Exercise Solutions

Question 1: Visible light has wavelengths in the range of 400 nm to 780 nm. Calculate the range of energy of the photons of visible light.

Solution:
We know, Energy of photon = hv = hc/λ

where h is plank's constant
v is the frequency
c is the speed of light and
λ is the wavelength.

Now,

Energy of photon of wavelength 400 nm = $[6.63\times10^{-34}\times3\times10^{8}]/[400\times10^{-9}]$

$= 4.97\times10^{-19}$ J

Energy of photon of wavelength 780 nm = $[6.63\times10^{-34}\times3\times10^{8}]/[780\times10^{-9}]$

$= 2.55 \times 10^{-19}$ J

The range from 2.55×10^{-19} J to 4.97×10^{-19} J

Question 2: Calculate the momentum of a photon of light of wavelength 500 nm.

Solution:
Momentum of photon of wavelength λ is, P = h/λ
Now, the momentum of photon of wavelength of 500 nm :
$= [6.63\times10^{-34}]/[500\times10^{-9}]$

$\Rightarrow P = 1.32 \times 10^{-27}$ kg m s^{-1}.

Question 3: An atom absorbs a photon of wavelength 500 nm and emits another photon of wavelength 700 nm. Find the net energy absorbed by the atom in the process.

Solution:

Energy of photon atom absorbed = hc/λ

where h is plank's constant
v is the frequency

c is the speed of light and
λ is the wavelength.

Energy of photon atom absorbed = $[6.63 \times 10^{-34} \times 3 \times 10^{8}]/[500 \times 10^{-9}]$

= 3.978×10^{-19} J

Energy of photon atom emit = $[6.63 \times 10^{-34} \times 3 \times 10^{8}]/[700 \times 10^{-9}]$

= 2.841×10^{-19} J

Now,
Net absorbed energy = [Energy of photon atom absorbed] – [Energy of photon atom absorbed]
= 3.978×10^{-19} J **-** 2.841×10^{-19} J
= 1.137×10^{-19}J.

Question 4: Calculate the number of photons emitted per second by a 10 W sodium vapour lamp. Assume that 60% of the consumed energy is converted into light. Wavelength of sodium light = 590 nm.

Solution:

60% of 10 W = 6 W is converted into light.

Energy of single photon = $[6.63 \times 10^{-34} \times 3 \times 10^{8}]/[590 \times 10^{-9}]$

= 3.371×10^{-19} J

Number of photons required to produce 6 W energy:
n = $6/[3.371 \times 10^{-19}] = 1.7 \times 10^{19}$

Question 5: When the sun is directly overhead, the surface of the earth receives 1.4×10^{3} W m^{-2} of sunlight. Assume that the light is monochromatic with average wavelength 500 nm and that no light is absorbed in between the sun and the earth is 1.5×10^{11} m.
(a) Calculate the number of photons falling per second on each square meter of earth's surface directly below the sun.
(b) How many photons are there is each cubic meter near the earth's surface at any instant?
(c) How many photons does the sun emit per second?

Solution:
(a) Intensity at the earth surface = 1.4×10^{3} W m^{-2} (Given)

Energy of one photon = $[6.63 \times 10^{-34} \times 3 \times 10^{8}]/[500 \times 10^{-9}] = 3.978 \times 10^{-19}$ J

The number of photons required to produce 1.4×10^3 J per square meter and per unit sec of earth's surface directly below the sun is

$= [1.4 \times 10^3]/[3.978 \times 10^{-19}]$

$= 3.519 \times 10^{21}$

(b) In a unit sec all the photons which are at height, 3×10^8 m fall in unit m^2.
The photons which fall from 1 m height will fall in $(1/3) \times 10^{-8}$ sec.

If there are 3.519×10^{21} photons which fall in 1 sec, then, the number of photons which fall in $(1/3) \times 10^{-8}$ sec is $(1/3) \times 10^{-8} \times 3.519 \times 10^{21} = 1.173 \times 10^{13}$, and these photons are in 1 m^3 near the earth surface.

(c) From above results, we can conclude that, the number of photons which fall on the surface of sphere of radius is the total no of photons which will be emitted by sun in 1 s.

Surface area of sphere is $4\pi(1.5 \times 10^{11})^2$ m square.
Therefore, the number of photons $= 4\pi(1.5 \times 10^{11})^2 \times 3.519 \times 10^{21}$

$= 9.89 \times 10^{44}$

Question 6: A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is 1.0×10^{19}. Calculate the force exerted by the light beam on the mirror.

Solution:
Wavelength of monochromatic light $= \lambda = 663$ nm $= 663 \times 10^{-9}$ m
The angle of incidence is 60° and the number of photons striking the mirror per second is 1.0×10^{19}.

We know, momentum of photon $= p = h/\lambda = [6.63 \times 10^{-34}]/[663 \times 10^{-9}] = 10^{-27}$

Now,
Force exerted on the wall $= F = n[p \cos\theta - (-p \cos \theta)] = 2 np \cos \theta$

$= 2 \times 1 \times 10^{19} \times 10^{-27} \times ½$

$= 10^{-8}$ N

Question 7: A beam of white light is incident normally on a plane surface absorbing 70% of the light and reflecting the rest. If the incident beam carries 10 W of power, find the force exerted by it on the surface.

Solution:
From statement, we have

Force, F = 7/10(absorbed) + 2x(3/10) (reflected) ...(1)

Using below relations,
$\lambda = h/p$ ...(a)
$E = hc/\lambda$ ...(b)
P(Power) = E/t ......(c)
F = p/t ...(d)
Where p = momentum

Divide each side of equation (a) and (b) by t

(a)=> $\lambda/t = h/pt$ or $p/t = h/\lambda t$ and

(b)=> $E/t = hc/\lambda t = pc/t$ (using above result)

Using above result in (c), we get

P = E/t = pc/t

=> P/c = p/t

(d)=> F = P/C

Now,

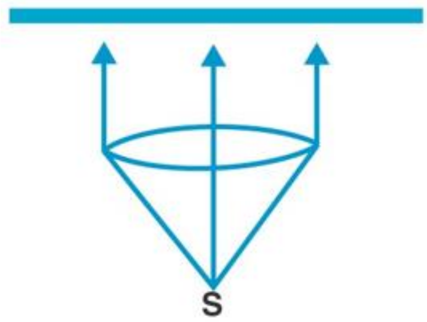
(1)=> F = 7/10 x P/c + 2x(3/10) x P/c

Where P = 10 watt (given) and c = speed of light = 3×10^8 m/s

=>F = 7/10 x 10/[3×10^8] + 2x(3/10) x 10/[3×10^8]

=>F = 4.33×10^{-8} N

Question 8: A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in figure. The mass of the mirror is 20g. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Find the power of the source needed to support the weight of the mirror. Take g = 10 m s^{-2}.



Solution:

The weight of the mirror will be balanced if the force exerted by proton = weight of the mirror.

From previous solution, we have F = P/c

As the light gets reflected normally, then

Force exerted = 2(Rate of change of momentum)

We know, Rate of change of momentum = Power/[speed of light] = P/c

Force exerted = 2 x P/c = 30% of (2P/c) = mg

=> Power, P = $[20 \times 10^{-3} \times 10 \times 3 \times 10^{8} \times 10]/[2 \times 3]$ = 100 MW

Question 9: A 100 W light bulb is placed at the center of a spherical chamber of radius 20 cm. Assume that 60% of the energy supplied to the bulb is converted into light and that the surface of the chamber is perfectly absorbing. Find the pressure exerted by the light on the surface of the chamber

Solution:
Power of light bulb = 100 W
Spherical chamber of radius 20 cm, say R = 20 cm = 0.2 m
Power of the light emitted by bulb, say P' = 60 W

We know, F = P/c = $60/[3 \times 10^{8}] = 2 \times 10^{-7}$ N

Again, Pressure = Force/Area

= $[2 \times 10^{-7}]/[4 \times 3.14 \times (0.2)^{2}]$

[Surface area of sphere = $4\pi r^{2}$]

=> Pressure = 4×10^{-7} Nm^{-2}.

Question 10: A sphere of radius 1.00 cm is placed in the path of a parallel beam of light of large aperture. The intensity of the light is 0.50 W cm^{-2}. If the sphere completely absorbs the radiation falling on it, find the force exerted by the light beam on the sphere.

Solution:

Intensity of light = I = 0.5 Wcm^{-2} and Radius of sphere = r = 1 cm

Force exerted by light beam on the sphere = F = P/c

Where P = IA; [I = intensity and A = area = πr^2]
and c = speed of light

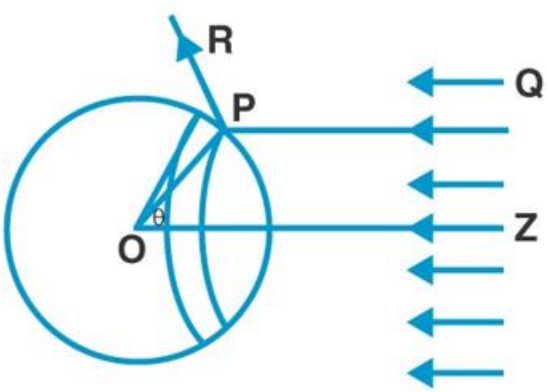
=> F = IA/c

=> F = $[0.5 \times 3.14 \times 1^2]/[3 \times 10^8]$

=> F = 5.2×10^{-9} N

Question 11: Consider the situation described in the previous problem. Show that the force on the sphere due to the light falling on it is the same even if the sphere is not perfectly absorbing.

Solution:



Advised to have a look on the previous Problem and its solution once again.

Let us rotate the radius about OZ to get another circle on the sphere.
Let dθ is the angle which forms with the center as origin (point O) and the breath of strip.

Area of strip = Length × Breath = $2\pi r^2 \sin\theta \, d\theta$

Consider a small part of area of ring at point P,
Energy in time Δt, $\Delta U = I\Delta t(\Delta A \cos\theta)$

As light is reflected by the sphere along PR, the change in momentum is

$\Delta p = 2\, \Delta U/c \cos\theta = 2/c\, I\Delta t\, (\Delta A \cos^2 \theta)$

So, the force will be $\Delta p/\Delta t = 2I/c\, (\Delta A \cos^2 \theta)$

The component of force along ZO is

$\Delta p/\Delta t \cos\theta = 2I/c\, (\Delta A \cos^3 \theta)$

Now, the net force on the ring is,

$dF = 2I/c\, (2\pi r^2 \sin\theta d\theta)\cos^3 \theta$

Integrate above equation from range 0 to π/2 to find the force on the entire sphere.

$$F = \int_0^{\pi/2} 2\frac{I}{c}\,(2\pi r^2 sin\theta d\theta\,)\,\cos^3\theta$$

$$= \int_0^{\pi/2} 2\frac{I}{c}\,2\pi r^2\, sin\theta\cos^3\theta\,\, d\theta$$

$$= \frac{\pi r^2 I}{c}$$

Question 12: Show that it is not possible for a photon to be completely absorbed by a free electron.

Solution:
In any collision of electron and photon the collision will be elastic. So, energy and momentum will be conserved.

By principle of conversation, $pc + m_0 c^2 = \sqrt{[p^2c^2+m_0^2c^4]}$

Squaring both the sides and solving above equation, we get

$2(pc)(m_o c^2) = 0$

=> $pc = 0$
[As m and c are non zero]

Which is not possible for a photon to be completely absorbed by a free electron.

Question 13: Two neutral particles are kept 1m apart. Suppose by some mechanism some charge is transferred from one particle to the other and the electric potential energy lost is completely converted into a photon. Calculate the longest and the next smaller wavelength of the photon possible.

Solution:
The energy between particles is transferred into photon form.

So, $kq^2/r = hc/\lambda$ ...(1)

Where, r is the distance between two particles = r = 1m
and kq^2/r be the potential energy between the particles if charge "q" appears on them.

(1)=> $\lambda = hrc/kq^2$

For maximum λ, charge should be electronic charge.

=> $\lambda_{max} = [6.6x10^{-34}x1x3x10^8]/[9x10^9x(1.6x10^{-19})^2] = 863$ m

For shortest wavelength, $\lambda = \lambda_{max}/4 = 863/4 = 215.7$ m

Question 14: Find the maximum kinetic energy of the photoelectrons ejected when light of wavelength 350 nm is incident on a cesium surface. Work function of cesium = 1.9 eV.

Solution:
From Einstein photoelectric equation, $hf = K_{max} + \phi$

where K_{max} = maximum kinetic energy
h = Planck's constant
f = frequency = c/λ
λ = wavelength of light

$K_{max} = hc/\lambda - \phi$

$= [6.63x10^{-34}x3x10^8]/[350x10^{-9}x1.6x10^{-19}] - 1.9$

= 1.65 eV

=> K_{max} = 1.65 eV

Question 15: The work function of a metal is 2.5×10^{-19} J.
(a) Find the threshold frequency for photoelectric emission.
(b) If the metal is exposed to a light beam of frequency 6.0×10^{14} Hz, what will be the stopping potential?

Solution:
(a) We know that, $W_o = hf$

Here h = Planck's constant, and f = threshold frequency

Given $W_o = 2.5 \times 10^{-19}$ J.

=> $2.5 \times 10^{-19} = 6.63 \times 10^{-34} \times f$

=> $f = 3.77 \times 10^{14}$ Hz

(b) From photoelectric equation, $eV_0 = hv - \phi$
Let V_o is the stopping potential, so
V = frequency of light
ϕ = work function

$V_o = [hv - \phi]/e$

$= [6.63 \times 10^{-34} \times 6 \times 10^{14} - 2.5 \times 10^{-19}]/[1.6 \times 10^{-19}]$

= 0.91 V

Question 16: The work function of a photoelectric material is 4.0 eV.
(a) What is the threshold wavelength?
(b) Find the wavelength of light for which the stopping potential is 2.5 V.

Solution:
(a) We know that, $W_o = hf = hc/\lambda$

Here h = Planck's constant, c = speed of light and λ = threshold wavelength

Given $W_o = 4$ eV.

=> $4 \times 1.6 \times 10^{-19} = [6.63 \times 10^{-34} \times 3 \times 10^{8}]/\lambda$

=> $\lambda = 3.1 \times 10^{-7}$ m

(b) From photoelectric equation, $hc/\lambda = eV_o + \phi$

Let V_o is the stopping potential, ϕ = word function so

=> $\lambda = [6.63x10^{-34}x3x10^{8}]/[6.5\ x1.6x10^{-19}]$

$= 1.9125 \times 10^{-7}$ = 191 nm

Question 17: Find the maximum magnitude of the linear momentum of a photoelectron emitted when light of wavelength 400 nm falls on a metal having work function 2.5 eV.

Solution:
From photoelectric equation, $hc/\lambda = K + \phi$
Where k = K.E., h = plank constant , ϕ = word function

Given:
λ = 400nm = 400×10^{-9} m and Work function = 2.5eV

=> $k + 2.5 = [6.63x10^{-34}x3x10^{8}]/[400x\ 10^{-9}x1.6x10^{-19}]$

=>K = 0.607 eV

Also, we know, $K = p^2/2m$
Here, p = linear momentum and m = mass of electron

=> $0.607\ x1.6\ x10^{-19}x2x9.1 \times 10^{-31} = p^2$

Or $p = 4.20 \times 10^{-25}$ kg m s^{-1}.

Question 18: When a metal plate is exposed to a monochromatic beam of light of wavelength 400 nm, a negative potential of 1.1 V is needed to stop the photocurrent. Find the threshold wavelength for the metal.

Solution:
From photoelectric equation, $hc/\lambda = hc/\lambda_o + eV_0$
Where c = speed of light, λ = wavelength of light, λ_0 = threshold wavelength, h = plank constant , V_0 = stop potential

Given:
λ = 400nm = 400×10^{-9} m and stopping potential = V_0= 1.1 V

Putting all the values, we have

=> $[6.63x10^{-34}x3x10^{8}]/[400x\ 10^{-9}] = [6.63x10^{-34}x3x10^{8}]/[\lambda_o] + 1.6 \times 10^{-19} \times 1.1$

$=>4.97 = [19.87 x 10^{-7}]/ \lambda_o + 1.76$

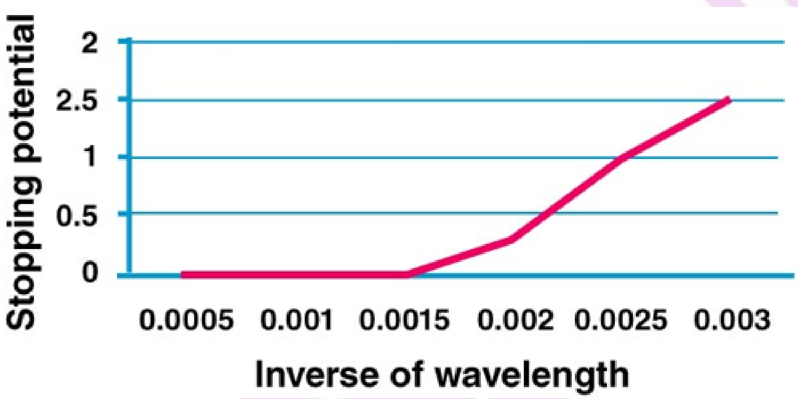
$=> \lambda_o$ 6.196×10^{-7} m = 620 nm

Question 19: In an experiment on photoelectric effect, the stopping potential is measured for monochromatic light beams corresponding to different wavelength. The data collected are as follows:

Wavelength(nm)	350	400	450	500	550
Stopping potential (V)	1.45	1.00	0.66	0.38	0.16

Plot the stopping potential against inverse of wavelength (1/λ) on a graph paper and find
(a) the Planck constant,
(b) the work function of the emitter and
(c) the threshold wavelength.

Solution:



From Einstein photoelectric equation, hc/λ = K + ϕ

where K = kinetic energy
h = Planck's constant
λ = wavelength of light
ϕ = work function

(a)from table, When λ = 350 nm then V = 1.45

=> hc/350 = 1.45 + ϕ ...(1)

When λ = 400 nm then V = 1

=> hc/400 = 1 + ϕ ...(2)

Now, (1) – (2)

=> hc(1/350 – 1/400) = 0.45 e

=> hc x 0.00035 = 0.45 e

=> h x 3 x 10^8 x 0.00035 = 0.45 e

=>h = 4.2 x 10^{-15} eVs

(b) Work function = ϕ = [4.2 x 10^{-15} x 3 x 10^8]/[350x10^{-9}] - 1.45

= 2.15 eV

(c) Again work function, ϕ = hc/ λ

=> λ = [6.63x10^{-34}x3x10^8]/[2.15] = 578.8 nm

Question 20: The electric field associated with a monochromatic beam becomes zero 1.2 × 10^{15} times per second. Find the maximum kinetic energy of the photoelectrons when this light falls on a metal surface whose work function is 2.0 eV.

Solution:
If electric field becomes zero 1.2 × 10^{15} times per second, then number of oscillations per second = [1.2x10^{15}]/2 = 0.6 x 10^{15}, also the frequency of the monochromatic light.
From Einstein's photoelectric equation, hf = K_{max} + ϕ

=>[6.63x10^{-34}x0.6x10^{15}] = k + (2x1.6x 10^{-19}]

=> K_{max} = 0.77 x 10^{-19} J