 = 300K

Since the volume has not been changed: $v_1 = v_2$
Number of moles will also be same = $n_1 = n_2 = n$

Let T_2 be the temperature at which the cylinder will break.

$n_1 = P_1V_1/RT_1$ and $n_2 = P_2V_2/RT_2$

=> $P_1/ P_2 = T_1/T_2$

=>$T_2 = P_2T_1/P_1 = [10^6x300]/[8x10^5] = 375$ K

Question 8: 2g of hydrogen is sealed in a vessel of volume 0.02 m^3 and is maintained at 300K. Calculate the pressure in the vessel.

Solution:
We know ideal gas equation, PV = nRT
R = gas constant and n = number of moles
Pressure = P and V= volume of gas
Temperature = T

Given, V = 0.02 m^3, T = 300K
Mass of hydrogen gas =M = 2g
m = 2g

Here n = m/M ; m = given mass and M = molar mass

So, PV = (m/M) RT

=> P x 20 = (2/2) x 0.082 x 300

=> P = 1.23 atm = 1.23 x 10^5 Pa (approx)

Question 9: The density of an ideal gas is 1.25 × 10^{-3} g cm^{-3} at STP. Calculate the molecular weight of the gas.

Solution:
We know ideal gas equation, PV = nRT
R = gas constant and n = number of moles
Pressure = P and V= volume of gas
Temperature = T

Given: P = 101.325 x 10^3 Pa and T = 273.15K
Density of ideal gas = ρ = 1.25 × 10^{-3} g cm^{-3} = 1.25 kgm^3
and density = m/V
Also, n = m/M ; m = given mass and M = Molecular weight of gas

So, PV = (m/M) RT

=> M = (ρ RT)/P = 1.25 x 8.31 x 300 x 10^{-5}

= 2.38 x 10^{-2} g mol^{-1}

Question 10: The temperature and pressure at Shimla are 15.0°C and 72.0 cm of mercury and at Kalka these are 35.0°C and 76.0 cm of mercury. Find the ratio of air density at Kalka to the air density at Shimla.

Solution:
We know ideal gas equation, PV = nRT
R = gas constant and n = number of moles
Pressure = P and V= volume of gas
Temperature = T

Given: Pressure of Shimla = P_1 = 72.0 cm of mercury

Temperature of Shimla = T_1 = 15.0°C = 15+273.15 = 288.15K

Pressure of Kalka = P_1 = 76.0 cm of mercury
Temperature of Kalka = T_2 = 35.0°C =35+273.15=308.15K

Also, n = m/M ; m = given mass and M = Molar mass
and density = ρ = m/V ; v = volume

So, PV = (m/M) RT

Now, PV = nRT = (m/M)RT = (ρv/M) RT

=> ρ = PM/RT

$\rho_1 = P_1M/RT_1$ ...(1) and
$\rho_2 = P_2M/RT_2$ ...(1)

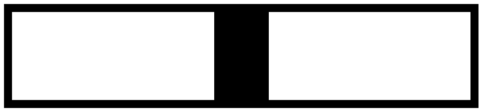
$\rho_1 / \rho_2 = [P_1M/RT_1]/[P_2M/RT_2] = [P_1T_2]/[P_2T_1]$

=> ρ_1 / ρ_2 = [0.76 x 308.15]/[0.76x288.15] = 1.013

And ρ_2 / ρ_1 = 1/1.013 = 0.987

The ratio of air density at Kalka to the air density at Shimla is 0.987.

Question 11: Figure (below) shows a cylindrical tube with adiabatic walls and fitted with a diathermic separator. The separator can be slid in the tube by an external mechanism. An ideal gas is injected into the two sides at equal pressures and equal temperatures. The separator remains in equilibrium at the middle. It is now slid to a position where it divides the tube in the ratio of 1: 3. Find the ratio of the pressures in the two parts of the vessel.



Solution:

We know ideal gas equation: PV = nRT

Volume of first part=V and Volume of second part=3V

from question. $n_1 = n_2 = n$

Since the walls of separator is diathermic, the temperature of both the parts will always be same i.e. $T_1 = T_2 = T$

Now,
$P_1 = nRT/V$ and $P_2 = nRT/3V$

$P_1/P_2 = 3:1$

Question 12: Find the rms speed of hydrogen molecules in a sample of hydrogen gas at 300 K. Find the temperature at which the rms speed is double the speed calculated in the previous part.

Solution:
Temperature = T = 300K
Molar mass of hydrogen = 2g = 0.002 kg

We know that rms speed of gas is $v_{rms} = \sqrt{(3RT/M)}$
Where R= gas constant

$= \sqrt{(3x8.31x300/0.002)}$

= 1932.6 m/s

Let the temp at which the speed is doubed, i.e. 2 x 1932.6 is T'

$2 \times 1932.6 = \sqrt{(3x8.31xT'/0.002)}$

=> T' = 1199.98 K = 1200 K(approx.)

Temperature of the gas when speed is doubled is 1200K which is 4 times the pervious temperature.

Question 13: A sample of 0.177 g of an ideal gas occupies 1000 cm^3 at STP. Calculate the rms speed of the gas molecules.

Solution:
Mass = 0.177g = 0.177 x 10^{-3} kg
Volume = 1000 cm3 = 10^{-3} m^3

Also, density = ρ = mass/volume = 0.177 kg/m^3

Temperature = T= 273.15K
Pressure = P = 101.325 x 10^3 Pa

We know that rms speed of gas is v_rms = √(3RT/M) ...(1)
Where R= gas constant = 8.31J/molK

The ideal gas equation: PV = nRT

or RT = PV/n

(1)=> v_{rms} = √(3PV/nM)
Where, nM = total mass of gas

Above equation can be written in the form of density as,

v_{rms} = √(3P/ρ)

= √[(101.325 x 10^3)/0.177]

= 1310.4 m/s

Question 14: The average translational kinetic energy of air molecules is 0.040 eV (1 eV = 1.6 × 10^{-19}J). Calculate the temperature of the air. Boltzmann constant κ = 1.38 × 10^{-23} J K^{-1}.

Solution:
Average kinetic energy = (3/2) KT
Where k= Boltzmann constant

=> $(3/2) \times 1.38 \times 10^{-23} \times T = 0.04 \times 1.6 \times 10^{-19}$

=> T = 309.178

Question 15: Consider a sample of oxygen at 300 K. Find the average time taken by a molecule of travel a distance equal to the diameter of the earth.

Solution:
In kinetic theory of ideal gas, the average energy: $v_{avg} = \sqrt{(8RT/\pi M)}$
Molar mass of oxygen = M = 32amu = 32 g/mol = 32×10^{-3} kg/mol

Given, Temperature = T = 300K

Now, $v_{avg} = \sqrt{[(8 \times 8.31 \times 300)/(3.14 \times 32 \times 10^{-3})]}$

= 445.25 m/s

Also, Time =Distance/Speed
Here, Distance = diameter of earth = 2 x 6400000 m

Time = (2 x 6400000)/445.25

= 28747.83 sec

= 8 hour (approx)

Question 16: Find the average magnitude of linear momentum of a helium molecule in a sample of helium gas at 0°C. Mass of a helium molecule = 6.64×10^{-27} kg and Boltzmann constant = 1.38×10^{-23} J K^{-1}.

Solution:

In kinetic theory of ideal gas, the average energy: $v_{avg} = \sqrt{(8RT/\pi M)}$

Where M = Molar mass of gas molecule = N_A x m ...(a)

Given, Temperature = T = 0+273.15=273.15K
Mass of helium molecule = m =6.64×10^{-27} kg

We know, Gas constant =R = $k_B N_A$

Where k_B = Boltzmann constant = 1.38×10^{-23} J K^{-1} and
N_A = Avogadro number = 6.023 x 10^{-23} mol^{-1}

Now,

$v_{avg} = \sqrt{(8k_B N_A T/\pi M)}$

Using (a), we get

$v_{avg} = \sqrt{(8k_B T/\pi m)}$

$= \sqrt{[(8x1.38x10^{-23}x273.15)/(3.14x6.64 \times 10^{-27})]}$

= 1202.31 m/s

Momentum = mass x v_avg

$= 6.64 \times 10^{-27}$ x 1202.31

= 8 x 10^{-24} kgm/s

Question 17: The mean speed of the molecules of a hydrogen sample equals the mean speed of the molecules of a helium sample. Calculate the ratio of the temperature of the hydrogen sample to the temperature of the helium sample.

Solution:
In kinetic theory of ideal gas, the average energy: $v_{avg} = \sqrt{(8RT/\pi M)}$

Given, v_{mean} (H) = v_{mean} (He)

Molar mass of hydrogen gas = 2amu and
Molar mass of helium gas = 4amu

Now,

$$\sqrt{\frac{8RT(H)}{\pi M(H)}} = \sqrt{\frac{8RT(He)}{\pi M(He)}}$$

$$\sqrt{\frac{T(H)}{T(He)}} = \sqrt{\frac{M(H)}{M(He)}}$$

$$= \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}}$$

Squaring both the sides, we get

T(H)/T(He) = ½

Question 18: At what temperature the mean sped of the molecules of hydrogen gas equals the escape speed from the earth?

Solution:
In kinetic theory of ideal gas, the average energy: $v_{mean} = \sqrt{(8RT/\pi M)}$

Molar mass of hydrogen = 2amu = 2 g/mol = 2 x 10^{-3} kg/mol

Escape speed of earth = $V_e = \sqrt{2gr}$

=> $\sqrt{(8RT/\pi M)} = \sqrt{2gr}$

Squaring both sides,

=> $(8RT/\pi M) = 2gr$

or $T = (2gr\pi M)/8R$

$= [2 \times 9.8 \times 6400000 \times 3.14 \times 2 \times 10^{-3}]/[8 \times 8.3]$

= 11800 m/s (approx)

Question 19: Find the ratio of the mean speed of hydrogen molecules to the mean speed of nitrogen molecules in a sample containing a mixture of the two gases.

Solution:
In kinetic theory of ideal gas, the average energy: $v_{mean} = \sqrt{(8RT/\pi M)}$

Molar mass of hydrogen molecule =M(H)=2 amu and Molar mass of nitrogen molecule= M(N) = 28 amu

Now,

Mean speed of hydrogen molecule= $v_{mean}(H) = \sqrt{(8RT/\pi M(H))}$ and

Mean speed of nitrogen molecule= $v_{mean}(N) = \sqrt{(8RT/\pi M(N))}$

Since, Temperature of both the gases is same.

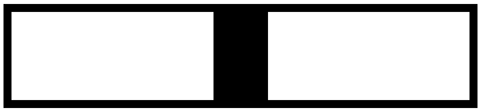
On dividing $v_{mean}(H)$ by $v_{mean}(N)$, we get

$v_{mean}(H)/V_{mean}(N) = \sqrt{[M(N)/M(H)]}$

$= \sqrt{28/2}$

= 3.74

Question 20: Figure shows a vessel partitioned by a fixed diathermic separator. Different ideal gases are filled in the two parts. The rms speed of the molecules in the left part equals the mean speed of the molecules in the right part. Calculate the ratio of the mass of a molecule in the left part to the mass of a molecule in the right part.



Solution:

In kinetic theory of ideal gas, the average energy: $v_{avg} = \sqrt{(8RT/\pi M)}$

We know, Gas constant = $R = k_B N_A$

Where k_B= Boltzmann constant = 1.38×10^{-23} J K^{-1}.

N_A = Avogadro number = 6.023×10^{-23} mol^{-1}

=> $v_{avg} = \sqrt{(8k_B N_A T/\pi M)}$

Using $M = N_A \times m$

=> $v_{avg} = \sqrt{(8k_B T/\pi m)}$

And, Rms speed of gas molecule = $v_{rms} = \sqrt{(3RT/M)}$

$v_{rms} = \sqrt{[3(k_B N_A)T/M]}$

Using $M = N_A \times m$

=> $v_{rms} = \sqrt{[3(k_B)T/m]}$

Given: the rms speed of the molecules in the left part equals the mean speed of the molecules in the right part.

Let m_1 and m_2 masses of molecule in left part and in the right part respectively

so, $\sqrt{[3(k_B)T/m_1]} = \sqrt{(8k_B T/\pi m_2)}$

Solving above equation, we have

$m_1/m_2 = 1.17$

Question 21: Estimate the number of collisions per second suffered by a molecule in a sample of hydrogen at STP. The mean free path (average distance covered by a molecule between successive collisions) = 1.38×10^{-8} cm.

Solution:

Molar mass of hydrogen = 2 amu = 2×10^{-3} kg/mol
In kinetic theory of ideal gas, the average energy: $v\_avg = \sqrt{(8RT/\pi M)}$

=> $v_{avg} = \sqrt{(8x8.31x273.15/(3.14x2x10^{-3})} = 1700$ m/s

Distance between successive collision = $\lambda = 1.38 \times 10^{-8}$ (Given)

Time between collision = t = distance/velocity

=> $t = \lambda/v_{avg}$

$= [1.38 \times 10^{-8}]/1700$

$= 8x10^{-12}$ sec

And, Frequency of collision = $1/t = 1.23 \times 10^{11}$

Question 22: Hydrogen gas is contained in a closed vessel at 1 atm (100 kPa) and 300 K

(a) Calculate the means speed of the molecules.
(b) Suppose the molecules strike the wall with this speed making an average angle of 45° with it.
How many molecules strike each square meter of the wall per second?

Solution:
In kinetic theory of ideal gas, the average energy: $v_{mean} = \sqrt{(8RT/\pi M)}$

(a)
Molar mass of hydrogen gas =2amu = 2g/mol = 2×10^{-3} kg/mol

$v_{mean} = \sqrt{[(8 \times 8.31 \times 300)/(3.14 \times 2 \times 10^{-3})]}$ = 1780 m/s (approx)

(b) when the molecules strike at an angle 45 degrees

Force exerted = $mV \cos 45^o - (-mv \cos 45^o) = 2\ mV(1/\sqrt{2}) = \sqrt{2}\ mv$

Number of molecules striking per unit area = Force/[Force exerted x Area]

= Pressure/($\sqrt{2}$mv)

Mass of 6.023×10^{23} of hydrogen molecule = 2×10^{-3} kg
Mass of 1 hydrogen molecule = m = 3.3×10^{-27} kg

Therefore,
$n = 10^5/[\sqrt{2} \times 3.3 \times 10^{-27} \times 1780] = 1.2 \times 10^{28}$

Question 23: Air is pumped into an automobile tyre's tube up to a pressure of 200 kPa in the morning when the air temperature is 20°C. During the day the temperature rises to 40°C and the tube expands by 2%. Calculate the pressure of the air in the tube at this temperature.

Solution:
Ideal gas equation, PV = nRT
Given:
Volume at temperature =$V_1 = 20^0C$

Pressure at temperature = $P_1 = 20^0C = 200 \times 10^3$ Pa
$T_1 = 20^0C = 293$ K and $T_2 = 40\ ^0C = 313$ K

Increase in volume = V_1 = 2%

So, Volume at temperature 40 ^0C, say $V_2 = V_1 + 2\%\ V_1$

=> $V_2 = V_1 + 0.02\ V_1 = 1.02\ V_1$

From ideal gas equation,

$nR = P_1V_1/T_1 = P_2V_2/T_2$
This implies,

$$\frac{200 \times 10^3 \times V_1}{293.15} = \frac{P_2 \times 1.02V_1}{313.15}$$

$$P_2 = \frac{313.15 \times 200 \times 10^3}{293.15 \times 1.02} = 209.45kPa$$

Question 24: Oxygen is filled in a closed metal jar of volume 1.0×10^{-3} m^3 at a pressure of 1.5×10^5 Pa and temperature 400 K. The jar has a small leak in it. The atmospheric temperature is 300 K. Find the mass of the gas that leaks out by the time the pressure and the temperature inside the jar equalize with the surrounding.

Solution:
We know ideal gas equation: PV=nRT

Volume, temperature and Pressure inside the jar :

$V_1 = 1.0 \times 10^{-3}$ m^3, T_1 = 400K and $P_1 = 1.5 \times 10^5$ Pa

Temperature and Pressure surrounding of jar:

T_2 = 300K and P_2 = 1atm = 1.0×10^5 Pa

Let volume of oxygen at T_2 and P_2 = V_2

From question,

$nR = P_1V_1/T_1 = P_2V_2/T_2$
On substituting the values, this implies,

$V_2 = 1.125 \times 10^{-3}$ m^3

If we consider leak,
Volume of gas leaked = $V_2 - V_1$ = (1.125-1) x$10^{-3}m^3$ = 1.25 x 10^{-4} m^3

If n_2 are number of moles leaked out, then
Mass of the gas leaked out = n_2 x molar mass of oxygen molecule

=> n_2 = [P_2 x volume of leaked gas]/T_2

= [1x10^5x1.25x10^{-4}]/300

= 0.005 mol

Now, Molar mass of oxygen molecule = 32g/mol
Mass of gas leaked out = 0.005 x 32 = 0.16 g

Question 25: An air bubble of radius 2.0 mm is formed at the bottom of a 3.3 m deep river. Calculate the radius of the bubble as it comes to the surface. Atmospheric pressure = 1.0 × 10^5 Pa and density of water = 1000 kg m^{-3}.

Solution:
Pressure at depth inside a fluid is related to atmospheric pressure by relation

$P_1 = P_a + hg\rho$

Where P_1 = Pressure at depth h
P_a = atmospheric pressure = 1.0 × 10^5 Pa
ρ = density of fluid
g = acceleration due to gravity = 9.8 m/s^2

Depth of the river = h = 3.3m (Given)
Density of water = 1000 kg m^{-3} (Given)

So, P_1 = 1.0 × 10^5 + 3.3 x 9.8 x 1000

=> P_1 = 1.32 x 10^5 Pa

Let V_1 and V_a be the volume of air bubble at bottom of deep river and volume of air bubble at surface of river respectively.

$V_1 = (4/3)\, \pi\, R_1^3$ and $V_a = (4/3)\, \pi\, R_a^3$

According to Boyle's law: $P_1 V_1 = P_a V_a$

$\Rightarrow 1.32 \times 10^5 \times (4/3)\, \pi\, (2 \times 10^{-3})^3 = 1 \times 10^5 \times (4/3)\, \pi\, R_a^3$

$\Rightarrow R_a = 2.2 \times 10^{-3}$ m

Therefore, Radius of the air bubble at the surface of river is 2.2×10^{-3} m.