Exercise Solutions

Question 1: The Bohr radius is given by $a_0 = (\epsilon_o h^2)/(\pi m e^2)$ .Verify that the RHS has dimensions of length.

Solution:
Dimensions of $\epsilon_o = [A^2T^2]$
Dimensions of $\pi = [L^2MLT^{-2}]$
Dimensions of $e = [AT]$
Dimensions of $h = [ML^2T^{-1}]$

Bohr's radius = $[M^2L^4T^{-2}]/[M^2L^3T^{-2}]$

Or $a_0 = L$, which is in the dimension of length.

Question 2: Find the wavelength of the radiation emitted by hydrogen in the transitions (a) n = 3 to n = 2, (b) n=5 to n=4 and (c) n=10 to n=9

Solution:
We know the formula to find the wavelength:

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

Where n = quantum number of final state and m = quantum number of state

Here **Z = 1** [As, atom is hydrogen]

(a) n=2 and m=3

$1/\lambda = 1.1 \times 10^7[1/2^2 - 1/3^2]$

or $\lambda = 6.54 \times 10^{-7} = 654$ nm

(b) n=4, m=5

$1/\lambda = 1.1 \times 10^7[1/4^2 - 1/5^2]$

or $\lambda = 400/[1.1 \times 10^7 x9] = 4050$ nm

(c) n=9, m=10

$1/\lambda = 1.1 \times 10^7[1/9^2 - 1/10^2]$

or $\lambda = [81 \times 100]/[1.1 \times 10^7 \times 19] = 38849$ nm

Question 3: Calculate the smallest wavelength of radiation that may be emitted by (a) hydrogen (b) He^+ and (c) Li^{++}

Solution:
For smallest wavelength, energy should be maximum. So, electron will jump from ground state to the state where energy is maximum.

Let us say: ground state is n=1 and the final stage is $m = \infty$

we know, $1/\lambda = RZ^2 (1/n^2 - 1/m^2)$

(a) Here Z = 1, for Hydrogen atom

=> $1/\lambda = 1.1 \times 10^7[1/1^2 - 1/\infty]$

Or $\lambda = 91$ nm

(b) Here Z = 2, for He atom

=> $1/\lambda = 1.1 \times 10^7 \times 2^2[1/1^2 - 1/\infty]$

Or $\lambda = 23$ nm

(b) Here Z = 3, for Li atom

=> $1/\lambda = 1.1 \times 10^7 \times 3^2[1/1^2 - 1/\infty]$

Or $\lambda = 10$ nm

Question 4: Evaluate Rydberg constant by putting the values of the fundamental constants in its expression.

Solution:
Rydberg constant = $R = me^4/8\epsilon_0^2h^3c$

Where,
Charge of electron =e = 1.6×10^{-19} C
Mass of electron = m = 9.1×10^{-31} kg
Velocity of light = c = 3×10^8 m/s

Planks constant = h = 6.63×10^{-34} J-s
Permittivity = $\epsilon_o = 8.85 \times 10^{-12}$

Substituting all the above values, we have

$R = [9.1 \times 10^{-31} x (1.6 \times 10^{-19})^4]/[8x(6.63 \times 10^{-34})^3 x3 \times 10^8 x(8.85 \times 10^{-12})^2]$

$= 1.097 \times 10^7$

Question 5: Find the binding energy of a hydrogen atom in the state n=2.

Solution:
Energy of hydrogen atom = $E = -13.6/n^2$ eV

Where 13.6 is the binding energy of the hydrogen atom

In this case, initial state of the atom will be m=2

If we consider transition state from infinity to n = 2

$E = -13.6/n^2 - (-13.6/m^2)$

or = $13.6(1/\infty^2 - 1/2^2)$

or $E = -13.6 \times 1/4 = -3.4$ eV

Question 6: Find the radius and energy of He^+ ion in the states

(a) n=1 (b) n=4 (c) n=10

Solution:

We know, radius of an atomic orbit = $r = n^2 a_o/Z$

Here a_o = 0.053 nm

or $r = (0.053xn^2)/Z$

The energy of the atomic orbit = $E = (-13.6xZ^2)/n^2$

Here Z = 2, As atomic number of He is 2

(a) n = 1

$r = 0.053x1^2/2 = 0.265$ angstrom

$E = -13.6x2^2/1^2 = -54.4$ eV

(b) n = 4

$r = 0.053x4^2/2 = 4.24$ angstrom

$E = -13.6x2^2/4^2 = -3.4$ eV

(c) n = 10

$r = 0.053x10^2/2 = 26.5$ angstrom

$E = -13.6x2^2/10^2 = -0.544$ eV

Question 7: A hydrogen atom emits ultraviolet radiation of wavelength 102.5 nm. What are the quantum numbers of the states involved in the transition?

Solution:
we know, $1/\lambda = RZ^2(1/n^2 - 1/m^2)$

Given, λ = 102.5nm , n = 1 and Z=1 (atomic number of hydrogen is 1)

$=>1/[102.5x10^{-9}] = 1.1 \text{ x } 10^7(1/1^2 – 1/m^2)$

$=>10^9/[102.5x1.1 \text{ x } 10^9] = 1 – 1/m^2$

=>m = 2.97 = 3 (approx.)

Question 8: (a) Find the first excitation potential of He^+ ion.
(b) Find the ionisation potential of Li^{++} ion.

Solution:

(a) Using relation, $E = -13.6xz^2/n^2$

As He^+ makes first transition, so n=1 and m=2.

Energy at stage 1, $E_1 = -13.6x2^2/1^2 = -54.4$ eV

Energy at stage 2, $E_2 = -13.6x2^2/2^2 = -13.6$ eV

The difference of the two energies will give us the excitation potential of He^+. So

$E = E_2 - E_1 = -13.6 - (-54.4) = 40.8$ V

(b) For Li^{++}, atomic number, Z = 3.

Ionisation potential = $13.6 \times Z^2 = 13.6 \times 9 = 122.4$

So, ionisation potential of Li^+ is 122.4 V

Question 9: A group of hydrogen atoms are prepared in n=4 states. List the wavelengths that are emitted as the atoms makes the transitions and return to n=2 states.

Solution: There will be 3 wavelengths:

we know, $1/\lambda = RZ^2(1/n^2 - 1/m^2)$

As atomic number of hydrogen = 1, so Z = 1.

Here $R = 1.1 \times 10^7\ m^{-1}$

For the first case, n=3 and m=4

$1/\lambda = 1.1 \times 10^7 \times (1/9 – 1/16)$

Or λ = 1875 nm

For the second case, n=2 and m=3

$1/\lambda = 1.1 \times 10^7 \times (1/4 – 1/9)$

Or λ = 655 nm

For the third case, n=2 and m=4

$1/\lambda = 1.1 \times 10^7 \times (1/4 – 1/16)$

Or λ = 487 nm

Question 10: A positive ion having just one electron ejects it if a photon of wavelength 228Å or less is absorbed by it. Identify the ion.

Solution:
Since the electron is ejected, so it makes a transition from its ground state n=1 to the immediate excited state m=2.

we know, $1/\lambda = RZ^2(1/n^2 - 1/m^2)$ and $E = hc/\lambda$

From both the equations, we have

$-13.6xZ^2 (1/2^2 – 1/1^2) = 0.0872 \times 10^{-16}$ eV

Or $-13.6xZ^2(-3/4) = 0.0872 \times 10^{-16} \times 1.6 \times 10^{-19}$ V

Or $Z^2 = 5.3$

Or Z = 2.3 (approx.)

The ion may be He^+.

Question 11: Find the maximum Coulomb force that can act on the electron due to the nucleus in a hydrogen atom.

Solution:
Using Coulomb force, F, is

$F = q_1q_2/4\pi\epsilon_o r^2$

Here r = distance between nucleus and the electron = Bohr's radius = 0.053 angstrom

Also value of $1/4\pi\epsilon_o = 9 \times 10^9$

$q_1 = q_2 = q = 1.6 \times 10^{-19}$ C

So, $F = 9 \times 10^9 \times (1.6 \times 10^{-1})^2/(0.053 \times 10^{-10})^2 = 8.22 \times 10^{-8}$ N

Question 12: A hydrogen atom in a state having a binding energy of 0.85eV makes transition to a state with excitation energy 10.2eV.
(a) Identify the quantum number n of the upper and the lower energy states involved in the transition.
(b) Find the wavelength of the emitted radiation.

Solution:

We know, binding energy = $E = 13.6Z^2/n^2 = 13.6/n_1^2$

Given: binding energy in the first case 0.85eV and second case 10.2eV

$0.85 = 13.6/n_1^2 \Rightarrow n_1 = 4$

Again $10.2 = 13.6/n_2^2 \Rightarrow n_2 = 2$

Thus, the states are 4 and 2.

(b) using part (a) results

$1/\lambda = 1.1 \times 10^7[1/2^2 - 1/4^2]$

Atomic number of Hydrogen = Z = 1

$\Rightarrow \lambda = 487$ nm

Question 13: Whenever a photon is emitted by hydrogen in Balmer series, it is followed by another photon in Lyman series. What wavelength does this latter photon correspond to?

Solution:
Given: photon is emitted in Balmer series and then next in Lyman series.

Let n = 1 and m=2, then we obtain the wavelength as,

$1/\lambda = 1.1 \times 10^7[1/1^2 - 1/2^2]$

Atomic number of Hydrogen = Z = 1

$\Rightarrow \lambda = 122$ nm

Question 14: A hydrogen atom in state n=6 makes two successive transitions and reaches the ground state. In the first transition a photon of 1.13eV is emitted.
(a) Find the energy of the photon emitted in the second transition.
(b) What is the value of n in the intermediate state?

Solution:

(a) Energy at n = 6

$E = -13.6 x z^2/n^2 = -13.6 x 1^2/6^2 = -0.37777$

Energy for hydrogen in the ground state = E = -13.6

Energy in the second transition = [Energy for hydrogen in the ground state] - [Energy at sixth state (n = 6)] + [first transition energy]

= -13.6 + (-0.37777) + 1.13 = -12.1

-ve sign here indicates that energy is being given for making the transitions from 6th state to the ground state.

=>E = 12.1 eV

(b)
Energy in the intermediate state =1.13+0.3777777 = 1.507 eV

We know, $E = 13.6xz^2/n^2 = 13.6x1^2/n^2$

$=>1.507 = 13.6x1^2/n^2$

=> n = 3 (approx)

Question 15: What is the energy of a hydrogen atom in the first excited state if the potential energy is taken to be zero in the ground state?

Solution:
We know, hydrogen requires 10.2 eV of energy to get excited from the ground state.
As the energy of hydrogen atom in the ground state is 13.6 eV. So, the energy needed to make the hydrogen atom reach the first excited state is,

E = 13.6 + 10.2 = 23.8 eV

Question 16: A hot gas emits radiation of wavelengths 46 nm,82.8 nm and 103.5 nm only. Assume that the atoms have only two excited states and the difference between consecutive energy levels decreases as energy is increased. Taking the energy of the highest energy state to be zero, find the energies of the ground state and the first excited state.

Solution:
Energy of the ground state will be the energy acquired in the transition of 2 excitation state to ground state.

As per given statement, higher energy state to be zero, so n=2 is zero in terms of energy, as atom makes transition to n=2 from ground state.

Energy of the ground state = $E = hc/\lambda_1 = [6.634 \times 10^{34} \times 3 \times 10^8]/[46 \times 10^{-9}]$ = 27 eV

Energy of the first excited state = E = hc/ λ_2 = [6.634 x 10^{34} x 3 x 10^8]/[103.5 x 10^{-9}] = 12 eV

Question 17: A gas of hydrogen like ions is prepared in a particular excited state A. It emits photons having wavelength equal to the wavelength of the first line of the Lyman series together with photons of five other wavelengths. Identify the gas and find the principal quantum number of the state A.

Solution:
We know, (n(n-1))/2 = 6 ...(1)

where n =principal quantum number

As gas emits total of 6 wavelengths.

=> Solving (1), we have n = 4

And gas is in the 4th excited state. Gas is He^+.

Question 18: Find the maximum angular speed of the electron of a hydrogen atom in a stationary orbit.

Solution:
Let m be the mass of the electron, v be the linear speed, ω angular speed and r be the radius of hydrogen's first stationary orbit.

We know, mvr = nh/2π

Where n = any integer

and V = rω

So, from both the equations, we have

$mr^2\omega$ = nh/2π

=> ω = nh/2πmr^2 = [1x(6.63 x 10^{34})] / [2x3.14x(9.1093 x 10^{-31})x(0.53 x 10^{-10})^2]

=> ω = 0.413 x 10^{17}

= 4.13 x 10^16 rad s^{-1}.

Question 19: A spectroscopic instrument can resolve two nearby wavelengths λ and λ+ Δλ if λ/Δλ is smaller than 8000.This is used to study the spectral lines of the Balmer series of hydrogen. Approximately how many lines will be resolved by the instrument?

Solution:
The range of Balmer series is from 656.3nm to 365 nm. (known)

Now,
Number of wavelengths in the given range= [656.3-365]/8000 = 36

Two lines will be extra for the 1st and last wavelength, so total number of lines = 36+2 = 38

Question 20: Suppose, in certain conditions only those transitions are allowed to hydrogen atoms in which the principal quantum number n changes by 2.
(a) Find the smallest wavelength emitted by hydrogen.
(b) List the wavelengths emitted by hydrogen in the visible range (380 nm to 780 nm).

Solution:
Let n = 1 and m = 3
n = for minimum wavelength and m = for given condition

Using Einstein-Planck equation:

$E = hc/\lambda$ and

Also, we know, $E = 13.6 (1/n^2 - 1/m^2)$

$\Rightarrow 13.6 (1/n^2 - 1/m^2) = hc/\lambda$

$\Rightarrow 13.6 (1/1^2 - 1/3^2) = [6.64\times10^{-34}\times3\times10^8]/\lambda$

$\Rightarrow 13.6 (1/1 - 1/9) = [4.14\times10^{-15}\times3\times10^8]/\lambda$

$\Rightarrow \lambda = 1.027 \times 10^{-7}$ m = 103 nm

(b) Wavelength obtained in above result does not fall in visible range so let us consider another transition possible that is from n = 2 to m = 4

$\Rightarrow 13.6 (1/2^2 - 1/4^2) = [6.64\times10^{-34}\times3\times10^8]/\lambda$

$\Rightarrow \lambda = 487$ nm
Above wavelength falls in visible range. This is the wavelength emitted by hydrogen in the visible range.

Question 21: According to Maxwell's theory of electrodynamics, an electron going in a circle should emit radiation of frequency equal to its frequency of revolution. What should be the wavelength of the radiation emitted by a hydrogen atom in ground state if this rule is followed?

Solution:
Let r be the radius of the ground state and v be the velocity of the electron moving in ground state.
Frequency of the revolution of electron in the circle, $f = v/2\pi r$

Also, $f = c/\lambda$

Where f = frequency of radiation emitted by hydrogen atom in ground state

=> $\lambda = 2\pi cr/v = [2x3x10^8x3.14x0.53x10^{-10}]/[2187x10^3] = 45.686$ nm

Question 22: The average kinetic energy of molecules in a gas at temperature T is 1.5 kT. Find the temperature at which the average kinetic energy of the molecules of hydrogen equals the binding energy of its atoms. Will hydrogen remain in molecular from at this temperature? Take $k = 8.62 \times 10^{-5}$ eV K^{-1}.

Solution:
As, Binding energy is equal to the average kinetic energy of the molecules.

(3/2)KT = 13.6 eV

=> $T = 13.6/[1.5x8.62x10^{-5}] = 1.05 \times 10^5$ K

No, it is not possible for hydrogen molecules to remain in molecular state at such a high temp.

Question 23: Find the temperature at which the average thermal kinetic energy is equal to the energy needed to take a hydrogen atom from its ground state to n = 3 state. Hydrogen can now emit red light of wavelength 653.1 nm. Because of Maximillian distribution of speeds, a hydrogen sample emits red light at temperatures much lower than that obtained from this problem. Assume that hydrogen molecules dissociate into atoms.

Solution:
Energy needed to take hydrogen atom from ground to second exited state is

$E = 13.6(1/1^2 - 1/3^2) = 12.08$ eV

The average thermal kinetic energy = 3/2 KT

As average thermal kinetic energy is equal to the energy.

So, 3/2 KT = 12.08 e V

$1.5 \times 8.62 \times 10^{-5} \times T = 12.08$

$\Rightarrow T = 12.08/[1.5 \times 8.62 \times 10^{-5}] = 9.4 \times 10^4$ kelvin

Question 24: Average lifetime of a hydrogen atom excited to n = 2 state is 10^{-8} s. Find the number of revolutions made by the electron on the average before it jumps to the ground state.

Solution:
From Bohr's model, frequency is

$f = me^4/4\varepsilon^2 n^3 h^3$

Where n = 2, $h = 6.63 \times 10^{-34}$ and $m = 9.1 \times 10^{-31}$ kg

Time period (T) = $1/f = 4\varepsilon^2 n^3 h^3/me^4$

$= [4 \times 8.85^2 \times 2^3 \times 6.63^3]/[9.1 \times 1.6^4] \times 10^{-19}$

$= 1.224 \times 10^{-15}$ seconds
Now,
Number of revolution = [Average lifetime]/[Time period]

Given: Average lifetime = 10^{-8} s

so, Number of revolution = $10^{-8}/[1.224 \times 10^{-15}] = 8.16 \times 10^6$ revolutions

Question 25: Calculate the magnetic dipole moment corresponding to the motion of the electron in the ground state of a hydrogen atom.

Solution:
Since hydrogen atom is in ground state, this implies n = 1

Dipole moment of the electron = $\mu = niA = qfA$

$= e \times me^4/[4\epsilon_o^2 h^3 n^3] \times \pi r^2 n^2$

$= [me^5 \pi r^2 n^2]/[4\epsilon_o^2 h^3 n^3]$

$$\mu = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^5 \times 3.14 \times (0.53 \times 10^{-10})^2}{4 \times (8.85 \times 10^{-12})^2 \times (6.64 \times 10^{-34})^3 \times 1}$$

$$= 9.17 \times 10^{-24} A/m^2$$