Exercise Solutions

Question 1: Find the energy, the frequency and the momentum of an X-ray photon of wavelength 0.10 nm.

Solution:
Wavelength of X-rays = 0.10 nm.

We know, E = hc/λ

Where,
Velocity of light = c = 3×10^8 m/s
Planks constant = h = 6.63×10^{-34} J-s
Wavelength of X-rays = λ = 0.10×10^{-9} m (given)

E = $[6.63 \times 10^{-34} \times 3 \times 10^8]/[0.10 \times 10^{-9}] = 1.98 \times 10^{-15}$ Joules/photon

Also, the frequency can be calculated using below formula:

$\gamma = c/\lambda = [3 \times 10^8]/[0.10 \times 10^{-9}] = 3 \times 10^{18}$ Hertz

Find momentum:
$p = h/\lambda = [6.63 \times 10^{-34}]/[0.10 \times 10^{-9}] = 6.626 \times 10^{-24}$ kg ms^{-1}.

Question 2: Iron emits K_α X-ray of energy 6.4 keV and calcium emits K_α X-ray of energy 3.69 keV. Calculate the times taken by an iron K_α photon and a calcium K_α photon to cross through a distance of 3 km.

Solution:
Let "t" be the time taken each of those rays to travel a distance of 3 Km.

We know, time = distance/speed

=> $t = [3\times10^3]/[3\times10^8] = 10^{-5}$ s = 10 μs

Both the K_α photon and X-ray photon will take the same i.e. 10μs

Question 3: Find the cutoff wavelength for the continuous X-rays coming from an X-ray tube operating at 30 kV.

Solution:
Cutoff wavelength = λ = hc/eV = [1242 eV-nm]/[$e\times30\times10^3$] = 414×10^{-4} nm

Question 4: What potential difference should be applied across an X-ray tube to get X-ray of wavelength not less than 0.10 nm? What is the maximum energy of a photon of this X-ray in joule?

Solution:

We know, λ = hc/eV

Or V = hc/eλ = $[6.63 \times 10^{-34} \times 3 \times 10^{8}]/[1.6 \times 10^{-19} \times 10^{-10}]$ = 12.4 KV

Now,
Maximum energy of photon of wavelength λ = 0.10 nm is

E = hc/λ = $[6.63 \times 10^{-34} \times 3 \times 10^{8}]/[0.10 \times 10^{-9}] = 1.98 \times 10^{-15}$ Joules/photon

Question 5: The X-ray coming from a Coolidge tube has a cutoff wavelength of 80 pm. Find the kinetic energy of the electrons hitting the target.

Solution:
We know, E = hc/λ = $[1242 \text{ eV-nm}]/[80 \times 10^{-3}] = 15.525 \times 10^{3}$ eV = 15.5 KeV(approx..)

Question 6: If the operating potential in an X-ray tube is increased by 1%, by what percentage does the cut-off wavelength decrease?

Solution:
If the operating voltage is increased by 1%, then the new operating voltage, say v' will be,

V' = V + V/100 = 1.01 V

Cutoff wavelength (λ') in increasing the operation voltage is

λ' = (h/1.01) V = λ/1.01

Therefore, difference in wavelength is λ' - λ/1.01 = (0.01)/1.01λ

Now,
Percentage change in the wavelength:

(0.01)/1.01λ x λ x 100 = 1/1.01 = 0.9901 = 1 % (approx)

Question 7: The distance between the cathode (filament) and the target in an X-ray tube is 1.5 m. If the cutoff wavelength is 30 pm, find the electric field between the cathode and the target.

Solution:

The distance between the cathode (filament) and the target in an X-ray tube is 1.5 m. If the cutoff wavelength is 30 pm.

We know, E = hc/ λ

= [6.63 x 10^{-34} x 3 x10^{8}]/[30x10^{-12}] = 41.4 x 10^{3} eV

Now,
Electric field = V/d = [41.4 x 10^{3}]/1.5 = 27.6 kVm^{-1}.

Question 8: The short-wavelength limit shifts by 26 pm when the operating voltage in an X-ray tube is increased to 1.5 times the original value. What was the original value of the operating voltage?

Solution:
When the operating voltage of the X-ray tube is increased to 1.5 times

we have, λ' = λ - 26
where λ = initial wavelength and λ' = new wavelength

Also, we have E = hc/λ

or eV = hc/λ

Where V = operating initial potential

λ = hc/eV

Consider V' be the new operating voltage, then

λV = λ'V' = (λ - 26) x 1.5 V

=> 0.5λ = 26 x 1.5

=> λ = 78 pm

Question 9: The electron beam in a color TV is accelerated through 32 kV and then strikes the screen. What is the wavelength of the most energetic X-ray photon?

Solution:

E = hc/ λ

Or $\lambda = hc/E$

Given: Potential across the X-ray Tube, V = 32kV

=> $\lambda = hc/E = [1242 \times 10^{-9}]/[32 \times 10^{3}] = 38.8$ pm

Question 10: When 40 kV is applied across an X-ray tube, X-ray is obtained with a maximum frequency of 9.7×10^{18} Hz. Calculate the value of Planck constant from these data.

Solution:
Wavelength of X-ray = $\lambda = hc/eV$ ….(1)

We know, $\gamma = c/\lambda$, using in (1), we get

$h = eV/\gamma = [40 \times 10^{3}]/[9.7 \times 10^{18}] \times e = 4.12 \times 10^{-15}$ eVs

Question 11: An X-ray tube operates at 40 kV. Suppose the electron converts 70% of its energy into a photon at each collision. Find the lowest three wavelengths emitted from the tube. Neglect the energy imparted to the atom with which the electron collides.

Solution:
Energy utilized by the electron = E = 70% energy into a photon

=> $E = 70/100 \times 40 \times 10^{3}$ eV $= 28 \times 10^{3}$ eV

Wavelength of the X-ray = $\lambda = hc/eV$

$= [1242 \times 10^{-9}]/[28 \times 10^{3}]$

= 44.35 pm

For the 2nd wavelength (which is 70% of leftover energy)

$E = 70/100 \times (40-28) \times 10^{3}$ eV $= 8.4 \times 10^{3}$ eV

Wavelength of the X-ray = $\lambda = hc/eV$

$= [1242 \times 10^{-9}]/[8.4 \times 10^{3}]$

= 148 pm

Similarly, For the 3rd wavelength:

E = 70/100 x (12-8.4) x 10^3 eV = 25.2 x 10^2 eV

Wavelength of the X-ray = λ = hc/eV

= [1242x10^{-9}]/[25.2x10^2]

= 493 pm

Question 12: The wavelength of K_α X-ray of tungsten is 21.3 pm. It takes 11.3 keV to knock out an electron from the L shell of a tungsten atom. What should be the minimum accelerating voltage across an X-ray tube having tungsten target which allows production of K_α X-ray?

Solution:
Energy required to knock out an electron from L-shell = E_L = 11.3 KeV
Wavelength of X-ray = 21.3 pm
Voltage = 11.3 kV

The energy gap between K and L shell,

E_K - E_L = [1242x10^{-9}]/[21.3x10^{-12}] = 58.309 KeV

E_k = 11.3 + 58.309 = 69.609 KeV

The accelerating voltage across the X-ray tube for the production of K_α X-ray is V_k = 69.609 KeV

Question 13: The K_β X-ray of argon has a wavelength of 0.36 nm. The minimum energy needed to ionize an argon atom is 16 eV. Find the energy needed to knock out an electron from the K shell of an argon atom

Solution:
Energy of K_β x-ray of argon, E = [1242x10^{-9}]/[0.36x10^{-9}] = 3450 eV

[Given: Wavelength of K_β X-ray of argon = 0.36 nm]

Energy needed to knock out an electron from K shell can be calculated as:

E_k = 3450 + 16 eV

E_k = 3.47 KeV (approx.)

Question 14: The K_α X-rays of aluminum (Z = 13) and zinc (Z = 30) have wavelengths 887 pm and 146 pm respectively. Use Moseley's law $\sqrt{v} = a(Z - b)$ to find the wavelength of the K_α X-ray of iron (Z = 26).

Solution:

Let γ_1 and γ_2 are the frequencies of K_α X-rays of aluminum and zinc respectively

So, $\gamma_1 = [3\text{x}10^8]/[887 \text{ x } 10^{-12}] = 33.82 \text{ x } 10^{16}$ Hz

and $\gamma_2 = [3\text{x}10^8]/[146 \text{ x } 10^{-12}] = 2.055 \text{ x } 10^{18}$ Hz

Again,

$\sqrt{\gamma} = a(Z - b)$

For aluminum:

$5.815 \text{ x } 10^8 = a(13 - b)$ ...(a)

For Zinc:

$1.4331 \text{ x } 10^9 = a(30 - b)$ ...(a)

Dividing above equations, we get

$(13-b)/(30-b) = [5.815 \text{ x } 10^8]/[1.4331 \text{ x } 10^9] = 0.4057$

=> $(13-b)/(30-b) = 0.4057$

=> $(13-b)0.4057 = (30-b)$

=> $b = 1.3949$

(a)=> $a = [5.815 \text{ x } 10^8]/(13-1.3949) = 5 \text{ x } 10^7$

Now, let us find the wavelength of the K_α X-ray of iron:

Frequency of the iron is given as $\sqrt{\gamma} = (5 \text{ x } 10^7)(26 - 1.39) = 123.05 \text{ x } 10^7$

or $\gamma = 5.1413 \text{ x } 10^{14}$

We know, $\gamma = c/\lambda$

or $\lambda = c/\gamma = [3\text{x}10^8]/[5.1413 \text{ x } 10^{14}] = 198$ pm

Question 15: A certain element emits K_α X-ray of energy 3.69 keV. Use the data from the previous problem to identify the element.

Solution:
Energy of K_α X-rays= 3.69 KeV (Given)

Using formula to find the Wavelength, or λ = hc/eV

=> $\lambda = [1242 \times 10^{-9}]/[3690] = 0.34 \times 10^{-9}$ m

[Because 3.69 KeV = 3690 eV]

Using Moseley's law, $\sqrt{\gamma} = a(Z - b)$ ....(1)

Where $\gamma = c/\lambda$

$\sqrt{\gamma} = \sqrt{c/\lambda} = \sqrt{[3 \times 10^8]}/\sqrt{[0.34 \times 10^{-9}]} = 9.39 \times 10^8$

(1)=> $9.39 \times 10^8 = (5 \times 10^7)(Z - 1.39)$

[Note: Used values of a and b from previous problem]

=> z = 20.17 = 20 (approx)

Atomic number (Z) = 20

Therefore, element with Atomic number 20 is calcium (Ca).

Question 16: The K_β X-rays from certain elements are given below. Draw a Moseley-type plot of $\sqrt{v}$ versus Z for K_β radiation.

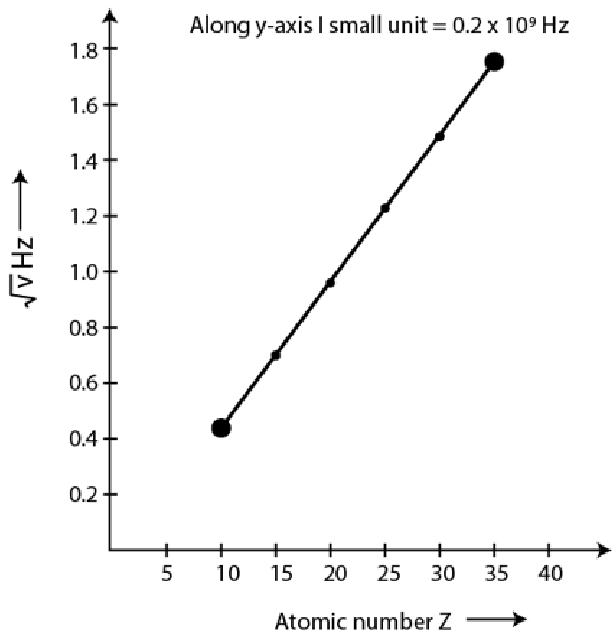
Element	Ne	P	Ca	Mn	Zn	Br
Energy(KeV)	0.858	2.14	4.02	6.51	9.57	13.3

Solution:
Using energy frequency relation to find the value of v.

i.e. Frequency = Energy/h

Where h = Planck's constant
and energy in KeV.

The required graph is:



Question 17: Use Moseley's law with b = 1 to find the frequency of the K_α X-ray of La (Z = 57) if the frequency of the K_α X-ray of Cu (Z = 29) is known to be 1.88×10^{16} Hz.

Solution:
Using Moseley's law, $\sqrt{\nu} = a(Z - b)$ ....(1)

Where $\nu = c/\lambda$

Given b = 1

Now,

$$\frac{\nu_{La}}{\nu_{Cu}} = (\frac{Z_{La} - 1}{Z_{Cu} - 1})^2$$

$$\nu_{La} = 1.88 \times 10^{18}(\frac{Z_{La} - 1}{Z_{Cu} - 1})^2$$

Given: $Z_{La} = 57$ and $Z_{Cu} = 29$

=> $\nu_{La} = 1.88 \times 10^{18} ((57-1)/(29-1))^2 = 7.52 \times 10^{18}$

=> $\nu_{La} = 7.52 \times 10^{18}$ Hz

Question 18: The K_α and K_β X-rays of molybdenum have wavelengths 0.71 angstrom and 0.63 angstrom respectively. Find the wavelength of L_α X-ray of molybdenum.

Solution: The K_α and K_β X-rays of molybdenum have wavelengths 0.71 angstrom and 0.63 angstrom respectively.

Let λ_1 = 0.71 angstrom and λ_2 = 0.63 angstrom

To find: Wavelength of L_α

Energy of K_α x-rays is given by: $K_\alpha = E_K - E_L$

Energy of K_β is given by: $K_\beta = E_K - E_M$ and

Energy of L_α is given by: $L_\alpha = E_K - E_M$

Relating above equations, we have

$L_\alpha = K_\alpha - K_\beta$

$= [3x10^8]/[0.63x10^{-10}] - [3x10^8]/[0.71x10^{-10}]$

or $L_\alpha = 0.536 \times 10^{18}$ Hz

We know, $\lambda = c/v = (3x10^8)/(0.536x10^{18})$ = 5.6 angstrom.

Question 19: The wavelengths of K_α and L_α X-rays of a material are 21.3 pm and 141 pm respectively. Find the wavelength of K_β X-ray of the material.

Solution:
The wavelengths of K_α and L_α X-rays of a material are 21.3 pm and 141 pm respectively.

Let E_1, E_2 and E_3 are the energies of K_α, L_α and K_β.

Using all three relations, we have

$E_3 = E_1 + E_2$

$\Rightarrow E_3 = 1242/[21.3x10^{-3}] + 1242/[141x10^{-5}]$

$= 58.309 \times 10^3 + 8.8085 \times 10^3$

= 67.118 x 10^3 eV

Now, Find the wavelength of K_β X-ray of the material:

Using relation,

λ = hc/E = (1242)/(67.118 x 10^3) = 18.5 pm.

Question 20: The energy of a silver atom with a vacancy in K shell is 25.31 keV, in L shell is 3.56 keV and in M shell is 0.530 keV higher than the energy of the atom with no vacancy. Find the frequency of K_α, L_β and L_α X-rays of silver.

Solution:

Let E_1, E_2 and E_3 are the energies of K shell, L shell and M shell.

So, E_1 = 25.31 KeV , E_2 = 3.56 keV and E_3 = 0.530 keV

Let v_1, v_2 and v_3 be the frequencies of K_α, L_β and L_α x-rays respectively.

K_α x-ray emitted when the transition takes place between l and k shells.

=> $K_\alpha = E_1 - E_2 = h v_1$

or $v_1 = (E_1 – E_2)/h$ = (25.31-3.56)/(6.63x10^{-34}) x 1.6 x 10^{-19} x 10^3

= 5.25 x 10^{18} Hz

k_β x-ray is emitted when the transition takes place between k and m shells

=> $K_\beta = E_1 – E_3 = h v_2$

or $v_2 = (E_1 – E_3)/h$ = (25.31-0.53)/(6.63x10^{-34}) x 1.6 x 10^{-19} x 10^3

= 5.985 x 10^{18} Hz

L_α x-ray is emitted when the transition takes place between l and m shells

=> $L_\alpha = E_2 – E_3 = h v_3$

or $v_3 = (E_2 – E_3)/h$ = (3.56-0.53)/(6.63x10^{-34}) x 1.6 x 10^{-19} x 10^3

= 7.32 x 10^{17} Hz